

FINAL EXAMINATION

Due: 16 December 2021

Time: 23:59

Please read the following instruction carefully.

1. There are 9 pages (including this cover page) and 5 questions in this exam.
2. Write answers in the provided space, you may add additional page(s) to this PDF file if your work requires more space.
3. This exam is open book, open notes and open internet. You are however not allowed to get help from other human being on exam via any means (Ex: stackoverflow, discord, chegg, Line, etc.). (Ex: Talking to a cat for moral support is allowed.)
4. You must turn in the exam paper before the deadline on specified on canvas. Late submission will not be accepted.
5. You must not communicate with any other person by any means. Students found cheating during the examination will be penalized according to the university regulation.

Good luck!

FOR INSTRUCTOR USED

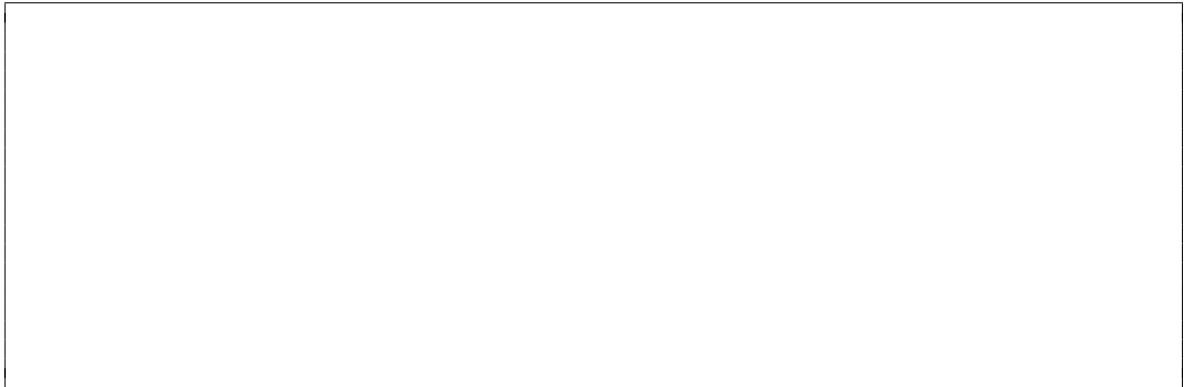
Distribution of Marks

| | | | | | | |
|-----------|-----|-----|-----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points: | 100 | 100 | 100 | 90 | 60 | 405 |
| Score: | | | | | | |

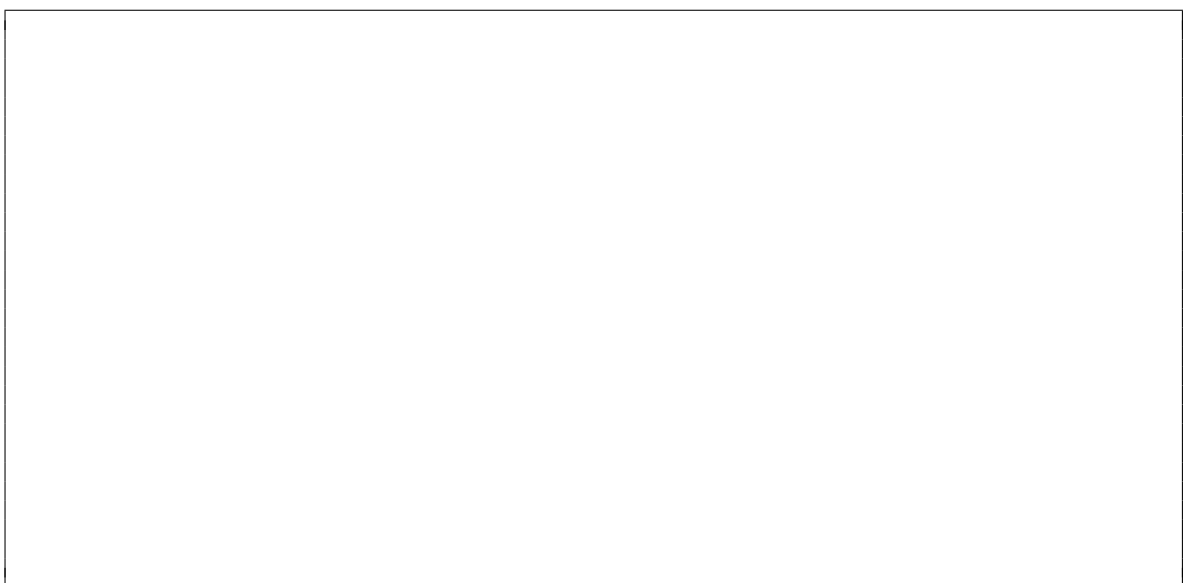
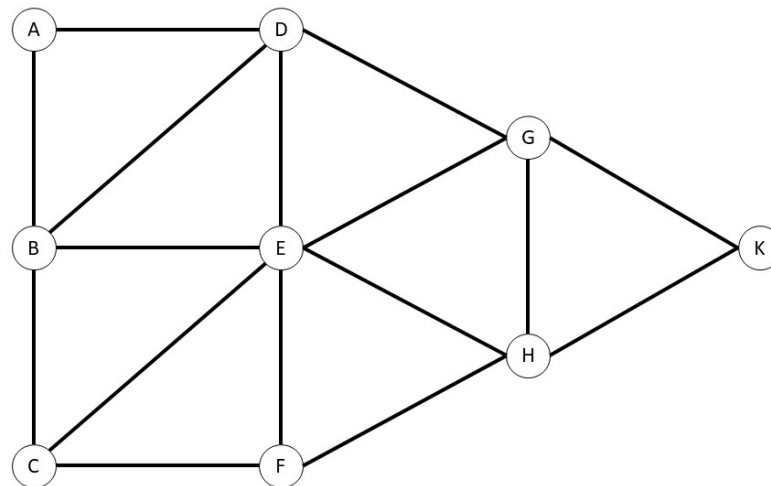
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1. (100 points) Easy problem.

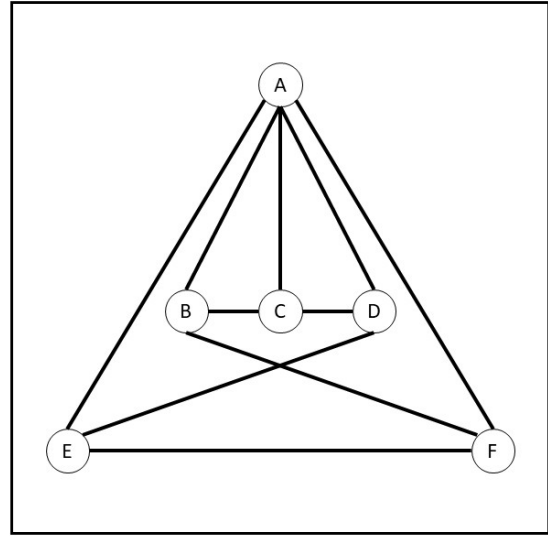
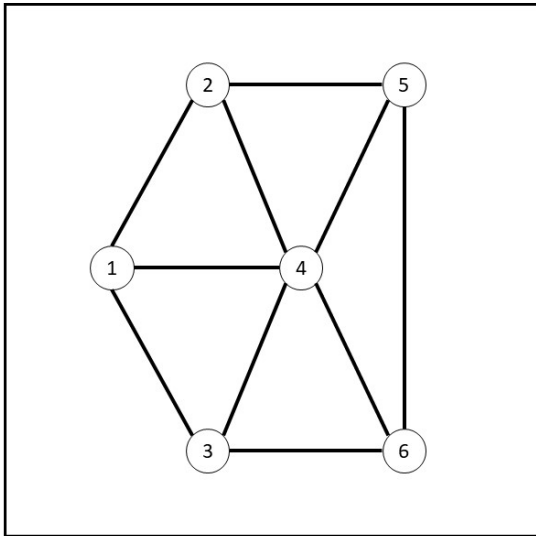
(a) (20 points) Draw K_6 which is a graph with 6 vertices and each vertex has degree 5.



(b) (20 points) Consider the following graph
Is it possible to walk on each path exactly once?
Write the path if it is possible.

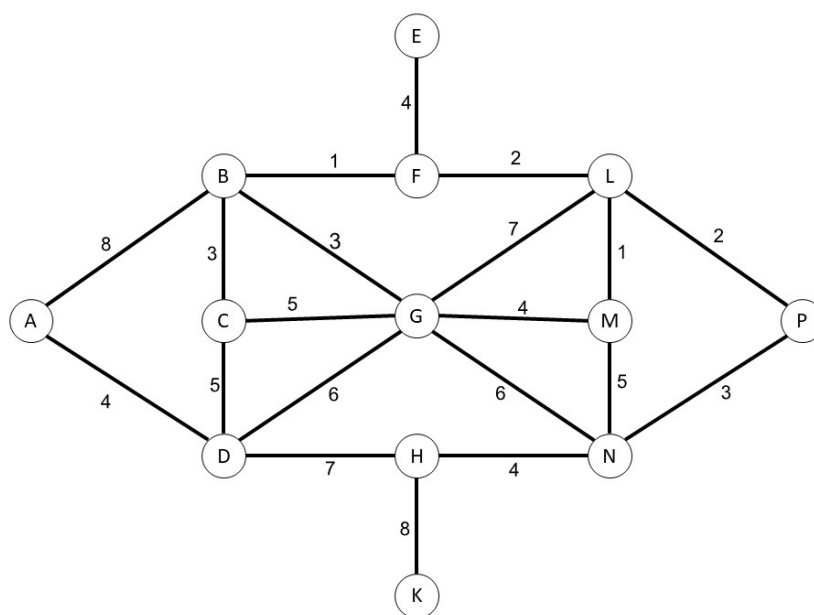


(c) (20 points) Show that the two graphs are isomorphic.

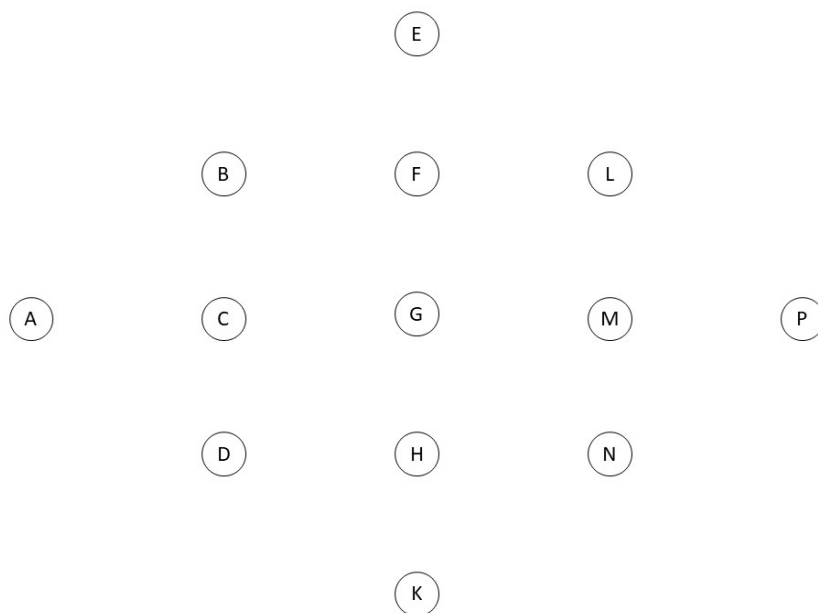


(d) (20 points) Aj. Wasakorn collects PS4 games. Currently, he has 35 distinct PS4 game titles. He wants to arrange his games on a 3-layer shelf. How many ways can the games be placed on the shelf if each shelf layer is considered distinct and some shelf may be empty? (You don't need to get numerical value)

(e) (20 points) Consider the following weighted graph. Find the minimum spanning tree.



Total weight = _____



2. (100 points) For the next semester, MUIC offers 10 courses (4-credit) for second-year students. Aj. Wasakorn is in charge of setting the course timetable. Each course must be taught in 2 sessions of a two-hour period and the classes must not be allocated during the weekend.

| | 8-10 | 10-12 | 12-14 | 14-16 | 16-18 |
|----|------|-------|-------|-------|-------|
| M | | | | | |
| T | | | | | |
| W | | | | | |
| Th | | | | | |
| F | | | | | |

- (a) (30 points) Suppose that the requirement is that each time slot can be filled by only one course so that every second-year student can choose any courses without time conflicts. What is the number of ways to schedule 10 courses?

- (b) (30 points) Aj. Piti told Aj. Wasakorn that courses must be allocated in pair blocks, such that
- if the first session of a subject is scheduled on Monday, then the second session must also be scheduled at the same time on Wednesday,
 - if the first session of a subject is scheduled on Tuesday, then the second session must also be scheduled at the same time on Thursday,
 - if the first session of a subject is scheduled on Friday, then the second session must be scheduled on Friday.

What is the number of ways to allocate these 10 courses? (The rule from the previous part where each timeslot can hold exactly one class still holds.)

- (c) (40 points) Aj. Piti conducted a survey for student intention to register the 10 courses and the result is shown below.

Discrete Math = {Tassapon, Chayapon, Eric, Pavanpreet, Harsh, Karan}
 Database = {Tassapon, Priya, Nawat}
 Calculus = {Priya, Nawat, Karan}
 Data Structure = {Harsh, Karan, Pongpat}
 Linear Algebra = {Saran, Witsanu, Zhiyu}
 Network = {Tassapon, Chayapon, Eric}
 Probability = {Witsanu, Zhiyu, Karan}
 OS = {Witsanu, Zhiyu, Puriwat}
 Statistics = {Suphawit, Nopbovone, Robert}
 Data Mining = {Tassapon, Priya, Eric, Saran, Harsh, Karan}

Now, the course timetable can be done in a much efficient way because two courses that are not enrolled by the same students can be assigned to the same time slot.

Find the minimum number of two-hour periods for the course timetable design.

(Don't forget to multiply by 2 as each course requires 2 periods.)

For example, the following timetable uses 4 periods: Monday 8-10, Monday 12-14, Wednesday 8-10, and Wednesday 12-14.

| | 8-10 | 10-12 | 12-14 | 14-16 | 16-18 |
|----|-----------------------|-------|----------|-------|-------|
| M | OS, Calculus, Network | | Database | | |
| T | | | | | |
| W | OS, Calculus, Network | | Database | | |
| Th | | | | | |
| F | | | | | |

3. (100 points) This is not a medical advice. Consider the following data. Out of 1000 COVID-19 death around mid July 2021(refer to hereafter as Die from Covid) the Ministry of Public Health found that

- 100 dead people were vaccinated.
- 900 dead people did not received any vaccination.

Some may or may not be useful data:

- At that time, only 25% of the population were vaccinated.
- There are 10^7 people in Thailand.

- (a) (20 points) Find $P(\text{Die from Covid})$ and $P(\text{Vaccinated})$

- (b) (20 points) Find $P(\text{Vaccinated}|\text{Die from Covid})$

- (c) (20 points) Find $P(\text{Die from Covid}|\text{Not Vaccinated})$

- (d) (20 points) Are the event “Die from Covid” and “Vaccinated” independent? Justify your answer with calculation.

- (e) (20 points) Dr.K¹ looks at the data and claim that vaccination can reduce the death rate by 9 times(900/100). He is wrong. Find the more correct number. (Yep, you need to figure out what to calculate.)

¹Really good manga reference

4. (90 points) Consider two gamblers in a Casino. If you win you win 1 Baht and if you lose you lose 1 Baht. They both start out with 20 Baht in their pocket.

- Mr. A bet 20 Baht on 1 fair coin toss.
- Mr. B bet 1 Baht each on 20 fair coin tosses. Each winning/losing are independent of each other.

After both finish the game, they walk out from the casino.

(a) (15 points) What is the probability that Mr. A will lose all 20 Baht?

(b) (15 points) What is the probability that Mr. B will lose all 20 Baht?

(c) (15 points) Find the expected profit of Mr. A.

(d) (15 points) Find the expected profit of Mr. B.

(e) (15 points) Find the standard deviation of Mr. A profit.

(f) (15 points) Find the standard deviation Mr. B profit.

5. (60 points) Consider a vial of virus. There are 500 viruses in the test vial. If we incubate it for 1 hour, each **pair** of virus has a 1 in 10000 chance of merging together and become a **super virus**(yet can't merge anymore.).

(a) (60 points) What is the expected number of **super virus** after 1 hour of incubation?