Homework 1 Extra

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Problem 1 A number x is a perfect square if and only if

$$\exists m \in I \text{ such that } x = m^2$$

Prove that if x and y are both perfect squares, then xy is also a perfect square.

Problem 2 If $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

Problem 3 Show that if x is a rational number and y is a rational number then, $x + y$ is a rational number.

Problem 4 Show that if x is irrational then $1/x$ is irrational.

Problem 5 (This one is a bit tricky)Use contradiction to show that

$$r^3 + r + 1 = 0$$

has no rational solution.

Hint: assume $r = a/b$ then get equation involving b^3 and consider even/oddness of the equation.

Problem 6 Show that the last digit of the perfect square number is either $0, 1, 4, 5, 6$ or 9.

Problem 7 Given two positive real numbers(no zero). Arithmetic mean is given by

$$\text{Arithmatic Mean} = \frac{x + y}{2}$$

Geometric mean is given by

$$\text{Geometric Mean} = \sqrt{xy}$$

Which one is greater? Then show that the inequality holds for all numbers.

Problem 8 (Engel.) **Hard** Consider a group of n people such that every subset of 4 people has at least one person who know the other three people in the subset. Show that for such group has at least one person who knows every one in the group.