Homework 7

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Problem 1 More counting.

1.1) Let $x_1, x_2, x_3, \ldots x_k \in \{1, 2, 3, \ldots\}$. How many solutions are there for

$$\sum_{i=1}^{k} x_i = n$$

For example, n=4, k=3, there solutions are 1+2+1, 1+1+2, 2+1+1.

Answer: Have n stones and place $k-1$ sticks between n stones to create k partitions.

$$\binom{n-1}{k-1}$$

1.2) Let $x_1, x_2, x_3, \ldots x_k \in \{1, 2, 3, \ldots\}$. How many solutions are there for

$$\sum_{i=1}^{k} x_i \leq n$$

Answer: Have n stones and place k sticks between n stones. The last stick can be at the right of n stones for the case where the sum is equal to n.

$$\binom{n}{k}$$

1.3) Let $x_1, x_2, x_3, \ldots x_k \in \{0, 1, 2, 3, \ldots\}$. How many solutions are there for

$$\sum_{i=1}^{k} x_i = n$$

Note that we allow zero here.

Answer: Add $k-1$ stones and place then right on top of the $n+(k-1)$ stones to create k partition.

$$\binom{n+k-1}{k-1}$$

1.4) Let $x_1, x_2, x_3, \ldots x_k \in \{0, 1, 2, 3, \ldots\}$. How many solutions are there for

$$\sum_{i=1}^{k} x_i \leq n$$

Note that we allow zero here.

Answer: Add k stones and place sticks right one the stone. The last partition takes care of $\geq$.

$$\binom{n+k}{k}$$

Problem 2 Suppose you collect the data whether AJ Piti end the class late and whether it rains on that day. You collect the data for 100 class. Here is your data:

	Late	Not Late
Rain	10	15
Not Rain	50	25

2.1) What is the probability that it rains?

Answer:

$$\frac{25}{100}$$

2.2) What is the probability that I let you out late?

Answer:

$$\frac{60}{100}$$

2.3) What is the probability that I let you out late *given* that it rains?

Answer:

$$\Pr(\text{Late}|\text{rain}) = \frac{\Pr \text{Late} \cap \text{rain}}{\Pr(\text{rains})} = \frac{10/100}{25/100} = \frac{2}{5}$$

2.4) What is the probability that I let you out late *and* that it rains?

Answer:

$$\frac{25}{100}$$

2.5) What is the probability that it rains today given that I let you out on time?

Answer:

$$\Pr(\text{rain}|\text{ontime}) = \frac{\Pr \text{ontime} \cap \text{rain}}{\Pr(\text{ontime})} = \frac{15/100}{40/100} = \frac{3}{8}$$

2.6) Are the event that I let you out late and the event that it rains independent?

Answer:

$$\Pr(\text{Late}|\text{rain}) \neq \Pr(\text{late})$$

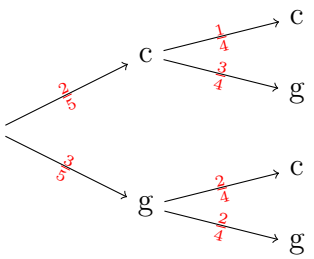
So, they are not independent.

Problem 3 Consider a bag of candy. Initially there are 2 cola yoyo and 3 grape yoyo inside the bag. Some of the questions below are trick question. Be confident in your reasoning.

3.1) If we draw two yoyo from the bag, one at a time without returning the one we draw to the bag.

a) What is the probability of getting 1 cola and 1 grape?

Answer: Here is what the tree looks like.



So,

$$\Pr(1 \text{ Cola } 1 \text{ Grape}) = \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4} = \frac{6}{10}$$

b) What is the probability of getting 2 cola?

Answer: $\frac{2}{5} \times \frac{1}{4}$

c) What is the probability of getting cola as the first yoyo?

Answer: $\frac{2}{5}$

d) Given that the second yoyo is grape, what is the probability that the first yoyo is cola?

Answer:

$$\begin{aligned}\Pr(\text{First Cola}|\text{Second Grape}) &= \frac{\Pr(\text{First Cola and Second grape})}{\Pr(\text{Second Grape})} \\ &= \frac{\frac{2}{5} \times \frac{3}{4}}{\frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4}}\end{aligned}$$

e) Given that the first yoyo is cola, what is the probability that the two yoyo we get are of different flavor?

Answer:

$$\begin{aligned}\Pr(\text{Different} \mid \text{First Cola}) &= \frac{\Pr(\text{Different and First Cola})}{\Pr(\text{First Cola})} \\ &= \frac{\frac{2}{5} \times \frac{3}{4}}{\frac{2}{5}}\end{aligned}$$

f) Given that the two yoyo we draw are of different flavors, what is the probability that the first yoyo is cola? **Answer:**

$$\begin{aligned}\Pr(\text{ First Cola} \mid \text{Different}) &= \frac{\Pr(\text{Different and First Cola})}{\Pr(\text{Diffferent})} \\ &= \frac{\frac{2}{5} \times \frac{3}{4}}{\underset{\text{From a)}}{\underbrace{\frac{6}{10}}}}\end{aligned}$$

g) Given that the two yoyo we draw are of the same flavor, what is the probability that the first yoyo is cola?

$$\begin{aligned}\Pr(\text{ First Cola} \mid \text{Same}) &= \frac{\Pr(\text{Same and First Cola})}{\Pr(\text{Same})} \\ &= \frac{\frac{2}{5} \times \frac{1}{4}}{1 - \frac{6}{10}}\end{aligned}$$

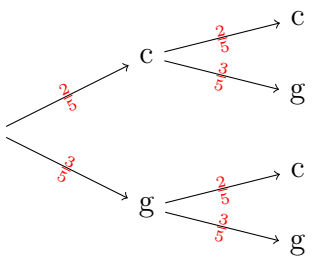
3.2) Suppose that we draw two yoyo simultaneously.

Answer: Same tree as above.

a) What is the probability of getting 1 cola and 1 grape?
b) What is the probability of getting 2 cola?

3.3) Suppose that we draw the first yoyo, look at it then put it back into the bag. Then draw the second yoyo.

Answer: Here is the tree. Figure out the rest yourself.



a) What is the probability of getting 1 cola and 1 grape?
b) What is the probability of getting 2 cola?
c) What is the probability of getting cola as the first yoyo?
d) Given that the second yoyo is grape, what is the probability that the first yoyo is cola?
e) Given that the first yoyo is cola, what is the probability that the two yoyo we get are of different flavor?
f) Given that the two yoyo we draw are of different flavor, what is the probability that the first yoyo is cola?
g) Given that the two yoyo we draw are of the same flavor, what is the probability that the first yoyo is cola?

Problem 4 In the class we discuss the three strange dice where the faces on the dice are

- $A = \{2, 6, 7\}$
- $B = \{1, 5, 9\}$
- $C = \{3, 4, 8\}$.

We found in class that A beats B, B beats C and C beats A.

Let us consider a modified rule where we select a dice then we throw it twice and take the sum as our score. Find out which dice beats which dice.

Answer:. The order gets reversed. Draw the tree.

Problem 5 Consider the game where we have 3 identical boxes. Each box has two yoyos. Two of the box has 1 cola and 1 grape and one of the box has 2 cola. Consider the following game:

A) First we pick a box.

B) Then we draw a yoyo for that box.

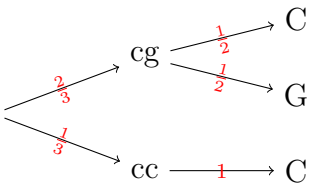
C) If it is a grape yoyo, we select the box again.

D) If it is a cola yoyo then we can continue the game.

E) We then draw another yoyo from the box. If the next yoyo is cola AJ wins. If the next yoyo is grape, you win.

Is this a fair game? Explain your reasoning.

Answer:



The thing we want to calculate is

$$\Pr[cg|C] = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3}} = \frac{1}{2}.$$

Fair game.

Problem 6 Suppose that AJ Piti has one super rare banana flavor yoyo. He decides to give it to either Bossy, Tow+ or John. Each one of the had equal chance in getting the yoyo. AJ Piti already know which one to give the yoyo to using a python program he wrote.

Bossy is very excited about the news of the super rare yoyo. So, he asks AJ Piti to tell the name of one of his friend who won't get the yoyo. The way AJ Piti decides which name to tell Bossy is the following

- If Bossy doesn't get the yoyo, then AJ will just tell the name of the other person who doesn't get the yoyo
- If Bossy gets the yoyo, then AJ will flip a fair coin to decide whether to tell the name of John or Tow+.

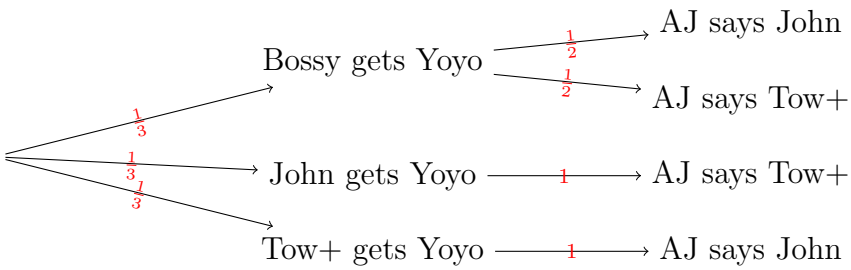
AJ Piti told Bossy that John won't get the yoyo.

- With the information, Bossy was so excited because he reasons that the his chance of getting yoyo is 1/2 since it is either him or Tow+ that get the yoyo.
- Bossy then tell Tow+ that AJ Piti told him that John won't get the yoyo. Tow+ reasons that since the probability of Bossy getting the yoyo has nothing to do what what AJ Piti told so it is still 1/3. That means since he knows that John won't get the yoyo, Tow+'s chance of getting the yoyo is now 2/3.

Which one is correct? Given all these information, find the probability that Bossy will get the yoyo and find the probability that Tow+ will get the yoyo.

Hint: Draw the tree.

Answer:



What we want to calculate is

$$\Pr(\text{Bossy get Yoyo} \mid \text{AJ says John}) = \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{1}{2} + \frac{1}{3}} = \frac{1}{3}$$

and

$$\Pr(\text{Tow+ get Yoyo} \mid \text{AJ says John}) = \frac{\frac{1}{3}}{\frac{1}{3} \times \frac{1}{2} + \frac{1}{3}} = \frac{2}{3}$$