# Informal Bids → Formal Round: Threshold/Interval Framework (Meeting Notes)

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## 1 Scope and context

**Meeting date:** 18 Dec (per handwritten notes).
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This document records the threshold/interval framework discussed for modeling admission from the informal-bid stage to the formal round in an M&A-style auction process. The key observable outcome is whether each bidder is *admitted* or *rejected* after submitting an informal bid. The central modeling device is a (possibly bidder-adjusted) *deterministic screening threshold* that is latent to the econometrician and is characterized by inequality/interval restrictions implied by observed bids and admission decisions. Likelihood-based estimation and MCMC-style data augmentation were discussed as implementation routes.

**Data context.** A hand-collected sample of approximately 20 auctions was mentioned in the discussion (with a reference to Matvos and Seru, 2014, *RFS*, "conglomerates"). This dataset has not been shared with the author as of the date of this note due to the holiday season.

## 2 Process timeline and information structure

The discussion treats the auction process as proceeding along the following timeline:

1. **Start.**
2. **NDA / limited information access.** Bidders sign an NDA and obtain access to limited target information.
3. **Informal bids.** Each bidder $j$ submits an informal bid $b^I_{ij}$ in auction $i$.
4. **Admission decision (A/R).** The target admits a subset of bidders to the formal round and rejects the remainder.
5. **Formal bids.** Admitted bidders submit formal bids (and potentially other terms).
6. **Deal announcement.**

# 3 Notation

Indexing:

- $i$ indexes an **auction/deal process**.
- $j$ indexes a **bidder**.

Observed objects:

- $b^I_{ij}$: bidder $j$'s **informal bid** in auction $i$.
- $X_i$: **auction/target characteristics**. The discussion noted that $X_i$ may include *moments of the informal-bid vector* within an auction (e.g., mean, variance, maximum), in addition to target/auction states observed outside the bidding data.
- $y_{ij}$: **bidder-side characteristics** (e.g., bidder type indicators, financing certainty proxies, reputation, "seriousness" proxies such as number of interactions/bids and bid dispersion).

Admission sets:

- $\mathcal{A}_i$: set of bidders **admitted** to the formal round in auction $i$.
- $\mathcal{R}_i$: set of bidders **rejected** after the informal round, with $\mathcal{R}_i = \{1, \ldots, J\} \setminus \mathcal{A}_i$ when $J$ bidders participate.

Latent objects:

- $b^{I*}_i$: an **auction-level screening threshold/cutoff**.
- Optionally, $b^{I*}_{ij}$: a **bidder-adjusted threshold** in auction $i$.

# 4 Case (1): simplest threshold screening (auction-level cutoff)

## 4.1 Deterministic admission rule

A baseline deterministic admission rule discussed is:

$$j \in \mathcal{A}_i \quad \Longleftrightarrow \quad b^I_{ij} \geq b^{I*}_i.$$

The threshold $b^{I*}_i$ is not observed by the econometrician.

### 4.2 Interval restriction from admits/rejects

If both admitted and rejected bidders are observed in auction $i$, then the data imply an interval restriction on $b_i^{I*}$:

$$L_i := \max_{j \in \mathcal{R}_i} b_{ij}^I, \tag{1}$$

$$U_i := \min_{j \in \mathcal{A}_i} b_{ij}^I, \tag{2}$$

so that

$$b_i^{I*} \in [L_i, U_i].$$

Toy example from the discussion: informal bids $\{18, 20, 22\}$ with admitted bidders $\{20, 22\}$ imply $L_i = 18$, $U_i = 20$, hence $b_i^{I*} \in [18, 20]$.

**One-sided cases.**

- If $\mathcal{R}_i = \emptyset$ (everyone admitted), then only an upper bound is available: $b_i^{I*} \leq U_i$.
- If $\mathcal{A}_i = \emptyset$ (no one admitted), then only a lower bound is available: $b_i^{I*} \geq L_i$.

## 5 Contrast: probit/logit versus threshold/interval

A probabilistic admission model (e.g., probit/logit) would specify an index with an idiosyncratic error, such as

$$\mathbf{1}\{j \in \mathcal{A}_i\} = \mathbf{1}\{\text{index}(X_i, y_{ij}) + \varepsilon_{ij} \geq 0\},$$

and estimate admission probabilities directly.

The threshold/interval approach instead treats admission as driven by a deterministic screening rule. Randomness in observed admission outcomes (from the econometrician's perspective) arises from unobserved components of the screening rule and/or latent objects that are not observed in the data.

## 6 Parametric representation of the latent cutoff and estimation

### 6.1 Auction-level latent cutoff

A parametric representation discussed is:

$$b_i^{I*} = \beta^\top X_i + \nu_i^I,$$

where $\nu_i^I$ collects unobserved screening components. Combining this with the interval restriction yields:

$$\nu_i^I \in [L_i - \beta^\top X_i,\ U_i - \beta^\top X_i] \qquad \text{(for auctions with both admits and rejects).}$$

### 6.2 Likelihood-based estimation (one convenient specification)

A common starting point is to impose a distribution on $\nu_i^I$ (e.g., $\nu_i^I \sim \mathcal{N}(0, \sigma^2)$). Under that choice, the likelihood contribution for an auction with two-sided bounds is:

$$\Pr(L_i \le b_i^{I*} \le U_i \mid X_i) = \Phi\left(\frac{U_i - \beta^\top X_i}{\sigma}\right) - \Phi\left(\frac{L_i - \beta^\top X_i}{\sigma}\right),$$

with one-sided analogues when only an upper or lower bound is available. Other parametric families can be substituted without changing the interval logic.

### 6.3 MCMC/data-augmentation approach (outline recorded from the discussion)

A data-augmentation approach was discussed for settings with small samples and/or extensions beyond the simplest likelihood. The recorded outline is:

1. Initialize parameter values (e.g., $\beta_0$).
2. For each auction, simulate latent unobservables (e.g., $\nu_i^I$) subject to satisfying the observed inequality restrictions implied by admits/rejects, and compute the corresponding latent thresholds $b_i^{I*}$.
3. Update/estimate $\beta$ given the simulated thresholds $\{b_i^{I*}\}$ (and observed $X_i$).
4. Draw new parameter values (or propose/accept updates) and iterate.

This outline is compatible with standard Gibbs or Metropolis–Hastings implementations once specific distributional assumptions and conditional sampling steps are fixed.

## 7 Case (2): highest informal bid may fail to advance (bidder-adjusted screening)

The discussion emphasized that the highest informal bid need not be admitted to the formal round. A convenient way to represent this is bidder-adjusted screening:

$$j \in \mathcal{A}_i \quad \Longleftrightarrow \quad b_{ij}^I \ge b_{ij}^{I*}, \qquad b_{ij}^{I*} = b_i^{I*} + \delta^\top y_{ij} + \rho_{ij}^I,$$

where $y_{ij}$ includes bidder-specific attributes that can shift the effective screening bar, and $\rho_{ij}^I$ collects unobserved bidder-auction components.

An equivalent representation (when $\rho_{ij}^I$ is omitted or structured) is to define a bidder-adjusted bid:

$$\widetilde{b}_{ij}^I(\delta) := b_{ij}^I - \delta^\top y_{ij},$$

and use an auction-level threshold:

$$j \in \mathcal{A}_i \iff \widetilde{b}_{ij}^I(\delta) \geq b_i^{I*}.$$

Then the implied interval restriction becomes:

$$b_i^{I*} \in \Big[\max_{j \in \mathcal{R}_i} \widetilde{b}_{ij}^I(\delta),\ \min_{j \in \mathcal{A}_i} \widetilde{b}_{ij}^I(\delta)\Big].$$

**Bidder types.** The discussion used two bidder types labelled $S$ and $F$ and noted that bidder "seriousness" and related attributes (e.g., number of interactions/bids, dispersion of bids) may enter $y_{ij}$. One possible interpretation of $S/F$ is strategic versus financial bidders, but the formal development uses the labels $S$ and $F$ generically.

# 8 Case (3): screening depends on the target's inference about bidder valuations

A further extension discussed is that the target's admission decision can depend on its inference about bidders' valuations, and in particular on *moments of the bidder valuation distribution* (or moments of bids as proxies).

One way to represent this is to let the cutoff depend on target characteristics that include valuation moments:

$$b_i^{I*} = \beta^\top X_i + \nu_i^I, \qquad X_i = \big(\text{target/auction states},\ \text{moments of bidder valuations or bids}\big).$$

The notes also recorded a reduced-form representation for valuations as a function of auction/target state variables (schematically),

$$v_{ij}^I = \delta^\top X_i + \varepsilon_{ij},$$

with the understanding that the target can form (estimated) moments of $\{v_{ij}^I\}_j$ (or of $\{b_{ij}^I\}_j$) given $X_i$ and observed bidding objects.

**MCMC outline recorded for this case.** The recorded algorithmic sketch is:

1. Initialize parameter values (e.g., $\delta_0$, $\beta_0$).
2. Simulate latent components (e.g., $\nu_i^I$ and bidder-level shocks) for each auction.
3. Given current $\delta$, compute implied moments of bidder valuations (or bid-based proxies) within each auction.
4. Update/estimate $\beta$ and $\delta$ using the computed valuation moments as controls.
5. Draw new $(\beta, \delta)$ and iterate.

## 9 Simulation tasks (feasibility and sensitivity checks)

The simulation tasks discussed were intended to validate whether the threshold/interval framework can recover cutoffs (and, in extensions, type-specific cutoffs) under limited observability. The numeric values below are illustrative baseline values recorded from the handwritten notes; the tasks explicitly require sensitivity analysis around these values.

### 9.1 Task A (Case 1 simulation): single constant cutoff

**Goal.** Estimate a constant cutoff $b^{I*}$ when only admission/rejection decisions are observed (together with the simulated bids/valuations, depending on the simulation design).

**Baseline DGP (illustrative).**

1. Simulate $N$ auctions. Each auction has $J = 3$ bidders.
2. Simulate bidder valuations at the informal stage:

$$v_{ij} = 1.3 + \epsilon_{ij}, \qquad \epsilon_{ij} \sim \mathcal{N}(0, 0.2^2), \qquad b^I_{ij} = v_{ij}.$$

3. Set a constant admission cutoff (illustrative baseline):

$$b^{I*} = 1.4.$$

4. Apply admission:

$$j \in \mathcal{A}_i \iff b^I_{ij} \geq b^{I*}.$$

5. Some simulated auctions will be "incomplete" (all admit or all reject). The notes recorded dropping incomplete auctions in a first pass.

**Sensitivity requirements.** Vary the illustrative numbers and assess how recovery changes:

1. Vary $b^{I*}$ over a grid (e.g., percentiles of the simulated bid distribution).
2. Vary the dispersion of valuations (e.g., change 0.2).
3. Vary the number of auctions $N$, including a small-sample stress test around $N \approx 20$.
4. Track the frequency of incomplete auctions and quantify the selection effect of dropping them.

### 9.2 Task B (Case 2 simulation): two types, two cutoffs

**Goal.** Estimate type-specific cutoffs when bidders fall into two types labelled $S$ and $F$.

**Baseline DGP (illustrative).**

1. Simulate $N$ auctions. Each auction has $J = 3$ bidders.

2. Assign bidder types $T_{ij} \in \{S, F\}$ (either fixed shares or bidder-specific rules).
3. Simulate valuations/bids as in Task A (or allow type-dependent distributions as an extension).
4. Set type-specific admission cutoffs (illustrative baseline recorded in the notes):

$$b_S^{I*} = 1.45, \qquad b_F^{I*} = 1.35.$$

5. Apply admission:

$$j \in \mathcal{A}_i \iff b_{ij}^I \geq b_{T_{ij}}^{I*}.$$

6. As in Task A, record and (initially) drop incomplete auctions.

**Sensitivity requirements.**

1. Vary the levels of $(b_S^{I*}, b_F^{I*})$ and their gap.
2. Vary the type share to assess identification for minority types.
3. Compare cases where $T_{ij}$ is observed versus latent (must be inferred) and document identification issues in the latent-type case.

### 9.3 Deliverables

1. A short write-up (1–2 pages) reporting recovery accuracy for cutoffs/parameters, frequency of informative intervals, and sensitivity to cutoff magnitudes and type separations.
2. Figures/tables showing estimator bias/MSE versus cutoff levels, cutoff gaps, sample size $N$, and type share.
3. A recommendation on whether the empirical implementation should begin with a single cutoff, observed-type cutoffs, or a latent-type extension.

## Addendum (January 9, 2026)

- Simulation now allows auction covariates: $b_i^{I*} = X_i'\beta + \epsilon_i$ (default remains intercept-only).
- Performance metrics compare against $b^{I*}(\bar{X})$ (cutoff at mean covariates) when covariates are used.
- Sample size $N$ is interpreted as $N_{\text{observed}}$ (conditional on reaching the formal stage).
- Planned incomplete-auction handling updates:
  - Task A: drop all-reject auctions; include all-accept auctions as one-sided upper bounds.
  - Task B: drop auctions with zero admitted overall; include type-specific one-sided bounds (including type-specific all-reject lower bounds).