Comparing the strength of alternative formulations (more abstractly)

Recall the following formulation for the uncapacitated facility location problem:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} && && (1) \\
& x_{ij} \le y_i, && \forall i, j && (2) \\
& \sum_{i=1}^{n} x_{ij} = 1, && \forall j && (3) \\
& x_{ij} \ge 0, && \forall i, j && (4) \\
& 0 \le y_i \le 1, && \forall i && (5) \\
& y_i \text{ integer}, && \forall i. && (6)
\end{aligned}
$$

Here, y_i represents the decision to open facility i, and x_{ij} represents the proportion of customer j's demand that is fulfilled by facility i. For simplicity, we write $\forall i$ instead of the more tedious $\forall i \in \{1, \dots, n\}$. Similarly, $\forall j$ means for all $j \in \{1, \dots, m\}$.

An alternative formulation replaces the nm coupling inequalities (2) with only n constraints:

$$
\sum_{j=1}^{m} x_{ij} \le m y_i, \qquad \forall i. \tag{7}
$$

Note that this formulation is correct as well; both formulations model the same feasible region.

Which formulation is stronger? Is it strictly better?

To formalize our discussion, let P_1 denote the initial LP relaxation's feasible region, i.e., P_1 is the set of $(x, y) \in \mathbb{R}^{n \times m} \times \mathbb{R}^n$ that satisfies constraints (2), (3), (4), (5).

Similarly, let P_2 be the set of all $(x, y) \in \mathbb{R}^{n \times m} \times \mathbb{R}^n$ satisfying (7), (3), (4), (5).

First question: is $P_1 \subseteq P_2$? In other words, is P_1 at least as strong as P_2?

Perhaps the formulations are equally strong?

Second question: is $P_2 \subseteq P_1$?