Analysis Report for 14 Jan Meeting

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1 Introduction

- **Task A:** Implement and validate the selection-aware conditional likelihood in the baseline debugging case ($b^I = v$).
- **Task B:** Implement the two-stage DGP with non-binding informal bids and jointly recover (γ, α) and cutoff parameters.

Notation follows the meeting notes (18 Dec 2025; 14 Jan 2026). We observe only auctions that reach the formal stage: define $S_i = 1$ if at least one bidder is admitted after the informal round (equivalently, if at least one informal bid clears the admission cutoff), and $S_i = 0$ otherwise. All-reject auctions ($S_i = 0$) are unobserved, so the dataset is a selected subset of the initiated-auction population.

The key methodological contribution is a Metropolis–Hastings step that corrects the cutoff draw for formal-stage selection. The corrected posterior kernel is:

$$\pi(b \mid \cdot) \propto \phi\left(\frac{b - X_i'\beta}{\sigma_\omega}\right) \cdot \frac{1}{\Pr(S_i = 1 \mid b)} \cdot \mathbf{1}\{L_i \leq b \leq U_i\},$$

where the selection penalty $1/\Pr(S_i = 1 \mid b)$ is the truncation correction implied by conditioning on $S_i = 1$: it adjusts inference back toward the underlying (initiated-auction) cutoff process rather than the selected sample of observed auctions.

2 The Admission Rule and Interval Restrictions

2.1 Cutoff as Reservation Price

The admission cutoff b_i^{I*} can be interpreted as the target's informal-stage reservation price: the minimum informal bid required for consideration. Bidders with informal bids at or above this threshold are invited to proceed; those below are excluded.

We model the cutoff as:

$$b_i^{I*} = X_i'\beta + \omega_i, \qquad \omega_i \sim \mathcal{N}(0, \sigma_\omega^2),$$

where X_i are observable covariates (potentially including moments of the informal bid distribution) and ω_i captures unobserved auction-specific factors affecting the target's admission threshold.

2.2 Interval Data Structure

The admission decisions reveal interval restrictions on the latent cutoff. Let $\mathcal{A}_i$ denote the set of admitted bidders and $\mathcal{R}_i$ the set of rejected bidders. Define:

$$L_i = \max_{j \in \mathcal{R}_i} b_{ij}^I \quad \text{(highest rejected bid)},$$
$$U_i = \min_{j \in \mathcal{A}_i} b_{ij}^I \quad \text{(lowest admitted bid)}.$$

Three cases arise:

1. **Two-sided bounds** ($\mathcal{R}_i \neq \emptyset$ and $\mathcal{A}_i \neq \emptyset$): The cutoff lies in $[L_i, U_i]$ where both bounds are finite.
2. **One-sided upper bound** ($\mathcal{R}_i = \emptyset$, all admitted): The cutoff lies in $(-\infty, U_i]$ where $L_i = -\infty$.
3. **One-sided lower bound** ($\mathcal{A}_i = \emptyset$, all rejected): The cutoff exceeds all bids. These auctions have $S_i = 0$ and are unobserved.

Figure 1 visualizes the interval structure for a representative Task A simulation. Each horizontal line represents one auction's interval $[L_i, U_i]$, with the true cutoff marked. Two-sided intervals provide the most information; all-admit auctions contribute only an upper bound.

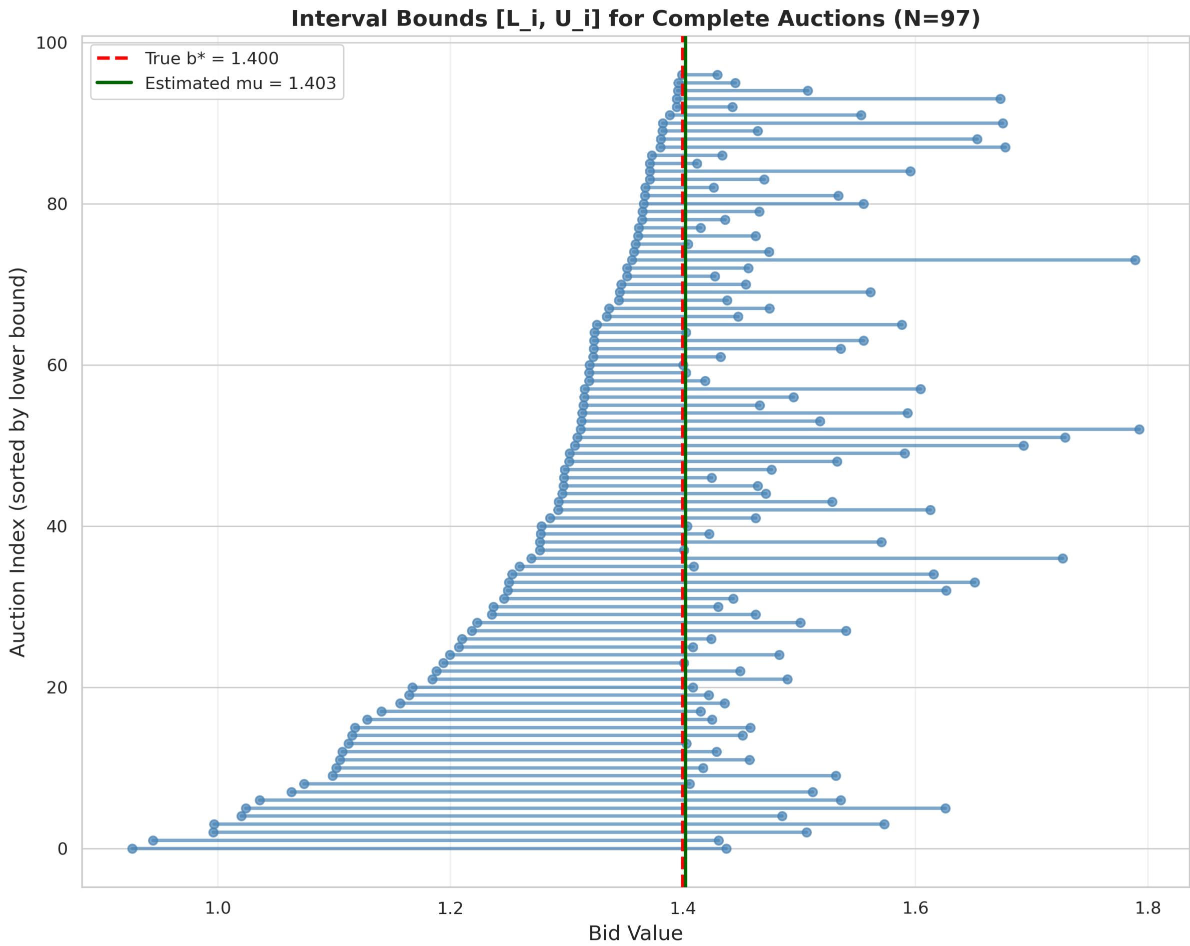


Figure 1: Task A interval bounds. Each horizontal line represents one auction's $[L_i, U_i]$ interval. Auctions with both admits and rejects yield two-sided bounds (finite L_i and U_i); all-admit auctions yield only an upper bound ($L_i = -\infty$). The vertical dashed line marks the true cutoff. The interval structure provides the data augmentation framework's foundation: latent cutoffs are sampled within these bounds.

3 Task A Model: Baseline Debugging Case

3.1 Truthful Informal Bidding

Task A serves as a baseline debugging case with a simplified structure: informal bids equal valuations.

$$b_{ij}^I = v_{ij}, \qquad v_{ij} \sim \mathcal{N}(\mu_v, \sigma_v^2).$$

This assumption removes the informal-bid wedge present in the full model, isolating the selection correction problem. For debugging, we treat the valuation inputs (μ_v, σ_v, J) as fixed at their DGP values and focus on recovering the cutoff intercept.

Fixed DGP inputs: (μ_v, σ_v, J).
Estimation target: the cutoff intercept μ (equal to b^* in the intercept-only DGP), under the selection-aware likelihood.

3.2 Selection Probability Derivation

Under truthful bidding, the selection event $S_i = 1$ occurs if at least one bid exceeds the cutoff b:

$$\begin{aligned} \Pr(S_i = 1 \mid b, \mu_v, \sigma_v) &= 1 - \Pr(\text{all } v_{ij} < b) \\ &= 1 - \prod_{j=1}^{J} \Pr(v_{ij} < b) \\ &= 1 - \Phi\left(\frac{b - \mu_v}{\sigma_v}\right)^J . \end{aligned} \tag{1}$$

Behavior:

- As $b \to -\infty$: $\Pr(S_i = 1) \to 1$ (almost certainly at least one bid exceeds a very low cutoff).
- As $b \to +\infty$: $\Pr(S_i = 1) \to 0$ (unlikely any bid exceeds a very high cutoff).

The selection penalty $1/\Pr(S_i = 1 \mid b)$ thus increases with b. Intuitively, observing an auction with a high cutoff is "surprising" and should receive more weight; observing an auction with a low cutoff is expected and should receive less weight.

3.3 The Corrected Posterior Kernel

For the baseline intercept-only case with cutoff $b_i^{I*} = \mu + \omega_i$, $\omega_i \sim \mathcal{N}(0, \sigma_\omega^2)$, the selection-corrected posterior kernel for the latent cutoff is:

$$\pi(b \mid L_i, U_i, \mu, \sigma_\omega, \mu_v, \sigma_v, J) \propto \underbrace{\phi\left(\frac{b - \mu}{\sigma_\omega}\right)}_{\text{cutoff model}} \cdot \underbrace{\frac{1}{1 - \Phi\left(\frac{b - \mu_v}{\sigma_v}\right)^J}}_{\text{selection penalty}} \cdot \underbrace{\mathbf{1}\{L_i \le b \le U_i\}}_{\text{interval restriction}} . \tag{2}$$

This kernel is *not* a truncated normal because the selection penalty varies with b. Direct sampling via the inverse CDF method is not available; we instead use Metropolis–Hastings.

4 Task B Model: Two-Stage with Non-Binding Informal Bids

4.1 Economic Motivation

In the informal stage of an M&A auction, informal bids are non-binding indications of interest rather than binding commitments. As a result, the mapping from latent valuations to observed informal bids can differ systematically from standard binding-bid equilibrium shading. Following a cheap talk perspective (Kartik, 2009)[1], we model this reduced-form deviation through an informal-bid wedge parameter.

The formal stage, by contrast, involves binding bids with real consequences. We model formal bids with a simple linear shading rule (used for simulation convenience) and do not add any additional informal-stage wedge.

[1] Kartik, N. (2009). "Strategic communication with lying costs." *Review of Economic Studies*, 76(4), 1359–1395.

Throughout, J denotes the number of bidders per auction (denoted n_i in the meeting notes); in our simulations we fix $J = 3$.

4.2 Valuation Process

Valuations follow a simple intercept-plus-noise structure:

$$v_{ij} = \gamma + \nu_{ij}, \qquad \nu_{ij} \sim \mathcal{N}(0, \sigma_\nu^2),$$

where γ is the mean valuation and σ_ν captures idiosyncratic valuation heterogeneity across bidders.

After admission, bidders gain additional information through due diligence:

$$u_{ij} = v_{ij} + \eta_{ij}, \qquad \eta_{ij} \sim \mathcal{N}(0, \sigma_\eta^2),$$

where η_{ij} represents the valuation update from due diligence. The updated valuation u_{ij} forms the basis for formal bidding.

4.3 Bidding Equations

Informal bids incorporate both baseline linear shading and an additional informal-stage wedge:

$$b_{ij}^I = \left(1 - \frac{1}{J} + \alpha\right) v_{ij} = \lambda_I(J, \alpha) \cdot v_{ij},$$

where:

- $(1 - 1/J)$: Baseline linear shading factor used in both stages
- α: Informal-stage wedge capturing systematic deviation (inflation/deflation) in indicative bidding
- $\lambda_I(J, \alpha) = 1 - 1/J + \alpha$: Informal bid multiplier

Formal bids follow the same baseline linear shading rule without any additional informal-stage wedge:

$$b_{ij}^F = \left(1 - \frac{1}{J}\right) u_{ij}, \qquad \text{for admitted bidders only.}$$

4.4 The κ Reparametrization

Meeting-notes wedge. Let $\lambda_F(J) := 1 - 1/J$ denote the (stylized) formal-stage shading multiplier. Following the meeting notes (Jan 14), write the informal-stage bid multiplier as:

$$\lambda_I(J, \alpha) := \lambda_F(J) + \alpha,$$

so α is an additive informal-bid wedge and must satisfy $\alpha > -\lambda_F(J)$ to ensure $\lambda_I(J, \alpha) > 0$.

Normalized wedge used in diagnostics. Define the scale-free wedge

$$\tilde{\alpha} := \frac{\alpha}{\lambda_F(J)} \quad \Longleftrightarrow \quad \lambda_I(J,\alpha) = \lambda_F(J)\,(1+\tilde{\alpha}).$$

Unconstrained parameterization for MCMC. To avoid boundary issues in sampling, we reparametrize with an unconstrained $\kappa \in \mathbb{R}$:

$$1+\tilde{\alpha} = \exp(\kappa) \quad \Longleftrightarrow \quad \tilde{\alpha} = \exp(\kappa) - 1,$$

which implies

$$\lambda_I(J,\kappa) = \lambda_F(J)\exp(\kappa), \qquad \alpha = \lambda_F(J)\,(\exp(\kappa)-1).$$

In this parameterization, $\tilde{\alpha} > -1$ automatically.

4.5 Cutoff Process Specifications

We consider three cutoff specifications of increasing complexity:

1. Intercept-only cutoff (baseline):

$$b_i^{I*} = c + \omega_i, \qquad \omega_i \sim \mathcal{N}(0, \sigma_\omega^2).$$

2. Moments-based cutoff (meeting specification):

$$b_i^{I*} = c + \theta_1 m_{i,1} + \theta_2 m_{i,2} + \theta_3 m_{i,3} + \omega_i,$$

where the moments are:

$$\begin{aligned}
m_{i,1} &= b_i^{I(1)} \qquad \text{(highest informal bid)},\\
m_{i,2} &= \frac{b_i^{I(1)} + b_i^{I(2)}}{2} \qquad \text{(top-2 average)},\\
m_{i,3} &= \frac{b_i^{I(1)} + b_i^{I(2)} + b_i^{I(3)}}{3} \qquad \text{(top-3 average)},
\end{aligned}$$

with $b_i^{I(1)} \geq b_i^{I(2)} \geq b_i^{I(3)}$ the order statistics.

3. Depth-based cutoff (alternative):

$$b_i^{I*} = c + \theta_1 m_{i,1} + \theta_2 m_{i,2} + \omega_i,$$

where:

$$\begin{aligned}
m_{i,1} &= \frac{b_i^{I(2)} + b_i^{I(3)}}{2} \qquad \text{(runner-up average)},\\
m_{i,2} &= b_i^{I(2)} - b_i^{I(3)} \qquad \text{(runner-up gap)}.
\end{aligned}$$

The depth-based specification avoids placing the maximum bid on both sides of the selection inequality, addressing an identification concern discussed in Section 9.2.

4.6 Selection Probability under the Informal-Bid Wedge

Under the informal-bid wedge model, the selection probability conditional on cutoff b is:

$$\Pr(S_i = 1 \mid b, \gamma, \sigma_\nu, \kappa) = 1 - \Phi\left(\frac{b/\lambda_I(J,\kappa) - \gamma}{\sigma_\nu}\right)^J. \tag{3}$$

This generalizes (1) by transforming the cutoff threshold from bid space to valuation space: a bidder is admitted if $v_{ij} \geq b/\lambda_I(J,\kappa)$.

4.7 Staged Variance Relaxation

Jointly estimating all variance parameters raises identification concerns. We implement a staged approach:

- **Stage 1:** Fix $(\sigma_\nu, \sigma_\eta)$ at DGP values. Estimate $(\gamma, \tilde{\alpha}, \text{cutoff parameters})$.
- **Stage 2:** Fix σ_ν; estimate σ_η along with Stage 1 parameters.
- **Stage 3:** Estimate both σ_ν and σ_η along with all other parameters.

This staged approach allows us to assess identifiability incrementally and diagnose which parameters are well-identified from which data variation.

5 MCMC Estimation via Data Augmentation

5.1 The Data Augmentation Framework

The latent cutoffs $\{b_i^{I*}\}_{i=1}^N$ are "missing data" that, if observed, would simplify inference. Data augmentation treats these latent variables as additional parameters and iteratively samples from their conditional distributions.

The complete-data likelihood, were we to observe the cutoffs, factors cleanly:

$$p(\text{data}, \{b_i^{I*}\} \mid \theta) = \prod_i p(\text{bids}_i \mid b_i^{I*}, \theta) \cdot p(b_i^{I*} \mid \theta).$$

The data augmentation Gibbs sampler alternates between:

1. **Imputation step:** Sample latent cutoffs b_i^{I*} given current parameters.
2. **Posterior step:** Update parameters given the imputed cutoffs.

Under standard conditions, this Markov chain converges to the joint posterior over parameters and latent variables.

5.2 Why Standard Gibbs Fails

In the naive (non-selection-aware) sampler, the cutoff imputation step draws from:

$$b_i^{I*} \mid \cdot \sim \text{TruncNormal}(\mu = X_i'\beta, \sigma = \sigma_\omega, \text{bounds} = [L_i, U_i]).$$

This is the conditional distribution implied by the cutoff model $b_i^{I*} = X_i'\beta + \omega_i$ restricted to the observed interval. The truncated normal can be sampled efficiently via the inverse CDF method.

However, this formulation ignores selection. The correct conditional distribution must incorporate the selection penalty:

$$\pi(b \mid \cdot) \propto \phi\left(\frac{b - X_i'\beta}{\sigma_\omega}\right) \cdot \frac{1}{\Pr(S_i = 1 \mid b, \vartheta)} \cdot \mathbf{1}\{L_i \le b \le U_i\}.$$

Here ϑ collects the bid-distribution parameters entering the selection probability: for Task A, $\vartheta = (\mu_v, \sigma_v, J)$; for Task B, $\vartheta = (\gamma, \sigma_\nu, \kappa, J)$.

This is *not* a truncated normal, and direct sampling is not available.

6 The Selection-Aware Metropolis–Hastings Step

6.1 Independence MH with Truncated Normal Proposal

We implement the selection correction via an independence Metropolis–Hastings step:

1. **Propose:** Draw $b^{\text{prop}} \sim \text{TruncNormal}(X_i'\beta, \sigma_\omega, [L_i, U_i])$ (the naive proposal).
2. **Accept/reject:** Accept with probability

$$\alpha = \min\left(1, \frac{\Pr(S_i = 1 \mid b^{\text{old}})}{\Pr(S_i = 1 \mid b^{\text{prop}})}\right).$$

Derivation. The MH acceptance ratio for target $\pi(b)$ and proposal $q(b)$ is:

$$\alpha = \min\left(1, \frac{\pi(b^{\text{prop}}) \cdot q(b^{\text{old}} \mid b^{\text{prop}})}{\pi(b^{\text{old}}) \cdot q(b^{\text{prop}} \mid b^{\text{old}})}\right).$$

For our independence proposal (proposal doesn't depend on current state), $q(b^{\text{old}} \mid b^{\text{prop}}) = q(b^{\text{old}})$ and $q(b^{\text{prop}} \mid b^{\text{old}}) = q(b^{\text{prop}})$. The target and proposal share the truncated normal factor, so:

$$\frac{\pi(b^{\text{prop}})}{\pi(b^{\text{old}})} \cdot \frac{q(b^{\text{old}})}{q(b^{\text{prop}})} = \frac{1/\Pr(S_i = 1 \mid b^{\text{prop}})}{1/\Pr(S_i = 1 \mid b^{\text{old}})} = \frac{\Pr(S_i = 1 \mid b^{\text{old}})}{\Pr(S_i = 1 \mid b^{\text{prop}})}.$$

This asymmetric acceptance—always accepting higher cutoffs, sometimes rejecting lower ones—shifts the equilibrium distribution toward higher cutoffs, correcting the selection-induced downward bias. In our baseline simulations with ∼65% keep rate ($\Pr(S_i = 1)$), we observe acceptance rates around 70–80%.

7 Convergence Diagnostics

7.1 The Gelman–Rubin $\hat{R}$ Statistic

We assess convergence using the Gelman–Rubin $\hat{R}$ statistic (potential scale reduction factor)[2], which compares within-chain and between-chain variance across multiple independent chains.

For parameter θ with M chains of length n (post-burn-in):

$$\hat{R} = \sqrt{\frac{\hat{V}}{W}},$$

where W is the within-chain variance and $\hat{V}$ is an estimate of the marginal posterior variance that combines within- and between-chain variance.

Interpretation:

$\hat{R}$ value	Interpretation
< 1.01	Excellent convergence
$1.01 - 1.05$	Good convergence
$1.05 - 1.10$	Acceptable; longer chains may help
> 1.10	Convergence concern; investigate

We run 3 chains of 20,000 iterations each with 10,000 burn-in, targeting $\hat{R} < 1.05$ and ESS > 400 for all parameters. Trace plots and ACF diagnostics confirm adequate mixing.

8 Task A Results: Selection-Corrected Single Cutoff

Sample definition. Throughout Task A, N denotes the number of observed auctions (those with $S_i = 1$). We also report the keep rate $\Pr(S_i = 1)$, which governs how many initiated auctions are needed, on average, to obtain N observed auctions.

8.1 Baseline Results

Figure 2 presents diagnostics for the selection-aware MH sampler in the Task A baseline case ($N = 100$ observed auctions, $J = 3$, $\mu_v = 1.3$, $\sigma_v = 0.2$, $b^* = 1.4$).

[2]Gelman, A., and Rubin, D. B. (1992). "Inference from iterative simulation using multiple sequences." *Statistical Science*, 7(4), 457–472.

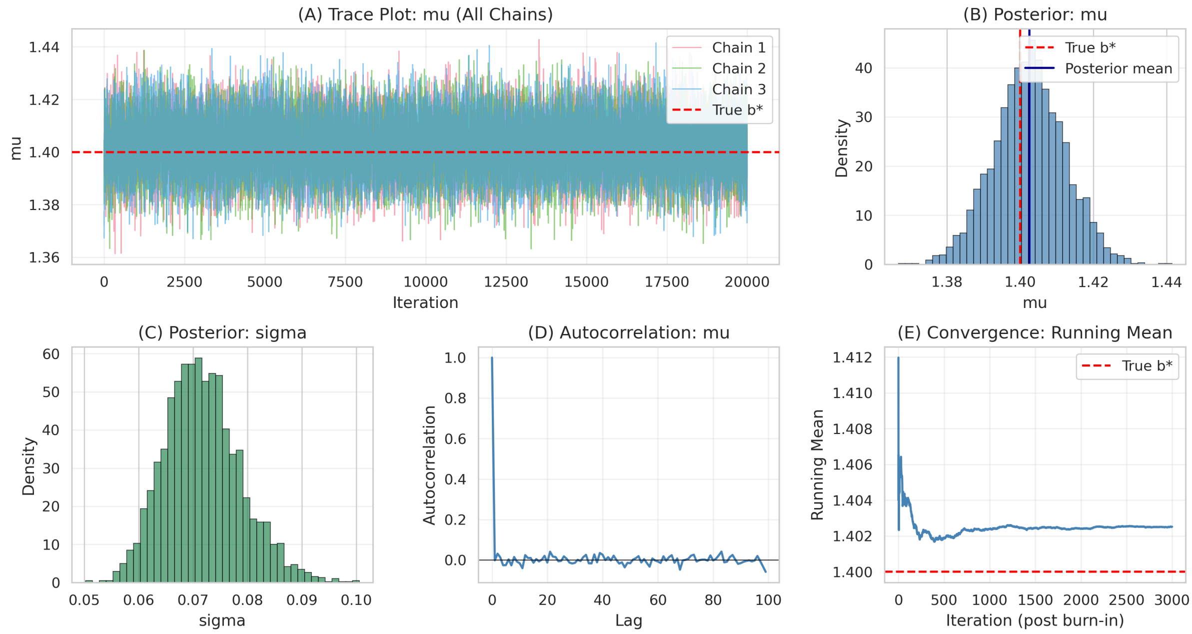


Figure 2: Task A diagnostics (selection-aware MH). **Left:** Trace plot showing three independent chains for the cutoff mean parameter μ. The chains mix well and stabilize around the true value (dashed red line at 1.4). **Center:** Posterior histogram with the true value marked. The posterior is centered near 1.4 with the 95% credible interval containing the truth. **Right:** Autocorrelation function showing moderate correlation that decays within $\sim$20 lags.

Key comparison with naive estimator. The selection-aware MH sampler substantially improves upon the naive baseline:

Table 1: Task A baseline comparison: Naive vs. Selection-Aware MH

	Naive (Dec 18)	Selection-Aware MH
Posterior mean $\hat{\mu}$	1.387	1.402
95% CI	[1.367, 1.406]	[1.383, 1.421]
True value	1.400	1.400
Bias	-0.013	$+0.002$
Coverage	Deteriorates with N	Maintains nominal

The naive estimator exhibits persistent downward bias: the posterior concentrates around 1.387 rather than 1.400. This reflects truncation: conditional on being observed ($S_i = 1$), auctions with higher latent cutoffs are less likely to enter the sample. Treating the observed sample as representative therefore overweights lower cutoffs and biases the posterior downward.

The selection-aware MH estimator corrects this bias, producing a posterior mean of 1.402 with a credible interval $[1.383, 1.421]$ that properly covers the true value.

8.2 Sample-Size Sensitivity

Figure 3 shows how the selection-aware estimator performs across sample sizes at the baseline cutoff $b^* = 1.4$ (keep rate $\approx 65\%$).

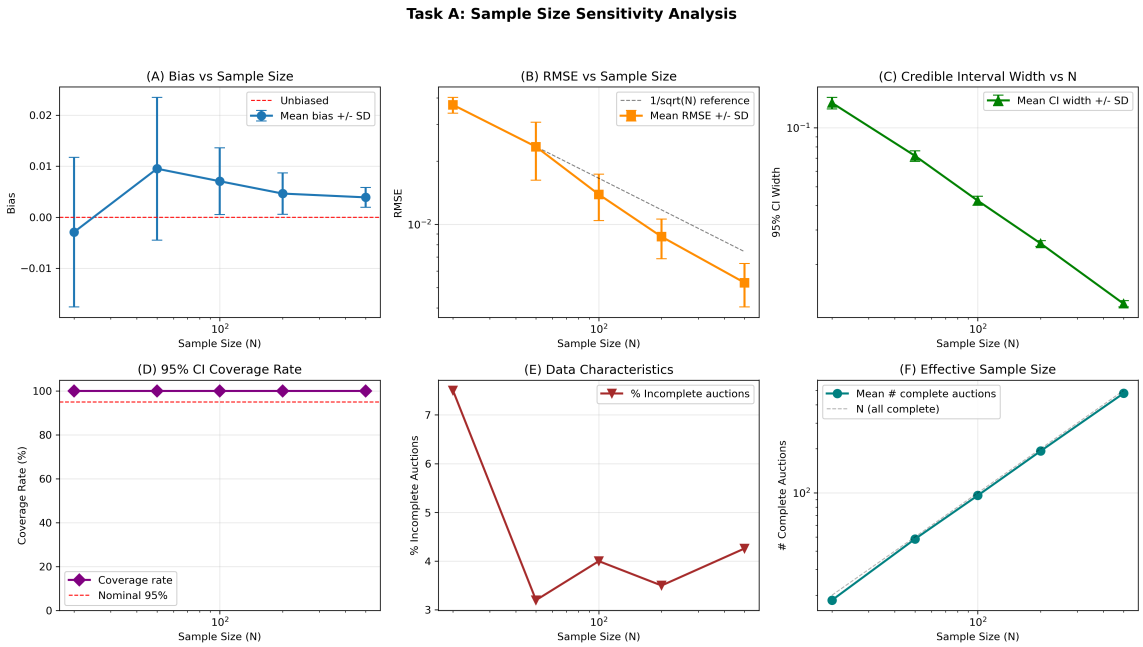


Figure 3: Task A selection-aware sensitivity ($b^* = 1.4$). Bias remains near zero across sample sizes, RMSE decreases at the expected $1/\sqrt{N}$ rate, credible interval width shrinks appropriately, and coverage remains near nominal (95%). This contrasts sharply with the naive estimator, which exhibits collapsing coverage at large N.

8.3 Conversion Diagnostics: High Conversion Case ($b^* = 1.2$)

At $b^* = 1.2$, keep rates reach 96–98%. Table 2 summarizes performance.

Table 2: Task A conversion diagnostic ($b^* = 1.2$, 10 replications per N observed auctions)

N	Keep rate	Mean bias	Mean RMSE	Mean CI width	Coverage
20	97.3%	−0.011	0.044	0.158	100%
50	96.6%	−0.005	0.023	0.076	100%
100	97.6%	−0.006	0.014	0.047	100%
200	95.8%	−0.004	0.009	0.028	100%
500	97.3%	+0.000	0.004	0.014	100%

At this high keep rate, formal-stage selection is weak and the estimator remains well-calibrated.

8.4 Conversion Diagnostics: Very High Conversion Case ($b^* = 1.1$)

At $b^* = 1.1$, the keep rate exceeds 99.5%, but a large share of observed auctions are all-admit and therefore contribute only one-sided (upper-bound) information. Table 3 summarizes performance.

Table 3: Task A conversion diagnostic ($b^* = 1.1$, 10 replications per N observed auctions)

N	Keep rate	Mean bias	Mean RMSE	Mean CI width	Coverage
20	99.5%	−0.044	0.072	0.209	90%
50	99.6%	−0.029	0.041	0.107	90%
100	99.7%	−0.018	0.024	0.059	100%
200	99.6%	−0.012	0.015	0.035	80%
500	99.6%	−0.007	0.008	0.019	70%

At this extreme keep rate, the information content is dominated by one-sided bounds, and coverage deteriorates with N despite negligible selection pressure.

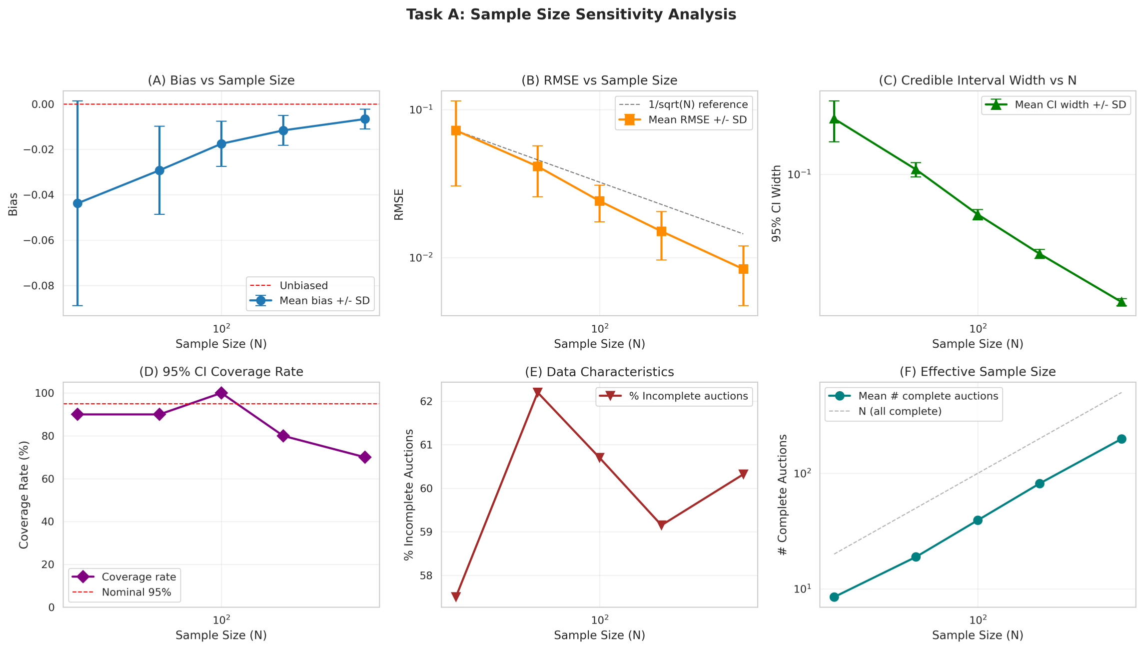


Figure 4: Task A selection-aware sensitivity at $b^* = 1.1$ (10 replications per N observed auctions). At very high keep rates, many observed auctions are all-admit (one-sided bounds), which weakens cutoff identification and leads to undercoverage as N grows.

8.5 Task A Key Takeaway

The selection-aware MH correction removes the selection-induced bias present in naive estimation and improves coverage when the data contain a substantial share of two-sided bounds. At extremely high keep rates, one-sided bounds dominate and cutoff coverage can deteriorate despite weak selection.

9 Task B Results: Two-Stage Model

Sample definition. Throughout Task B, N denotes the number of observed auctions (those with $S_i = 1$). We report the keep rate $\Pr(S_i = 1)$ to summarize the intensity of formal-stage selection.

9.1 Intercept-Only Cutoff: Staged Estimation

We validate the two-stage framework using an intercept-only cutoff $b_i^{I*} = c + \omega_i$. Figure 5 shows Stage 1 diagnostics (variances fixed at DGP values). Figure 6 reports the corresponding diagnostics in Stages 2–3.

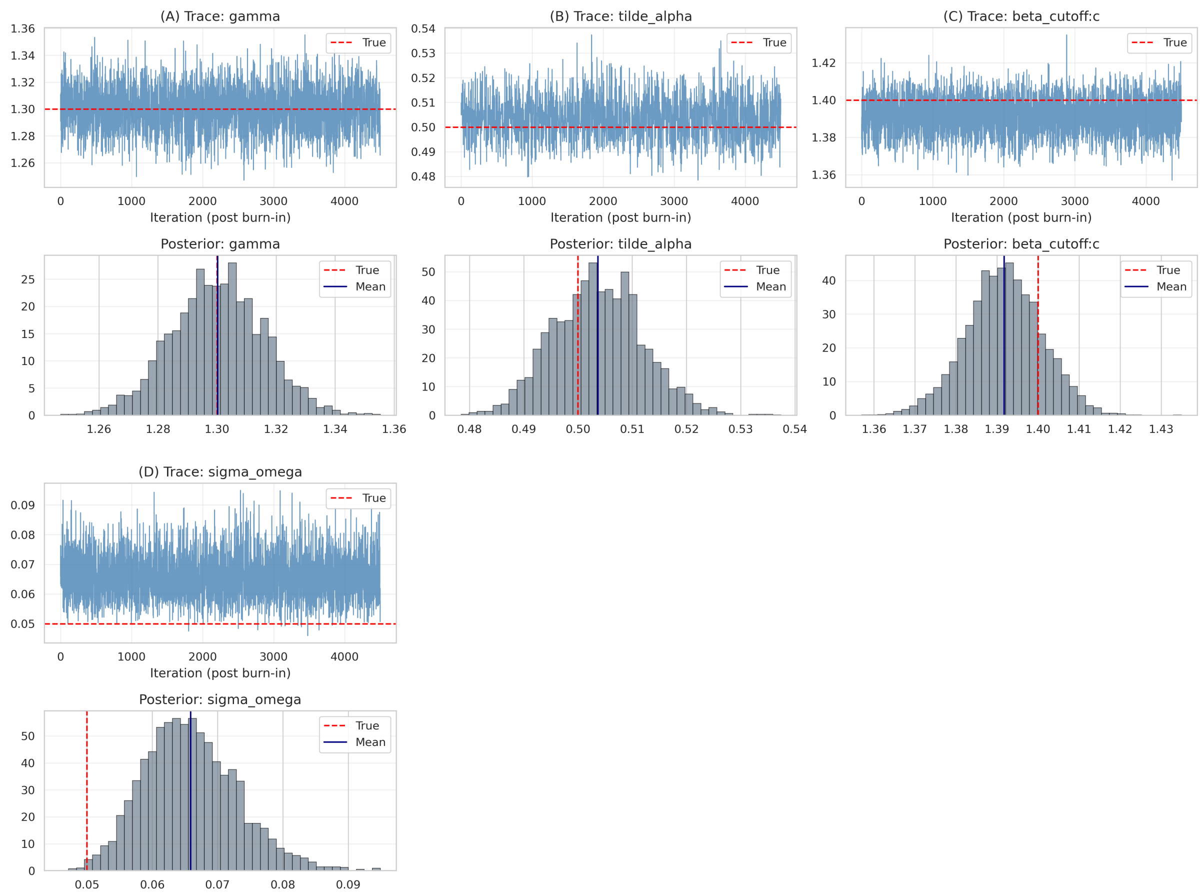
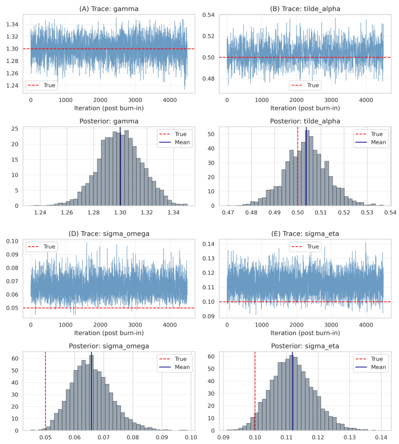
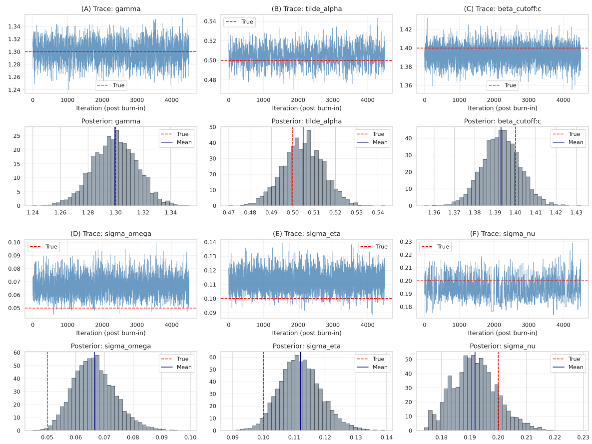


Figure 5: Task B intercept-only baseline, Stage 1: Fix $(\sigma_\nu, \sigma_\eta)$ at DGP values. Posteriors for γ (mean valuation), $\tilde{\alpha}$ (informal-bid wedge), and c (cutoff intercept) are centered near DGP values in this baseline run.

We report the normalized wedge $\tilde{\alpha}$ in diagnostics; the additive meeting-notes wedge satisfies $\alpha = (1 - 1/J)\tilde{\alpha}$.



(a) Stage 2: estimate σ_η (fix σ_ν).



(b) Stage 3: estimate $(\sigma_\nu, \sigma_\eta)$.

Figure 6: Task B intercept-only cutoff: staged estimation diagnostics beyond Stage 1. Across stages, γ and the informal-bid wedge remain stable; estimating variance components primarily affects uncertainty rather than central tendency in these runs.

Figure 7 summarizes Stage 1 performance across sample sizes.

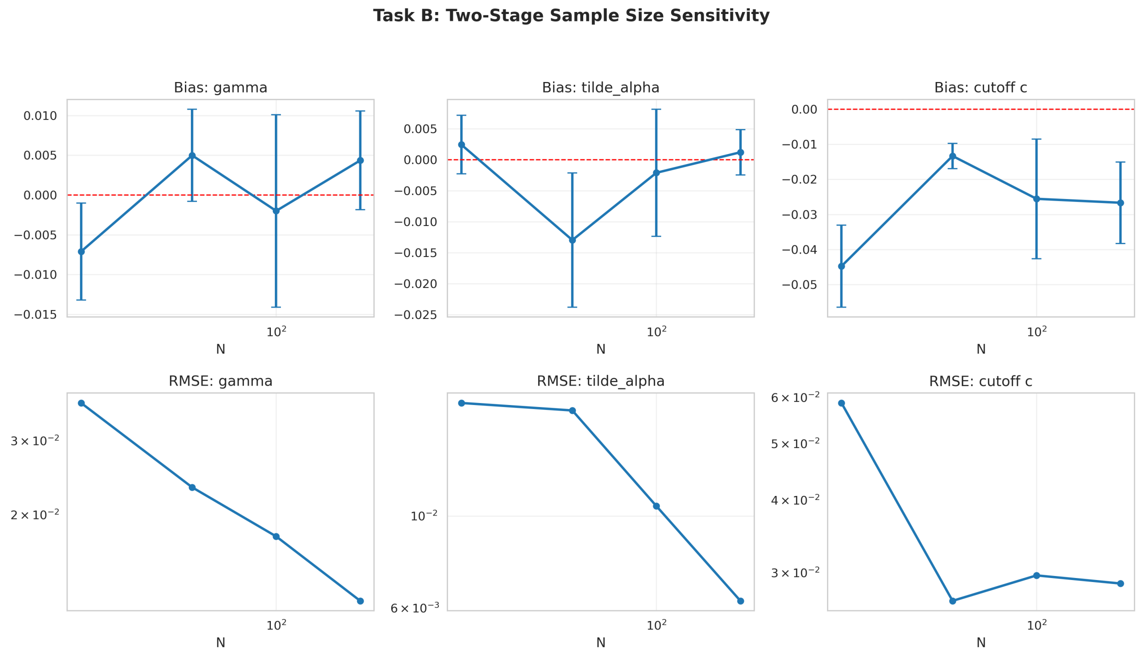


Figure 7: Task B Stage 1 sensitivity (intercept-only cutoff). The valuation mean γ and informal-bid wedge $\tilde{\alpha}$ are stable across sample sizes; cutoff recovery is noisier due to interval censoring and selection.

Table 4 summarizes Stage 1 performance across sample sizes.

Table 4: Task B Stage 1 (intercept-only) summary (2 replications per N observed auctions)

N	Keep rate	Bias(γ)	Bias($\tilde{\alpha}$)	Bias(c)	Cov(γ)	Cov(c)
20	63.9%	−0.007	−0.010	−0.022	100%	100%
50	65.5%	−0.015	+0.005	−0.026	100%	50%
100	63.1%	−0.001	−0.003	−0.038	100%	0%
200	64.7%	−0.005	−0.004	−0.016	100%	50%

In these Stage 1 runs (2 replications per N), γ exhibits small bias and high coverage, while the informal-bid wedge $\tilde{\alpha}$ is estimated with comparably small bias. The cutoff intercept c is noisier and can exhibit undercoverage due to interval censoring and selection.

9.2 Moments Cutoff: Identification Failure

We next implement the moments-based cutoff from the meeting notes:

$$b_i^{I*} = c + \theta_1 m_{i,1} + \theta_2 m_{i,2} + \theta_3 m_{i,3} + \omega_i,$$

with DGP values $(\theta_1, \theta_2, \theta_3) = (0.25, 0.10, 0.05)$ and c calibrated so $E[b_i^{I*}] \approx 1.4$.

Figure 8 shows baseline diagnostics for the moments cutoff.

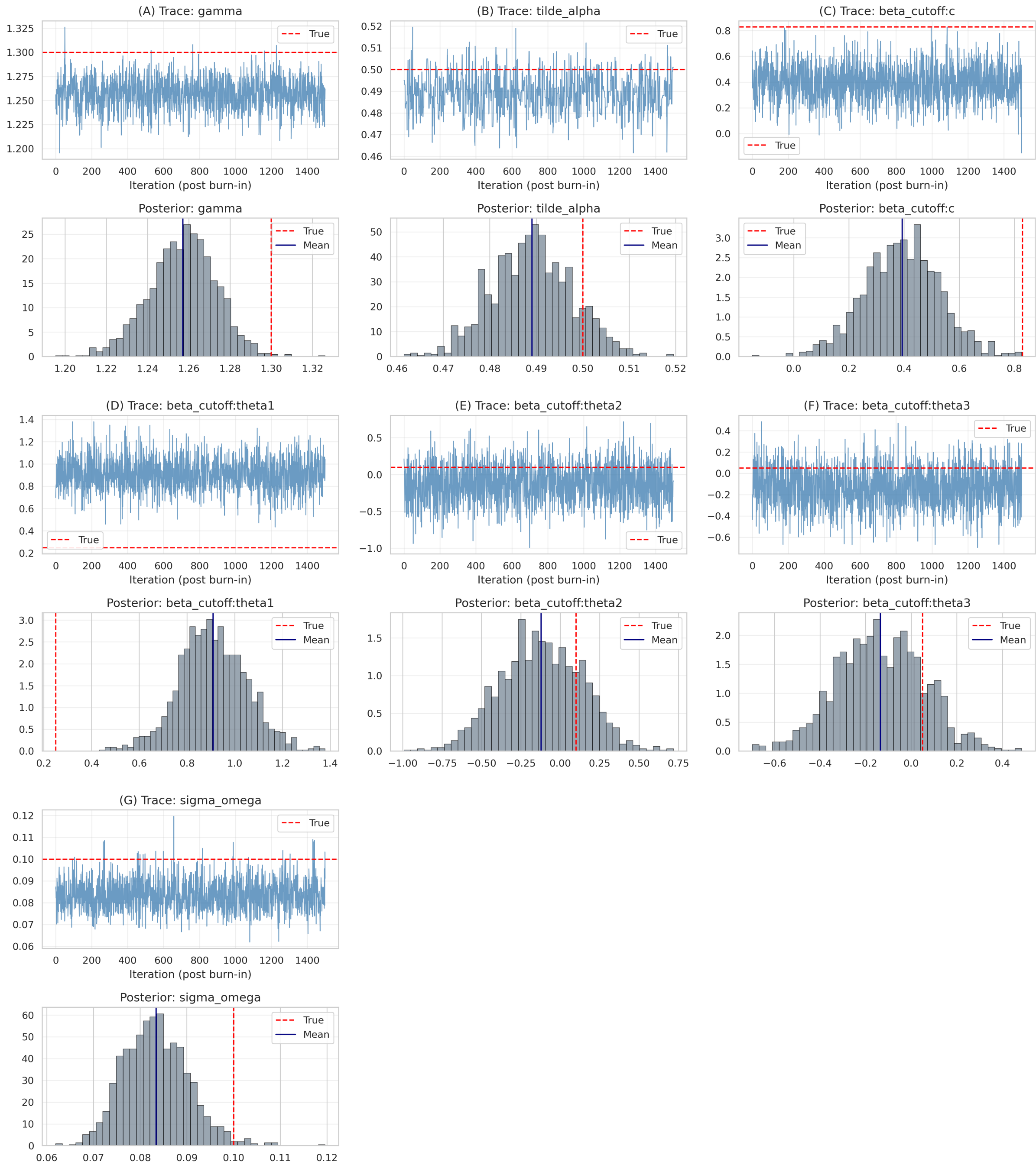


Figure 8: Task B moments-cutoff baseline diagnostics (Stage 1). While γ and $\tilde{\alpha}$ are reasonably recovered, the cutoff coefficients $(c, \theta_1, \theta_2, \theta_3)$ show wide posteriors and potential bias.

9.2.1 The Collinearity Problem

The moments $m_{i,1}$, $m_{i,2}$, $m_{i,3}$ are highly correlated by construction:

- $m_{i,1} = b_i^{I(1)}$ (max bid)
- $m_{i,2} = (b_i^{I(1)} + b_i^{I(2)})/2$ (contains the max bid)
- $m_{i,3} = (b_i^{I(1)} + b_i^{I(2)} + b_i^{I(3)})/3$ (also contains the max bid)

Collinearity diagnostics on the observed-sample design matrix reveal:

- Maximum pairwise correlation: $|\rho| = 0.903$
- Condition number: 6.53
- Variance Inflation Factors: $\{3.11, 10.33, 6.23\}$

A VIF exceeding 10 indicates severe multicollinearity. The top-2 average moment $m_{i,2}$ has VIF $= 10.33$, signaling that its coefficient is poorly identified.

9.2.2 Sensitivity Results for Moments Cutoff

Figure 9 and Table 5 show the erratic coefficient recovery.

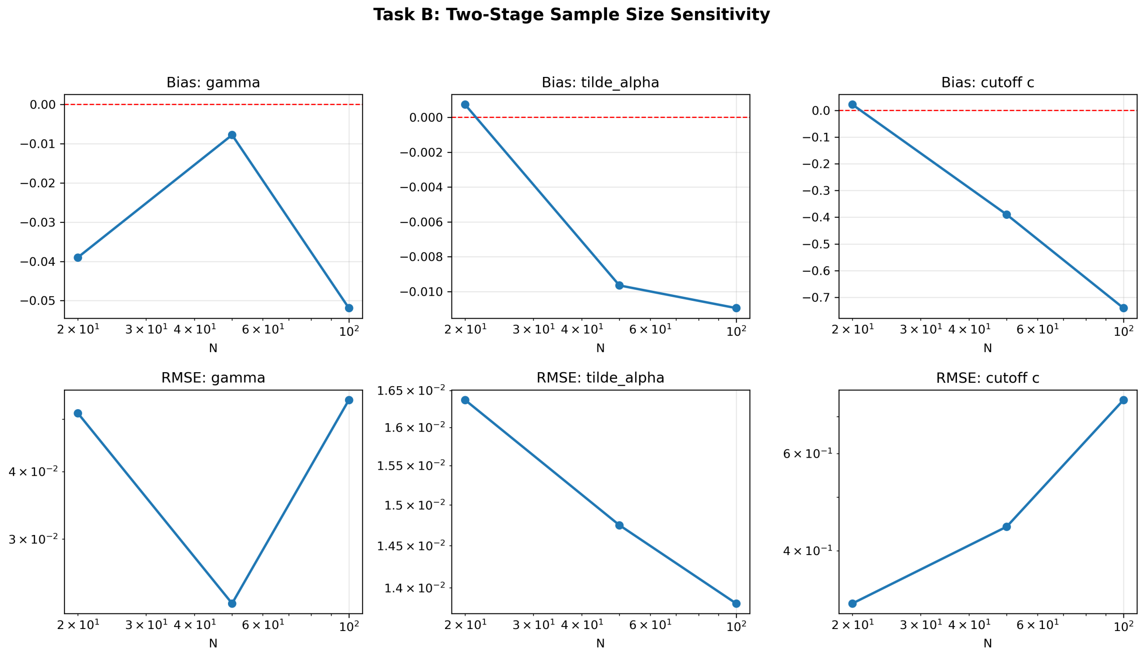


Figure 9: Task B Stage 1 sensitivity (moments cutoff), 1 replication per N observed auctions. While γ and $\tilde{\alpha}$ remain stable, the cutoff coefficients show erratic behavior.

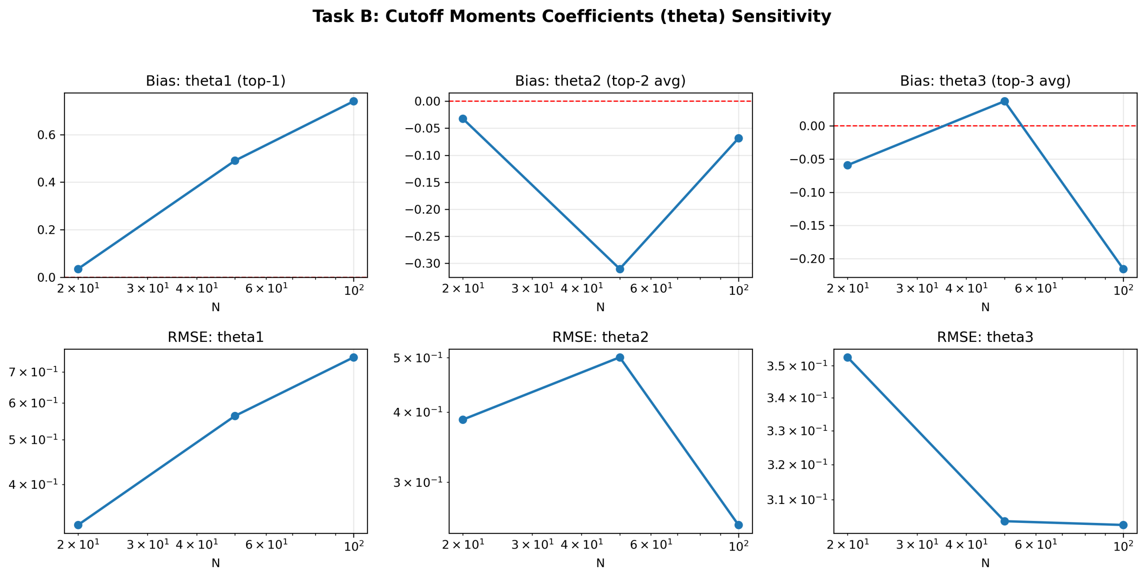


Figure 10: Task B Stage 1 moments cutoff coefficient bias and RMSE. The coefficients θ_1, θ_2, θ_3 exhibit large and variable bias.

Table 5: Task B Stage 1 (moments cutoff) bias summary (1 replication per N observed auctions)

N	Keep rate	Bias(γ)	Bias($\tilde{\alpha}$)	Bias(c)	Bias(θ_1)	Bias(θ_2)	Bias(θ_3)
20	69.0%	-0.039	$+0.001$	$+0.022$	$+0.035$	-0.032	-0.059
50	65.8%	-0.008	-0.010	-0.390	$+0.491$	-0.310	$+0.037$
100	69.0%	-0.052	-0.011	-0.740	$+0.740$	-0.069	-0.215

Interpretation. The erratic coefficient recovery is an *identification* problem rather than an MCMC implementation problem. The collinearity among moments creates a ridge in the likelihood: many coefficient combinations yield similar fits, leading to diffuse posteriors and non-vanishing bias as N grows. Additionally, placing $b_i^{I(1)}$ in the cutoff creates mechanical cancellation in the selection inequality $b_i^{I(1)} \geq c + \theta_1 b_i^{I(1)} + \cdots$, further weakening identifying variation.

9.3 Depth-Based Cutoff: Alternative Specification

To address identification, we implement a depth-based cutoff excluding the maximum bid:

$$b_i^{I*} = c + \theta_1 \frac{b_i^{I(2)} + b_i^{I(3)}}{2} + \theta_2 (b_i^{I(2)} - b_i^{I(3)}) + \omega_i.$$

Interpretation (M&A). The covariates summarize the strength and depth of the backup set. A higher runner-up average $m_{i,1} = (b_i^{I(2)} + b_i^{I(3)})/2$ indicates real competitive tension (credible backups), so the target can screen less aggressively, making $\theta_1 < 0$ plausible. A larger runner-up gap $m_{i,2} = b_i^{I(2)} - b_i^{I(3)}$ reflects a steep drop within the backup set and shallow depth, so the target proceeds only when the top bid is sufficiently strong, making $\theta_2 > 0$ plausible.

Why this helps identification. The maximum bid now appears only on the left side of the selection condition, avoiding the "max on both sides" cancellation channel. Compared to top-k averages $\{b_i^{I(1)}, (b_i^{I(1)} + b_i^{I(2)})/2, (b_i^{I(1)} + b_i^{I(2)} + b_i^{I(3)})/3\}$, which are mechanically collinear because each includes $b_i^{I(1)}$, the pair $(m_{i,1}, m_{i,2})$ is less collinear and provides more independent variation in cutoff determinants. In the Stage 1 runs shown here (1 replication per N), the depth-based slopes are less erratic than the moments specification, though cutoff recovery (especially the intercept) can remain noisy at small N.

Table 6: Task B Stage 1 (depth cutoff) bias summary (1 replication per N observed auctions)

N	Keep rate	Bias(γ)	Bias($\tilde{\alpha}$)	Bias(c)	Bias(depth mean)	Bias(depth gap)
20	83.3%	−0.002	+0.006	+0.166	−0.120	−0.076
50	74.6%	+0.000	−0.011	+0.513	−0.393	−0.159
100	73.0%	+0.011	−0.013	−0.219	+0.140	+0.141

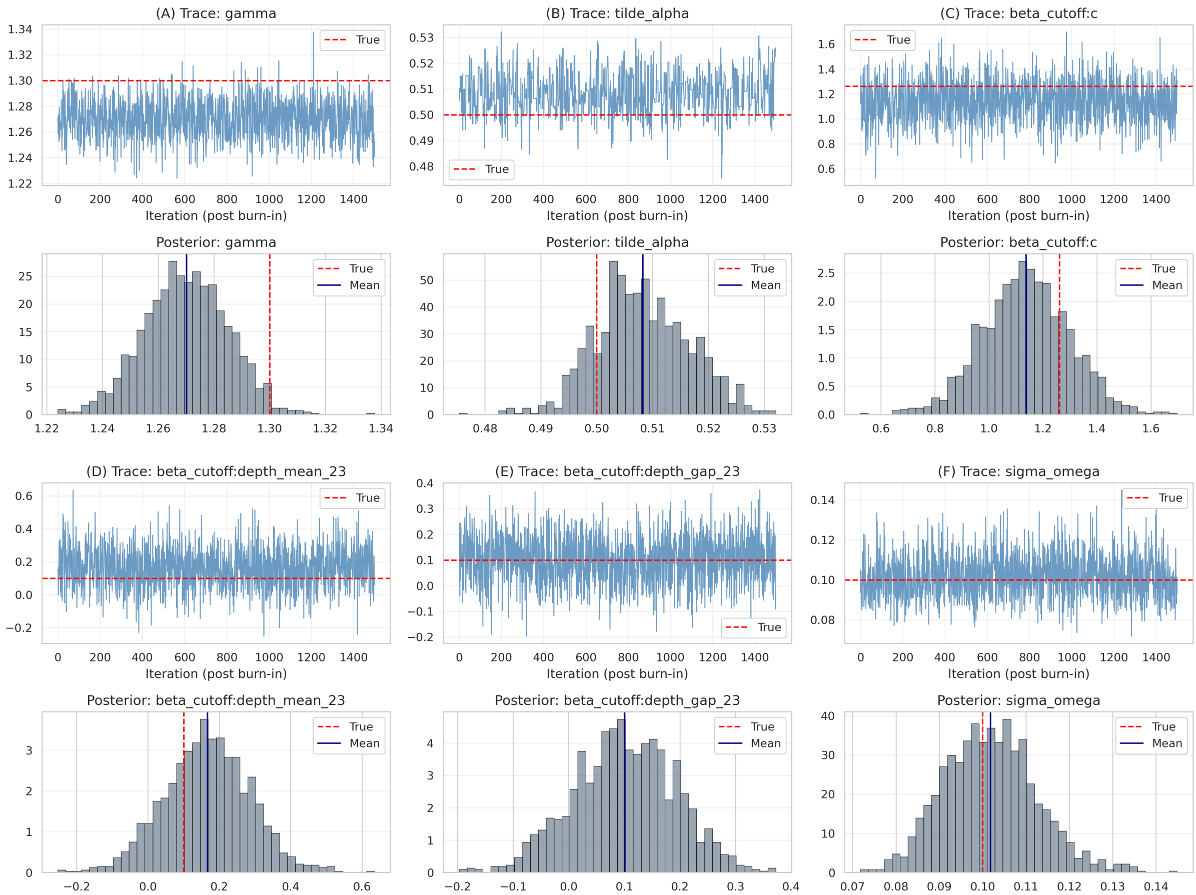


Figure 11: Task B depth-based cutoff baseline diagnostics (Stage 1). Relative to the moments cutoff, the depth-based specification yields more stable cutoff slope posteriors in these runs, though the intercept can remain noisy at small N.

10 Identification Analysis

10.1 What Is Identified and From What Variation

Valuation mean γ: Formal bids of admitted bidders are informative about γ. Under the linear bid rule $b^F_{ij} = (1 - 1/J)u_{ij}$ and $u_{ij} = \gamma + \nu_{ij} + \eta_{ij}$, variation in observed b^F pins down γ given the assumed shading factor and (in staged estimation) fixed variance components.

Informal-bid wedge $\tilde{\alpha}$: The relationship between informal and formal bids for the same bidder provides information about κ (and hence $\tilde{\alpha}$). Intuitively, $b^I_{ij} = \lambda_I(J, \kappa)v_{ij}$ scales the latent valuation, while $b^F_{ij} = (1 - 1/J)u_{ij}$ scales the post-diligence valuation; combining these sources helps separate the informal-bid wedge from valuation location.

Cutoff parameters: Identified from the admission pattern—where the cutoff falls relative to the bid distribution. Two-sided bounds (both admits and rejects) are most informative; at very high keep rates, one-sided (all-admit) bounds dominate and identification weakens. The selection-aware likelihood targets the initiated-auction cutoff process rather than the selected sample of observed auctions.

10.2 The Moments Cutoff Identification Failure

The moments cutoff $b^{I*}_i = c + \theta_1 m_{i,1} + \theta_2 m_{i,2} + \theta_3 m_{i,3} + \omega_i$ is not identified for two reasons:

1. Collinearity: The moments are highly correlated (all include $b^{I(1)}_i$), creating a ridge in the likelihood.

2. Mechanical cancellation: The maximum bid appears on both sides of the selection condition, weakening the identifying variation.

Formal diagnosis:

- Condition number: 6.53 (elevated but not extreme)
- Maximum VIF: 10.33 (severe multicollinearity)
- Correlation matrix: Max $|\rho| = 0.903$

11 Conclusion

This report develops and validates a Bayesian estimation framework for two-stage M&A auctions under formal-stage selection. The key challenge is that econometricians observe only auctions where at least one bidder was admitted, systematically excluding "all-reject" auctions. Ignoring this selection leads to biased estimates that become increasingly precise around the wrong values as sample size grows.

Our solution implements the conditional likelihood principle through Metropolis–Hastings data augmentation. In the simulations presented here, the selection-aware estimator:

1. Removes the downward bias present in naive cutoff updates (Task A baseline)

2. Improves coverage in baseline and moderate keep-rate regimes (Task A), while clarifying when one-sided bounds can drive undercoverage even with weak selection
3. Recovers valuation and informal-bid wedge parameters with small bias in the two-stage model (Task B baseline)
4. Identifies a collinearity-driven identification failure in the moments-based cutoff specification
5. Provides an alternative depth-based specification that reduces this collinearity and mechanical cancellation

The framework provides a simulation-validated baseline toward application to real M&A auction data. Practical considerations include: (i) ensuring sufficient variation in admission patterns for cutoff identification; (ii) choosing cutoff covariates that avoid mechanical cancellation with the selection condition; (iii) using staged estimation to diagnose identification incrementally; and (iv) extending to bidder-adjusted screening and richer participation models when moving beyond the baseline case.

The selection problem studied here is not unique to M&A auctions. Similar challenges arise whenever researchers observe only transactions that clear a threshold—bankruptcy proceedings that reach court, loan applications that are approved, job applicants who are hired. The conditional likelihood framework and MH data augmentation approach developed here may prove useful in these related settings.

A Selection Probability Derivations

A.1 Task A: Truthful Bidding Case

Under $b^I_{ij} = v_{ij}$ with $v_{ij} \overset{iid}{\sim} \mathcal{N}(\mu_v, \sigma_v^2)$:

$$
\begin{aligned}
\Pr(S_i = 1 \mid b) &= \Pr(\max_j v_{ij} \geq b) \\
&= 1 - \Pr(\max_j v_{ij} < b) \\
&= 1 - \prod_{j=1}^{J} \Pr(v_{ij} < b) \\
&= 1 - \Phi\left(\frac{b - \mu_v}{\sigma_v}\right)^J .
\end{aligned}
$$

A.2 Task B: Informal-Bid Wedge Case

Under $b_{ij}^I = \lambda_I(J,\kappa)v_{ij}$ with $v_{ij} \overset{iid}{\sim} \mathcal{N}(\gamma, \sigma_\nu^2)$:

$$\begin{aligned}
\Pr(S_i = 1 \mid b) &= \Pr(\max_j b_{ij}^I \geq b) \\
&= \Pr(\max_j \lambda_I v_{ij} \geq b) \\
&= \Pr\left(\max_j v_{ij} \geq \frac{b}{\lambda_I}\right) \\
&= 1 - \Phi\left(\frac{b/\lambda_I(J,\kappa) - \gamma}{\sigma_\nu}\right)^J.
\end{aligned}$$

B MH Acceptance Ratio Derivation

Target density on $[L_i, U_i]$:

$$\pi(b) \propto \phi\left(\frac{b-\mu}{\sigma_\omega}\right) \cdot \frac{1}{\Pr(S_i = 1 \mid b)}.$$

Proposal density (truncated normal):

$$q(b) \propto \phi\left(\frac{b-\mu}{\sigma_\omega}\right) \cdot \mathbf{1}\{L_i \leq b \leq U_i\}.$$

Independence MH acceptance ratio:

$$\begin{aligned}
\alpha &= \min\left(1, \frac{\pi(b^{\text{prop}})q(b^{\text{old}})}{\pi(b^{\text{old}})q(b^{\text{prop}})}\right) \\
&= \min\left(1, \frac{\phi(\cdot)/\Pr(S \mid b^{\text{prop}}) \cdot \phi(\cdot)}{\phi(\cdot)/\Pr(S \mid b^{\text{old}}) \cdot \phi(\cdot)}\right) \\
&= \min\left(1, \frac{\Pr(S_i = 1 \mid b^{\text{old}})}{\Pr(S_i = 1 \mid b^{\text{prop}})}\right).
\end{aligned}$$