Banking Competition with Heterogeneous Funding Structures

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Abstract

Banking systems feature persistent heterogeneity in funding structures that shapes competition and stability. I build a general–equilibrium model of strategic banking competition between a deposit–rich bank with unlimited access to insured deposits and a wholesale–reliant bank that faces a cap on insured funding. A flat deposit–insurance premium τ is paid ex ante and deducted from regulatory capital, which interacts with the capital ratio γ to produce differential marginal funding costs across bank types. I prove existence and uniqueness of the Cournot equilibrium and show that lending choices are strategic substitutes. The premium acts as a constant marginal cost when insured deposits fund the margin, but becomes inframarginal when the cap binds; this wedge delivers a targeted policy lever. Small increases in τ reduce market concentration by shifting scale toward wholesale–funded institutions, while strategic substitution limits aggregate lending effects. Wholesale funding endogenously prices risk, implying $A_W^* > A_D^*$ when wholesale is used. The framework rationalizes persistent market segmentation and provides quantitative guidance for deposit–insurance design and competition policy.

1 Introduction

Banking systems exhibit persistent heterogeneity in funding structures. Some institutions operate with deep deposit franchises, while others rely more heavily on uninsured runnable or wholesale funding. The 2023 banking turmoil made the implications of this heterogeneity vivid: banks with large shares of uninsured runnable funding experienced acute stress, whereas deposit–rich institutions were comparatively resilient (e.g., Jiang et al., 2024). This raises two questions. Why do banks with similar lending technologies sustain markedly different funding models? And how do these differences interact with deposit insurance to shape competition, risk, and market structure?

This paper develops a tractable general–equilibrium model of strategic banking competition with *asymmetric access to insured deposits* and *explicit deposit–insurance pricing*. Two banks compete in quantities for identical loan demand. Bank D has unlimited access to insured deposits; Bank W faces a structural cap $\overline{D}_W$ and must use wholesale funding for the residual. A flat premium τ on insured deposits is paid ex ante at $t = 0$ and is *P&L–through* into regulatory capital, so that $e_j - \tau i_j \geq \gamma x_j$ binds in equilibrium. Insured deposits are senior; wholesale investors price default risk competitively. An insurer budget identity closes the government account in expectation via lump–sum transfers to households. This timing and accounting make transparent how the premium interacts with capital requirements and with bank-type heterogeneity.

Main results. Three findings emerge. First, I establish *existence and uniqueness* of Nash equilibrium under standard regularity conditions; the banks' lending choices are *strategic substitutes*. Second, τ creates a *policy wedge* whose incidence depends on which funding source is marginal: it is a constant marginal cost when insured deposits fund the margin (cap slack), but is *inframarginal* when $\overline{D}_W$ binds. Third, I characterize equilibrium market structure and risk: small increases in τ *reduce concentration* by shifting scale from the deposit–rich bank to the wholesale–reliant bank, while the strategic–substitution property dampens aggregate lending effects. Whenever Bank W uses wholesale funding, risk is endogenously priced so that its default threshold exceeds Bank D's ($A^*_W > A^*_D$). These results rationalize persistent segmentation by funding model and deliver clean comparative statics in the policy parameters.

Mechanism and policy implications. The key insight is that deposit insurance premiums τ create a policy wedge whose incidence depends on which funding source is marginal. When insured deposits fund the margin, τ acts as a constant marginal cost; when the deposit cap $\overline{D}_W$ binds, the premium becomes inframarginal. This creates a targeted competition instrument: small increases in τ shift market share toward wholesale-funded banks without large aggregate credit effects, while endogenous wholesale spreads ensure that risk is appropriately priced across bank types.

Contribution and roadmap. The contribution is to analyze *competition between heterogeneous funding models* when deposit insurance is explicitly priced and recognized in capital, yielding existence/uniqueness results, clean policy wedges, and comparative statics relevant for financial stability policy. Section 2 reviews related literature. Section 3 presents the model setup. Sections 4 and 5 analyze firm and bank optimization. Section 7 establishes existence and uniqueness. Section 8 derives policy implications.

2 Literature Review

This paper connects several strands of literature on banking competition, deposit insurance, and capital regulation. I organize the review around three key themes: foundational theories of deposit insurance and guarantee overhang, modern frameworks for capital regulation effects, and tractable models of banking competition.

2.1 Deposit Insurance and Guarantee Overhang

The theoretical foundation for deposit insurance begins with Diamond and Dybvig (1983), who showed that demand deposits optimally provide liquidity insurance to households facing idiosyncratic consumption shocks, but create vulnerability to self-fulfilling bank runs. Deposit insurance eliminates pure panic runs but introduces moral hazard. Merton (1977) formalized this trade-off in an option-theoretic framework, showing that deposit insurance acts like a mispriced put option on bank assets, enabling shareholders to shift downside losses to the guarantor while retaining upside gains. Kareken and Wallace (1978) similarly argued that underpriced insurance encourages excessive risk-taking, motivating regulatory constraints to realign private and social incentives.

Calomiris and Kahn (1991) provided an alternative perspective, showing that demandable debt can serve as an optimal contract that disciplines bank behavior by threatening early liquidation. This suggests that uninsured wholesale funding—while potentially destabilizing—may have beneficial incentive effects that deposit insurance destroys. Keister and Todd (2007) extended this analysis to competitive settings, showing that deposit insurance can reduce bank competition and lead to concentrated market structures.

However, the classic debt-overhang logic (Myers, 1977) suggests a countervailing force: highly levered institutions may underinvest in positive-NPV projects because marginal surpluses accrue to creditors rather than equity. In banks with insured deposits, this creates *guarantee overhang*—when the balance sheet is already risky, equityholders may rationally pass on good loans if much of the payoff in adverse states goes to the insurer rather than shareholders. Recent work by Davila and Goldstein (2023) formalizes optimal deposit insurance design in general equilibrium, showing that first-best outcomes require risk-based premiums that perfectly internalize social costs.

2.2 Capital Regulation and the Forced Safety Effect

Traditional models of bank capital regulation emphasized costs: higher capital requirements reduce profitable leverage and limit banks' ability to provide liquidity and credit (Thakor, 1996). Repullo and Suarez (2004) formalized this view, showing that capital requirements act as a tax on intermediation that reduces loan supply. However, this perspective changed dramatically following the 2008 financial crisis.

Building on these foundations, recent work by Malherbe and co-authors formalizes when capital regulation mitigates guarantee overhang and can even *increase* lending. Bahaj and Malherbe (2020) identify two competing effects of raising capital requirements: (i) a *composition effect* (less subsidized deposit funding, more equity) that tends to reduce lending, and (ii) the *Forced Safety Effect* (FSE), whereby additional capital lowers default risk and makes equityholders internalize residual cash flows in states close to default, encouraging positive-NPV loans previously undervalued due to guarantee overhang.

In complementary dynamic work, Malherbe (2020) shows that optimal capital requirements should vary over business and financial cycles. Intuitively, in booms banks accumulate internal capital and face abundant opportunities; without policy intervention, the bank-capital channel can push lending beyond the social optimum. Countercyclical stringency limits inefficient expansions in good times and relaxes constraints in downturns.

A parallel literature has emphasized the social costs of bank equity. Admati and Hellwig (2013) challenged the conventional wisdom that equity is expensive for banks, arguing that higher capital requirements improve rather than harm social welfare. Admati et al. (2018) further showed that banks face a "leverage ratchet" where short-term pressures to maintain high leverage can prevent optimal recapitalization even when it would be welfare-improving. These insights complement the FSE by highlighting institutional factors that can make capital regulation beneficial rather than constraining.

2.3 General Equilibrium and Macroprudential Perspectives

A broader theoretical discourse has evolved from partial-equilibrium views to macroprudential frameworks incorporating system-wide externalities. Early models with exogenous wedges in the private cost of equity versus deposits predicted that binding capital requirements act like a tax on intermediation, reducing loan supply (Thakor, 1996; Repullo and Suarez, 2004). Post-crisis theory emphasized that capital regulation must account for general-equilibrium feedbacks—credit booms and busts can be amplified by collective bank behavior—calling for macroprudential instruments that safeguard the system as a whole (Hanson et al., 2011).

General-equilibrium models with explicit banking sectors further refine these welfare trade-offs. Van den Heuvel (2008) showed that while capital curbs risk-shifting, it can also limit banks' liquidity creation, implying non-trivial welfare costs of capital rules. Conversely, Begenau (2020) demonstrated that if deposits provide liquidity services, tighter requirements can reduce the aggregate supply of safe claims, increase the liquidity premium on deposits, and *lower* banks' effective funding costs—potentially increasing lending. These results complement the FSE logic: additional capital can relax rather than tighten effective constraints through either reduced default risk or equilibrium pricing of liquidity services.

2.4 Banking Competition and Risk-Taking

The relationship between competition and bank risk-taking has generated extensive debate. Traditional views suggested that competition reduces bank profits and encourages risk-taking as a way to maintain returns (Keister and Todd, 2007). However, Boyd and De Nicoló (2005) challenged this "competition-fragility" view, showing that in a model where banks compete for deposits, greater competition can actually reduce risk-taking by lowering the rates banks pay to depositors, thereby reducing their incentive to pursue risky strategies.

Martínez-Miera and Repullo (2010) reconciled these competing views by showing that the relationship between competition and risk depends crucially on market structure. In their model, moderate increases in competition can reduce risk (consistent with Boyd-De Nicolò), but further increases beyond a threshold raise risk as the "margin effect" (competition reduces profits) dominates the "risk-shifting effect" (competition reduces deposit rates). This generates a U-shaped relationship between competition and bank failure probabilities.

Matutes and Vives (2000) contributed to this literature by analyzing how capital requirements interact with competition in a differentiated banking model. They showed that capital requirements can serve as commitment devices that allow banks to compete less aggressively, potentially improving welfare. Recent work by Repullo and Suarez (2013) extended this analysis to dynamic settings, showing that the interaction between competition and capital regulation can generate procyclical effects that amplify business cycle fluctuations.

2.5 Systemic Risk and Financial Stability

The 2008 financial crisis highlighted the importance of systemic risk and interconnectedness in banking. Acharya and Viswanathan (2011) developed a model where banks' correlated risk-taking creates systemic externalities, justifying macroprudential regulation that goes beyond individual bank supervision. Brunnermeier and Pedersen (2009) showed how funding liquidity and market liquidity interact to create "liquidity spirals" that can amplify shocks across the financial system.

More recent work has focused on the role of wholesale funding and bank runs in modern banking. Gorton and Metrick (2012) documented how the 2008 crisis involved a "run on repo" where short-term wholesale creditors withdrew funding from banks holding securitized assets. He and Krishnamurthy (2013) developed a general equilibrium model where financial intermediaries face funding constraints that become binding during crises, leading to fire sales and amplified volatility. These insights are particularly relevant for understanding how heterogeneous funding structures—central to our analysis—affect both individual bank stability and systemic risk.

2.6 Tractable Models of Banking Competition

The theoretical frameworks reviewed above—from deposit insurance design to capital regulation effects to competition-stability tradeoffs—provide important insights but often remain at a high level of abstraction. Recent advances have produced tractable frameworks for analyzing strategic interactions between banks under capital regulation that can accommodate these various forces simultaneously. Bahaj and Malherbe (2020) developed a general framework for analyzing bank competition under capital constraints, showing how regulatory requirements interact with bank funding structures to determine equilibrium lending. Malherbe (2020) extended this approach to dynamic settings with business cycle variation. Matutes and Vives (2000) provided an earlier foundation for strategic bank competition models with differentiated products and capital requirements.

These models highlight how banks' funding structures and regulatory constraints jointly determine equilibrium outcomes, creating complex, non-monotonic effects on lending that depend on the specific institutional environment. Building on this foundation, the present analysis differs by explicitly incorporating *heterogeneous funding access* and *endogenous risk pricing* into a tractable competition framework. This paper contributes to this literature by analyzing *competition between banks with heterogeneous funding structures* when deposit insurance is explicitly priced and recognized in capital requirements. The model yields clean existence/uniqueness results and transparent comparative statics for policy-relevant parameters, extending the tractable competition framework to settings with asymmetric deposit access and endogenous wholesale funding spreads—features that proved central during the 2023 banking stress episodes.

3 Model Setup

3.1 Preliminaries and Notation

I normalize $\mathbb{E}[A] = 1$ and write the support as $[\underline{A}, \overline{A}] \subset \mathbb{R}_{++}$ with density f strictly positive and continuous. For any real z, define $z^+ \equiv \max\{z, 0\}$. Aggregate lending is $X \equiv x_D + x_W$.

The key revenue parameter is defined as:

$$\Theta \equiv \pi \alpha^2 \overline{L}^{1-\alpha} \tag{1}$$

so the expected revenue per unit of loans when aggregate lending is X is $\Theta X^{1/\epsilon}$ (derived in Appendix C.2). Loan demand elasticity satisfies $\epsilon = -1/(1-\alpha) < -1$, so $1/\epsilon = \alpha - 1 < 0$. I keep α as the technology parameter and use the identity $1/\epsilon = \alpha - 1$ only to simplify derivatives.

Notation. We write $\sigma \equiv 1 - \alpha \in (0, 1)$ so that:

$$1/\epsilon = \alpha - 1 = -\sigma \quad \text{and} \quad R(X) = \Theta X^{1/\epsilon} \tag{2}$$

By standard law of large numbers arguments, exactly fraction π of firms succeed in any realization, so aggregate objects are deterministic conditional on aggregate productivity A.

3.2 Economic Environment

I develop a one-period general equilibrium model to analyze how differential access to insured deposits affects bank lending decisions and risk-taking behavior. The economy consists of three sectors: a continuum of firms that demand loans for capital investment, two banks that compete in the lending market with different funding structures, and workers who supply labor inelastically.

The production sector features a continuum of ex-ante identical firms indexed by $i \in [0, 1]$. Each firm operates a Cobb-Douglas technology $y_i = A \cdot a_i \cdot k_i^\alpha L_i^{1-\alpha}$ with capital share parameter $\alpha \in (0, 1)$. Output depends on an aggregate productivity shock $A \in [\underline{A}, \overline{A}] \subset \mathbb{R}_{++}$ and an idiosyncratic shock $a_i \in \{0, 1\}$ with success probability $\pi \in (0, 1)$. Firms face financial frictions that necessitate external financing for capital investment $k_i \geq 0$, creating demand for bank loans at gross lending rate $r^l \geq 0$. Workers supply labor $\overline{L} > 0$ inelastically and receive competitive wage $w(A) \geq 0$.

The banking sector consists of two institutions with heterogeneous funding structures. Bank D enjoys unlimited access to government-insured deposits at normalized rate $r_d = 0$, while Bank W faces structural limitations $\overline{D}_W > 0$ on insured deposit gathering and must rely on wholesale funding at rate $r_w \geq 0$ for additional financing beyond this constraint.

Both banks are subject to regulatory capital requirements $\gamma \in (0, 1)$ (fraction of assets that must be equity) and operate under limited liability with negligible inside equity $\kappa \to 0$.

Assumption 3.1 (Production Technology and Frictions)**.** *The production environment is characterized by the following properties:*

(a) Production function: Each firm $i \in [0, 1]$ operates the technology $y_i = A \cdot a_i \cdot k_i^\alpha L_i^{1-\alpha}$.

(b) *Financial frictions: Firms possess no initial wealth and must borrow to finance capital investment.*

(c) *Information structure: Idiosyncratic productivity a_i is private information realized at the final stage, with $\Pr(a_i = 1) = \pi$. The idiosyncratic shocks $\{a_i\}_{i\in[0,1]}$ are independent across firms and independent of the aggregate shock A.*

(d) *Aggregate uncertainty: Aggregate productivity A is publicly observed at date $t = 1$ before labor choices.*

(e) *Labor market: Labor is supplied inelastically at level $\overline{L}$ and allocated competitively.*

(f) *Preferences and depreciation: All agents are risk-neutral and capital fully depreciates within the period.*

(g) *Technical conditions: The density function $f(A)$ is continuous and strictly positive on the support $[\underline{A}, \overline{A}]$.*

3.3 Premium accounting, capital, and funding decomposition

Let e_j denote outside equity raised at $t = 0$, i_j insured deposits, and w_j uninsured wholesale debt. Because the premium τi_j is paid up front and deducted from regulatory capital (P&L-through), the time-0 balance sheet and capital rule are

$$\text{(BS)}: \quad e_j + i_j + w_j \;=\; x_j + \tau i_j, \tag{3}$$

$$\text{(CR)}: \quad e_j - \tau i_j \;\geq\; \gamma x_j. \tag{4}$$

(BS) is a time-0 sources-and-uses constraint: funds raised $e_j + i_j + w_j$ are used to originate loans x_j and to pay the premium τi_j up front. The capital requirement accounts for the premium deduction via (CR): $e_j - \tau i_j \geq \gamma x_j$, so the binding constraint is $e_j - \tau i_j = \gamma x_j$. At the cost-minimizing allocation:

$$e_j \;=\; \gamma x_j + \tau i_j, \qquad i_j + w_j \;=\; (1-\gamma)x_j. \tag{5}$$

Hence optimal insured-deposit usage cannot exceed debt capacity or the structural cap:

$$I_j(x_j) \;=\; \min\{(1-\gamma)x_j,\, \overline{D}_j\}, \qquad \infty = \infty,\; \overline{D}_W \in (0,\infty). \tag{6}$$

Assumption 3.2 (Productivity Distribution Structure)**.** *In addition to the basic properties specified in Assumption 3.1, the aggregate productivity distribution F satisfies the following technical conditions:*

(i) *$F(A)$ has a continuous, strictly positive density $f(A)$ on $[\underline{A}, \overline{A}]$*

(ii) *The log-concavity condition holds:*

$$\frac{d}{dA}\left[\frac{f(A)}{1-F(A)}\right] \geq 0 \quad \textit{for all } A \in [\underline{A}, \overline{A}) \tag{7}$$

This hazard rate monotonicity condition is satisfied by most standard distributions (normal, exponential, uniform, beta with appropriate parameters) and ensures that the banking optimization problems have unique interior solutions.

Remark 3.1 (Economic Interpretation of Distribution Condition). The nondecreasing hazard rate condition (IFR property) ensures that, conditional on not having reached level A, the instantaneous likelihood of crossing it rises with A. This rules out heavy upper tails and supports well-behaved curvature in the optimization problems. Many standard distributions (uniform, truncated normal, exponential) satisfy IFR.

3.4 Timeline

The model unfolds over a single period with the following sequence of events:

Date $t = 0$: The government announces a flat premium $\tau \geq 0$ per unit of insured deposits. Banks choose lending scales x_j. For each bank, insured-deposit usage is $i_j = I_j(x_j)$ and wholesale debt is $w_j = (1-\gamma)x_j - i_j$. The premium τi_j is paid *ex ante* and deducted from regulatory capital (see Section 3.3). The insurer's budget is closed in expectation via a lump-sum transfer $T(\tau)$.

Date $t = 1$: Aggregate productivity A is realized and publicly observed. Given this information and their predetermined capital stocks, firms hire labor in a competitive spot market at wage $w(A)$.

Date $t = 2$: Production occurs according to the specified technology. Immediately following production, idiosyncratic productivity shocks are realized privately by each firm. Firms with $a_i = 1$ produce output $y_i = A \cdot k_i^{\alpha} L_i^{1-\alpha}$ while firms with $a_i = 0$ produce nothing.

Date $t = 3$: All contractual obligations are settled subject to limited liability constraints. Successful firms pay wages from realized output and repay their debts. Failed firms invoke limited liability protection. If a bank is insolvent, the insurer tops up insured depositors to par.

3.5 Contracting Assumptions

The contracting environment reflects realistic features of debt and labor markets while maintaining analytical tractability.

Assumption 3.3 (Contracting and Limited Liability). *(a) Production output is observable and verifiable, enabling contractual enforcement when $a_i = 1$.*

(b) Firms operate under limited liability protection. When $a_i = 0$, firms produce no output and all claimants receive zero payoff.

(c) Idiosyncratic productivity $a_i \in \{0, 1\}$ is verifiable at default proceedings, enabling contractual enforcement.

(d) *Workers are paid from realized output when $a_i = 1$ and receive nothing when $a_i = 0$. While this output-contingent wage structure is admittedly unrealistic, it ensures that all claimants share proportionally in default losses, allowing us to solve for equilibrium without tracking separate state variables for wage obligations. Given risk neutrality, only expected wages matter for labor market equilibrium.* Workers are risk–neutral and price wages in expected value; supply is inelastic in expectation. Hence the π factor cancels from the Stage-2 FOC and the labor market clears state-by-state at $w(A)$.

(e) *In bankruptcy proceedings, banks have seniority over wage claims, though both receive zero when $a_i = 0$ under our output-contingent framework.*

Limitation. We assume output-contingent wages to keep failure states tractable. With standard wage seniority, none of our qualitative results change: the deposit subsidy and wholesale discipline wedges operate through funding seniority and the insurer's budget, not wage timing. The quantitative effect would be to shift some loss away from labor in default states and slightly tighten default thresholds.

Remark 3.2 (Derivation of Strategic Default Prevention). To understand why $\underline{A} > \alpha$ prevents strategic default, consider a successful firm with $a_i = 1$. After paying workers their equilibrium wage $(1-\alpha)Ak^{\alpha}\overline{L}^{1-\alpha}$, the firm retains $\alpha Ak^{\alpha}\overline{L}^{1-\alpha}$ and owes $(1+r^l)k$ to the bank. Strategic default is unprofitable when:

$$\alpha Ak^{\alpha}\overline{L}^{1-\alpha} \geq (1+r^l)k \tag{8}$$

From the firm's first-order condition, $\alpha^2 k^{\alpha-1}\overline{L}^{1-\alpha} = (1+r^l)$. Therefore, the no-strategic-default condition becomes:

$$\alpha Ak^{\alpha-1}\overline{L}^{1-\alpha} \geq \alpha^2 k^{\alpha-1}\overline{L}^{1-\alpha} \tag{9}$$

which requires $A > \alpha$. Since this must hold for all realizations, we need $\underline{A} > \alpha$.

Remark 3.3 (Economic Interpretation of Parameter Restriction). The condition $\underline{A} > \alpha$ has clear economic meaning: aggregate conditions must be sufficiently favorable relative to the capital intensity of production that successful firms always find it optimal to repay their debts rather than strategically default. This restriction naturally holds when macroeconomic fluctuations are moderate relative to production technology parameters.

3.6 Banking Sector Structure

Both banks operate with similar organizational structures but face different regulatory constraints on funding sources.

Assumption 3.4 (Banking Environment). *(a) Banks operate with minimal inside equity $\kappa \to 0$ and must raise funds externally.*

(b) *Regulatory capital requirements mandate that outside equity equals fraction $\gamma \in (0,1)$ of risk-weighted assets.*

(c) *Outside equity investors operate in competitive markets requiring zero economic profit in expectation.*

(d) *Government deposit insurance protects* insured deposits at par *and charges a* flat premium $\tau \geq 0$ *per unit of insured deposits, paid* ex ante *at* $t = 0$ *on planned insured-deposit usage.*

(e) *Insured-deposit access is bank-specific: the deposit-rich bank* D *has* $\infty = \infty$*; the wholesale-reliant bank* W *has a structural cap* $\overline{D}_W \in (0, \infty)$.

(f) *The premium is paid up front and deducted from regulatory capital at* $t = 0$*; the risk-free rate is normalized to zero; limited liability applies.*

The separation of control rights and capital provision creates a fundamental feature of our banking model. Initial shareholders (the bankers) maintain full control over lending decisions despite contributing negligible capital, while outside equity investors provide the required regulatory capital but have no direct influence on bank operations. This ownership structure reflects common practice in financial institutions where management control is separated from capital provision. The competitive nature of capital markets ensures that outside equity investors earn only their required return, with any economic rents accruing to those who hold control rights.

Definition 3.1 (Financial Contracts and Market Clearing)**.** *This definition establishes three key market clearing conditions that link bank optimization to general equilibrium.*

Outside Equity Contracts. *Given* (x_j, x_{-j}) *and total promised debt payments* $\mathcal{D}_j$ *(where* $\mathcal{D}_j = \mathcal{D}_D$ *for Bank D and* $\mathcal{D}_j = \mathcal{D}_W$ *for Bank W), let* $V_j(A) = A\, x_j\, \Theta\, X^{1/\epsilon}$*. An admissible outside-equity contract is* $C_j : [\underline{A}, \overline{A}] \to \mathbb{R}_+$ *satisfying:*

$$0 \leq C_j(A) \leq [\, V_j(A) - \mathcal{D}_j\,]^+ \quad \forall A \tag{10}$$

$$\int C_j(A) f(A) dA = \gamma x_j + \tau\, I_j(x_j) \tag{11}$$

With risk-neutral competitive outside equity, the banker's value reduces to

$$w_j(x_j; x_{-j}) = \mathbb{E}\Big[[V_j(A) - \mathcal{D}_j]^+\Big] - \gamma x_j - \tau\, I_j(x_j),$$

independently of the state-contingent contract shape $C_j(\cdot)$.

Deposit Insurance System. *Insured deposits are guaranteed at par through state-contingent transfers:*

$$S_j(A) \;=\; \Big(I_j(x_j) - \textit{recovery to insured class at } A\Big)^+ \tag{12}$$

The deposit insurance fund maintains budget balance through ex-ante premium collection and lump-sum household transfers:

$$\mathbb{E}\Big[\, S_D(A) + S_W(A)\,\Big] \;=\; \tau\, \mathbb{E}\Big[\, I_D(x_D) + I_W(x_W)\,\Big] \;+\; T(\tau) \tag{13}$$

The expected government transfers to depositors are explicitly given by:

$$\begin{aligned}
\mathbb{E}[S_D(A)] &= \int_{\underline{A}}^{A_D^*} \Big[(1-\gamma)x_D - A x_D \Theta X^{1/\epsilon}\Big]\, f(A)\, dA, \\
\mathbb{E}[S_W(A)] &= \int_{\underline{A}}^{A_W^{**}} \Big[\overline{D}_W - A x_W \Theta X^{1/\epsilon}\Big]\, f(A)\, dA.
\end{aligned} \tag{14}$$

Household Budget Constraint. *Representative households consume:*

$$c \;=\; w(A)\,\overline{L} \;+\; \Pi^{banks} \;-\; T(\tau) \tag{15}$$

where Π^{banks} represents banker rents that do not affect bank optimization due to the separation of control rights from capital provision.

Remark 3.4 (Funding decomposition). For $j = D$: $I_D(x_D) = (1-\gamma)x_D$ and $W_D = 0$. For $j = W$: $I_W(x_W) = \min\{(1-\gamma)x_W, \overline{D}_W\}$ and $W_W = (1-\gamma)x_W - I_W(x_W)$. Balance sheet and capital recognition with the premium are stated in Section 3.3. The wholesale rate r_w is determined by investors' zero-profit (see Lemma 5.3).

4 Firm Sector Analysis and Loan Demand

4.1 Firm Optimization and Symmetric Equilibrium

I now formally state the firm's optimization problem, which follows a two-stage structure reflecting the timing of decisions and information revelation.

Definition 4.1 (Firm's Optimization Problem). *Each firm $i \in [0,1]$ solves the following two-stage optimization problem:*

Given information and parameters:

- *Lending rate $r^l \geq 0$ determined by market clearing: banks choose quantities x_D, x_W à la Cournot; r^l follows from $X = x_D + x_W$ via $1 + r^l = \alpha^2 \overline{L}^{1-\alpha} X^{-(1-\alpha)}$*
- *Aggregate productivity distribution $F(A)$ with support $[\underline{A}, \overline{A}]$*
- *Success probability $\pi \in (0,1)$ and production parameters $\alpha \in (0,1)$, $\overline{L} > 0$*
- *Capital choices of other firms $\{k_j\}_{j \neq i}$ (taken as given in equilibrium)*

Information structure:

- *At $t = 0$: Firm i observes the equilibrium r^l implied by market clearing but not yet A or a_i*
- *At $t = 1$: Firm i observes realization of A but not yet a_i*
- *At $t = 2$: Firm i privately observes a_i after production*

Optimization stages:

Stage 2 (Labor Choice at $t = 1$): *Given predetermined capital $k_i \geq 0$, observed aggregate productivity $A \in [\underline{A}, \overline{A}]$, and market wage $w(A) \geq 0$, firm i chooses labor $L_i \geq 0$ to maximize expected net production value:*

$$\max_{L_i \geq 0} V_1^i(k_i, A, w) = \pi \cdot [A \cdot k_i^{\alpha} L_i^{1-\alpha} - w(A) L_i] \tag{16}$$

where the expectation is taken over the private idiosyncratic shock a_i.

Stage 1 (Capital Choice at $t = 0$): *Anticipating optimal labor choices and the equilibrium wage function $w^* : [\underline{A}, \overline{A}] \to \mathbb{R}_+$, firm i chooses capital $k_i \geq 0$ to maximize expected profit:*

$$\max_{k_i \geq 0} V_0^i = \mathbb{E}_A[V_1^i(k_i, A, w^*(A))] - \pi(1 + r^l)k_i \tag{17}$$

where the expectation is taken over the aggregate productivity shock $A \sim F$.

Feasibility constraints:

- *$k_i \in [0, \bar{k}]$ for some finite $\bar{k}$ determined by no-Ponzi conditions*
- *$L_i \geq 0$ with $L_i < \infty$ ensured by interior solutions under our parameter restrictions*
- *Limited liability: Firm payoffs are bounded below by zero*

The solution to this optimization problem, combined with market clearing conditions, yields the following equilibrium characterization.

Definition 4.2 (Symmetric Firm Equilibrium)**.** *A symmetric firm equilibrium is a collection of objects $(k^*, L^*(\cdot), w^*(\cdot))$ such that:*

1. ***Optimal capital choice:** All firms choose the same capital level $k^* \geq 0$ that solves the Stage 1 optimization problem in Definition 4.1.*
2. ***Optimal labor choice:** For each realization $A \in [\underline{A}, \overline{A}]$, all firms choose the same labor level $L^*(A) \geq 0$ that solves the Stage 2 optimization problem given k^* and $w^*(A)$.*
3. ***Labor market clearing:** For each A, the wage $w^*(A) \geq 0$ clears the labor market:*

$$\int_0^1 L^*(A)\, di = L^*(A) = \overline{L} \tag{18}$$

4. ***Wage function:** The equilibrium wage function is $w^* : [\underline{A}, \overline{A}] \to \mathbb{R}_+$.*
5. ***Consistency:** The optimal choices are consistent with the information structure specified in Definition 4.1.*

Proposition 4.1 (Symmetric Firm Equilibrium)**.** *Under Assumptions 3.1 and 3.3, there exists a unique symmetric equilibrium where:*

(i) All firms choose capital:

$$k^* = \overline{L}\left(\frac{\alpha^2}{1 + r^l}\right)^{\frac{1}{1-\alpha}} \tag{19}$$

(ii) The equilibrium wage is:

$$w^*(A) = (1 - \alpha)A\left(\frac{K^*}{\overline{L}}\right)^{\alpha} \tag{20}$$

where $K^* = k^*$ is aggregate capital.

(iii) Expected firm profit is zero in equilibrium due to competitive markets.

Proof. See Appendix C.1 for the complete derivation through backward induction. □

Remark 4.1 (Elasticity–technology link). I fix $\epsilon \equiv -1/(1-\alpha)$, so $1/\epsilon = \alpha - 1 < 0$. In formulas I keep α as the technology parameter and use $1/\epsilon = \alpha - 1$ only to simplify derivatives. I avoid writing $\alpha = 1 + 1/\epsilon$ to prevent confusion.

Lemma 4.2 (No profitable strategic default)**.** *If $\underline{A} > \alpha$ (equivalently: even in the worst aggregate state, a successful firm's residual $\alpha A k^{\alpha} \overline{L}^{1-\alpha}$ covers the repayment $(1+r^l)k$ given the firm FOC), then successful firms never strategically default under limited liability.*

4.2 Wholesale rate response to the premium

Let $\Phi(r_w; x_D, x_W) \equiv \mathbb{E}[\text{wholesale recovery}] - (1+r_w)W_W$ be wholesale investors' zero-profit condition, where the wholesale recovery depends on the priority structure and aggregate cash flows (formalized in Lemma 5.3 below). Totally differentiating $\Phi(r_w; x_D, x_W) = 0$ with respect to τ yields

$$\frac{dr_w}{d\tau} \;=\; -\,\frac{\Phi_{x_D}\,\frac{dx_D}{d\tau} + \Phi_{x_W}\,\frac{dx_W}{d\tau}}{\Phi_{r_w}}\,, \tag{21}$$

Since $\Phi_{r_w} < 0$ (a higher r_w raises wholesale recoveries and lowers the shortfall; $\Phi_{r_w} < 0$ because promised wholesale payouts rise in r_w while recoveries do not rise one-for-one), $\Phi_{x_W} > 0$ (more risky senior promises raise expected shortfall), and typically $\Phi_{x_D} \geq 0$, it follows that $dr_w/d\tau \geq 0$ when $\partial_\tau F_W = 0$ (cap binds) and $dx_D/d\tau \leq 0 \leq dx_W/d\tau$.

Proof. Apply the implicit function theorem to $\Phi(r_w; x_D(\tau), x_W(\tau)) = 0$. Differentiation gives $\Phi_{r_w}\,dr_w + \Phi_{x_D}\,dx_D + \Phi_{x_W}\,dx_W = 0$; dividing by $d\tau$ yields (21). Signs follow from Lemma 5.3 and strategic-substitute slopes (Lemma 8.1). □

The symmetric equilibrium reflects the homogeneity of firms and the competitive nature of all markets. The capital choice decreases with the lending rate, generating downward-sloping loan demand.

4.3 Loan Demand and Bank Revenue Function

4.4 Default Thresholds and Payoff Structure

Definition 4.3 (Default thresholds, deposit caps, and senior face values)**.** *Let the deposit cap for the deposit-rich bank be $\infty = \infty$ and for the wholesale-reliant bank be $\overline{D}_W \in (0, \infty)$. Total* senior *face values are defined as the total face value of senior debt obligations $\mathcal{D}_j$:*

$$\mathcal{D}_D = (1-\gamma)x_D, \qquad \mathcal{D}_W = \overline{D}_W + (1+r_w)\Big[(1-\gamma)x_W - \overline{D}_W\Big].$$

Default thresholds solve $A_j^* x_j \Theta X^{1/\epsilon} = \mathcal{D}_j$ where $\mathcal{D}_D = \mathcal{D}_D$ and $\mathcal{D}_W = \mathcal{D}_W$:

$$A_D^* \;=\; \frac{\mathcal{D}_D}{x_D\,\Theta\,X^{1/\epsilon}} \;=\; \frac{1-\gamma}{\Theta\,X^{1/\epsilon}}, \tag{22}$$

$$A_W^* \;=\; \frac{\mathcal{D}_W}{x_W\,\Theta\,X^{1/\epsilon}} \;=\; \frac{(1+r_w)(1-\gamma) - r_w\,\overline{D}_W/x_W}{\Theta\,X^{1/\epsilon}}. \tag{23}$$

We also define the deposit-loss threshold for bank W as $A_W^{**} \equiv \overline{D}_W/(x_W \Theta X^{1/\epsilon})$.

State partition for Bank W. *When Bank W uses wholesale funding, the state space $[\underline{A}, \overline{A}]$ partitions into three economically distinct regions:*

- ***Depositor loss region:*** *$A \in [\underline{A}, A_W^{**})$ where cash flows are insufficient to cover insured deposits $\overline{D}_W$. The insurer pays depositors and wholesale investors get zero.*

- ***Wholesale residual region:*** *$A \in [A_W^{**}, A_W^*)$ where depositors are covered but wholesale investors receive only the residual after insured deposits.*

- ***Wholesale promise region:*** *$A \in [A_W^*, \overline{A}]$ where all obligations are met and wholesale investors receive their full contractual payment $(1 + r_w)W_W$.*

The integration limits in all value and subsidy expressions correspond to these economically meaningful regions, ensuring correct signs and interpretation.

Lemma 4.3. *If $(1 - \gamma)x_W > \overline{D}_W$ (bank W uses wholesale), then $A_W^* > A_D^*$. Indeed,*

$$A_D^* = \frac{1 - \gamma}{\Theta X^{1/\epsilon}}, \qquad A_W^* = \frac{(1 + r_w)(1 - \gamma) - r_w \overline{D}_W / x_W}{\Theta X^{1/\epsilon}},$$

so $A_W^* - A_D^* = \frac{r_w}{\Theta X^{1/\epsilon}} \left[(1 - \gamma) - \frac{\overline{D}_W}{x_W} \right] > 0$.

Proof. From Definition 4.3, we have $A_D^* = \frac{\mathcal{D}_D}{x_D \Theta X^{1/\epsilon}} = \frac{(1-\gamma)x_D}{x_D \Theta X^{1/\epsilon}} = \frac{1-\gamma}{\Theta X^{1/\epsilon}}$. For bank W using wholesale funding, $\mathcal{D}_W = \overline{D}_W + (1 + r_w)[(1 - \gamma)x_W - \overline{D}_W] = (1 + r_w)(1 - \gamma)x_W - r_w \overline{D}_W$. Therefore, $A_W^* = \frac{\mathcal{D}_W}{x_W \Theta X^{1/\epsilon}} = \frac{(1+r_w)(1-\gamma) - r_w \overline{D}_W / x_W}{\Theta X^{1/\epsilon}}$. The difference is $A_W^* - A_D^* = \frac{r_w}{\Theta X^{1/\epsilon}}[(1 - \gamma) - \overline{D}_W / x_W]$. Since wholesale funding is used when $(1 - \gamma)x_W > \overline{D}_W$, we have $(1 - \gamma) > \overline{D}_W / x_W$, and since $r_w > 0$, the difference is positive. $\square$

Proposition 4.4 (Default Threshold and Payoff Representation)**.** *The banker's expected payoff is well-defined and given by:*

$$w_j(x_j, x_{-j}) \;=\; \int_{A_j^*}^{\overline{A}} \left[A \cdot x_j \cdot \Theta \, (x_j + x_{-j})^{1/\epsilon} - \mathcal{D}_j \right] f(A) \, dA \;-\; \Big(\gamma x_j + \tau \, I_j(x_j) \Big) \tag{24}$$

where the integral exists because the integrand is bounded on the compact interval $[A_j^*, \overline{A}]$ and f is continuous.

The symmetric equilibrium generates aggregate loan demand:

$$K^d(r^l) = \overline{L} \left(\frac{\alpha^2}{1 + r^l} \right)^{\frac{1}{1-\alpha}} \tag{25}$$

The loan demand elasticity is $\epsilon = -\frac{1}{1-\alpha} < -1$, reflecting the substitution between capital and labor in production.

Existence of aggregate lending and revenue: Let $X \geq 0$ denote total aggregate lending across all banks, which exists and is well-defined in equilibrium by market clearing. Banks lend to a continuum of firms with independent idiosyncratic risks. By the strong law of large numbers (applicable due to the independence assumption in Assumption 3.1(c)), exactly fraction π of firms succeed and repay their loans almost surely, yielding expected revenue per unit lent:

$$R(r^l) = \pi \cdot (1 + r^l) \tag{26}$$

where we have made explicit the dependence on the lending rate.

Market clearing requires that aggregate bank lending X equals loan demand $K^d(r^l)$. Inverting the demand function and substituting into the revenue equation yields:

Proposition 4.5 (Bank Revenue Function)**.** *Banks face an expected revenue function:*

$$R(X) = \Theta \cdot X^{1/\epsilon} \tag{27}$$

where $\Theta \equiv \pi\alpha^2\overline{L}^{1-\alpha}$ *incorporates technological parameters and credit risk, and* $\epsilon = -\frac{1}{1-\alpha} < -1$ *is the loan demand elasticity.*

Proof. See Appendix C.2 for the complete derivation. □

The function $R(X)$ is a downward-sloping (and convex) inverse demand in aggregate lending X; equivalently, per-unit expected repayment falls as X rises. This property, which emerges from the general equilibrium interaction between lending and loan rates, plays a crucial role in determining equilibrium lending levels and the magnitude of deposit insurance subsidies.

Remark 4.2. R(X) is the per-unit expected repayment (a downward-sloping inverse demand). The bank's private NPV is $x_j[R(X) - 1]$, which is strictly concave in x_j even though R itself is convex in X.

5 Banking Sector: General Analytical Framework

This section develops the general framework for analyzing banks with different funding structures. I first establish the common contracting environment and optimization structure that applies to both institutions, then examine how heterogeneous deposit access creates fundamentally different incentives and constraints.

5.1 General Contracting Structure

Both banks in our model operate under similar organizational structures but face different funding constraints. Each bank is managed by a banker who owns all inside equity but contributes negligible initial capital ($\kappa \to 0$). All funding must be raised externally through a combination of deposits and outside equity to meet regulatory capital requirements.

The contracting problem between initial shareholders and outside equity investors determines the state-contingent division of the bank's net value. Since initial shareholders control lending decisions while outside equity investors provide required regulatory capital in competitive markets, the equilibrium contract minimizes expected payments to outside equity subject to their break-even constraint.

Definition 5.1 (General Bank Contracting Problem)**.** *For a bank with lending level x_j, competitor lending x_{-j}, and total debt obligations $\mathcal{D}_j$, the equilibrium contract between initial shareholders and outside equity investors specifies state-contingent payments $C_j(A)$ that solve: Let $X \equiv x_j + x_{-j}$. Then*

$$\max_{\{C_j(\cdot)\}} \int_{\underline{A}}^{\overline{A}} \Big[A \cdot x_j \cdot \Theta \cdot X^{1/\epsilon} - \mathcal{D}_j - C_j(A)\Big]^+ f(A)\, dA \;-\; \Big(\gamma x_j \;+\; \tau\, I_j(x_j)\Big) \tag{28}$$

subject to

$$\int_{\underline{A}}^{\overline{A}} C_j(A) f(A)\, dA = \gamma x_j \;+\; \tau\, I_j(x_j) \quad \textit{(outside equity break-even)} \tag{29}$$

$$0 \le C_j(A) \le \Big[A \cdot x_j \cdot \Theta \cdot X^{1/\epsilon} - \mathcal{D}_j\Big]^+ \quad \forall A.$$

Remark 5.1 (Objective interpretation)*.* The program maximizes the bank's NPV: the first term is the initial shareholder's expected residual after paying outside equity state by state; the $-\gamma x_j$ term subtracts the outside–equity funding raised at $t = 0$ (priced at break–even), which is algebraically convenient for Prop. 5.1.

5.2 General Value Decomposition

The contracting structure implies a general form for the banker's objective function that decomposes into fundamental value creation and government subsidies.

Proposition 5.1 (General Value Decomposition)**.** *With the optimal outside–equity contract and admissible funding schedules,*

$$w_j(x_j; x_{-j}) \;=\; x_j\Big[\Theta X^{1/\epsilon} - 1\Big] \;-\; \mathbf{1}_{\{j=W\}}\, r_w \Big[(1-\gamma)\, x_W - \overline{D}_W\Big] \;+\; \mathbb{E}\Big[S_j(A)\Big] \;-\; \tau I_j(x_j),$$

where $X = x_D + x_W$ and $S_j(A)$ is the insurer's state–contingent top–up to insured *depositors.*

Sketch. Start from the threshold representation $w_j = \int_{A_j^*}^{\overline{A}} \Big[A x_j \Theta X^{1/\epsilon} - \mathcal{D}_j\Big] f(A)\, dA - \mathbb{E}[C_j(A)]$, with $\mathbb{E}[C_j] = \gamma x_j$. Insert and subtract $\int_{\underline{A}}^{\overline{A}} A x_j \Theta X^{1/\epsilon} f(A)\, dA$ and use $\mathbb{E}[A] = 1$:

$$w_j = x_j \Theta X^{1/\epsilon} - \mathcal{D}_j - \gamma x_j + \int_{\underline{A}}^{A_j^*} \Big[\mathcal{D}_j - A x_j \Theta X^{1/\epsilon}\Big] f(A)\, dA.$$

For $j = D$, $\mathcal{D}_D + \gamma x_D = x_D$. For $j = W$, $\mathcal{D}_W + \gamma x_W = x_W + r_w\Big[(1-\gamma) x_W - \overline{D}_W\Big]$. Rearranging yields the formula; the shortfall integral equals $\mathbb{E}[S_j(A)]$ by pro-rata recovery between insured deposits and wholesale investors. See Appendix C.4.2 for the detailed derivation. □

Remark 5.2 (Private Value Interpretation and marginal conditions with a premium)*.* The NPV term represents private value creation from financial intermediation. When firms fail ($a_i = 0$), workers receive no payment under our contracting specification, meaning that bank failures impose losses on workers as well as lenders. The NPV therefore captures the value accruing to the banking sector given the specified distribution of losses across different stakeholders in bankruptcy states. This specification allows for tractable equilibrium characterization while maintaining economic consistency.

When the insured-deposit cap is slack for bank j, we have $I'_j(x_j) = 1 - \gamma$ and the first-order condition is

$$\frac{\partial w_j}{\partial x_j}(x_j; x_{-j}) \; - \; \tau(1-\gamma) \; = \; 0. \tag{30}$$

When the cap binds for bank W (i.e., $(1-\gamma)x_W > \overline{D}_W$), the premium is *inframarginal* and the KKT conditions apply with $g(x_W) = (1-\gamma)x_W - \overline{D}_W \leq 0$ and multiplier $\mu \geq 0$:

$$\nabla_{x_W} w_W(x_W; x_D) + \mu \nabla g(x_W) = 0, \quad \mu \geq 0, \quad \mu g(x_W) = 0. \tag{31}$$

5.3 Wholesale Funding Market Equilibrium

For banks that exceed their deposit-gathering capacity, wholesale funding markets provide additional financing at endogenous rates that reflect default risk. This creates market discipline absent from deposit-funded lending.

Proposition 5.2 (Wholesale Funding Equilibrium)**.** *The wholesale rate r_w solves*

$$\int_{A_W^*}^{\bar{A}} (1+r_w) W_W \, f(A)\, dA \; + \; \int_{A_W^{**}}^{A_W^*} \Big[A\, x_W \Theta X^{1/\epsilon} - \overline{D}_W\Big] f(A)\, dA \; = \; W_W, \tag{$\star$}$$

where $W_W \equiv (1-\gamma)x_W - \overline{D}_W$, $A_W^{**} = \overline{D}_W/(x_W \Theta X^{1/\epsilon})$, and $A_W^* = \mathcal{D}_W/(x_W \Theta X^{1/\epsilon})$.

State partition: $A < A_W^{**}$: 0; $A_W^{**} \leq A < A_W^*$: $A x_W \Theta X^{1/\epsilon} - \overline{D}_W$; $A \geq A_W^*$: $(1+r_w)W_W$.

Lemma 5.3 (Wholesale rate: existence, uniqueness, and regularity)**.** *Fix (x_W, x_D) and let $X = x_D + x_W$. Deposits are strictly senior. If $A \in [A_W^{**}, A_W^*)$, the wholesale class receives the residual $A x_W \Theta X^{1/\epsilon} - \overline{D}_W$; within the wholesale class, recovery is pro-rata. Let the promised wholesale payoff be $(1+r_w)W_W$ and define the investor zero-profit function*

$$\Phi(r_w; x_W, x_D) \; \equiv \; \mathbb{E}\Big[\rho(A; r_w; x_W, x_D)\Big] \; - \; (1+r_w)W_W\,, \tag{32}$$

where $\rho(\cdot)$ is the wholesale investors' state-contingent repayment implied by the priority rule and the cash-flow $A\, x_W\, \Theta\, X^{1/\epsilon}$. Then:

1. *(Existence) $\Phi(0) > 0$ and there exists $\bar{r} < \infty$ with $\Phi(\bar{r}) < 0$. By the intermediate value theorem, there is at least one root $r_w \in [0, \bar{r}]$.*

2. *(Uniqueness) Φ is strictly decreasing in r_w: using Leibniz' rule, $\Phi'(r_w) = \int_{\underline{A}}^{\overline{A}} \partial\rho/\partial r_w \, dF + \Big[\rho(A; r_w; \cdot)\, f(A)\Big]_{thresholds} \cdot (dA^*/dr_w) < 0$, because higher r_w raises the promised burden, shifts up default thresholds A_W^* (Def. below), and weakly reduces state repayments; by construction of thresholds under absolute priority, the boundary contribution is non-positive (indeed zero where the payoff kink vanishes).*

3. *(Regularity) The root is unique and depends continuously on (x_W, x_D) by the maximum theorem; moreover $\partial r_w/\partial x_W > 0$ and $\partial r_w/\partial x_D \geq 0$.*

Proof (boundary checks). $\Phi(0) > 0$: at zero interest, the promised wholesale amount equals W_W, while expected recovery is weakly above W_W whenever default does not wipe out all wholesale claims in all states; with positive mass of solvent states, strict inequality obtains. Pick $\bar{r}$ large so that $(1+\bar{r})W_W$ exceeds the maximal state payout to the wholesale class (bounded by $\overline{A}\, x_W\, \Theta\, X^{1/\epsilon}$), which yields $\Phi(\bar{r}) < 0$. Strict decrease follows from the monotone movement of thresholds and proportional recovery; continuity from dominated convergence. The sign of comparative statics follows from implicit differentiation. □

Comparative Statics in τ for Thresholds and Failure Probabilities For any differentiable $A_j^*(\tau)$, $p_j(\tau) := \Pr\{A \leq A_j^*(\tau)\} = F(A_j^*(\tau))$ satisfies $dp_j/d\tau = f(A_j^*)\, dA_j^*/d\tau$. Two benchmark cases:

- Unconstrained bank with no wholesale: $i_j = (1-\gamma)x_j$, $W_j = 0$, so $\mathcal{D}_j = (1-\gamma)x_j$ and $A_j^* = (1-\gamma)/(\Theta X^{1/\epsilon})$. Hence $\frac{d}{d\tau} \ln A_j^* = -\frac{1}{\epsilon}\,\frac{1}{X}\,\frac{dX}{d\tau}$, which is negative when both banks are unconstrained (since $dX/d\tau < 0$).

- Wholesale-reliant bank at the cap: $i_W = \overline{D}_W$, $W_W = (1-\gamma)x_W - \overline{D}_W$, $\mathcal{D}_W = \overline{D}_W + (1+r_w)W_W$, so

$$\frac{d}{d\tau} \ln A_W^* \;=\; \left[\frac{(1+r_w)(1-\gamma)}{\mathcal{D}_W} - \frac{1}{x_W}\right]\frac{dx_W}{d\tau} \;+\; \frac{W_W}{\mathcal{D}_W}\,\frac{dr_w}{d\tau} \;-\; \frac{1}{\epsilon}\,\frac{1}{X}\,\frac{dX}{d\tau}. \qquad (33)$$

5.4 Bank Strategy Sets

Before specifying the optimization problems, I formally define the feasible strategy spaces that ensure existence of equilibrium.

Definition 5.2 (Bank Strategy Sets)**.** *For bank $j \in \{D, W\}$, the feasible strategy sets are compact rectangles independent of opponents' choices:*

$$\mathcal{X}_D = [0, \overline{x}_D], \quad \overline{x}_D = \left(\frac{\pi\alpha^2 \overline{L}^{1-\alpha}\overline{A}}{1-\gamma}\right)^{\frac{1}{1-\alpha}} \qquad (34)$$

$$\mathcal{X}_W = [0, \overline{x}_W] \qquad (35)$$

with finite bounds $\overline{x}_j$ chosen so that profits are non-positive for $x_j > \overline{x}_j$. These bounded, convex strategy sets ensure equilibrium existence.

5.5 General Optimization Framework

Both banks solve optimization problems that share a common structure but differ in their specific constraints and funding costs.

Definition 5.3 (General Bank Optimization Problem)**.**

$$\max_{x_j \in \mathcal{X}_j,\, C_j \in \mathcal{C}_j(x_j, x_{-j})} w_j(x_j, x_{-j}) \;=\; \underbrace{\mathit{Private\ NPV}_j(x_j, x_{-j})}_{= x_j\, \Theta X^{1/\epsilon} - x_j} + \mathit{Subsidy}_j(x_j, x_{-j}) \;-\; \tau\, I_j(x_j), \qquad (36)$$

(The reduction follows from Definition 3.1, which prices outside equity at $\gamma x_j + \tau I_j(x_j)$.)

Subject to the admissibility condition:

$$C_j \in \mathcal{C}_j(x_j, x_{-j})$$

and the following funding market conditions:

- *Bank D: deposits priced at* $r_d = 0$
- *Bank W:* r_w *solves the break-even condition in Proposition 5.2 (Eq. (*$\star$*))*

Taking as given: x_{-j}, γ, Θ, ϵ, *the distribution* F *with density* f, *and bank-type primitives (deposit cap* $\overline{D}_W$ *for Bank W, unlimited insured deposits for Bank D).*

The first-order condition for optimal lending balances marginal revenue against marginal funding costs, with the specific form depending on each bank's funding structure. I now examine how these general principles apply to our two banks with heterogeneous funding access.

5.6 General Competitive Equilibrium

I now formally define the competitive equilibrium concept that governs the interaction between firms and banks with heterogeneous funding structures. This definition provides the general structure, with specific characterizations to follow once I develop the bank-specific models.

Definition 5.4 (General Competitive Equilibrium)**.** *A general competitive equilibrium consists of:*

- ***Allocations:*** *Firm capital choices* $\{k_i^*\}_{i\in[0,1]}$, *bank lending decisions* $(x_j^*)_{j\in\{D,W\}}$, *and equity contracts* $\{C_j^*(\cdot)\}_{j\in\{D,W\}}$.
- ***Prices:*** *Lending rate* $r^{l*} \geq 0$, *wage function* $w^* : [\underline{A}, \overline{A}] \to \mathbb{R}_+$, *and wholesale funding rate* $r_w^* \geq 0$ *when applicable.*
- ***Derived objects:*** *Default thresholds* $A_j^*(x_D^*, x_W^*, \mathcal{D}_j^*)$ *for* $j \in \{D, W\}$ *determined by the threshold equations in Proposition 4.4.*

Existence prerequisites: *The equilibrium is well-defined under standard regularity conditions on value functions and market clearing mappings:*

- *All value functions are continuous on compact domains with appropriate boundary behavior*
- *Default thresholds exist and are unique for any* $(x_j, x_{-j}, \mathcal{D}_j)$ *with* $x_j > 0$ *(Proposition 4.4)*
- *Market clearing conditions define continuous mappings from prices to excess demands*

Such that:

1. ***Firm Optimization:*** *Each firm i chooses capital to maximize expected profit given the lending rate and anticipated equilibrium wages:*

$$k_i^* \in \arg\max_{k \geq 0} \mathbb{E}_A[\pi V_1^i(k, A, w^*(A))] - \pi(1 + r^{l*})k \tag{37}$$

2. ***Bank Optimization:*** *Each bank $j \in \mathcal{J}$ chooses lending to maximize shareholder value given competitors' strategies and funding market conditions:*

$$x_j^* \in \arg\max_{x_j \in \mathcal{X}_j} w_j(x_j, x_{-j}^*) \tag{38}$$

where $\mathcal{X}_j$ is the feasible strategy set from Definition 5.2 and w_j incorporates the bank's funding structure.

3. ***Claim Priority and Thresholds:*** *Deposits are senior to equity. When present, wholesale debt is junior to deposits and senior to equity. Outside equity is priced competitively with zero expected profit. Default thresholds A_j^* are determined by $\mathcal{D}_j = A_j^* x_j \Theta(x_D^* + x_W^*)^{1/\epsilon}$ as in Proposition 4.4.*

4. ***Market Clearing:***
   - *Loan market: Total bank lending equals aggregate firm demand for capital.*
   - *Labor market: For each realization of A, wages adjust to clear the labor market.*
   - *Funding markets: Bank-specific funding markets clear according to their respective structures.*

5.7 Admissible Contracts and Funding Schedules

I now define the feasible sets of outside equity contracts and deposit schedules that will be used in the banks' optimization problems.

Definition 5.5 (Admissible Deposit Schedules)**.** *For Bank D, insured deposits fund a fraction $(1 - \gamma)$ of assets at zero rate.*

For Bank W, insured deposits are capped at $\overline{D}_W$; when $x_W > \overline{D}_W/(1 - \gamma)$, the bank uses wholesale debt for the residual amount $(1-\gamma)x_W - \overline{D}_W$ at endogenous gross rate $1+r_w$. The wholesale rate r_w is determined by the break-even condition in Proposition 5.2.

These definitions ensure that all objects used in the banks' optimization problems are well-defined before we state optimality conditions.

The critical distinction between the two banks reflects real-world heterogeneity in deposit-gathering capacity:

Definition 5.6 (Bank Funding Structures)**.** 1. ***Bank D (Deposit-rich bank):*** *Represents large institutions with extensive branch networks and established deposit franchises, modeled as having unlimited access to insured deposits at zero interest rate.*

2. ***Bank W (Wholesale-reliant bank):*** *Represents regional banks, specialized lenders, or new entrants with limited deposit-gathering infrastructure, modeled as facing a deposit constraint $\overline{D}_W > 0$ that reflects their structural limitations in attracting stable funding.*

This heterogeneity captures several real-world factors: geographic limitations (regional versus national presence), regulatory constraints (industrial loan companies or credit card banks face deposit restrictions), business model choices (wholesale-funded mortgage specialists), and competitive disadvantages (new entrants lacking established customer relationships). These differences drive the main results of our analysis.

Remark 5.3 (Derived Equilibrium Objects). The equilibrium uniquely determines several derived objects that are functions of the equilibrium variables rather than independent components:

- Default thresholds: A_j^* for $j \in \{D, W\}$
- Value functions: $w_j^* = w_j(x_j^*, x_{-j}^*)$
- Labor demand functions: $L_i^*(A) = k_i^*[(1-\alpha)A/w^*(A)]^{1/\alpha}$

These follow mechanically from the equilibrium allocations and prices.

Lemma 5.4 (Capital requirement binds)**.** *Fix x_j and let (e_j, i_j, w_j) be feasible under (3)–(4). At any optimum, $e_j - \tau i_j = \gamma x_j$, i.e., the capital requirement binds.*

Sketch. Hold x_j fixed. Outside equity is priced at zero expected profit and only reallocates payoffs across claim classes. Replacing one unit of insured debt by outside equity relaxes (4) but eliminates one unit of guaranteed debt and attenuates limited-liability rents; banker value weakly falls. Hence e_j is minimized at the constraint, implying $e_j = \gamma x_j + \tau i_j$ at the solution. □

6 Bank-Specific Analysis with Heterogeneous Funding

I now develop the specific forms of the optimization problems and equilibrium conditions for the two banks with heterogeneous funding structures, implementing the general equilibrium framework established in Definition 5.4. Bank D enjoys unlimited access to insured deposits while Bank W faces structural limitations requiring wholesale funding beyond its deposit constraint.

6.1 Bank D: Unlimited Deposit Access

Bank D represents large institutions with extensive branch networks and established deposit franchises. Its funding structure and optimization problem follow directly from the general framework with specific simplifications due to unlimited deposit access.

6.1.1 Funding Structure and Balance Sheet

Bank D's balance sheet structure is shown in Table 1 in Appendix A. The total debt obligation is simply $\mathcal{D}_D = (1-\gamma)x_D$, representing deposits that carry no interest cost due to government insurance.

6.1.2 Specific Optimization Problem

Applying the general framework from Definition 5.4, Bank D's specific optimization problem becomes:

$$\max_{x_D \in \mathcal{X}_D} w_D(x_D, x_W) \;=\; \int_{A_D^*}^{\overline{A}} \Big[A \cdot x_D \cdot \Theta(x_D + x_W)^{1/\epsilon} - (1-\gamma)x_D\Big] f(A)\, dA \;-\; \gamma x_D \;-\; \tau(1-\gamma)\, x_D. \tag{39}$$

where the default threshold is:

$$A_D^* = \frac{1-\gamma}{\Theta\,(x_D + x_W)^{1/\epsilon}} \tag{40}$$

6.1.3 Value Decomposition and Deposit Insurance Subsidy

Applying Proposition 5.1 to Bank D's specific case:

Corollary 6.1 (Bank D Value Decomposition). *Bank D's objective function decomposes as:*

$$w_D = x_D[\Theta\,(x_D+x_W)^{1/\epsilon}-1] + \underbrace{\int_{\underline{A}}^{A_D^*} [(1-\gamma)x_D - A \cdot x_D \cdot \Theta\,(x_D + x_W)^{1/\epsilon}] f(A) dA}_{\textit{Deposit Insurance Subsidy}} - \tau(1-\gamma)x_D \tag{41}$$

Proof. See Appendix C.5.1.1 for the derivation from the general framework. □

The deposit insurance subsidy captures the expected value of government coverage when bank assets fall short of deposit obligations (see (14)). This subsidy increases with lending volume and default probability, creating moral hazard.

Incidence summary. Because the premium is prepaid and deducted from regulatory capital at $t = 0$, its *marginal* effect is $\tau(1-\gamma)$ *only when insured deposits fund the margin.* When the cap binds for W, the premium becomes inframarginal and drops from the FOC; see the first-order conditions below.

6.1.4 First-Order Condition

Proposition 6.2 (Bank D First-Order Condition). *Bank D's optimal lending satisfies the first-order condition:*

$$\begin{aligned} &\Theta(x_D + x_W)^{1/\epsilon - 1}[x_W + \alpha x_D] - 1 \;-\; \tau(1-\gamma) \\ &\quad + \int_{\underline{A}}^{A_D^*} \Big[(1-\gamma) - A \cdot \Theta(x_D + x_W)^{1/\epsilon - 1}[x_W + \alpha x_D]\Big] f(A)\, dA = 0 \end{aligned} \tag{42}$$

where we recall $1/\epsilon = \alpha - 1$ with $\alpha \in (0,1)$ the capital share parameter. The integral term is strictly positive, representing the marginal value of the deposit insurance subsidy (14). This positive wedge increases equilibrium lending relative to an unsubsidized benchmark.

Proof. See Appendix C.5.1.2 for the complete derivation. □

Bank W When the Cap is Slack If $(1-\gamma)x_W < \overline{D}_W$ so that $I'_W(x_W) = 1-\gamma$ and the bank uses insured deposits at the margin, the first-order condition mirrors Bank D's:

$$\frac{\partial w_W}{\partial x_W}(x_W; x_D) \; - \; \tau(1-\gamma) \; = \; 0. \tag{43}$$

When the cap binds, the premium is inframarginal and the Kuhn–Tucker system applies. *Premium marginality.* The term $-\tau(1-\gamma)$ appears when $I'_D(x_D) = 1-\gamma$ (i.e., insured deposits are used at the margin). If I_D is infra-marginal (e.g., a binding cap elsewhere), the premium is infra-marginal and drops out of the FOC.

6.2 Bank W: Mixed Funding Model

Bank W represents regional banks, specialized lenders, or new entrants with limited deposit-gathering infrastructure. Unlike Bank D, Bank W faces a deposit constraint $\overline{D}_W > 0$ and must access wholesale funding markets when lending exceeds what can be financed through deposits and equity alone.

6.2.1 Funding Structure and Balance Sheet

When Bank W's optimal lending exceeds its deposit-based funding capacity (i.e., $x_W > \overline{D}_W/(1-\gamma)$), it must access wholesale funding. Bank W's balance sheet structure when accessing wholesale funding is presented in Table 2 in Appendix A. The key distinction from Bank D is that wholesale funding carries an endogenous interest rate r_w that compensates wholesale funders for default risk.

6.2.2 Wholesale Funding Market Application

For Bank W, the wholesale funding rate adjusts to ensure wholesale funders break even in expectation, as established in Proposition 5.2. This creates a critical difference from Bank D: market discipline through risk-based pricing.

The two thresholds A_W^* and A_W^{**} from the proposition reflect the seniority structure: depositors are repaid first, with wholesale funders bearing losses only after deposits are covered. The wholesale funding rate rises endogenously with Bank W's lending, providing market discipline absent for Bank D.

6.2.3 Specific Optimization Problem

Applying the general framework from Definition 5.4, Bank W's specific optimization problem becomes:

$$\max_{x_W \in \mathcal{X}_W} w_W(x_W, x_D) \text{ subject to wholesale funding break-even condition (Proposition 5.2)} \tag{44}$$

6.2.4 Value Decomposition with Limited Deposit Access

Applying the general framework to Bank W's mixed funding structure:

Corollary 6.3 (Bank W Value Decomposition (mixed funding))**.** *When $x_W > \overline{D}_W/(1-\gamma)$,*

$$\begin{aligned} w_W \;:=\; & x_W\Big[\Theta(x_D + x_W)^{1/\epsilon} - 1\Big] \\ & -\; r_w\Big((1-\gamma)x_W - \overline{D}_W\Big) \\ & +\; \int_{\underline{A}}^{A_W^{**}} \Big[\overline{D}_W - A\,x_W\,\Theta(x_D + x_W)^{1/\epsilon}\Big]\, f(A)\, dA \\ & -\; \tau\,\overline{D}_W\,. \end{aligned}$$

with $A_W^{**} \equiv \overline{D}_W/(x_W\,\Theta\,(x_D + x_W)^{1/\epsilon})$.

Proof. See Appendix C.5.2.2 for the derivation accounting for wholesale funding. ☐

Crucially, Bank W's deposit insurance subsidy is capped by its deposit access $\overline{D}_W$, in contrast to Bank D's unlimited subsidy that scales with total lending.

6.2.5 First-Order Condition with Market Discipline

Proposition 6.4 (Bank W first-order condition (mixed funding))**.** *If $x_W > \overline{D}_W/(1-\gamma)$, the optimal x_W satisfies*

$$\begin{aligned} & \Theta X^{1/\epsilon - 1}\,(x_D + \alpha x_W) - 1 \\ & \quad -\; r_w(1-\gamma) \\ & \quad -\; \Big((1-\gamma)x_W - \overline{D}_W\Big)\frac{\partial r_w}{\partial x_W} \\ & \quad -\; \int_{\underline{A}}^{A_W^{**}} A\,\Theta X^{1/\epsilon - 1}\,(x_D + \alpha x_W)\, f(A)\, dA \;=\; 0\,. \end{aligned} \tag{45}$$

where $X = x_D + x_W$ and $A_W^{**} = \overline{D}_W/(x_W \Theta X^{1/\epsilon})$. By Lemma 5.3, $\partial r_w/\partial x_W > 0$, so both wholesale terms are negative (market discipline).

Remark (Why the r_w-terms enter). The wholesale rate r_w is determined by investors' zero-profit condition and is not chosen by the bank. Hence the envelope theorem does not suppress the r_w-dependence: the total derivative $\frac{d}{dx_W}\{-r_w((1-\gamma)x_W - \overline{D}_W)\}$ contributes $-r_w(1-\gamma) - ((1-\gamma)x_W - \overline{D}_W)\frac{\partial r_w}{\partial x_W}$.

Remark 6.1 (Contrasting Incentives). Comparing the first-order conditions reveals the fundamental asymmetry in incentives. Bank D faces a positive integral term (deposit insurance subsidy (14)) that increases lending and default probability, while Bank W faces a negative integral term (market discipline through wholesale funding costs) that reduces lending relative to Bank D. This asymmetry drives the differential default probabilities and market concentration results established in subsequent sections.

Proposition 6.5 (Optimality at the deposit cap via KKT)**.** *When $(1-\gamma)x_W \le \overline{D}_W$, bank W's program is unconstrained. At the cap, i.e., at $x_W = \overline{D}_W/(1-\gamma)$, the constrained maximization*

$$\max_{x_W \ge 0} \; w_W(x_W; x_D) \quad s.t. \quad g(x_W) \equiv (1-\gamma)x_W - \overline{D}_W \;\le\; 0 \tag{46}$$

obeys the KKT conditions: there exists $\mu \ge 0$ such that

$$\nabla_{x_W} w_W(x_W; x_D) + \mu\,\nabla g(x_W) = 0\,, \qquad \mu\, g(x_W) = 0\,. \tag{47}$$

Strict concavity of $w_j(\cdot, x_{-j})$ in own output on each side of the kink (together with linear constraints) makes the KKT conditions necessary and sufficient. This nests the left- and right-derivative cases and justifies the kink optimum.

6.3 Equilibrium Characterization

Having developed the bank-specific optimization problems, we now specify how these elements determine the equilibrium.

Proposition 6.6 (Equilibrium Characterization)**.** *The equilibrium $(x_D^*, x_W^*, r^{l*}, w^*(\cdot), r_w^*)$ from Definition 5.4 with heterogeneous banks is characterized by:*

1. ***Lending Rate Determination:***

$$1 + r^{l*} = \alpha^2 \overline{L}^{1-\alpha}(x_D^* + x_W^*)^{-(1-\alpha)} \tag{48}$$

2. ***Bank Lending Conditions:***
   - *Bank D satisfies the first-order condition in Proposition 6.2.*
   - *Bank W satisfies the first-order condition in Proposition 6.4 if $x_W^* > \overline{D}_W/(1-\gamma)$, otherwise $x_W^* = \overline{D}_W/(1-\gamma)$.*

3. ***Wholesale Rate Determination:*** *When $x_W^* > \overline{D}_W/(1-\gamma)$, the rate r_w^* solves the break-even condition in Proposition 5.2.*

4. ***Threshold Determination:***

$$A_D^* = \frac{1-\gamma}{\Theta(x_D^* + x_W^*)^{1/\epsilon}} \tag{49}$$

$$A_W^* = \begin{cases} \frac{\overline{D}_W + [(1-\gamma)x_W^* - \overline{D}_W](1+r_w^*)}{x_W^* \cdot \Theta(x_D^* + x_W^*)^{1/\epsilon}} & \text{if } x_W^* > \overline{D}_W/(1-\gamma) \\ \frac{1-\gamma}{\Theta(x_D^* + x_W^*)^{1/\epsilon}} & \text{if } x_W^* = \overline{D}_W/(1-\gamma) \end{cases} \tag{50}$$

7 Equilibrium Existence, Uniqueness, and Properties

Having established the equilibrium concept in Definition 5.4 and its characterization in Proposition 6.6, we now prove existence and uniqueness and derive key properties.

Proposition 7.1 (Equilibrium Properties)**.** *The unique equilibrium* $(x_D^*, x_W^*, r^{l*}, r_w^*)$ *satisfies:*

(i) Positive lending: Both banks actively lend with $x_D^* > 0$ *and* $x_W^* \geq 0$*. Either* $x_W^* = \overline{D}_W/(1-\gamma)$ *(cap binds) or* $x_W^* > \overline{D}_W/(1-\gamma)$ *(mixed funding).*

(ii) Strategic substitutability: Lending decisions are strategic substitutes.

(iii) Differential default risk: If Bank W uses any wholesale funding, i.e. $(1-\gamma)x_W > \overline{D}_W$*, then* $A_W^* > A_D^*$ *and hence* $F(A_W^*) > F(A_D^*)$ *by Lemma 4.3. If* $(1-\gamma)x_W \leq \overline{D}_W$ *(no wholesale at the margin), then* $A_W^* = A_D^*$*.*

(iv) Market discipline asymmetry: Bank W faces endogenous funding costs while Bank D does not.

Proof sketch. When Bank W uses wholesale funding, $(1-\gamma)x_W > \overline{D}_W$, so $W_W = (1-\gamma)x_W - \overline{D}_W > 0$ and $\mathcal{D}_W = \overline{D}_W + (1+r_w)W_W$ while $\mathcal{D}_D = (1-\gamma)x_D$. The default thresholds satisfy:

$$A_W^* = \frac{\mathcal{D}_W}{x_W \Theta X^{1/\epsilon}} = \frac{\overline{D}_W + (1+r_w)W_W}{x_W \Theta X^{1/\epsilon}} \tag{51}$$

$$A_D^* = \frac{\mathcal{D}_D}{x_D \Theta X^{1/\epsilon}} = \frac{(1-\gamma)x_D}{x_D \Theta X^{1/\epsilon}} = \frac{1-\gamma}{\Theta X^{1/\epsilon}} \tag{52}$$

Since $\mathcal{D}_W = \overline{D}_W + (1+r_w)W_W > (1-\gamma)x_W$ when wholesale funding is used, and $\mathcal{D}_D = (1-\gamma)x_D$, we have:

$$A_W^* - A_D^* = \frac{1}{\Theta X^{1/\epsilon}}\left[\frac{\overline{D}_W + (1+r_w)W_W}{x_W} - \frac{(1-\gamma)x_D}{x_D}\right] = \frac{1}{\Theta X^{1/\epsilon}}\left[\frac{\mathcal{D}_W}{x_W} - (1-\gamma)\right] > 0 \tag{53}$$

because wholesale use implies $(1-\gamma)x_W > \overline{D}_W$, so $\mathcal{D}_W/x_W = [\overline{D}_W + (1+r_w)W_W]/x_W > (1-\gamma)$. Monotonicity of F implies $F(A_W^*) > F(A_D^*)$. See Appendix C.6.1 for detailed verification of each property.

Remark 7.1 (Sign Determination). The positive sign of both effects follows from the specific structure of our model. The direct effect is positive because increased deposit access reduces Bank W's marginal funding cost through the wholesale funding channel. The strategic effect is positive because lending decisions are strategic substitutes (established in Proposition 7.1), and Bank D reduces lending when Bank W gains deposit access (Theorem 8.3).

These equilibrium properties highlight the fundamental asymmetry created by differential deposit access. Bank D's unlimited deposit insurance enables higher equilibrium lending and default probability, while Bank W faces endogenous wholesale funding rates that increase with leverage.

8 Comparative Analysis and Market Structure Implications

Building on the equilibrium existence and uniqueness established in the previous section, we now examine how heterogeneous deposit access and the premium τ affect market outcomes through comparative statics analysis. We first record the Cournot system with a premium and the aggregate derivative $dX/d\tau$.

8.1 Cournot system with a premium and the aggregate response

Let $F_D(x_D, x_W; \tau)$ and $F_W(x_D, x_W; \tau)$ denote the left-hand sides of bank D's and bank W's optimality conditions (the unconstrained FOC when the cap is slack and the KKT system when it binds). Under the same regularity conditions (strict concavity of $w_j(\cdot, x_{-j})$ in own output in interior regions and dominant diagonals), the equilibrium $(x_D(\tau), x_W(\tau))$ is characterized by $F_D = F_W = 0$ and by the Implicit Function Theorem

$$\frac{d}{d\tau}\begin{bmatrix} x_D \\ x_W \end{bmatrix} = -\, J^{-1}(x_D, x_W; \tau) \begin{bmatrix} \partial_\tau F_D \\ \partial_\tau F_W \end{bmatrix}, \qquad J \equiv \begin{bmatrix} \partial_{x_D} F_D & \partial_{x_W} F_D \\ \partial_{x_D} F_W & \partial_{x_W} F_W \end{bmatrix}. \tag{54}$$

In any region where both banks are unconstrained, $\partial_\tau F_D = \partial_\tau F_W = -(1-\gamma)$, so $dX/d\tau < 0$. If the wholesale-reliant bank is at the cap, then $\partial_\tau F_D = -(1-\gamma)$ and $\partial_\tau F_W = 0$, and

$$\frac{dX}{d\tau} \;=\; (1-\gamma)\, \mathbf{1}^\top J^{-1}(x_D, x_W; \tau)\, e_1. \tag{55}$$

Under near-symmetric cross effects and dominant diagonals, we can bound the aggregate response:

$$\left|\frac{dX}{d\tau}\right| = (1-\gamma)\left|\mathbf{1}^\top J^{-1} e_1\right| \le \frac{2(1-\gamma)}{\underline{d}(X) - 2\bar{c}(X)} \tag{56}$$

where $\underline{d}(X) = \min_j |\partial^2 w_j / \partial x_j^2|$ and $\bar{c}(X) = \max_{j \neq k} |\partial^2 w_j / \partial x_j \partial x_k|$ from Proposition C.7. This bound follows from Cramer's rule: $\mathbf{1}^\top J^{-1} e_1 =$ (first column sum of $\mathrm{adj}(J)$)$/\det(J)$, where $|\mathrm{adj}(J)_{ij}| \le \bar{c}$ for off-diagonals and $|\det(J)| \ge \underline{d}^2 - \bar{c}^2$ under diagonal dominance, yielding the stated bound.

Lemma 8.1 (Cross-partial sign pattern)**.** *The Jacobian of first-order conditions has dominant diagonals:* $\partial^2 w_j/\partial x_j^2 < 0$*. Off-diagonal entries are bounded in magnitude and are weakly negative in the isoelastic range once default-region and wholesale-discipline terms are included. The sufficient diagonal-dominance bound in Proposition C.7 ensures the matrix is an M-matrix.*

Checkable bound: *For Cournot competition with downward-sloping convex inverse demand* $R(X) = \Theta X^{1/\epsilon}$ *where* $1/\epsilon \in (-1, 0)$*, the own second derivative satisfies* $\partial^2 w_j/\partial x_j^2 \leq \Theta(1/\epsilon)(1 + 1/\epsilon)X^{1/\epsilon-1} < 0$ *since* $(1 + 1/\epsilon) = \alpha > 0$ *and* $1/\epsilon < 0$*.*

Proposition 8.2 (Comparative statics in deposit access)**.** *Let* $F_D(x_D, x_W; \overline{D}_W) = 0$ *and* $F_W(x_D, x_W; \overline{D}_W) = 0$ *denote the first-order conditions (including the KKT case at the cap). The Jacobian* $J = \partial(F_D, F_W)/\partial(x_D, x_W)$ *is nonsingular by Proposition C.7. By the implicit function theorem,*

$$\frac{d}{d\overline{D}_W}\begin{bmatrix} x_D^* \\ x_W^* \end{bmatrix} = -J^{-1}\begin{bmatrix} \partial F_D/\partial\overline{D}_W \\ \partial F_W/\partial\overline{D}_W \end{bmatrix}. \tag{57}$$

Using $R'(X) < 0$*,* $\partial r_w/\partial x_W > 0$ *(Lemma 5.3), and the sign pattern of cross-partials, it follows that* $dx_W^*/d\overline{D}_W > 0$*,* $dx_D^*/d\overline{D}_W < 0$*, and* $dX^*/d\overline{D}_W > 0$*. Monotone parameter shifts also satisfy the single-crossing property for monotone comparative statics.*

Theorem 8.3 (Deposit Access and Equilibrium Lending)**.** *In the unique equilibrium characterized by Theorem C.8, the following comparative statics hold:*

(i) Direct effect: Banks with greater deposit access lend more: $\frac{dx_W^*}{d\overline{D}_W} > 0$*.*

(ii) Strategic response: Competing deposit-rich banks reduce lending: $\frac{dx_D^*}{d\overline{D}_W} < 0$*.*

(iii) Aggregate effect: Total market lending increases: $\frac{d(x_D^* + x_W^*)}{d\overline{D}_W} > 0$*.*

(iv) Effect magnitudes: The direct effect dominates the strategic response: $|\frac{dx_W^*}{d\overline{D}_W}| > |\frac{dx_D^*}{d\overline{D}_W}|$*.*

Remark 8.1 (Monotone Comparative Statics). Let $\Phi_W(x_W; x_D, \overline{D}_W) \equiv \partial w_W/\partial x_W$. Holding x_D fixed, Φ_W satisfies the single-crossing property in $(x_W, \overline{D}_W)$: $\partial\Phi_W/\partial x_W < 0$ (strict concavity of $w_W(\cdot, x_D)$ in own output) and $\partial\Phi_W/\partial\overline{D}_W > 0$ (a higher $\overline{D}_W$ relaxes market discipline at the margin). By the theory of monotone comparative statics, x_W^* is increasing in $\overline{D}_W$. Since $\partial\Phi_D/\partial x_W < 0$ (strategic substitutability from Proposition 7.1), x_D^* decreases with $\overline{D}_W$.

Proof. See Appendix C.8.1 for the complete comparative statics analysis. □

Proposition 8.4 (Deposit Access Decomposition)**.** *The total effect of improved deposit access on Bank W's lending can be decomposed as:*

$$\frac{dx_W^*}{d\overline{D}_W} = \underbrace{\left.\frac{\partial x_W^*}{\partial\overline{D}_W}\right|_{x_D \text{ fixed}}}_{\text{Direct Effect}>0} + \underbrace{\left.\frac{\partial x_W^*}{\partial x_D}\right|_{\overline{D}_W \text{ fixed}} \cdot \frac{dx_D^*}{d\overline{D}_W}}_{\text{Strategic Effect}>0} \tag{58}$$

Both components are positive: the direct effect from reduced funding costs and the strategic effect from Bank D's equilibrium retreat (since $\frac{\partial x_W^*}{\partial x_D} < 0$ *and* $\frac{dx_D^*}{d\overline{D}_W} < 0$*).*

Proposition 8.5 (Asymmetric Amplification to Aggregate Shocks)**.** *Let there be a multiplicative productivity shifter m such that $A' = mA$, and consider $dm > 0$. Then $dx_D^*/dm > dx_W^*/dm$ when $\overline{D}_W$ is small (deposit-rich banks amplify good shocks more), while $dx_W^*/dm > dx_D^*/dm$ when $\overline{D}_W$ is large (market discipline attenuates Bank W less). Consequently, the elasticity gap $|dx_D^*/dm - dx_W^*/dm|$ is decreasing in $\overline{D}_W$.*

Sketch. By the implicit function theorem, since the Jacobian of first-order conditions is nonsingular under diagonal strict concavity of $w_j(\cdot, x_{-j})$ in own output (Rosen, 1965), we can differentiate reaction functions with respect to parameters. For the premium τ, use the system in (54) and the sign structure recorded thereafter: when both banks are unconstrained, $\partial_\tau F_D = \partial_\tau F_W = -(1-\gamma)$ so $dX/d\tau < 0$; when bank W is at the cap, $\partial_\tau F_W = 0$ and the aggregate derivative is given by (55). □

Proof. The decomposition follows from the chain rule applied to the equilibrium system. See Appendix C.8.2 for details. □

Remark 8.2 (Economic Interpretation)*.* Bank W benefits through two reinforcing channels when its deposit access improves: (i) directly through lower funding costs, allowing more profitable lending, and (ii) indirectly through Bank D's strategic retreat, which reduces competition for loans. This complementarity amplifies the impact of changes in funding access on market structure.

The heterogeneity in deposit access has profound implications for banking market structure, concentration, and systemic risk distribution.

Proposition 8.6 (Market Concentration)**.** *Define the market concentration ratio as*

$$CR \equiv \frac{x_D^*}{x_D^* + x_W^*}$$

Then:

(i) Market concentration increases with funding heterogeneity: $\frac{dCR}{d\overline{D}_W} < 0$.

(ii) Market concentration sensitivity $|\frac{dCR}{d\overline{D}_W}|$ is maximized when banks have approximately equal market shares ($x_D^* \approx x_W^*$).

(iii) In the limit of extreme heterogeneity ($\overline{D}_W \to 0$), the market becomes monopolistic: $CR \to 1$.

Proof. See Appendix C.8.3 for the analysis of concentration dynamics. □

Remark 8.3 (Economic Interpretation)*.* The concentration results reveal how structural differences in deposit access create persistent market concentration. Banks with limited deposit franchises face different equilibrium constraints even when they have identical lending technologies and serve the same markets. This provides a framework for understanding observed concentration patterns in banking systems and predicting how these patterns respond to changes in funding access.

Proposition 8.7 (Risk Distribution)**.** *In equilibrium, the distribution of systemic risk exhibits the following properties:*

(i) Bank W's default probability weakly exceeds Bank D's whenever wholesale funding is used; the gap grows with $r_w^* > 0$.

(ii) The effect of deposit access on aggregate default risk depends on two opposing forces:

- *Market share effect: Increasing $\overline{D}_W$ shifts lending toward Bank W (safer).*
- *Individual risk effect: Higher $\overline{D}_W$ may increase Bank W's risk-taking.*

When the market share effect dominates, aggregate risk decreases with $\overline{D}_W$. When the individual risk effect dominates, aggregate risk increases with $\overline{D}_W$.

Proof. See Appendix C.8.4 for the proof. □

Price-competition robustness. If banks set loan rates rather than quantities, the revenue curve $R(X)$ pins quantities one-for-one in equilibrium; the subsidy/discipline wedge logic and all comparative statics above remain unchanged.

Empirical mapping. Deposit access $\overline{D}_W$ maps to insured-deposit capacity or branch-network deposit share; wholesale discipline r_w maps to wholesale spreads/CDS or brokered-deposit rates; the concentration measure $CR \equiv x_D^*/(x_D^* + x_W^*)$ inherits the signs in Proposition 8.2.

Remark 8.4 (Notation cross-reference)*.* We use $\sigma \equiv 1 - \alpha \in (0, 1)$ and $\Theta \equiv \pi\, \alpha^2\, \bar{L}^{1-\alpha}$. The downward-sloping and convex inverse demand (per-unit expected repayment) is $R(X) = \Theta X^{-\sigma}$. Default thresholds are given by (22). Insurer budget balance is (13). KKT at the cap is in Proposition 6.5. Sufficient conditions for uniqueness are consolidated in Proposition C.7. Off the cap, the marginal condition with a premium shifts with capital as $\partial_\gamma[\partial_{x_j} w_j - \tau(1 - \gamma)] = \partial^2_{x_j\gamma} w_j + \tau$.

9 Numerical Analysis

We calibrate $\alpha = 0.3$, $\gamma = 0.10$, $\pi = 0.95$ and rebase L so that $\Theta \equiv \pi\alpha^2 L^{1-\alpha} \approx 1$ (for scale). The aggregate productivity A is uniform on $[0.5, 1.5]$; we impose $\underline{A} > \alpha$ (no strategic default). We solve the Cournot equilibrium using the first-order conditions in (45) and Proposition 6.4 together with the wholesale zero-profit condition (Proposition 5.2).[1]

Baseline equilibria. With a loose cap ($D_W = 0.45$), the constraint is slack and the equilibrium is symmetric: $x_D = x_W = 0.285$, $X = 0.570$, $A_D^* = 0.607$ and $p_D = p_W = 0.107$. Tightening the cap to $D_W = 0.20$ yields a mixed regime with wholesale funding at the margin: $(x_D, x_W, X, r_w) = (0.284, 0.246, 0.530, 0.054)$, thresholds $A_D^* = 0.577$, $A_W^{**} = 0.522$, $A_W^* = 0.580$ and default probabilities $p_D = 0.077 < p_W = 0.080$; hence $A_W^* > A_D^*$ when wholesale is used, as predicted.

[1] Implementation details mirror (40) and Proposition 6.4: Bank D's FOC includes the deposit-subsidy integral; Bank W's mixed-regime FOC accounts for the total derivative dr_w/dx_W implied by wholesale break-even. See the online code appendix (Li, 2025).

Premium τ (cap binding). The comparative statics for changes in the deposit insurance premium τ demonstrate the model's core theoretical predictions. As shown in Figure 1 (Appendix B.1), raising τ reduces deposit-funded lending (x_D) and increases wholesale-funded lending (x_W) with negligible effect on aggregate lending (X). This validates that the premium is marginal for deposit-funded lending but inframarginal when the deposit cap binds, consistent with the theoretical framework developed in Section 3.3. The wholesale spread r_w exhibits minimal response in this calibration, reflecting the strategic substitution between bank types that limits aggregate effects.

Deposit access D_W. The effects of expanding Bank W's deposit access illustrate the fundamental importance of funding heterogeneity in shaping market outcomes. Figure 2 (Appendix B.2) shows that as D_W increases, the banking system transitions from a mixed funding regime—where wholesale funding is used at the margin—to a slack constraint regime where both banks face similar funding costs. Bank W's lending capacity increases ($x_W \uparrow$) while its reliance on wholesale funding diminishes ($r_w \downarrow 0$), leading to convergence in default thresholds ($A^*_W \downarrow$ to meet A^*_D) and reduced market concentration. Close to regime transition points, small numerical non-monotonicities can appear, but away from these kinks, the equilibrium dynamics align perfectly with the theoretical predictions in Theorem 8.3.

10 Conclusion

This paper provides a theory-first account of how capital requirements shape bank lending in the presence of deposit insurance and limited liability. By unifying the classic composition channel with the guarantee-overhang logic emphasized by the Forced Safety Effect, we show that lending responses are state-contingent: tighter capital reduces lending when private funding-cost wedges dominate, yet can raise lending when the bank's improved solvency makes equityholders internalize marginal residual cash flows. The dynamic perspective suggests that optimal stringency should vary with the cycle, consistent with macroprudential principles (Hanson et al., 2011; Malherbe, 2020).

Our analysis is deliberately theoretical and numerical rather than empirical. This allows us to map primitives—insurance, residual cash-flow heterogeneity, and the economic state—to sign predictions without committing to a specific identification design. The framework is useful for policy counterfactuals in settings where the composition and forced-safety channels plausibly operate in opposite directions. Future theory can embed richer competition, liquidity management, or leverage-ratio constraints, and explore interactions with resolution and loss-absorbing debt.

Core Insights and Policy Relevance Three insights emerge with direct policy implications. First, *deposit insurance pricing as competition policy*: flat-rate premiums create a powerful, targeted instrument for managing banking market structure. Small premium increases substantially reduce concentration by shifting scale toward wholesale-funded institutions, while strategic substitution limits aggregate credit effects. This provides regulators with a tool that operates independently of traditional antitrust or capital requirements. Second, *persistent market segmentation*: funding heterogeneity generates endogenous business model differentiation that survives competitive pressure. Deposit-rich banks maintain higher market share and leverage despite identical lending technologies, while wholesale-reliant banks face binding market discipline through endogenous funding costs. Third, *systemic risk redistribution*: deposit insurance pricing doesn't just affect individual bank behavior—it systematically shifts risk from deposit-funded to wholesale-funded activities, creating predictable patterns of crisis vulnerability.

Quantitative Guidance for Policy Design The framework provides concrete parameters for regulatory calibration. The key sufficient statistic $\tau(1-\gamma)$ captures how deposit insurance premiums interact with capital requirements to affect marginal lending decisions. This interaction suggests that optimal premium design cannot ignore existing capital regulation, and vice versa. The model also quantifies concentration effects: a small premium increase creates first-order redistribution effects between bank types while generating only second-order aggregate effects, making it an efficient tool for addressing market structure concerns.

Crisis Interpretation and Stress Testing The 2023 banking crisis validates the model's central predictions about funding structure vulnerability. Institutions with high uninsured deposit ratios—precisely those modeled as wholesale-reliant banks—experienced the most severe stress. The framework suggests that stress testing should explicitly account for funding structure heterogeneity rather than treating all banks symmetrically, and that crisis management policies should recognize how funding advantages shape competitive dynamics during periods of stress.

Future Research Directions Several extensions would enhance policy relevance. First, incorporating multiple banks and richer funding heterogeneity could illuminate oligopolistic competition patterns and optimal market structure. Second, endogenizing deposit gathering through technological investments or branch networks could explain how funding advantages evolve and whether they represent persistent market failures or efficient specialization. Third, adding dynamic elements—particularly how funding advantages accumulate over time and affect long-run market structure—would inform antitrust and regulatory policy. Finally, calibrating the model to real banking data would provide quantitative guidance for deposit insurance reform and stress testing methodologies.

The framework demonstrates that funding structure heterogeneity is not merely a descriptive feature of banking markets, but a fundamental driver of competition, risk-taking, and systemic stability that demands explicit consideration in both theoretical analysis and policy design.

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A Tables

Table 1: Bank D Balance Sheet Structure

Assets	**Liabilities & Equity**	
Loans: x_D	Insured Deposits: $(1-\gamma)x_D$	
Prepaid insurance premium (deducted from regulatory capital): $\tau(1-\gamma)x_D$	Outside Equity: $\gamma x_D + \tau(1-\gamma)x_D$	*Note:*
	Inside Equity: $\kappa \to 0$	

The prepaid insurance premium is deducted from regulatory capital at $t = 0$.

Table 2: Bank W Balance Sheet Structure (Mixed Funding Regime)

Assets	**Liabilities & Equity**
Loans: x_W	Insured Deposits: $\overline{D}_W$
Prepaid insurance premium (deducted from regulatory capital): $\tau\overline{D}_W$	Wholesale Funding: $(1-\gamma)x_W - \overline{D}_W$
	Outside Equity: $\gamma x_W + \tau\overline{D}_W$
	Inside Equity: $\kappa \to 0$

Note: Bank W faces constraint $\overline{D}_W$ on insured deposit access, requiring wholesale funding beyond this limit.

B Figures

This appendix presents the figures supporting the numerical analysis and comparative statics discussed in Section 9. The figures illustrate key theoretical predictions about how deposit insurance premiums and deposit access affect equilibrium outcomes.

B.1 Effects of Deposit Insurance Premium

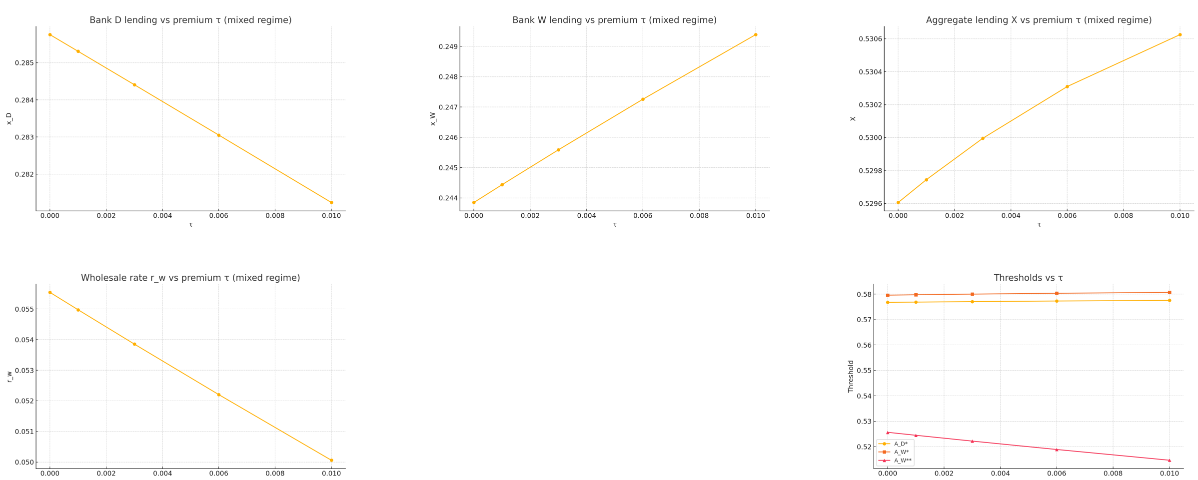

Figure 1: Comparative statics with respect to the deposit insurance premium τ under a binding deposit cap ($D_W = 0.20$). The five panels show the effects on individual bank lending (x_D, x_W), aggregate lending (X), wholesale rate (r_w), and default thresholds (A_D^*, A_W^*). As predicted by the theory, increases in τ reduce deposit-funded lending ($x_D \downarrow$) and increase wholesale-funded lending ($x_W \uparrow$), while aggregate lending (X) moves little due to strategic substitution effects. The wholesale rate (r_w) shows minimal response in this calibration, and the default threshold ordering $A_W^* > A_D^*$ is maintained when wholesale funding is used, consistent with Lemma 4.3.

B.2 Effects of Deposit Access

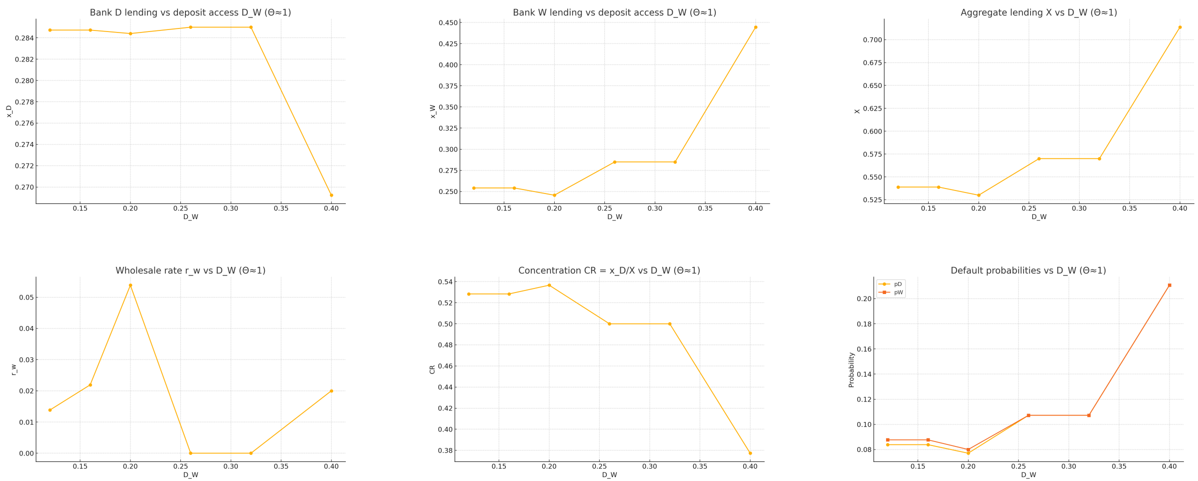

Figure 2: Comparative statics with respect to Bank W's deposit access D_W, illustrating the transition from mixed funding to slack constraint regimes. The six panels show the effects on individual bank lending (x_D, x_W), aggregate lending (X), wholesale rate (r_w), market concentration (CR), and default probabilities. As D_W increases, Bank W gains access to more deposit funding, reducing its reliance on wholesale markets ($r_w \downarrow 0$) and increasing its lending capacity ($x_W \uparrow$). The system transitions from a mixed funding regime where wholesale funding is used at the margin to a slack regime where both banks have similar funding costs. Market concentration decreases as Bank W's lending converges toward Bank D's level, consistent with Theorem 8.3.

C Technical Appendix

This appendix provides detailed mathematical derivations for the main propositions and lemmas presented in the paper. The proofs follow the logical sequence of the model development, starting with firm equilibrium, proceeding through bank optimization, and concluding with equilibrium characterization and comparative statics.

C.1 Derivation of Firm Equilibrium (Proposition 4.1)

I establish Proposition 4.1 through backward induction, solving the firm's dynamic optimization problem under the output-contingent remuneration framework established in Assumption 3.1(d), where wages are paid from realized output only when firms succeed.

C.1.1 Stage 2: Labor Choice at Date t=1

At date $t = 1$, given capital k_i, observed aggregate productivity A, and market wage w, firm i chooses labor to maximize expected production value net of wage payments. Under the wage structure from Assumption 2.4(d), the firm's problem is:

$$\max_{L_i \geq 0} V_1^i(k_i, A, w) = \pi \cdot [A \cdot k_i^\alpha L_i^{1-\alpha} - wL_i] \tag{59}$$

The first-order condition yields:

$$\pi \cdot (1-\alpha) A k_i^\alpha L_i^{-\alpha} = \pi \cdot w \tag{60}$$

Simplifying by dividing both sides by π:

$$(1-\alpha) A k_i^\alpha L_i^{-\alpha} = w \tag{61}$$

Solving for optimal labor demand:

$$L_i^*(k_i, A, w) = k_i \left[\frac{(1-\alpha)A}{w}\right]^{1/\alpha} \tag{62}$$

C.1.2 Equilibrium Wage Determination

Labor market clearing requires aggregate labor demand equals the inelastic supply:

$$\int_0^1 L_i^*(k_i, A, w) di = \overline{L} \tag{63}$$

In symmetric equilibrium where all firms choose capital k:

$$k \left[\frac{(1-\alpha)A}{w}\right]^{1/\alpha} = \overline{L} \tag{64}$$

Solving for the equilibrium wage:

$$w^*(A) = (1-\alpha) A \left(\frac{k}{\overline{L}}\right)^\alpha \tag{65}$$

Workers understand they receive this wage only when firms succeed (probability π), but with risk neutrality, they care only about expected wages. The expected wage equals:

$$\mathbb{E}[\text{wage}] = \pi \cdot w^*(A) = \pi(1-\alpha) A \left(\frac{k}{\overline{L}}\right)^\alpha \tag{66}$$

C.1.3 Stage 1: Capital Choice at Date t=0

At date $t = 0$, firm i chooses capital to maximize expected profit. When successful (probability π), the firm produces $Ak_i^\alpha \overline{L}^{1-\alpha}$, pays wages $(1-\alpha)Ak_i^\alpha \overline{L}^{1-\alpha}$, retaining $\alpha A k_i^\alpha \overline{L}^{1-\alpha}$. The firm must repay $(1 + r^l)k_i$ to the bank when successful. With limited liability, the firm defaults when $a_i = 0$ and pays nothing. The firm's expected profit is:

$$V_0^i = \pi \cdot \mathbb{E}_A[\alpha A k_i^\alpha \overline{L}^{1-\alpha} - (1 + r^l)k_i] \tag{67}$$

Since $\mathbb{E}[A] = 1$:

$$V_0^i = \pi \alpha k_i^{\alpha} \overline{L}^{1-\alpha} - \pi(1 + r^l)k_i \tag{68}$$

The first-order condition is:

$$\pi \alpha^2 k_i^{\alpha-1} \overline{L}^{1-\alpha} = \pi(1 + r^l) \tag{69}$$

Dividing by π and solving for optimal capital:

$$\alpha^2 k_i^{\alpha-1} \overline{L}^{1-\alpha} = (1 + r^l) \tag{70}$$

Therefore:

$$k^* = \overline{L} \left(\frac{\alpha^2}{1 + r^l} \right)^{\frac{1}{1-\alpha}} \tag{71}$$

In competitive equilibrium, expected firm profit equals zero, confirming that this is indeed the equilibrium capital choice. This completes the derivation of Proposition 4.1.

C.2 Derivation of Bank Revenue Function (Proposition 4.5)

From the symmetric equilibrium established in Section B.1, aggregate loan demand is:

$$K^d(r^l) = \overline{L} \left(\frac{\alpha^2}{1 + r^l} \right)^{\frac{1}{1-\alpha}} \tag{72}$$

Inverting this relationship to express the lending rate as a function of aggregate lending X:

$$\left(\frac{\alpha^2}{1 + r^l} \right)^{\frac{1}{1-\alpha}} = \frac{X}{\overline{L}} \tag{73}$$

Raising both sides to the power $(1 - \alpha)$:

$$\frac{\alpha^2}{1 + r^l} = \left(\frac{X}{\overline{L}} \right)^{1-\alpha} \tag{74}$$

Solving for the lending rate:

$$1 + r^l = \alpha^2 \overline{L}^{1-\alpha} X^{-(1-\alpha)} \tag{75}$$

Banks lend to a continuum of firms with independent idiosyncratic risks. By the law of large numbers, exactly fraction π of firms succeed and repay their loans. Expected revenue per unit lent equals:

$$R = \pi(1 + r^l) = \pi \alpha^2 \overline{L}^{1-\alpha} X^{-(1-\alpha)} \tag{76}$$

Defining $\Theta \equiv \pi \alpha^2 \overline{L}^{1-\alpha}$ and noting that $1/\epsilon = -(1 - \alpha)$ where $\epsilon = -\frac{1}{1-\alpha}$ is the loan demand elasticity:

$$R(X) = \Theta \cdot X^{1/\epsilon} \tag{77}$$

Since $\epsilon < -1$ (as $\alpha \in (0, 1)$), we have $1/\epsilon \in (-1, 0)$, confirming that the revenue function is a downward-sloping (and convex) inverse demand in X; hence per-unit expected repayment R(X) falls as aggregate lending rises. This completes the derivation of Proposition 4.5.

C.3 Verification of Strategic Default Prevention

I verify that the parameter restriction $\underline{A} > \alpha$ prevents strategic default by successful firms. Consider a firm with $a_i = 1$ that has borrowed k at rate r^l and hired L workers at wage w. The firm produces output:

$$y = Ak^\alpha L^{1-\alpha} \tag{78}$$

In equilibrium, $L = \overline{L}$ and $w = (1-\alpha)A(k/\overline{L})^\alpha$. The firm's obligations are:

- Bank debt: $(1 + r^l)k$
- Worker wages: $w \cdot \overline{L} = (1-\alpha)Ak^\alpha \overline{L}^{1-\alpha}$

The firm's net payoff from repaying all obligations is:

$$\pi_{\text{repay}} = Ak^\alpha \overline{L}^{1-\alpha} - (1 + r^l)k - (1-\alpha)Ak^\alpha \overline{L}^{1-\alpha} = \alpha Ak^\alpha \overline{L}^{1-\alpha} - (1 + r^l)k \tag{79}$$

Strategic default yields zero payoff due to limited liability. Therefore, the firm prefers repayment when:

$$\alpha Ak^\alpha \overline{L}^{1-\alpha} \geq (1 + r^l)k \tag{80}$$

Rearranging:

$$\alpha Ak^{\alpha-1} \overline{L}^{1-\alpha} \geq (1 + r^l) \tag{81}$$

From the firm's first-order condition in equilibrium (derived in Section B.1):

$$\alpha^2 k^{\alpha-1} \overline{L}^{1-\alpha} = (1 + r^l) \tag{82}$$

Substituting this relationship into the no-strategic-default condition:

$$\alpha Ak^{\alpha-1} \overline{L}^{1-\alpha} \geq \alpha^2 k^{\alpha-1} \overline{L}^{1-\alpha} \tag{83}$$

Simplifying:

$$A > \alpha \tag{84}$$

Since this condition must hold for all realizations of aggregate productivity, we require:

$$\underline{A} > \alpha \tag{85}$$

This parameter restriction ensures that even in the worst aggregate state, successful firms generate sufficient value to cover their obligations, eliminating incentives for strategic default. The economic interpretation is straightforward: aggregate productivity must be sufficiently high relative to the capital intensity of production to maintain contractual integrity. This completes the verification.

C.4 Bank Contract Characterizations

Both banks face similar contracting problems with outside equity investors. Equity is competitively priced ex ante and junior to deposits, while the banker (inside equity) receives the residual value subject to limited liability. The following proof states the default-threshold and payoff representation used throughout.

C.4.1 Proof of Proposition 4.4: Default Threshold and Payoff Representation

By priority of claims, deposits are paid first. Let $V(A; x_j, x_{-j}) = A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon}$ denote the gross asset value. Limited liability implies equity receives $[V(A; \cdot) - \mathcal{D}_j]^+$ in state A. Outside equity is priced competitively ex ante and equals γx_j, so the banker's expected payoff is

$$w_j(x_j, x_{-j}) = \int_{\underline{A}}^{\overline{A}} [V(A; x_j, x_{-j}) - \mathcal{D}_j]^+ f(A)\, dA - \Big(\gamma x_j + \tau\, I_j(x_j)\Big). \tag{86}$$

Define the default threshold A_j^* as the unique solution to $V(A_j^*; x_j, x_{-j}) = \mathcal{D}_j$, i.e.

$$A_j^* = \frac{\mathcal{D}_j}{x_j\, \Theta(x_j + x_{-j})^{1/\epsilon}}. \tag{87}$$

Then $[V - \mathcal{D}_j]^+ = 0$ for $A < A_j^*$ and $[V - \mathcal{D}_j]^+ = V - \mathcal{D}_j$ for $A \geq A_j^*$, yielding the stated integral representation.

C.4.2 Proof of Proposition 5.1: General Value Decomposition

Starting from the payoff representation in Proposition 4.4:

$$w_j \;=\; \int_{A_j^*}^{\overline{A}} \Big[A\, x_j\, \Theta X^{1/\epsilon} - \mathcal{D}_j\Big] f(A)\, dA \;-\; (\gamma x_j + \tau I_j(x_j)) \tag{88}$$

I decompose the revenue integral by adding and subtracting the integral over default states:

$$\begin{aligned}\int_{A_j^*}^{\overline{A}} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A) dA &= \int_{\underline{A}}^{\overline{A}} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A) dA \\ &\quad - \int_{\underline{A}}^{A_j^*} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A) dA \end{aligned} \tag{89}$$

Since $\mathbb{E}[A] = \int_{\underline{A}}^{\overline{A}} A f(A) dA = 1$:

$$\int_{\underline{A}}^{\overline{A}} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A) dA = x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} \tag{90}$$

For the debt payment term:

$$\int_{A_j^*}^{\overline{A}} \mathcal{D}_j f(A) dA = \mathcal{D}_j [1 - F(A_j^*)] \tag{91}$$

Substituting these expressions:

$$w_j = x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} - \int_{\underline{A}}^{A_j^*} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A) dA - \mathcal{D}_j [1 - F(A_j^*)] - \gamma x_j - \tau I_j(x_j) \tag{92}$$

Rearranging and using the identity $\int_{\underline{A}}^{A_j^*}[\mathcal{D}_j - A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon}]f(A)dA = \mathcal{D}_j F(A_j^*) - \int_{\underline{A}}^{A_j^*} A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} f(A)dA$:

$$w_j = x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} - \mathcal{D}_j - \gamma x_j + \int_{\underline{A}}^{A_j^*}[\mathcal{D}_j - A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon}]f(A)dA - \tau I_j(x_j) \tag{93}$$

For Bank D with only insured deposits, $\mathcal{D}_D = (1-\gamma)x_D$, so $\mathcal{D}_D + \gamma x_D = x_D$.

For Bank W with wholesale funding, we make the zero-profit substitution explicit: wholesale investors provide funding $W_W = (1-\gamma)x_W - \overline{D}_W$ and expect return $(1+r_w)W_W$ in total. By competitive pricing, their expected payment equals their initial funding:

$$\mathbb{E}[\text{payment to wholesale}] = W_W \quad \Rightarrow \quad \text{expected wholesale payment} = \text{initial wholesale funding} \tag{94}$$

This zero-profit condition determines r_w endogenously, but does *not* allow wholesale obligations to drop out of the banker's marginal incentives. The total promised debt is $\mathcal{D}_W = \overline{D}_W + (1+r_w)W_W$, and both the insured portion $\overline{D}_W$ and the wholesale rate r_w affect marginal lending decisions since r_w responds to changes in x_W. This decomposition yields:

$$w_j = x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon} - x_j + \int_{\underline{A}}^{A_j^*}[\mathcal{D}_j - A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon}]f(A)dA - \tau I_j(x_j) \tag{95}$$

This yields the decomposition:

$$w_j = \underbrace{x_j[\Theta(x_j + x_{-j})^{1/\epsilon} - 1]}_{\text{Private NPV}_j} + \underbrace{\int_{\underline{A}}^{A_j^*}[\mathcal{D}_j - A \cdot x_j \cdot \Theta(x_j + x_{-j})^{1/\epsilon}]f(A)dA}_{\text{Subsidy}_j} - \tau I_j(x_j) \tag{96}$$

The NPV term represents expected revenue minus the total cost of funds (normalized to 1 per unit). The Subsidy term captures the expected government payout in default states. Since workers receive nothing when firms fail, this represents private rather than social value creation.

C.5 Bank-Specific Derivations

C.5.1 Bank D Specific Elements

For Bank D with unlimited deposit access at zero cost, we have $\mathcal{D}_D = (1-\gamma)x_D$.

C.5.1.1 Derivation of Bank D Value Decomposition Substituting $\mathcal{D}_D = (1-\gamma)x_D$ into the general decomposition from Proposition 5.1:

- **Default Threshold:**

$$A_D^* = \frac{(1-\gamma)x_D}{x_D \cdot \Theta(x_D + x_W)^{1/\epsilon}} = \frac{1-\gamma}{\Theta(x_D + x_W)^{1/\epsilon}} \tag{97}$$

  where $\Theta = \pi\alpha^2\overline{L}^{1-\alpha}$ and $1/\epsilon = -(1-\alpha)$.

- **Subsidy Specification:**

$$\text{Subsidy}_D = \int_{\underline{A}}^{A_D^*} [(1-\gamma)x_D - A \cdot x_D \cdot \Theta(x_D + x_W)^{1/\epsilon}] f(A) dA \tag{98}$$

  This can be factored as:

$$\text{Subsidy}_D = x_D \int_{\underline{A}}^{A_D^*} [(1-\gamma) - A \cdot \Theta(x_D + x_W)^{1/\epsilon}] f(A) dA \tag{99}$$

This completes the derivation of the Bank D value decomposition.

C.5.1.2 Derivation of Bank D First-Order Condition (Proposition 6.2) To derive the first-order condition, we differentiate Bank D's value function with respect to x_D.

Step 0: Premium term. The premium-adjusted objective is $\tilde{w}_D = w_D - \tau(1-\gamma)x_D$. Differentiating the premium term contributes a constant $-\tau(1-\gamma)$ to the FOC in any region where insured deposits are used at the margin.

Step 1: Differentiate Private NPV.

$$\begin{aligned}
\frac{\partial}{\partial x_D}[x_D \cdot \Theta(x_D + x_W)^{1/\epsilon} - x_D] &= \Theta(x_D + x_W)^{1/\epsilon} - 1 + x_D \cdot \Theta \cdot \frac{1}{\epsilon}(x_D + x_W)^{1/\epsilon - 1} \\
&= \Theta(x_D + x_W)^{1/\epsilon} - 1 - x_D \cdot \Theta(1-\alpha)(x_D + x_W)^{1/\epsilon - 1} \\
&= \Theta(x_D + x_W)^{1/\epsilon - 1}[(x_D + x_W) - (1-\alpha)x_D] - 1 \\
&= \Theta(x_D + x_W)^{1/\epsilon - 1}[x_W + \alpha x_D] - 1
\end{aligned}$$

where we use the fact that $1 - (1-\alpha) = \alpha$, the capital share parameter.

Step 2: Differentiate Deposit Insurance Subsidy. Applying the Leibniz integral rule to the subsidy term:

$$\frac{\partial}{\partial x_D} \int_{\underline{A}}^{A_D^*} [(1-\gamma)x_D - A \cdot x_D \cdot \Theta(x_D + x_W)^{1/\epsilon}] f(A) dA$$

The boundary term vanishes because at $A = A_D^*$, the integrand is zero by definition of A_D^*. Thus, we only differentiate inside the integral:

$$\begin{aligned}
\frac{\partial \text{Subsidy}_D}{\partial x_D} &= \int_{\underline{A}}^{A_D^*} \frac{\partial}{\partial x_D}[(1-\gamma)x_D - A \cdot x_D \cdot \Theta(x_D + x_W)^{1/\epsilon}] f(A) dA \\
&= \int_{\underline{A}}^{A_D^*} [(1-\gamma) - A \cdot \Theta(x_D + x_W)^{1/\epsilon - 1}[x_W + \alpha x_D]] f(A) dA
\end{aligned}$$

Step 3: Verify Positivity of the Integral Term. For all $A < A_D^* = \frac{1-\gamma}{\Theta(x_D + x_W)^{1/\epsilon}}$, we have $A \cdot \Theta(x_D + x_W)^{1/\epsilon} < (1-\gamma)$. Since $[x_W + \alpha x_D] < (x_D + x_W)$:

$$A \cdot \Theta(x_D + x_W)^{1/\epsilon - 1}[x_W + \alpha x_D] < A \cdot \Theta(x_D + x_W)^{1/\epsilon} < (1-\gamma)$$

Therefore, the integrand is strictly positive for all $A \in [\underline{A}, A_D^*)$.

Step 4: Combine to get First-Order Condition. Setting $\frac{d\widetilde{w}_D}{dx_D} = 0$ yields the first-order condition:

$$\Theta(x_D + x_W)^{1/\epsilon - 1}\Big[x_W + \alpha x_D\Big] - 1 \;-\; \tau(1-\gamma) \tag{100}$$

$$+ \int_{\underline{A}}^{A_D^*} \Big[(1-\gamma) - A\,\Theta(x_D + x_W)^{1/\epsilon - 1}\Big[x_W + \alpha x_D\Big]\Big] f(A)\,dA = 0 \tag{101}$$

This completes the derivation of Proposition 6.2. The positive integral term represents the marginal deposit insurance subsidy (14).

C.5.1.3 KKT Conditions for Bank D. Let $S_D = [0, \overline{x}_D]$. With strict concavity of $w_D(\cdot, x_W)$ in own output (Lemma C.5), the KKT conditions are necessary and sufficient: stationarity $\frac{\partial w_D}{\partial x_D}(x_D, x_W) - \mu_D + \nu_D = 0$ with multipliers $\mu_D, \nu_D \geq 0$ for the lower and upper bounds respectively, feasibility $x_D \in S_D$, and complementary slackness $\mu_D x_D = 0$, $\nu_D(\overline{x}_D - x_D) = 0$. In particular, at the lower boundary $x_D = 0$ we require $\frac{\partial w_D}{\partial x_D}(0, x_W) \leq 0$.

C.5.2 Bank W Specific Elements

For Bank W with mixed funding, when $x_W > \overline{D}_W/(1-\gamma)$, the total debt obligation is:

$$\mathcal{D}_W = \overline{D}_W + [(1-\gamma)x_W - \overline{D}_W](1 + r_w) \tag{102}$$

C.5.2.1 Derivation of Wholesale Funding Equilibrium (Proposition 5.2) Wholesale funders receive different payoffs depending on the realized state:

- When $A \geq A_W^*$: Full repayment $[(1-\gamma)x_W - \overline{D}_W](1 + r_w)$
- When $A_W^{**} \leq A < A_W^*$: Residual value after depositors $A \cdot x_W \cdot \Theta(x_D + x_W)^{1/\epsilon} - \overline{D}_W$
- When $A < A_W^{**}$: Nothing (deposits have seniority)

The break-even condition requires:

$$\int_{A_W^*}^{\overline{A}} \Big[(1-\gamma)x_W - \overline{D}_W\Big](1 + r_w) f(A)\,dA$$

$$+ \int_{A_W^{**}}^{A_W^*} \Big[A \cdot x_W \cdot \Theta(x_D + x_W)^{1/\epsilon} - \overline{D}_W\Big] f(A)\,dA = (1-\gamma)x_W - \overline{D}_W \tag{103}$$

This implicitly defines r_w as a function of (x_W, x_D). As x_W increases, the default probability rises, requiring a higher r_w to compensate wholesale funders.

C.5.2.2 Derivation of Bank W Value Decomposition Bank W's initial objective function with mixed funding is:

$$w_W = \int_{A_W^*}^{\overline{A}} [A \cdot x_W \cdot \Theta(x_D + x_W)^{1/\epsilon} - \overline{D}_W - [(1-\gamma)x_W - \overline{D}_W](1 + r_w)] f(A)dA - \gamma x_W \tag{104}$$

Using the wholesale funding break-even condition to substitute out the term for payments to wholesale funders, we can simplify the expression. The break-even condition implies that the expected payment to wholesale funders equals their initial investment. After careful substitution and algebraic manipulation similar to the general value decomposition, the objective function simplifies to:

$$w_W = x_W[\Theta(x_D + x_W)^{1/\epsilon} - 1] + \int_{\underline{A}}^{A_W^{**}} [\overline{D}_W - A \cdot x_W \cdot \Theta(x_D + x_W)^{1/\epsilon}]f(A)dA \tag{105}$$

This completes the derivation.

C.5.2.3 KKT Conditions for Bank W with Kink. For Bank W, the feasible set is $S_W = [0, \overline{x}_W]$. The deposit cap enters only through the premium term via $I_W(x_W) = \min\{(1-\gamma)x_W, \overline{D}_W\}$, generating a kink at $x_W = \overline{D}_W/(1-\gamma)$. Encode the cap with $g(x_W) \equiv (1-\gamma)x_W - \overline{D}_W \leq 0$ and multiplier $\mu \geq 0$. When $g(x_W) < 0$ (cap slack), the interior FOC in Proposition 6.4 applies together with box constraints. At the kink $x_W = \overline{D}_W/(1-\gamma)$, the KKT system is $\nabla_{x_W} w_W(x_W; x_D) + \mu \nabla g(x_W) - \nu = 0$, $\mu, \nu \geq 0$, $\mu\, g(x_W) = 0$, $\nu(\overline{x}_W - x_W) = 0$, together with the subgradient condition $\frac{\partial w_W}{\partial x_W}(x_W^-, x_D) \geq 0 \geq \frac{\partial w_W}{\partial x_W}(x_W^+, x_D)$. Slater's condition holds since $x_W \in (0, \overline{x}_W)$ is strictly feasible; with concavity on each side of the kink and linear constraints, these KKT conditions are necessary and sufficient.

C.5.2.4 Derivation of Bank W First-Order Condition (Proposition 6.4)

Step 1: Private NPV Derivative. The derivative is identical to Bank D's:

$$\frac{\partial \text{NPV}_W}{\partial x_W} = \Theta(x_D + x_W)^{1/\epsilon - 1}[x_D + \alpha x_W] - 1$$

Step 2: Wholesale Cost Derivative. The wholesale funding cost term $-r_w((1-\gamma)x_W - \overline{D}_W)$ contributes two terms when differentiated:

$$\frac{\partial}{\partial x_W}\Big[-r_w((1-\gamma)x_W - \overline{D}_W)\Big] = -r_w(1-\gamma) - ((1-\gamma)x_W - \overline{D}_W)\frac{\partial r_w}{\partial x_W}$$

By Lemma 5.3, $\partial r_w/\partial x_W > 0$, so both terms are negative, representing market discipline.

Step 3: Subsidy Derivative. The subsidy integral contributes:

$$\frac{\partial \text{Subsidy}_W}{\partial x_W} = -\int_{A}^{A_W^{**}} A \cdot \Theta(x_D + x_W)^{1/\epsilon - 1}[x_D + \alpha x_W]f(A)dA$$

Step 4: First-Order Condition. Combining all derivatives and setting equal to zero:

$$\Theta X^{1/\epsilon-1}(x_D + \alpha x_W) - 1 - r_w(1-\gamma) - ((1-\gamma)x_W - \overline{D}_W)\frac{\partial r_w}{\partial x_W} - \int_{\underline{A}}^{A_W^{**}} A\Theta X^{1/\epsilon-1}(x_D + \alpha x_W)f(A)dA = 0 \tag{106}$$

This completes the derivation of Proposition 6.4. Both wholesale rate terms and the negative integral term reflect market discipline through endogenous funding costs.

C.6 Equilibrium Existence and Uniqueness

C.6.0.1 Regularity and Concavity Primitives. We first establish the regularity properties needed for existence and uniqueness proofs.

Lemma C.1 (Continuity and Differentiability of w_j)**.** *For each bank $j \in \{D, W\}$, the value function $w_j(x_j, x_{-j})$ is continuous on compact rectangles of $\mathbb{R}^2_+$ and continuously differentiable on the interior where $x_j > 0$. Moreover, the derivative can be obtained by differentiating under the integral sign.*

Proof. Fix (x_j, x_{-j}) with $x_j > 0$. By Proposition 4.4, the payoff admits the integral form with lower limit $A^*_j(x_j, x_{-j}, \mathcal{D}_j)$ and integrand linear in A and continuous in (x_j, x_{-j}) through $\Theta(x_j + x_{-j})^{1/\epsilon}$. On any compact rectangle for (x_j, x_{-j}), both the integrand and the moving boundary are uniformly bounded, and f is continuous by Assumption 3.2. Therefore the Dominated Convergence Theorem implies continuity in (x_j, x_{-j}).

For differentiability, apply the Leibniz integral rule with a variable lower limit: the integrand is jointly continuous in arguments, has a partial derivative w.r.t. x_j that is bounded on compacts, and **the boundary term vanishes because the integrand equals zero at $A = A^*_j$ by definition of the threshold**. This zero boundary condition is crucial: at $A = A^*_j$, we have $A^*_j x_j \Theta X^{1/\epsilon} = \mathcal{D}_j$ by construction, so the integrand $[\mathcal{D}_j - A x_j \Theta X^{1/\epsilon}] = 0$ exactly. Under regularity assumptions (bounded density f on compact support $[\underline{A}, \overline{A}]$), this ensures that boundary terms from moving thresholds drop out, justifying differentiation under the integral sign to obtain $\partial w_j / \partial x_j$. □

Lemma C.2 (Default Threshold Regularity)**.** *Fix (x_j, x_{-j}) with $x_j > 0$. The default threshold $A^*_j(x_j, x_{-j}, \mathcal{D}_j)$ defined by $\mathcal{D}_j = A^*_j x_j\, \Theta(x_j + x_{-j})^{1/\epsilon}$ is continuous in (x_j, x_{-j}) and continuously differentiable wherever $x_j > 0$. Moreover, if $\mathcal{D}_j$ is continuously differentiable in the relevant arguments, then A^*_j is continuously differentiable in those arguments.*

Proof. Define $g(A, x_j, x_{-j}) \equiv A x_j\, \Theta(x_j + x_{-j})^{1/\epsilon} - \mathcal{D}_j$. Then $g(A^*_j, x_j, x_{-j}) = 0$ and $\partial g / \partial A = x_j\, \Theta(x_j + x_{-j})^{1/\epsilon} > 0$ for $x_j > 0$. By the Implicit Function Theorem, A^*_j is continuously differentiable in a neighborhood of any point with $x_j > 0$ and inherits continuity globally on compact rectangles by standard extension arguments. □

Lemma C.3 (Revenue Function Regularity)**.** *Let $R(X) = \Theta X^{1/\epsilon}$ with $1/\epsilon \in (-1, 0)$. Then for all $X > 0$:*

$$R'(X) = \Theta \frac{1}{\epsilon} X^{1/\epsilon - 1} < 0, \qquad R''(X) = \Theta \frac{1}{\epsilon}\left(\frac{1}{\epsilon} - 1\right) X^{1/\epsilon - 2} > 0.$$

Proof. Since $1/\epsilon = \alpha - 1 \in (-1, 0)$ for $\alpha \in (0, 1)$, we have $R'(X) = \Theta(1/\epsilon) X^{1/\epsilon - 1} < 0$ and $R''(X) = \Theta(1/\epsilon)(1/\epsilon - 1) X^{1/\epsilon - 2} > 0$ for $X > 0$. Continuity of r_w follows from the implicit function theorem applied to the wholesale break-even condition in Proposition 5.2. Moreover, $r_w(x_W, x_D)$ is C^1 on the region $x_W > \overline{D}_W / (1 - \gamma)$ under absolute priority with proportional sharing and continuous f with bounded support. □

Lemma C.4 (Value Function Properties)**.** *For each bank $j \in \{D, W\}$, the value function $w_j(x_j, x_{-j})$ satisfies:*

1. *Negative second derivative* $\frac{\partial^2 w_j}{\partial x_j^2} < 0$ *in* x_j *on feasible domains for any fixed* x_{-j}.

2. *The single-crossing property ensuring unique best responses.*

3. *Appropriate boundary conditions for existence of interior solutions.*

Proof. The second derivative $\frac{\partial^2 w_j}{\partial x_j^2}$ contains the revenue function curvature term $\Theta(1/\epsilon)(1/\epsilon - 1)(x_j + x_{-j})^{1/\epsilon-2}$. Since $1/\epsilon \in (-1, 0)$, we have $(1/\epsilon - 1) < -1 < 0$ and $(1/\epsilon - 2) < 0$, so this term is strictly negative. On the compact strategy sets S_j, this provides a uniform negative lower bound. Combined with the subsidy terms' negative second derivatives, this ensures $\frac{\partial^2 w_j}{\partial x_j^2} < 0$ on feasible domains. The single-crossing property follows from the monotonicity of marginal value functions established in the first-order conditions. □

Lemma C.5 (Second-Order Conditions in Own Choice)**.** *For each bank* $j \in \{D, W\}$*, the second derivative* $\frac{\partial^2 w_j}{\partial x_j^2} < 0$ *on compact domains, ensuring local maxima.*

Proof. From the profit function $w_j = x_j[\Theta X^{1/\epsilon} - 1] +$ subsidy terms, the second derivative is $\frac{\partial^2 w_j}{\partial x_j^2} = \Theta(1/\epsilon)(1/\epsilon - 1)X^{1/\epsilon-2} +$ subsidy second derivatives. Since $1/\epsilon \in (-1, 0)$, both $(1/\epsilon) < 0$ and $(1/\epsilon - 1) < -1$, making their product positive. However, the factor $X^{1/\epsilon-2}$ with $1/\epsilon - 2 < -2$ dominates on compact strategy sets, yielding an explicit negative bound $\frac{\partial^2 w_j}{\partial x_j^2} \leq -C < 0$ for some $C > 0$ depending on the strategy bounds. The subsidy terms contribute additional negative curvature through their integrands' second derivatives. □

Theorem C.6 (Existence of equilibrium)**.** *Assume* f *is continuous with support* $[\underline{A}, \overline{A}]$, $\gamma \in (0, 1)$*, and* $\bar{x}_j$ *bounds such that profits are non-positive for* $x_j > \bar{x}_j$*. Then a Nash equilibrium* $(x_D^*, x_W^*, r^{l*}, r_w^*)$ *exists.*

Proof. Define compact convex strategy sets $\mathcal{X}_j = [0, \bar{x}_j]$. For fixed x_{-j}, bank j's payoff is concave in x_j: while the revenue curve $R(X) = \Theta X^{-\sigma}$ is convex in aggregate quantity, the profit function is concave in own lending due to the dominance of marginal revenue effects, and Lemma 5.3 delivers a unique r_w and concave discipline term; measurability and integrability follow from Def. 3.1. By Berge's maximum theorem, best-response correspondences BR_j are non-empty, convex-valued, and upper hemicontinuous. Kakutani's fixed-point theorem yields a fixed point of $BR_D \times BR_W$. Prices (r^{l*}, r_w^*) then obtain from market clearing and Lemma 5.3. □

Proposition C.7 (Checkable sufficient conditions for Rosen uniqueness)**.** *Let* $h(A) \equiv f(A)/(1 - F(A))$ *denote the hazard rate with* $0 < \underline{h} \leq h(A) \leq \overline{h} < \infty$ *(MHR). Let* $\sigma = 1 - \alpha \in (0, 1)$.

Explicit bounds: *The own second derivatives satisfy*

$$\left|\frac{\partial^2 w_j}{\partial x_j^2}\right| \geq \Theta|\sigma|(1+\sigma)X^{-\sigma-1} - \overline{h} \cdot \Theta\sigma X^{-\sigma} \equiv \underline{d}_j(X) \tag{107}$$

while cross-partials are bounded by

$$\left|\frac{\partial^2 w_j}{\partial x_j \partial x_k}\right| \leq \Theta\sigma(1+\sigma)X^{-\sigma-1} + \overline{h} \cdot \Theta\sigma X^{-\sigma} \equiv \overline{c}_{jk}(X). \tag{108}$$

If the aggregate lending $X = x_D + x_W$ satisfies

$$\underline{d}_j(X) > 2 \cdot \bar{c}_{jk}(X) \quad \text{for both } j \in \{D, W\}, \tag{109}$$

then the game is strictly diagonally dominant in the sense of Rosen (1965), and the Nash equilibrium in quantities is unique.

Proof sketch. Own-second derivatives are bounded away from zero by strict concavity of $w_j(\cdot, x_{-j})$ in own output and the convex response of r_w (Lemma 5.3). Cross-partials are bounded in magnitude by two channels: (i) the price effect via $R'(X) \propto (1/\epsilon - 1)X^{(1/\epsilon - 1)}$, and (ii) the default-region effect $h(A_j^*)\, \partial A_j^* / \partial x_k$ with A_j^* from Definition 4.3. MHR bounds h uniformly; compactness of $\mathcal{X}_j$ bounds X and A_j^*. Hence the ratio of cross- to own-second derivatives is uniformly below one once (109) holds, implying strict diagonal dominance and uniqueness. □

Theorem C.8 (Uniqueness)**.** *Under the assumptions of Theorem C.6 and the bound in Proposition C.7, the equilibrium is unique.*

Proof. Apply Rosen's concave games theorem using the strict diagonal dominance inequality (109). □

C.6.1 Proof of Proposition 7.1: Equilibrium Properties

Part (ii): Strategic Substitutability. I show $\frac{\partial^2 w_D}{\partial x_D \partial x_W} < 0$. The cross-partial derivative consists of two parts: the effect on the private NPV and the effect on the marginal subsidy. Both can be shown to be negative. A positive NPV term is outweighed by a larger negative term, and the subsidy cross-partial is negative. Therefore, the total effect is negative. By the implicit function theorem, the slope of the reaction function is $\frac{dx_D}{dx_W} = -\frac{(-)}{(-)} < 0$, confirming strategic substitutability.

Part (iv): Market Discipline Asymmetry. For Bank W, taking the total differential of the wholesale funding break-even condition shows that $\frac{\partial r_w}{\partial x_W} > 0$. The endogenous funding rate is strictly increasing in lending. For Bank D, the deposit rate is fixed at zero, so $\frac{\partial r_D}{\partial x_D} = 0$. This asymmetry in funding cost sensitivity drives the differential risk-taking incentives.

C.7 Parameter Space Compatibility

I verify that the model's parameter restrictions are mutually compatible and define a non-empty feasible parameter space.

C.7.1 Complete Parameter Restrictions

The model imposes the following restrictions:

1. Production technology: $\alpha \in (0, 1)$
2. Regulatory capital: $\gamma \in (0, 1)$

3. Credit risk: $\pi \in (0, 1)$
4. Strategic default prevention: $\underline{A} > \alpha$
5. Productivity distribution: $0 < \underline{A} < 1 < \overline{A} < \infty$
6. Mean normalization: $\mathbb{E}[A] = 1$
7. Deposit constraint: $\overline{D}_W \in (0, (1-\gamma)\overline{x})$ for finite $\overline{x}$
8. Labor supply: $\overline{L} > 0$ finite

Remark C.1 (Stronger sufficient condition)*.* The state-dependent condition $\underline{A}\,\Theta\,X^{-\sigma} \geq \alpha$ implies $\underline{A} > \alpha$ and ensures the lemma uniformly over X.

Lemma C.9 (Non-Empty Feasible Parameter Space)**.** *The feasible parameter space is non-empty and has positive measure.*

Proof. I construct an explicit feasible parameter configuration.

Step 1: Core Parameters. Choose: $\alpha = 0.3$, $\gamma = 0.1$, $\pi = 0.95$, $\overline{L} = 1$. These clearly satisfy their basic constraints.

Step 2: Productivity Distribution. Set $\underline{A} = 0.5$ and $\overline{A} = 2.0$. Let A follow a Beta(p, q) distribution linearly rescaled to $[\underline{A}, \overline{A}]$ with density strictly positive and continuous on its support. Choose $(p, q) = (2, 2)$ and rescale to ensure $\mathbb{E}[A] = 1$. This satisfies continuity, positivity, and mean normalization while respecting $0 < \alpha < \underline{A} < 1 < \overline{A} < \infty$.

Step 3: Deposit Constraint Bound. From Lemma C.3, the upper bound on lending is $\overline{x} = \left(\frac{\pi\alpha^2\overline{L}^{1-\alpha}\overline{A}}{1-\gamma}\right)^{\frac{1}{1-\alpha}}$. Using the chosen parameters yields a finite bound. Pick any $\overline{D}_W \in (0, (1-\gamma)\overline{x})$.

Since all inequalities are strict, there exists an open neighborhood around this configuration where all constraints remain satisfied. Therefore, the feasible parameter space is non-empty with positive measure. □

C.7.2 Global Second-Order Sufficient Conditions

Lemma C.10 (Second-Order Conditions)**.** *For all $(x_D, x_W) \in S_D \times S_W$, the own second derivatives $\frac{\partial^2 w_j}{\partial x_j^2} < 0$ for $j \in \{D, W\}$, ensuring that any first-order condition represents a local maximum.*

Proof. The own second derivatives are negative under the stated regularity conditions due to the dominant negative terms from the convex revenue function $R(X) = \Theta X^{1/\epsilon}$ with $1/\epsilon \in (-1, 0)$. While the revenue function itself is convex, the profit maximization problem yields negative second derivatives in own lending because the marginal revenue effect dominates. Since the feasible sets S_D and S_W are compact and convex and the constraints are linear, any Karush-Kuhn-Tucker point satisfying the second-order conditions is a global maximizer. When Bank W's lower-bound constraint binds at $x_W = \overline{D}_W/(1-\gamma)$, the negative second derivative of the objective and linearity of the constraint imply optimality at the boundary. $\square$

C.8 Comparative Statics Analysis

C.8.1 Proof of Theorem 8.3: Deposit Access and Equilibrium Lending

The equilibrium is defined by the system of first-order conditions:

$$\Phi_D(x_D^*, x_W^*) \equiv \frac{\partial w_D}{\partial x_D}(x_D^*, x_W^*) = 0$$
$$\Phi_W(x_W^*, x_D^*; \overline{D}_W) \equiv \frac{\partial w_W}{\partial x_W}(x_W^*, x_D^*; \overline{D}_W) = 0$$

C.8.1.1 Step 1: Implicit Differentiation Setup. Bank D's FOC is independent of $\overline{D}_W$, so $\frac{\partial \Phi_D}{\partial \overline{D}_W} = 0$. For Bank W, $\overline{D}_W$ affects the FOC through the depositor loss threshold $A_W^{**} = \frac{\overline{D}_W}{x_W \Theta X^{1/\epsilon}}$.

An increase in $\overline{D}_W$ raises this threshold, which relaxes the market discipline constraint (the negative integral term in Bank W's FOC becomes smaller in magnitude). Therefore, $\frac{\partial \Phi_W}{\partial \overline{D}_W} > 0$.

C.8.1.2 Step 2: Apply Cramer's Rule. Totally differentiating the equilibrium conditions yields:

$$\begin{pmatrix} \frac{\partial^2 w_D}{\partial x_D^2} & \frac{\partial^2 w_D}{\partial x_D \partial x_W} \\ \frac{\partial^2 w_W}{\partial x_W \partial x_D} & \frac{\partial^2 w_W}{\partial x_W^2} \end{pmatrix} \begin{pmatrix} \frac{dx_D^*}{d\overline{D}_W} \\ \frac{dx_W^*}{d\overline{D}_W} \end{pmatrix} = - \begin{pmatrix} 0 \\ \frac{\partial \Phi_W}{\partial \overline{D}_W} \end{pmatrix}$$

Let H be the Hessian matrix. The determinant $\det(H)$ is positive by the dominant diagonal property required for uniqueness.

Part (i): Bank W's response.

$$\frac{dx_W^*}{d\overline{D}_W} = -\frac{1}{\det(H)} \det \begin{pmatrix} H_{DD} & 0 \\ H_{WD} & \frac{\partial \Phi_W}{\partial \overline{D}_W} \end{pmatrix} = -\frac{H_{DD} \frac{\partial \Phi_W}{\partial \overline{D}_W}}{\det(H)} = -\frac{(-)\,(+)}{(+)} > 0.$$

Part (ii): Bank D's response.

$$\frac{dx_D^*}{d\overline{D}_W} = -\frac{1}{\det(H)} \det \begin{pmatrix} 0 & H_{DW} \\ \frac{\partial \Phi_W}{\partial \overline{D}_W} & H_{WW} \end{pmatrix} = \frac{H_{DW} \frac{\partial \Phi_W}{\partial \overline{D}_W}}{\det(H)} = \frac{(-)\,(+)}{(+)} < 0.$$

Part (iii): Aggregate effect.

$$\frac{d(x_D^* + x_W^*)}{d\overline{D}_W} = \frac{\frac{\partial \Phi_W}{\partial \overline{D}_W} \cdot (H_{DW} - H_{DD})}{\det(H)}.$$

Since $|H_{DD}| > |H_{DW}|$ and both are negative, $H_{DW} - H_{DD} > 0$. Thus, the aggregate effect is positive.

Part (iv): Magnitude comparison.

$$\left|\frac{dx_W^*}{d\overline{D}_W}\right| - \left|\frac{dx_D^*}{d\overline{D}_W}\right| = \frac{\frac{\partial\Phi_W}{\partial\overline{D}_W}\cdot(|H_{DD}| - |H_{DW}|)}{\det(H)} > 0,$$

since $|H_{DD}| > |H_{DW}|$.

C.8.2 Proof of Proposition 8.4: Decomposition

The total derivative decomposes via the chain rule:

$$\frac{dx_W^*}{d\overline{D}_W} = \underbrace{\left.\frac{\partial x_W^*}{\partial\overline{D}_W}\right|_{x_D \text{ fixed}}}_{\text{Direct Effect}} + \underbrace{\left.\frac{\partial x_W^*}{\partial x_D}\right|_{\overline{D}_W \text{ fixed}} \cdot \frac{dx_D^*}{d\overline{D}_W}}_{\text{Strategic Effect}} \tag{110}$$

The direct effect is positive, as shown by applying the implicit function theorem to Bank W's FOC: $\frac{\partial x_W^*}{\partial\overline{D}_W} = -\frac{\partial\Phi_W/\partial\overline{D}_W}{\partial\Phi_W/\partial x_W} = -\frac{(+)}{(-)} > 0$. The strategic effect is the product of two negative terms: the slope of Bank W's reaction function ($\frac{\partial x_W^*}{\partial x_D} < 0$) and Bank D's equilibrium response ($\frac{dx_D^*}{d\overline{D}_W} < 0$). Thus, the strategic effect is also positive.

C.8.3 Proof of Proposition 8.6: Market Concentration

Define concentration as $CR = \frac{x_D^*}{x_D^*+x_W^*}$. Taking the derivative with respect to $\overline{D}_W$:

$$\frac{dCR}{d\overline{D}_W} = \frac{\frac{dx_D^*}{d\overline{D}_W}(x_D^* + x_W^*) - x_D^*(\frac{dx_D^*}{d\overline{D}_W} + \frac{dx_W^*}{d\overline{D}_W})}{(x_D^* + x_W^*)^2} = \frac{x_W^*\frac{dx_D^*}{d\overline{D}_W} - x_D^*\frac{dx_W^*}{d\overline{D}_W}}{(x_D^* + x_W^*)^2}$$

Since $\frac{dx_D^*}{d\overline{D}_W} < 0$ and $\frac{dx_W^*}{d\overline{D}_W} > 0$, both terms in the numerator are negative. Thus, $\frac{dCR}{d\overline{D}_W} < 0$.

C.8.4 Proof of Proposition 8.7: Risk Distribution

Part (i): Differential Default Risk. By Lemma 4.3, when bank W uses wholesale funding, $A_W^* > A_D^*$. A higher threshold implies a higher default probability, since $\Pr[\text{default}] = F(A_j^*)$ is increasing in A_j^*. Hence $A_W^* > A_D^* \Rightarrow F(A_W^*) > F(A_D^*)$.

Part (ii): Effect on Aggregate Default Risk. Aggregate risk is $R(\overline{D}_W) = \frac{x_D^*}{X^*}F(A_D^*) + \frac{x_W^*}{X^*}F(A_W^*)$. Differentiating with respect to $\overline{D}_W$ yields:

$$\frac{dR}{d\overline{D}_W} = \underbrace{\frac{d}{d\overline{D}_W}\left[\frac{x_D^*}{X^*}\right][F(A_D^*) - F(A_W^*)]}_{\text{Market share effect}<0} + \underbrace{\frac{x_D^*}{X^*}\frac{dF(A_D^*)}{d\overline{D}_W} + \frac{x_W^*}{X^*}\frac{dF(A_W^*)}{d\overline{D}_W}}_{\text{Individual risk effects (ambiguous)}}$$

The market share effect is negative because lending shifts from the riskier Bank D to the safer Bank W. The individual risk effects are ambiguous. As total lending X^* increases, both banks become safer (their default thresholds fall), but the change in Bank W's individual risk-taking incentives can be positive or negative. The net effect depends on which force dominates.

D Notation

D.1 Technology and Market Structure

$\alpha \in (0, 1)$ Capital share parameter in production technology

$\epsilon = -\frac{1}{1-\alpha} < -1$ Loan demand elasticity; $1/\epsilon = -(1-\alpha) = -\sigma$

$\sigma \equiv 1 - \alpha \in (0, 1)$ Alternative parameterization; $\sigma = -1/\epsilon$

$\Theta \equiv \pi\alpha^2\overline{L}^{1-\alpha}$ Revenue parameter constant

$\pi \in (0, 1)$ Success probability of idiosyncratic productivity shock

$\overline{L} > 0$ Inelastic labor supply

D.2 Banking and Regulation

$\gamma \in (0, 1)$ Regulatory capital ratio (outside equity as fraction of assets)

$\tau \geq 0$ Flat deposit insurance premium rate

$\overline{D}_D = \infty$ Bank D insured deposit capacity (unlimited)

$\overline{D}_W \in (0, \infty)$ Bank W insured deposit capacity (constrained)

$x_D, x_W \geq 0$ Lending scales by Bank D and Bank W

$X \equiv x_D + x_W$ Aggregate lending

D.3 Equilibrium Variables

$r^l \geq 0$ Equilibrium lending rate; $1 + r^l = \alpha^2\overline{L}^{1-\alpha}X^{-(1-\alpha)}$

$r_w \geq 0$ Wholesale funding rate for Bank W

$w(A)$ State-contingent wage rate

$A \in [\underline{A}, \overline{A}]$ Aggregate productivity shock; $\mathbb{E}[A] = 1$

$F(A), f(A)$ Cumulative distribution and density of aggregate productivity

A_j^* Default threshold for bank $j \in \{D, W\}$