Homework 4

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Problem 1 Let F_n be the n-th Fibbonacci number. That is

$$F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$$

Show that F_n is given by

$$F_n = \frac{1}{\sqrt{5}}(a^n - b^n) \text{ where } a = \frac{1+\sqrt{5}}{2} \text{ and } b = \frac{1-\sqrt{5}}{2}$$

Problem 2 Suppose that I bring 5 Cola yoyo and 6 Grape yoyo to the class and there are 10 students. Here is the rule for deciding which color each student get. The students get in line and pick out two yoyo randomly at a time.

a) If both yoyo are grape yoyo, the student get to eat a grape yoyo and then put the other one back into the bag.

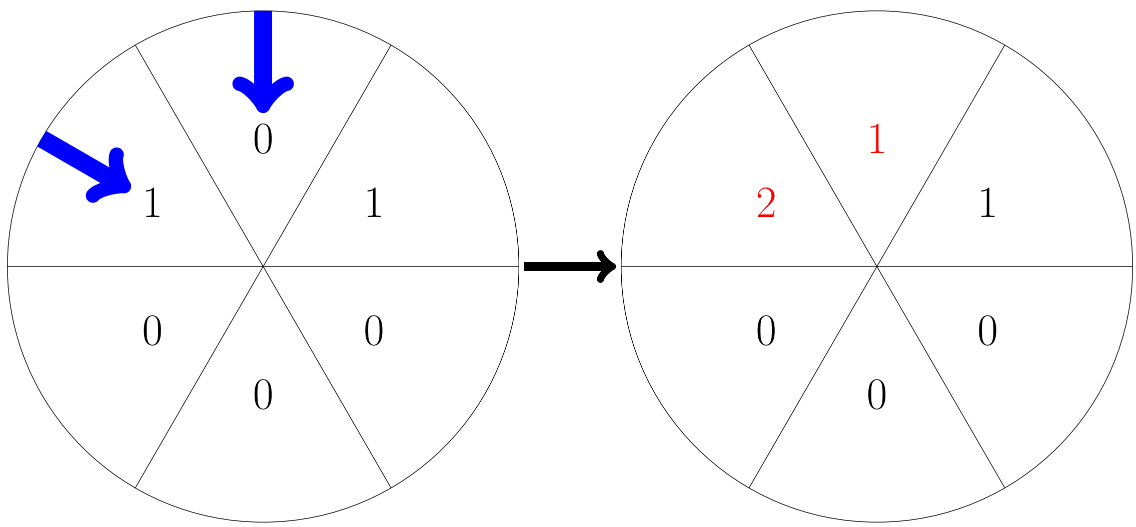
b) If both yoyo are cola yoyo, the student get to eat a cola yoyo and then put the other one back into the bag.

c) If the yoyo are of different color, the student get to eat grape yoyo and put the cola one back into the bag.

The last yoyo in the bag belong to AJ. Of course, I plan this deliberately to get the flavor I want. Which flavor of yoyo do I want? Prove that the rule always get me that flavor.

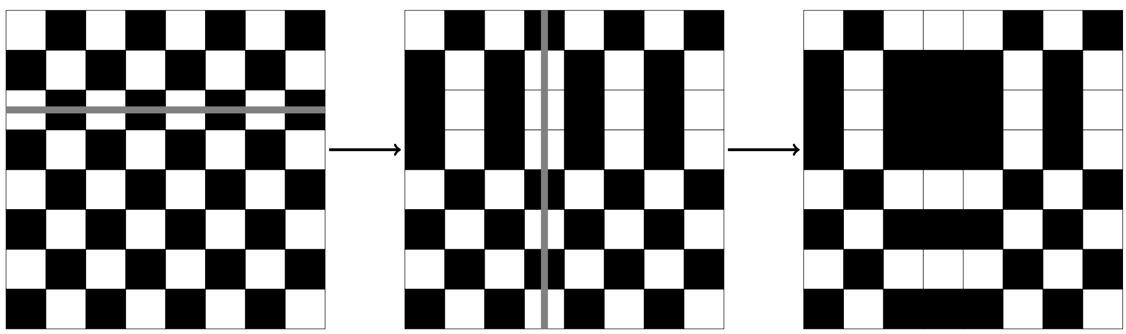
Problem 3 Consider the numbers 1,2,3,4,5,6. The game is played by picking any two numbers from the list (a,b). Then, we replace the two number with $|a - b|$. For example, if you pick 4 and 3, you erase the number 4 and 3 then add another 1 to the list(which means you will have 1,2,5,6,1). The process is then repeat until there is one number left. Prove that the last number is odd no matter how you pick it.

Problem 4 Let us play a game. A circle is divided into 6 sectors. The number 1,0,1,0,0,0 are written on each sector. You may pick two neighboring sectors and increase the number on both of them by 1. Can you make all the number in the circle equal? (If you want a hint, find me with at least 6 tries at the game.)



Problem 5 Consider an 8×8 chessboard with standard coloring. For each move you can switch all the color of *one row* or *one column* at a time. You can then repeat this as many times as you like and wherever you like. Can you reach the board with exacly 1 black square left?

Hint: Play it on 4×4 grid with the playing cards lying around in 1409.



If you are up for a challenge, look at the bonus problem of the previous midterm. Only Majeed got it last term.

Problem 6 Consider an island with 3 types of pokemon: blue pokemons, red pokemons and green pokemons.

a) At the beginning, there are 13 blue pokemons, 15 red pokemons, 17 green pokemons.
b) Whenever two pokemons meet, if the two pokemon are of different color, they will both transformed to the other color. For example, if a blue pokemon and a red pokemon meets, they both transform to a green pokemon resulting in 1 less blue pokemon, 1 less red pokemon and 2 more green pokemons.
c) Those that already transformed can meet and transform again.

Show that it is impossible for the island to be left with exactly one type of pokemon(ex: all blue) with any series of meeting and transformation.

Hint: Modulo 3