# 1-Page Course Summary

What good is IP? (Hint: $$$)

What is an IP?

How do we formulate IPs?

How can we solve IPs (to optimality) in practice?

1. Specialized polynomial-time algorithms for special classes of IPs (e.g., max flow and matching)
2. Branch-and-bound
3. Cutting plane methods
4. Branch-and-cut (the reigning champ used by Gurobi, CPLEX, Xpress, …)
5. Others: branch-and-price, Lagrangian relaxation, Lenstra.

We can solve IPs more quickly through "better" branching.

We can solve IPs more quickly through "better" formulations.

1. How do we choose between competing formulations?
    a. Size matters
    b. Strength matters
2. We can perform *preprocessing* procedures to improve a given formulation:
    a. Remove redundant inequalities
    b. Tighten existing inequalities
    c. Fix certain variables
3. We can add *additional inequalities* to our formulation in an attempt to tighten or strengthen it:
    a. How can we generate valid inequalities?
        i. Ad-hoc arguments
        ii. Chvatal-Gomory procedure
        iii. Based on the IP's structure, e.g., GMI and MIR inequalities
    b. How can we tell how strong/useful a valid inequality is?
        i. Does it cut off your optimal LP relaxation solution?
        ii. Is it irredundant?
        iii. Does it dominate an inequality that is currently in the formulation?
        iv. Is it "as tight as possible", i.e., is it *facet-defining*?
    c. How do we show that an inequality is facet-defining?
        i. Directly using the definition
        ii. Through lifting arguments
4. We can add *additional variables* to our formulation (with appropriate new constraints)
    a. in an attempt to reduce the number of constraints acting on the original variables
    b. in an attempt to strengthen the formulation
    c. This is called an extended formulation.
    d. How can we construct extended formulations?
    e. How few variables and constraints do we need to get a "perfect" formulation? (Extension Complexity)

Some formulations are "perfect" and adding valid inequalities will not help.

1. What does this mean? Why do I care?
2. How can I show a formulation is perfect?
    a. Using equivalent definitions of integral polyhedra
    b. Totally unimodular constraint matrix (and integral right-hand-side)
    c. Totally dual integral (TDI) system of linear inequalities