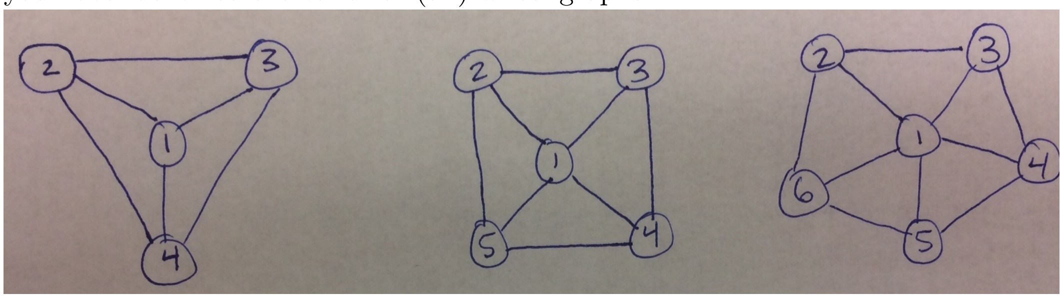
Homework 3 – Due 13 June 2022

1. Using the software PORTA [1], find (perfect) inequality descriptions for the stable set polytopes of the following "wheel" graphs. Submit your .poi and .ieq files to the dropbox. Do you see any (facet-defining) inequalities that we have not seen in class? Can you provide a general explanation for these inequalities? Ideally, give a (general) proof that the inequalities that you have identified are valid for (all) wheel graphs.



2. Solve the following pure integer program using Gomory fractional cuts.

$$\begin{aligned} \max\ & 2x_1 + x_2 \\ & -x_1 + x_2 \leq 0 \\ & 6x_1 + 2x_2 \leq 21 \\ & x_1, x_2 \geq 0 \text{ integer.} \end{aligned}$$

You can solve the LP relaxation however you want (e.g., graphically, or with Gurobi, or simplex by hand). However, I **do not** want to see your work for solving the LP. It is enough to report (for each iteration of the cutting plane method) an optimal LP solution and the Gomory fractional cut(s) that you add. If possible, always report the Gomory fractional cut in terms of the original variables (i.e., in terms of x_1 and x_2).

3. For each of the following sets, find a missing valid inequality, prove its validity, and verify graphically that its addition to the formulation gives $\text{conv}(X)$.

   (a) $X = \{(x, y) \in \mathbb{R}_+ \times \{0, 1\} \mid x \leq 20y,\ x \leq 7\}$.

   (b) $X = \{(x, y) \in \mathbb{R}_+ \times \mathbb{Z}_+ \mid x \leq 6y,\ x \leq 16\}$.

4. Recall the definition of linear independence:

   Definition 1. *Vectors $x^1, x^2, \ldots, x^k \in \mathbb{R}^n$ are* linearly independent *if $\sum_{i=1}^{k} \lambda_i x^i = \boldsymbol{0}$ implies $\lambda_i = 0$ for all $i \in [k]$. Here, each $\lambda_i \in \mathbb{R}$ is a scalar, and $\boldsymbol{0}$ is an n-dimensional vector of 0s.*

   Prove that the vectors $y^1, y^2, \ldots, y^n \in \mathbb{R}^n$ are linearly independent, where y^i is an n-dimesional vector of 1s but with a 0 in position i.

[1] `https://farkasdilemma.wordpress.com/2017/03/22/a-brief-tutorial-on-porta/`