**Ryerson University**
**Department of Electrical, Computer, and Biomedical Engineering**
**ELE709 - Real-Time Computer Control Systems**

**Project Answer Sheet 2**

**Name: Austin Luu**

---

2. Consider the graph for Task 2.3 (Basic PID Controller). Is the response of the system satisfactory? If not, explain why.

   Its not satisfactory as the PID controller does not fit the square function, this can be observed with the much over and undershooting done by the controller. A way to improve this is by implement an Anti-Windup controller as Anti-Windup PID controllers implement feedback loops to discharge the internal integrator when reaching saturation limits. This helps to prevent the overshooting/windup of the integral term and thus fit our square wave function more accurately.

3. Derive the difference equation to calculate the control $u_k := u(kT)$ required to implement the Basic PID control in Task 2.3

4. Derive the difference equation to calculate the control $u_k := u(kT)$ required to implement the Anti-Windup PID control in Task 2.4.

3. $u(t) = P(t) + I(t) + D(t)$

$\Rightarrow u(kt) = P(kt) + I(kt) + D(kt)$

$P(t) = K_p e(t) \Rightarrow \boxed{P(kt) = K_p e(kt)}$ ①

$\boxed{I(kT) = I((k-1)T) + \frac{k_p}{T_i} e((k-1)T)T}$ ②

$I(t) = \frac{k_p}{T_i}\int_0^t e(t)\,d\tau$

$\Rightarrow I(kt) = \frac{k_p}{T_i}\int_0^{kt} e(\tau)\,d\tau$

$= \frac{k_p}{T_i}\int_0^{(k-1)T} e(\tau) + \frac{k_p}{T_i}\int_{(k-1)T}^{kT} e(\tau)\,d\tau$

$D(t) + \frac{T_d}{N}\frac{dD}{dt} = k_p T_d \frac{de}{dt}$

$\Rightarrow \boxed{D(kT) + \frac{T_d}{N}\frac{D(kt) - D((k-1)T)}{T} = k_p T_d \frac{e(kT) - e((k-1)T)}{T}}$ ③

$\Rightarrow u(kT) = P(kT) + I(kT) + D(kT)$ ① ② ③

---

4. $u(kt) = P(kT) + I(kT) + D(kT)$

Stays the same

Changes for windup

$I(t) = \int_0^t \frac{k_p}{T_i} e(\tau)\,d\tau + \int_0^t \frac{1}{T_t} a(\tau)\,d\tau$

Where $a(t) = u(t) - v(t)$

$\Rightarrow I(kT) = \frac{k_p}{T_i}\int_0^{(k-1)} e(\tau)\,d\tau + \frac{k_p}{T_i}\int_{(k-1)T}^{kT} e(\tau)\,d\tau + \int_0^{(k-1)} a(\tau)\,d\tau + \int_{(k-1)T}^{kT} a(\tau)\,d\tau$

$\Rightarrow \boxed{I(kT) = I((k+1)T) + \frac{k_p}{T_i} e((k-1)T)T + \frac{1}{T_t} a((k-1)T)T}$

$\boxed{P(kT) = k_p e(kT)}$

$\boxed{D(kT) = \frac{T_d}{NT + T_d} D((k-1)T) + \frac{k_p T_d N}{NT + T_d}(e(kT) - e((k-1)T)T)}$

$\Rightarrow u(kT) = P(kT) + I(kT) + D(kT)$