Gurobi Example 1 – Chairs and Tables

Recall Question 3 from Homework 2:

> Jack Smith owns a small company that makes wooden tables and chairs. Jack is not good with his hands, and instead intends to hire two employees: one finisher and one carpenter. Each employee is expected to work a regular 40-hour week. It is known that making one table requires 4 hours of finishing labor and 2 hours of carpentry labor. Meanwhile, making one chair requires 3 hours of finishing labor and 2 hours of carpentry labor. Each chair requires \$15 in materials and is expected to sell for \$50. Each table requires \$25 in materials and is expected to sell for \$100. Formulate an LP that solves Jack's manufacturing problem (i.e., what to produce on average each week to maximize profit).

(Decision) Variables:

- x is the number of chairs produced each week (on average)
- y is the number of tables produced each week (on average)

Here is the linear program:

$$
\begin{aligned}
\max \quad & (50-15)x + (100-25)y && \text{(1a)}\\
\text{s.t.} \quad & 3x + 4y \le 40 && \text{(1b)}\\
& 2x + 2y \le 40 && \text{(1c)}\\
& x, y \ge 0. && \text{(1d)}
\end{aligned}
$$

The objective (1a) is to maximize profit. Constraint (1b) ensures that the finisher works at most 40 hours per week (on average). Constraint (1c) ensures that the carpenter works at most 40 hours per week (on average). The nonnegativity bounds (1d) ensure we do not produce a negative number of chairs and tables each week (on average).

Write Python code to solve this LP with Gurobi.

Gurobi Example 2 – Workforce Scheduling

Recall the Workforce Scheduling Problem.

> A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is given in the table below. Union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Monday to Friday must have Saturday and Sunday off. The post office wants to meet its daily requirements using only full-time employees. Formulate an integer program that the post office can use to minimize the number of full-time employees who must be hired.

#	Day	Number of full-time employees required
1	Monday	17
2	Tuesday	13
3	Wednesday	15
4	Thursday	19
5	Friday	14
6	Saturday	16
7	Sunday	11

This led to the following IP, where the variable x_i is the number of workers who *start* their 5-day shift on day i.

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{7} x_i && \text{(2a)}\\
\text{s.t.} \quad & x_1 + x_4 + x_5 + x_6 + x_7 \geq 17 && \text{(2b)}\\
& x_1 + x_2 + x_5 + x_6 + x_7 \geq 13 && \text{(2c)}\\
& x_1 + x_2 + x_3 + x_6 + x_7 \geq 15 && \text{(2d)}\\
& x_1 + x_2 + x_3 + x_4 + x_7 \geq 19 && \text{(2e)}\\
& x_1 + x_2 + x_3 + x_4 + x_5 \geq 14 && \text{(2f)}\\
& x_2 + x_3 + x_4 + x_5 + x_6 \geq 16 && \text{(2g)}\\
& x_3 + x_4 + x_5 + x_6 + x_7 \geq 11 && \text{(2h)}\\
& x \geq 0 \text{ integer.} && \text{(2i)}
\end{aligned}
$$

Write Python code to solve this *integer* program with Gurobi using a *vector* of variables x. First, number the variables, as above. In a second code, use intuitive names for the variables, like x[Monday].

Gurobi Example 3 – Transportation Problem

Consider the following problem.

> Wolfgang Drums manufactures drum sets in their 3 factories: Oklahoma City, Boulder, and Asheville. Each factory produces 2,000 sets per year. They are then shipped to 20 major cities. (Total demand is 6,000 sets per year.) You are tasked with fulfilling each city's demand while minimizing transportation costs. Suppose that 30 drum sets can fit on one truck, and that the rate is \$0.50 per mile.

Sets:

- S is the set of ship-from's (factory locations)
- T is the set of ship-to's (customer cities)

Indices:

- i is a ship-from (factory location)
- j is a ship-to (customer city)

Parameters:

- s_i is the supply at ship-from $i \in S$
- t_j is the demand at ship-to $j \in T$
- c is the cost per mile
- n is the number of drum sets that can fit on one truck
- d_{ij} is the distance (in miles) from ship-from $i \in S$ to ship-to $j \in T$.

Variables:

$$x_{ij} = \text{the number of drum sets shipped from } i \in S \text{ to } j \in T.$$

Here is the integer program:

$$\min \quad \sum_{i \in S} \sum_{j \in T} \left(\frac{c}{n}\right) d_{ij} x_{ij} \tag{3a}$$

$$\text{s.t.} \quad \sum_{j \in T} x_{ij} = s_i \qquad \forall i \in S \tag{3b}$$

$$\sum_{i \in S} x_{ij} = t_j \qquad \forall j \in T \tag{3c}$$

$$x_{ij} \geq 0 \ \text{ integer} \qquad \forall i \in S,\ j \in T. \tag{3d}$$

The objective (3a) is to minimize the transportation cost. Constraint (3b) ensures that each ship-from (factory) $i \in S$ sends out $s_i = 2,000$ drum sets. Constraint (3c) ensures that each ship-to (customer city) $j \in T$ receives t_j drum sets. Constraints (3d) ensure that the number of drum sets sent from ship-from $i \in S$ to ship-to $j \in T$ is nonnegative and integer.