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Portfolios Risk Optimize Solution

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Summary

This report provides an analysis of the cryptocurrency market. The study method includes the optimal solution of minimizing the risk of investments and two-way-sensitivities between the parameters, such as the budget and return rate. Other calculations include the beta of the portfolios, covariance matrix, weighted of securities. All calculation formulas are listed in the Appendix.

The data analysis analyzed the minimum risk of investing in cryptocurrency is high, even with the lowest expected return possible. Besides, the decision variables suggest we should invest only two out of three cryptocurrencies.

The two-way sensitivity analysis tells us that the more significant the return rate, the higher the risk, regardless of its budget. We don't suggest investing a lot in the cryptocurrency market. However, we recommend that the short-term investment will be more predictable based on the model set up.

Lastly, we paper traded on a simulator based on our outcome in the optimization method.

We noticed the result in the simulator with the actual data has similar outcomes in the report.

The model is conducted under some limitations:

- 1. The current economic conditions are not perfect, such as pandemics. Such a situation is hard to quantify the risk.
- 2. We do not have enough capacity to analyze more stocks.
- 3. The problem limits only to one type of market (Cryptocurrency).

Introduction

In recent years the Stock Market has gained popularity with the younger generation because of free trading platforms. Since they have started trading, the market volatility has increased in individual stocks like Tesla, Roku, Apple, but the most volatile is cryptocurrency. Cryptocurrency is a digital or virtual currency designed to work as a medium of exchange and is decentralized (no government control). The primary purpose of virtual currency was for transaction purposes without having to abide by government rules or regulations. However, many individuals use it for investment purposes because of the possibility of substantial returns. Unfortunately, the crypto market bubble burst in 2017, teaching investors how much riskier and extremely volatile investing in a non-backed currency stock can be. In July 2020, the Office of the Comptroller of the Currency (OCC) clarified national banks and federal saving associations to have custody services on cryptocurrency to their customers. Due to the news, our team believes the crypto market will have potential development. Thus, we want to create a cryptocurrency portfolio analysis based on three prevalent cryptocurrencies; Ethereum (ETH), Bitcoin (BTC), Monero (XMR-USD).

Define the Problem

The cryptocurrencies are known as sensitive to its volatility. Each trading of cryptocurrency because of its high risk would cause unexpected market change, leading to a sudden price move. Therefore, it is essential to understand the risk before trading.

Our objective function is to minimize our portfolio risk while also satisfying all constraints.

Our decision variable is to determine how much each stock in dollar amount (we define this as X_i) should be invested in the context of optimization methods.

Our constraints are below:

- 1. We set our rate of return (r) to be at 10%. We will adjust this rate in our sensitivity analysis.
- 2. We set our budget amount to be \$10,000 ($\sum_{i=1}^{3} x_i$). We will also adjust this amount in our sensitivity analysis.
- 3. All stocks are traded in positive real numbers.

And our assumptions are as follows:

- 1. There are no transaction or commission fees because we are using free trading platforms (i.e., WeBull, Robinhood, Fidelity).
- 2. No short sell is allowed while constructing the portfolio.
- 3. We buy and sell the whole portfolio at a specific time.
- 4. We can trade a continuum number of shares (i.e., can buy and sell shares in real number)
 When we do this analysis, there are several problems that we encountered.
- 1. Some stocks have null values. Therefore, we find the missing value and replace it with the average of the previous and after closing price.
- 2. Stock market is unpredictable We use the CAPM model to determine our stock rate of return.

Data Cleaning

Using Yahoo finance, we were able to collect data on Bitcoin (Bit), Ethereum (ETH), and Monero (XMR), which will make up our portfolio. We also collect the data of the S & P 500 and Treasury Bill. To ensure we had enough data to compute the analysis, we collected the closing price from October 4, 2015, to October 4, 2020. After reviewing the data, we realized cryptocurrency stocks were open 24/7 while S&P and the Treasury Bill was open on non-holiday

weekdays. Removing the NAN values was doable by the VLOOKUP function, IF, and SELECT functions that returned the closing cryptocurrency prices based on the S&P dates.

Solving Model

To think about how investors can achieve high return rates with lower risk when possible and what that means for the pricing of assets, we will use the CAPM (Capital Assets Pricing Model) formula to calculate the expected returns. The CAPM formula describes the relationship between the expected return values and the risk of investing in a particular security. The formula is:

$$E(R_i) = RFR + \beta_i * (R_m - RFR)$$

Formula Description:

 $E(R_i)$ = Required rate of return, or expected return of i stock

RFR = Risk-free rate

 β_i = Beta coefficient for the i stock

 R_m = Return expected from the market

 $(R_m - RFR) = Market risk premium$

The current Risk-free rate (RFR) based on <u>treasury.gov</u> (Oct. 2nd, 2020) is 0.28% under five years of T - Bill conditions and the 19.61% market return rate for the S&P 500.

To get the Beta coefficient (risk) for every stock, we need to calculate the covariance of each stock to one another. Covariance measures the direction and magnitude of the relationship between two variables. To calculate the covariance multiply two stocks closing price minus their mean closing price and then divide by sample size minus 1.

$$\frac{(x_i - \underline{x}) * (y_i - \underline{y})}{n - 1} = \text{Covariance}$$

	Bitcoin		XMR	S&P 500	T-Bill Five Years
Bitcoin	16454184.02	637349.7488	228375.7368	1363712.605	-63.23912103
ETH	637349.7488	48692.7806	16034.24876	39731.6193	52.73182549
XMR	228375.7368	16034.24876	5947.910522	12710.57172	23.3361186
S&P 500	1363712.605	39731.6193	12710.57172	161009.7696	-17.47715395
T-Bill Five Years	-63.23912103	52.73182549	23.3361186	-17.47715395	0.55812972

Above is our covariance matrix with five stocks. The covariance between the cryptocurrency stocks is all large and lively. However, stocks S&P 500 and T-Bill have smaller covariance, and some are negatively correlated with the three stocks.

Beta:		Rate of Return:	
Bitcoin	8.469750676	Bitcoin	166.0708954
ETH	0.24676527	ETH	4.841176005
XMR	0.07894286	XMR	1.550648447

The β is a measure of a stock's volatility to its stock market. If one stock is riskier than the market, it will have a beta greater than one. If beta is less than one, the formula assumes it will reduce a portfolio's risk. From the table above, we can see that Bitcoin has a much higher beta value of 8.47 than ETH 0.25 and XMR 0.08, which implies that it is the riskiest out of the three stocks. Beta is calculated by the covariance of the stock and its market divided by the variance of the market.

Beta Formula:

 $B_i = cov(Rate of return of stock ii, Rate of return of market) / var(rate of return of market)$

The CAPM formula aims to evaluate whether a stock is fairly valued when its risk and the time value of money are compared to its expected return. For example, an investor is considering

investing in Bitcoin while having beta calculated. The Bitcoin has a beta of 8.4698 compared to the market, which means it is riskier than a market portfolio.

Based on $E(Ri) = RFR + \beta i * (Rm - RFR)$. The expected return of the Bitcoin in the CAPM formula is 166.07 = 0.0028 + 8.4698*(19.61-0.0028).

After knowing the expected return E(Ri) rate for each asset, we could determine how much one portfolio returns. The investors need to use each weighted security, multiply each the expected return rate, and then add up each production. The weighted average here is each investment of a portfolio divided by the total amount of investment, \$10,000. The formula of portfolio returns will be:

$$\sum_{i=1}^{3} W_i R_i$$

Where: W_i = Weighted of Security i

 R_i = Expected Return i

We set up three portfolios, as we mentioned above, Bitcoin (BTC), Eutherum (ETH), Monero (XMR-US). For the weighted of Bitcoin (BTC), we indicated as W₁ is the BTC decision variable divided by the total investment amount, the same as the other two. The weighted securities calculation is all in table 1, and it shows we got approximated 0.0513575, 0, and 0.9486424 for BTC, ETH, and XMR, respectively.

	Stocks	Weight	Rate of Return
X1	Bitcoin	0.05135752735	166.0708954
X2	ETH	0	4.841176005
X 3	XMR	0.9486424727	1.550648447
	Total:	1	

Table 1: The Weighted Securities

So the equation of the total expected returns will be:

$$Total\ Expected\ Return = W_1R_1 + W_2R_2 + W_3R_3$$

We obtained the total expected return of 10 % / year, which equates to our RHS rate of return constraint. The expected return of each stock is obtained by using the historical data. We suggest that investors who ever consider investing in the stock market should take the expected return value into account that the likelihood of one stock achieving the historical return would give the same pattern in the future. However, there is a limit to the expected return value. Today's stock market is still volatile and unpredictable, considering the expected return of security only is a risk. Follow up, we will base on the covariance matrix to quantify the risk associated with a specific portfolio.

Objective Function (Portfolio Risk)

Below is the formula to calculate our portfolio risk where X_i and X_j are the amounts invested in stock i and stock j, and σ_{ij} is the variance/covariance between stock i and stock j.

$$\min \qquad \sum_{i=1}^{3} \sum_{j=1}^{3} x_i x_j \sigma_{ij}$$

Variability in stocks is a risk because the higher the variability of stock, the more challenging for us to predict which direction the stock is heading. Also, variance or covariance measures how far a set of numbers is spread out from their average value. Therefore, we want to determine the ideal combination of stocks in our portfolio, such that we have the optimal minimum portfolio risk.

Optimal Result.	(Optimal minimum)		
Portfolio Risk:	7100504390932.90		
Individual Stoc	k Risk:		
	Bitcoin	ETH	XMR
Bitcoin	4339948418057.73	0.00	1112645045401.97
ETH	0.00	0.00	0.00
XMR	1112645045401.97	0.00	535265882071.24

In our excel spreadsheet above, we calculate each stock risk before calculating our portfolio risk to avoid any calculation errors.

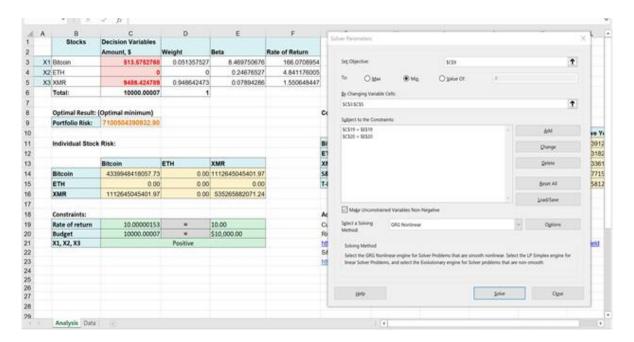
Analysis

Now that we have explained what each formula is for, we can start doing our analysis. We built our model as followed:

-4	Α	В	С	D	E	F	G	Н	1	J	К	L
1			Decision Variables									
2		Stocks	Amount, \$	Weight	Beta	Rate of Return						
3	X1	Bitcoin	513.5752768	0.05135752735	8.469750676	166.0708954						
4	X2	ETH	0	0	0.2467652702	4.841176005						
5	Х3	XMR	9486.424789	0.9486424727	0.07894286013	1.550648447						
6		Total:	10000.00007	1								
7												
8		Optimal Result:	(Optimal minimum)				Covariance Mat	rix:				
9		Portfolio Risk:	7100504390932.8									
10								Bitcoin	ETH	XMR	S&P 500	T-Bill Five Years
11		Individual Stoc	k Risk:				Bitcoin 16454184.02		637349.7488	228375.7368	1363712.605	-63.23912103
12							ETH	637349.7488	48692.7806	16034.24876	39731.6193	52.73182549
13			Bitcoin	ETH	XMR		XMR	228375.7368	16034.24876	5947.910522	12710.57172	23.3361186
14		Bitcoin	4339948418057.72	0.00	1112645045401.9		S&P 500	1363712.605	39731.6193	12710.57172	161009.7696	-17.47715395
15		ETH	0.00	0.00	0.00		T-Bill Five Years	-63.23912103	52.73182549	23.3361186	-17.47715395	0.5581297205
16		XMR	1112645045401.97	0.00	535265882071.24							
17												
18		Constraints:					Additional Infor	mation:				
19		Rate of return	10.00000153	=	10.00		Current (Oct 2):					
20		Budget	10000.00007	=	\$10,000.00		Risk Free Rate fi	rom: 0.0028				
21		X1, X2, X3		Positive			https://www.treas	sury.gov/resource	-center/data-char	t-center/interest-r	ates/pages/textvi	ew.aspx?data=yiel
22							S&P 500 Rate: 0	.1961				
23							https://ycharts.co	om/indicators/sp_	500_1_year_retu	<u>n</u>		
24												
25												

The numbers in red are our decision variables (amount of stocks to invest in dollar amounts), and the number in orange is our objective function (optimal minimum portfolio risk). Next to the decision variables, we have the weight, beta, and rate of return. The covariance matrix and the constraint tables are included in this spreadsheet too.

-Solver in Excel:



We set cell C9 as the minimum objective function, cell C3 to C5 as the variable cells, C19 = E19 and C20 = E20 as the two constraints (rate of return = 10% and budget = \$10,000.00) and made unconstrained variables non-negative. Finally, we used GRG nonlinear as our solving method. Below is the output.

20	A	В	С	D	E	
1 2		Stocks	Decision Variables Amount, \$	Weight	Beta	Rat
3	-X1	Bitcoin	513,5752768	0.051357527	8.469750676	
4	X2	ETH	0	0	0.24676527	
5	ХЗ	XMR	9486.424789	0.948642473	0.07894286	
6		Total:	10000.00007	1		
7				100		
8		Optimal Result:	(Optimal minimum)			
9		Portfolio Risk:	7100504390932.90			
10						
11		Individual Stoc	k Risk:			
12						
13			Bitcoin	ETH	XMR	
14		Bitcoin	4339948418057.73	0.00	1112645045401.97	
15		ETH	0.00	0.00	0.00	
16		XMR	1112645045401.97	0.00	535265882071.24	

We should invest \$513.58 in Bitcoin and \$9,486.42 in Monero to achieve an optimal minimum portfolio risk of 7,100,504,390,932.90. In other words, we are expecting a 10% return by accepting such a high amount of risk. Is this an ideal decision to invest in only cryptocurrency stocks? Probably not because of the following reasons:

1. The problem restricted us to construct a portfolio with only three types of cryptocurrency stocks (Bitcoin, Ethereum, and Monero). If we want to analyze more stocks, it would be very computation heavy. Imagine constructing a portfolio with ten stocks; we would need to build a 10x10 covariance matrix with 100 data.

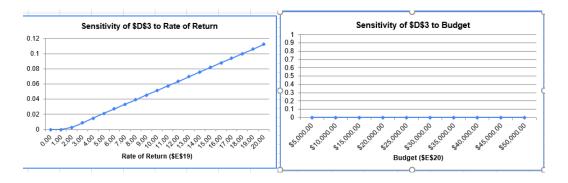
2. We focused only on cryptocurrency stocks. The diversification rules suggest that if we have negatively correlated stocks, we can lower our portfolio risk because the negative covariance will offset the positive covariance. All cryptocurrency stocks generally move in the same direction because they are in the same market. This explains why the covariance between these stocks is so high.

But what if we invest these stocks for the short term? The longer we invest in the stocks, the higher the variability in our portfolio. Therefore, we decided to run a simulation using Crytospaniards Simulator from cryptospaniards.com to consolidate our results. Before that, let's run a two-way sensitivity analysis of our results.

Two Way Sensitivity Analysis using Rate of Return and Budget:

-Bitcoin(BTC)

\$D\$3	\$5,000.0	0 \$10,000.00	\$15,000.00	\$20,000.00	\$25,000.00	\$30,000.00	\$35,000.00	\$40,000.00	\$45,000.00	\$50,000.00
0.00	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible 1	Not feasible	Not feasible
1.00	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible 1	Vot feasible	Not feasible
2.00	0.002	7 0.002	7 0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
3.00	0.008	8 0.008	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088	0.0088
4.00						0.0149		0.0149	0.0149	0.0149
5.00		_				0.0210		0.0210	0.0210	0.0210
6.00						0.0270		0.0270	0.0270	0.0270
7.00						0.0331		0.0331	0.0331	0.0331
8.00						0.0392		0.0392	0.0392	0.0392
9.00					_	0.0453		0.0453	0.0453	0.0453
10.00						0.0514		0.0514	0.0514	0.0514
11.00		_				0.0574		0.0574	0.0574	0.0574
12.00						0.0635		0.0635	0.0635	0.0635
13.00						0.0696		0.0696	0.0696	0.0696
14.00						0.0757		0.0757	0.0757	0.0757
15.00						0.0817		0.0817	0.0817	0.0817
16.00						0.0878		0.0878	0.0878	0.0878
17.00						0.0939		0.0939	0.0939	0.0939
18.00						0.1000		0.1000	0.1000	0.1000
19.00						0.1061		0.1061	0.1061	0.1061
20.00	0.112	1 0.112°	0.1121	0.1121	0.1121	0.1121	0.1121	0.1121	0.1121	0.1121



Using the two-way analysis with Rate of Return and Budget on Bitcoin, we can see that the Budget does not affect the proportion of the money we should invest in Bitcoin. It appears that no matter the budget at a rate of return under 2%, it is infeasible since we don't meet all constraints. One possibility is that our lowest stock rate of return is 1.55% from Monero (XMR). We cannot get a portfolio rate of return any lower than 1.55%. Similarly, we cannot create a portfolio rate of return with anything above 166% because our highest stock rate of return (Bitcoin) is 166%. Any rate of return between 1.55% and 166% can be achieved by using a different combination of stocks in our portfolio.

As our portfolio rate of return increases from 2% to 20% so does the weight proportion of Bitcoin from basically 0 to 11.21% of our portfolio. The increase in the weight of Bitcoin will result in a decrease in the weight of other stocks. We will see the reduction in the weight of different stocks in a later discussion.

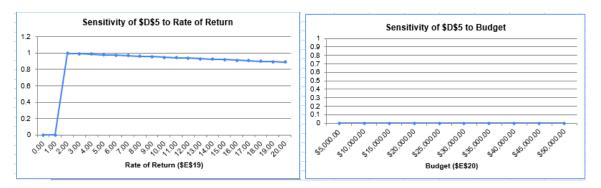
-Ethereum(ETH)

\$D\$4	\$5,000.00	\$10,000.0	\$15,000.00	\$20,000.00	\$25,000.00	\$30,000.00	\$35,000.00	\$40,000.00	\$45,000.00	\$50,000.00
0.00	Not feasible									
1.00	Not feasible									
2.00			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.00			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7.00	0.00				0.00	0.00	0.00	0.00	0.00	0.00
8.00					0.00	0.00		0.00	0.00	0.00
9.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10.00			0.00		0.00	0.00	0.00	0.00	0.00	0.00
11.00			0.00	0.00	0.00			0.00	0.00	0.00
12.00			0.00		0.00	0.00		0.00	0.00	0.00
13.00				0.00	0.00	0.00		0.00	0.00	0.00
14.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15.00			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16.00							0.00	0.00	0.00	0.00
17.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

For Ethereum Budget nor Rate of Return has any effect on the weight it has on our portfolio. This means we would not invest in this stock while our rate of return is between 2%-20% and our budget is \$5,000-\$50,000.

-Monero(XMR)

\$D\$5		\$5,000.00	\$1	0,000.00	\$15,	000.00	\$20,0	00.00	\$25,000.00		\$30,000.00		\$35,000.00		\$40,000.00	\$	45,000.00	\$5	00.000,00
0.00	Not feasible	,	Not feasible	,	Not feasible	,	Not feasible	•	Not feasible	Not feasible	,	Not feasible	•	Not feasible	,	Not feasible	•	Not feasible	
1.00	Not feasible		Not feasible	•	Not feasible		Not feasible	- 1	Not feasible	Not feasible	•	Not feasible		Not feasible	*	Not feasible		Not feasible	•
2.00		0.9973		0.9973		0.9973	0.	.9973	0.9973	1	0.9973		0.9973		0.9973		0.9973		0.9973
3.00		0.9912		0.9912		0.9912	0	.9912	0.9912	1	0.9912		0.9912		0.9912		0.9912		0.9912
4.00		0.9851		0.9851		0.985		.9851	0.9851	1	0.9851		0.9851		0.9851		0.9851		0.9851
5.00		0.9790		0.9790		0.9790	0.	.9790	0.9790	1	0.9790		0.9790		0.9790		0.9790		0.9790
6.00		0.9730		0.9730		0.9730		.9730	0.9730	1	0.9730		0.9730		0.9730		0.9730		0.9730
7.00		0.9669		0.9669		D.9669	0.	.9669	0.9669	1	0.9669		0.9669		0.9669		0.9669		0.9669
8.00		0.9608		0.9608		0.9608	0.	.9608	0.9608	1	0.9608		0.9608		0.9608		0.9608		0.9608
9.00		0.9547		0.9547		0.9547		.9547		1	0.9547		0.9547		0.9547		0.9547		0.9547
10.00		0.9486		0.9486		D.9486		9486		1	0.9486		0.9486		0.9486		0.9486		0.9486
11.00		0.9426		0.9426		0.9426		9426	0.9426	1	0.9426		0.9426		0.9426		0.9426		0.9426
12.00		0.9365		0.9365		0.9365		.9365	0.9365		0.9365		0.9365		0.9365		0.9365		0.9365
13.00		0.9304		0.9304		0.9304		.9304			0.9304		0.9304		0.9304		0.9304		0.9304
14.00		0.9243		0.9243		D.9243		.9243			0.9243		0.9243		0.9243		0.9243		0.9243
15.00		0.9183		0.9183		0.9183		.9183			0.9183		0.9183		0.9183		0.9183		0.9183
16.00		0.9122		0.9122		0.9122		.9122			0.9122		0.9122		0.9122		0.9122		0.9122
17.00		0.906		0.9061		0.906		.906			0.9061		0.906		0.906		0.906		0.9061
18.00		0.9000		0.9000		0.9000		.9000			0.9000		0.9000		0.9000		0.9000		0.9000
19.00		0.8939		0.8939		0.8939		.8939	0.8939		0.8939		0.8939		0.8939		0.8939		0.8939
20.00		0.8879		0.8879		0.8879 <mark>*</mark>	0.	.8879	0.8879	1	0.8879		0.8879		0.8879		0.8879		0.8879



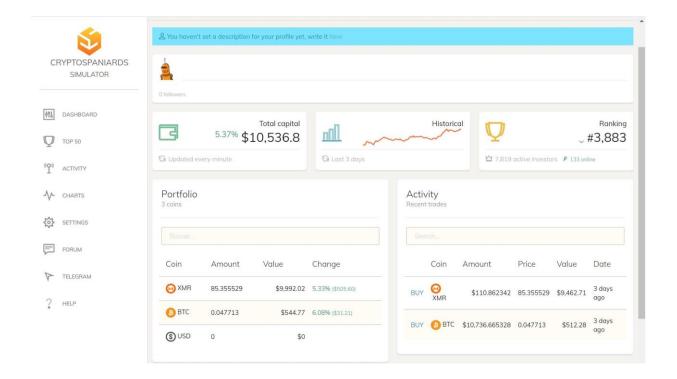
Just like the other decision variables, the Budget has no effect on the weight of this stock in our portfolio. For this, the stock rate of return has a negative effect on weight. As the rate of return increases from 2%-20%, the size of Monero in our portfolio decreases. This would mean the weight of another stock (Bitcoin as we have discussed earlier) would be increasing at the same time.

-Portfolio Risk

\$C\$9	\$5,000.00	\$10,000.00	\$15,000.00	\$20,000.00	\$25,000.00	\$30,000.00	\$35,000.00	\$40,000.00	\$45,000.00	\$50,000.00
0.00	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible
1.00	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible			Not feasible	Not feasible	Not feasible
2.00	182058022265.4	728232157487.84			4551450984299.04	6554089417390.62			14746701189128.90	18205803937196.20
3.00	277722258189.22	1110889149425.86	2499500613253.97	4443554833430.72	6943057225786.73	9998002548448.28	13608392023817.10	17774219336374.30	22495505145235.70	27772228945289.40
4.00	402949150380.23	1611796650665.00		6447186612436.24	10073729055750.70	14506169775305.00	19744502142028.30	25788746091725.40	32638870922795.60	
5.00	557738606515.9	2230954661205.27	5019647977585.90	8923818670146.56	13943466347689.40	20078591734725.90	27329194587652.20	35695274750900.50	45176832094536.20	55773865015669.50
6.00	742090770647.8	2968363181046.67	6678817204994.94	11873452740295.30	18552269676196.30	26715268997022.20	36362448434585.20	47493811017650.20	60109339138246.40	74209078425597.20
7.00	956005596207.97	3824022210189.20			23900133213386.50	34416200180519.00	46844261129206.80		77436449669629.70	
8.00	1199482896120.85	4797931748632.86	10795346499050.40	19191726944513.90	29987073037451.80	43181386245201.20	58774651691929.20	76766907711788.30	97158115583779.00	119948291618232.00
9.00	1472523156591.79	5890091796377.65	13252706608876.90		36813072942688.30	53010816395187.70	72153624294963.30		119274357085579.00	
10.00	1775125499976.97	7100502665369.80		28402009996017.30	44378140785164.40	63904522886211.60	86981156299159.00	113608041024007.00		177512564409402.00
11.00	2107290855062.51	8429164807557.01	18965617826995.40	33716654717299.00	52682273496055.60	75862472166519.30	103257252233051.00	134866621429512.00	170690564688460.00	210729087945234.00
12.00	2469018748527.46	9876076501455.70	22221169398902.70	39504302079403.50	61725472276352.10	88884681588011.00	120981917019015.00	158017211902645.00	199990535536949.00	246901889031098.00
13.00	2860309270349.89	11441238704655.50	25742783673559.80	45764948820554.50	71507733596689.20	102971135512188.00	140155162107318.00	183059795871553.00	231685064600371.00	286030925332372.00
14.00	3281162418924.74	13124651417156.50	29530462145704.80	52498600711500.60	82029061853211.60	118121849550956.00	160776960646724.00	209994405665013.00	265774156147003.00	328116247275891.00
15.00	3731578194804.89	14926314638958.50	33584204987668.30	59705252297362.10	93289458200157.60	134336821532782.00	182847340061376.00	238821029129775.00	302257845119287.00	373157837132939.00
16.00	4211556596048.56	16846226391022.80	37904009823615.60	67384907623492.00	105288918246357.00	151616040837500.00	206366279883070.00	269539633445256.00	341136104294743.00	421155675347646.00
17.00	4721097628605.90	18884390513176.70	42489878847028.40		118027446723612.00	169959514618589.00	231333791238585.00	302150241158899.00	382408910193729.00	472109794655933.00
18.00	5260201286688.98	21040805144631.60	47341812768838.10	84163224328097.60	131505041551747.00	189367254162885.00	257749874334502.00	336652907681563.00	426076340294252.00	526020181881246.00
19.00	5828867570442.47	23315470285387.70	52459808263754.20	93261882829076.00	145721688708219.00	209839233089125.00	285614520352380.00	373047533094806.00		582886750811827.00
20.00	6427096484424.46	25708385935444.90	57843868400078.90	102833542887423.00	160677412012451.00	231375474678702.00	314927726919139.00	411334170211297.00	520594834324770.00	642709648483120.00

There are no feasible solutions when the return rate is under 2% from the two-way sensitivity report. Nevertheless, Portfolio risk is significantly high when the return rate gets higher. Same as budget, the risk gets notably high when the investment amount goes up. Also, with the same amount of money, the greater the return rate, the more remarkable the risk.

Simulation on Cryptospaniards Simulator:



As seen above, we managed to invest these stocks for only three days because of our project's time constraint. We gained 5.37% with a \$537.80 return on our portfolio in just three days. It would have been higher if we held the position longer. The Bitcoin gained 6.08%, whereas the Monero gained 5.33% in stock price.

However, there might be other factors that we are not aware of that would affect the stock price. For example, we traded these stocks during the presidential debate and a pandemic. The stock price fluctuates unpredictably, and it can go up or down significantly in the next day. Our model is not complex enough to address these factors, so we can only use the historical stock price to predict the outcome.

Conclusion

After running the GRG nonlinear analysis, it is clear that the minimum risk is significantly high (over 7 trillion). Our Decision Variables are the dollar amount that we would invest in that particular stock. We would invest \$513.58 in Bitcoin, \$0 in Ethereum, and \$9,486.42 in Monero. The diversification is based on investing a positive amount, portfolio rate of return of 10%, and a total portfolio budget of \$10,000.00.

In our sensitivity analysis, we found out that an increase or decrease in the budget does not impact any of our three stocks. Also, Ethereum stock is insensitive to both the budget amount and portfolio rate of return.

Our suggestion is to diversify our portfolio with other stocks from markets that move in the opposite way, which would significantly reduce risk. If we are more risk-averse, we can also invest our stocks in the short term.

Bibliography

- a) Shabbir Ahmed (Fall 2002). The Optimization Process: An example of portfolio optimization from ISyE 6669 Deterministic Optimization
- b) Portfolio Return Formula: *Available online*. Retrieved Oct 10, 2020 from https://thismatter.com/money/investments/portfolios.htm
- c) Cryptocurrency Market Index change: Available online. Retrieved Oct 10,
 2020 from https://coinmarketcap.com
- d) Cryptocurrency: Available online. Retrieved Oct 10, 2020 from https://cointelegraph.com/bitcoin-for-beginners/what-are-cryptocurrencies
- e) Simulation: Available online. Retrieved Oct 10, 2020 from https://cryptospaniards.com
- f) Daily Treasury Yield Curve Rates: Available online. Retrieved Oct 10, 2020 from https://www.treasury.gov/resource-center/data-chart-center/interest-rates/pages/textview.aspx?data=yield
- g) S&P 500 1 Year Return: *Available online*. Retrieved Oct 10, 2020 from https://ycharts.com/indicators/sp_500_1_year_return

Appendix: Formulas

Inputs:

i - represents cryptocurrencies, i = 1, 2, 3

 R_i – represents expected returns, i = 1, 2, 3

RFR – represents Risk-Free rate

 β_i - represents Beta coefficient for i stock, i = 1, 2, 3

Wi – represents weighted of security i, i = 1, 2, 3

 σ_{ij} – represents covariance of return of stock i with stock j, i = 1, 2, 3, j = 1, 2, 3

Decision Variables:

 X_i = amount of investment for each portfolio i (i = 1, 2, 3)

Objective to minimize risks:

Min

$$\sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j \sigma_{ij}$$

Constraints:

Dollar invest in stock i:

$$\sum_{i=1}^{3} X_i \le 10000$$

Non-negative real values:

$$X_i >= 0, i = 1, 2, 3$$

Return of rate:

$$E(Ri) = RFR + \beta_i * (R_m - RFR)$$

Beta Formula:

 $\beta_i = cov(Rate of return of stock ii, Rate of return of market) / var(rate of return of market)$