Homework 3

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Problem 1 We discussed in class about breaking a bar of chocolate. Now consider the following game of you and your friend:

1) Your friend get the pick the size of the chocolate.(Ex: 5×2 or 11×11)
2) After your friend choose the chocolate size, you get to pick whether you or your friend get to do the first break.
3) Each player then take turns to break the chocolate bar until all the pieces are 1×1 block.
4) Whoever break the last piece wins the game.

Find the winning strategy. Explain why it is the winning strategy.

Bonus: Play this with your friends not in this class. Math should not be used for evil purposes.

Problem 2 Consider n-piece jigsaw puzzle. You solve it by, first, trying to find two pieces that fit together. Then, for the subsequent step, you piece together tow blocks each made of one or more jigsaw piece that have been assembled in previous steps. Prove that the number of steps required to put all the n pieces together is given by $n-1$.

Problem 3 Suppose Brew&Bev issues gift certificate of two types: 7 Baht and 4 Baht. Assume that you have infinitely many of the two types. Prove that we can by, with *exact* change, everything Brew&Bev sells with price $p \geq 18$. Ex: We can use 2 7Baht and 1 4Baht coupon for an 18Baht croissant.

Hint: How do you pay for 22 and 25 Baht?

Problem 4 Show that every positive integer is a product of a *power of two* and an *odd integer*. (Previous Midterm).

Problem 5 Let the sequence T_n be defined by $T_1 = T_2 = T_3 = 1$ and

$$T_n = T_{n-1} + T_{n-2} + T_{n-3} \text{ for } n \geq 4.$$

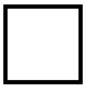
Show that

$$T_n < 2^n$$

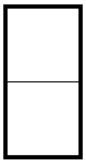
Problem 6 Show that every integer $n \geq 1$ can be written as a sum of *distinct* power of 2. For example,

$$1 = 2^0,\ 21 = 1 + 2^2 + 2^4,\ 100 = 2^2 + 2^5 + 2^6$$

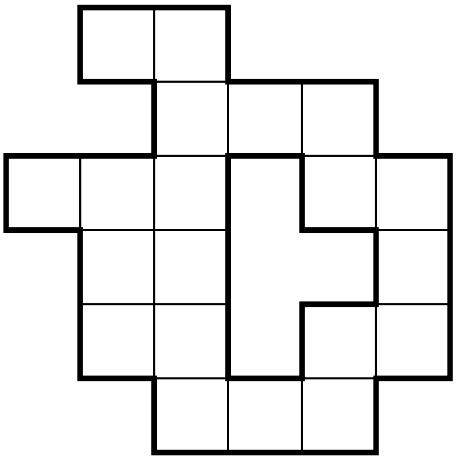
Problem 7 Consider an infinite grid. First we put a unit square on the grid. We will find that the length of the periphery is 4 which is even number. (count the length of thick line).



We then place another unit grid such that at least the new square share at least one grid with the already placed square. We will find again that the length of the periphery is 6. Another even number. What a coincidence!



Then, we keep placing one grid after another such that the next grid share at least one edge with the previous blob. But, no matter how we place it, we always find that the length of periphery is even.



What an amazing discovery! Your job is to prove it.

Problem 8 Is the following proposition true? If not, what is wrong with the proof?

Theorem: If we have n straight line in a 2 dimensional plane($n \geq 2$), and none of the line are parallel to each other, then all lines must intersect at exactly one and the same point.

Proof: by induction-ish?.

Inductive Predicate:$P(n)$ = every set of n straight lines intersect at exactly one point.

Base Case: $n = 2$. Every two lines intersect at one point. Therefore, $P(2)$ is true.

Inductive Step:

Assume that every n straight lines intersect at one point, we want to show that every $n + 1$ line intersect at one point.

- Let the set of $n + 1$ be $\{a_1, a_2, a_3, \ldots, a_n, a_{n+1}\}$
- From the inductive hypothesis $\{a_1, a_2, a_3, \ldots, a_n\}$ must intersect at one point since it is a set of n lines. Let us call the intersection point for these lines D.
- Similarly, $\{a_2, a_3, a_4 \ldots, a_n, a_{n+1}\}$ intersect at exactly one point since there are n lines in this set. Let us call this point E.
- Since both D and E are the point where a_2 intersects a_3. D and E are therefore the same point.
- Therefore, $\{a_1, a_2, a_3, \ldots, a_n, a_{n+1}\}$ intersect at exactly one point.■