Introduction to Data Exploration

Vector Spaces

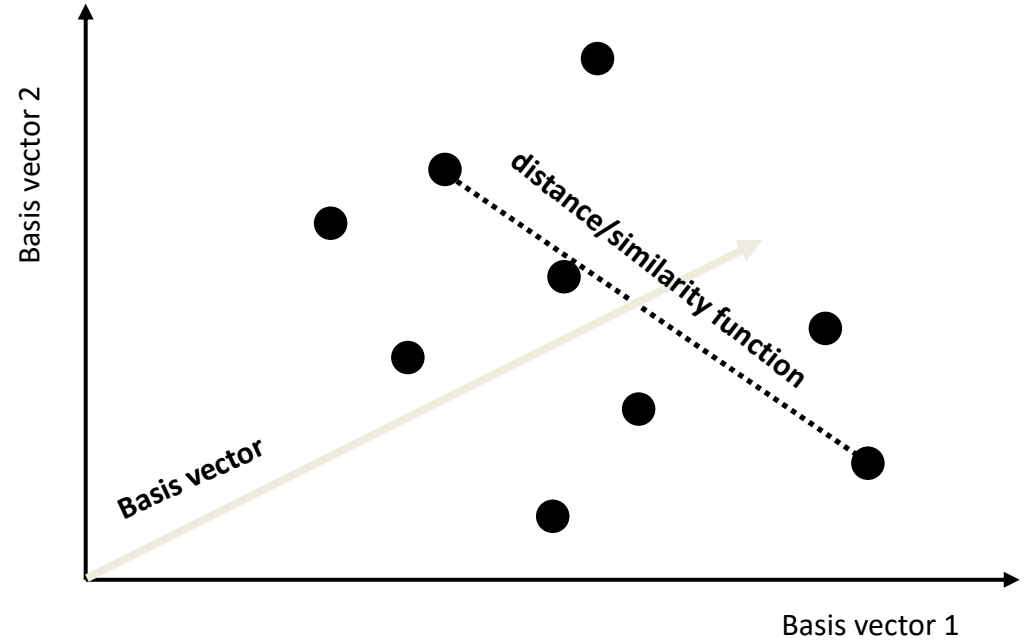
Ira A. Fulton Schools of Engineering
Arizona State University

Objectives

Objective

Identify the data types of elements within a dataset's data

Vector Spaces



- What are good features to use as basis vectors?

- How many features do we need as basis vectors?

Vector Spaces

Definition 3.1.1 (Vector space): *The set* $\mathbb{S}$ *is a vector space iff for all* $\vec{v}_i$, $\vec{v}_j$, $\vec{v}_k \in \mathbb{S}$ *and for all* $c, d \in \mathbb{R}$, *the following axioms hold:*

- $\vec{v}_i + \vec{v}_j = \vec{v}_j + \vec{v}_i$
- $(\vec{v}_i + \vec{v}_j) + \vec{v}_k = \vec{v}_j + (\vec{v}_i + \vec{v}_k)$
- $\vec{v}_i + \vec{0} = \vec{v}_i$ *(for some* $\vec{0} \in \mathbb{S}$*)*
- $\vec{v}_i + (-\vec{v}_i) = \vec{0}$ *(for some* $-\vec{v}_i \in \mathbb{S}$*)*
- $(c + d)\vec{v}_i = (c\vec{v}_i) + (d\vec{v}_i)$
- $c(\vec{v}_i + \vec{v}_j) = c\vec{v}_i + c\vec{v}_j$
- $(cd)\vec{v}_i = c(d\vec{v}_i)$
- $1.\vec{v}_i = \vec{v}_i$

The elements of $\mathbb{S}$ *are called vectors.*

Basics of a Vector Space

Definition (Linear independence and basis): *Let* $V = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ *be a set of vectors in a vector space* $\mathbb{S}$. *The vectors in* V *are said to be* linearly independent *if*

$$\left(\sum_{i=1}^{n} c_i \vec{v}_i = \vec{0}\right) \longleftrightarrow c_1 = c_2 = \cdots = c_n = 0.$$

non-redundant

The linearly independent set V *is said to be a basis for* $\mathbb{S}$ *if for every vector,* $\vec{u} \in \mathbb{S}$, *there exist constants* c_1 *through* c_n *such that*

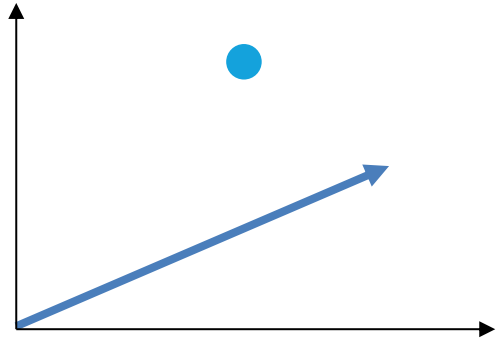
$$\vec{u} = \sum_{i=1}^{n} c_i \vec{v}_i.$$

complete

How many features we need?

…just 1?

…or 3?



…maybe 2?

…or, many many more (100s, 1000s) ???