Simple Example of Branch and Bound

One approach for solving IPs is *branch and bound* (B&B). All good IP solvers use branch-and-bound (in conjunction with other techniques, e.g., cutting planes, preprocessing, heuristics).

$$\text{maximize} \quad 12x_1 + 9x_2 + 7x_3 + 5x_4 + 3x_5$$

$$\text{subject to} \ \ 9x_1 + 9x_2 + 8x_3 + 6x_4 + 5x_5 \leq 14$$

$$x_i \in \{0,1\}, \quad i = 1, \dots, 5.$$

The first step in B&B is to *relax* the integrality constraints to get the LP relaxation:

Now solve the LP relaxation. What is its solution and objective value?

What does this tell you about the objective value of the IP?

One of our variables is *fractional*, but we only allow integer values. So, we *branch* on this variable, creating two subproblems:

What is the solution and objective value of the first subproblem? What does this tell us?

What about for the second subproblem? What do we know now?

Neither of these subproblems is solved (to IP optimality), and either could contain an optimal IP solution. So, we continue branching and bounding.

Draw the *branch-and-bound tree* here.