ID: _____ Name: _____

Discrete Mathematics

Final Exam T3 2021

Instruction

- Write your name
- This exam is take home based on your honor code. Open book open notes. But do not communicate with another human being via any means regarding exam. The violations includes but not limited to asking questions on stack overflow, chegg or similar services, or asking your friends via online or offline channel.
- Read the questions carefully.
- There are 4 problems. 520 points in total. You only need to get 460 points to get full score.
- Attempt all problems, state your reasons *clearly* and *legibly*, because partial credits will be given.

Question	Full Score	Your Score
1	100	
2	120	
3	160	
4	140	

Total: /460

ID: _____ Name: _____

Useful Formulas

Sum

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$
$$1 + 3 + 5 + 7 + \ldots + (2n-1) = n^2$$

Euler's Formula

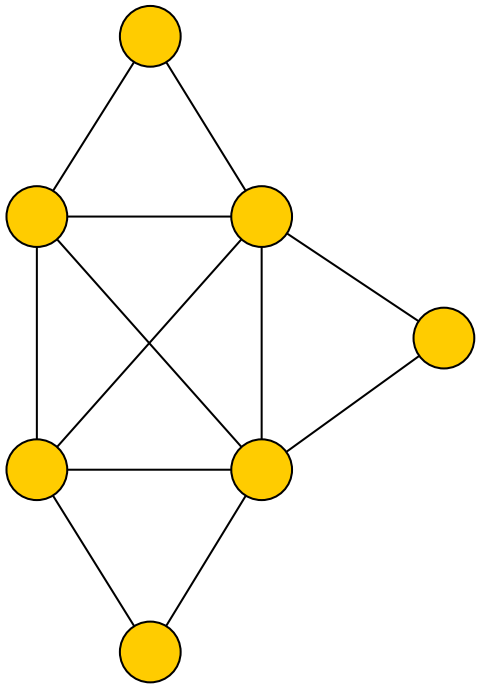
$$e + 2 = v + f$$

ID: _____ Name: _____

1. Graph Theory(100 points. 20 each)

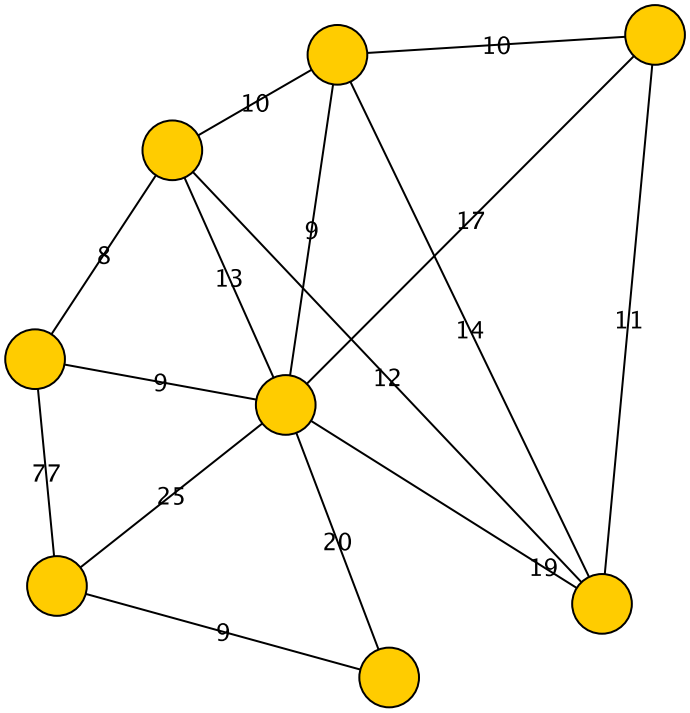
   (a) Drawing Graph Draw a **tree** with exacty 5 edges and exactly 3 leaves.

   (b) Find an Euler walk for the following graph. Label the edges with *numbers* so I can follow.



ID: _____ Name: _____

(c) The following graph shows the cost of electric wire to connect one city to another(in millions). If your job is to connect every city to electricity grid at the lowest cost, how much money do you need?

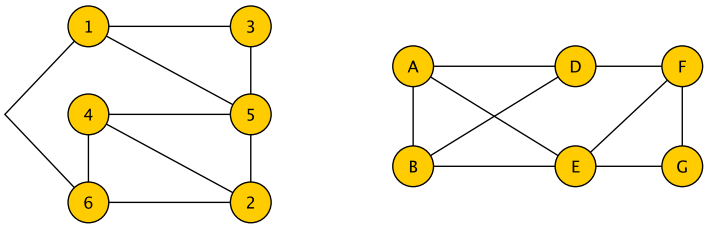


ID: _____ Name: _____

(d) Suppose AJ wants to do a secured take home exam. He came up with the following scheme. He first list out the list of students and find out who is friend with whom. If they are friend and AJ give out the same version of the exam to both there is a chance that they will copy each other. AJ could give out different exam for everyone but that would interfere with AJ gaming schedule too much. Use something you learn from graph theory to figure out the number of minimum different version exam paper he needs to make. (1 means friend 0 means not friend)

	A	B	C	D	E	F	G	H
A	-	0	1	1	0	1	1	0
B	-	-	0	0	0	0	1	1
C	-	-	-	1	1	1	0	1
D	-	-	-	-	0	1	0	0
E	-	-	-	-	-	0	1	1
F	-	-	-	-	-	-	1	1
G	-	-	-	-	-	-	-	1
H	-	-	-	-	-	-	-	-

(e) Find an isomorphism between these two graphs.



ID: _____ Name: _____

2. (20 each) Consider a popular ramen stall. Before the shop opens, there are 20 people waiting in front of the restaurant. (Make sure to write a short description for each term.)

   (a) How many ways are there to form a single line from these 20 people?

   (b) How many ways are there to form two **equal** lines(order matters) from 20 people? (The two lines are indistinguishable).

   (c) How many ways are there to form two *indistinguishable* lines from 20 people? Each line may contain from 0 to 20 people but the total must add up to 20 and the order of people in each line matters.

   (d) The ramen shop only serve 10 bowls a day. The shop owner pick only 10 from 20 people and kick the rest out. How many combinations of people who gets to eat are there? (Order doesn't matter)

   (e) The ramen shop has only 2 menus: Shoyu Ramen and Seafood Ramen. Given that the shop only serves 10 bowls per day. If each person can only order exactly one bowl from the menu, how many combination of ramen orders(how each customer order) are there? (Each customer is distinguishable.)

(f) Given that the shop only serves 10 bowls per day. If the maximum number of each menu the customers can order is 7 which means if the first 7 people order Shoyu Ramen, the rest must order Seafood Ramen. How many combination of ramen orders(how each customer order and who got kicked out) are there?

Ex: People can still order 5 Shoyu and 5 Seafood if they wish. But, no, the 10 people who gets in can't order 8 Shoyu and 2 Seafood.

3. (20 each)

   Each medical test has pro and cons. Extremely specific(very few false positive) medical test is typically very expensive while a less accurate one costs less. So typically what we do is to use less accurate and less expensive one as a screening test administered to wider population.

   In this problem, we are going to examine a two-step medical test for a disease that 20% of the population have the disease. **First we screen the people using less expensive test and for those whose test come out positive we are going to get them to a more expensive test. If the first test says no, then no more test is required. The person is confirmed positive if *both* tests say yes.**

   Here are the information about the two tests.

   ## Screening Test

   - Costs 100 Baht per person
   - Specification
     - If you have the disease, there is a 90% chance the test says yes.
     - if you do not have the disease, there is a 30% chance the test says yes.

   ## Expensive Test

   - The result is independent to screening test result.
   - Costs 5000 Baht per person.
   - Specification
     - If you have the diesease there is a 95%, chance the test says yes.
     - If you do not have the disease, there is a 5% chance the test says yes.

   (a) If you have the disease, what is the probability that you are confirmed postive.

   (b) If you do not have the disease what is the probability that you are confirmed positive.

(c) Given that you are confirmed positive what is the probability that you have the disease?

(d) Given that you are *not* confirmed positive what is the probability that you *do not* have the disease?

(e) If you have the diesase, what is the amount of money you are expected to pay as testing fee?

(f) If you have the disease what is the *variance* of total medical test fee?

(g) If you do not have the diesease what is the amount of money you are expected to pay as testing fee?

(h) If we switch the two tests, that is to have people go through expensive test first then cheaper one. Find the new false negative rate. (False negative is the probability that you have the disease but the process says no.)

ID: _____ Name: _____

4. (20 each) Consider two lottery players(Appy and Gordon) with different strategies. The lottery itself has number from 00-99 (total of 100 numbers). One winning number is selected randomly from that 100 numbers. Each lottery cost 80 Baht and if it's the winning number then the buyer with that number wins 2,000 Baht. (This is the actual number for Thai lotto)

   - Appy buys 50 numbers independently and the numbers are selected from uniform probability. Yes, he may have some duplicates.
   - Gordon buys 50 lotteries of exact same number.

   (a) Find expected value of Gordon profit/loss (Winning Amount - Cost).

   (b) Find expected value of Appy profit/loss.

   (c) Find the standard deviation of Gordon's profit/loss.

   (d) Find the standard deviation of Appy profit/loss.

   (e) What is the probability that Appy will buy all 50 lotteries of the same number **and** that number happens to win?

(f) What is the probability that Gordon will make profit?

(g) What is the probability that Appy will make profit? (You may need to think outside the box for this one.)