NAME ______________________ ID ____________ Seat No. ________

ICCS 206 Discrete Mathematics T.1/2021-2022

MIDTERM EXAMINATION

Please read the following instruction carefully.

1. There are 11 pages (including this cover page) and 6 questions in this exam.
2. Write answers in the provided space, you may add additional page(s) to this PDF file if your work requires more space.
3. This exam is open book, open notes and open internet. You are however not allowed to get help from other human being on exam via any means (Ex: stackoverflow, discord, chegg, Line, etc.). (Ex: Talking to a cat for moral support is allowed.)
4. You must turn in the exam paper before the deadline on specified on canvas. Late submission will not be accepted.
5. You must not communicate with any other person by any means. Students found cheating during the examination will be penalized according to the university regulation.

Good luck!

FOR INSTRUCTOR USED

Distribution of Marks

Question:	1	2	3	4	5	6	Total
Points:	100	100	100	100	100	100	540
Score:							

Mahidol University International College
Mahidol University

Useful Formula and Definitions

Asymptotics

Definiton	Definition			Intuition
Asym. Equal	$f \sim g$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$	$f \underbrace{\equiv}_{x\to\infty} g$
Big Oh	$f \in O(g)$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty$	$f \underbrace{\leq}_{x\to\infty} g$
Little Oh	$f \in o(g)$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$	$f \underbrace{<}_{x\to\infty} g$
Little Omega	$f \in \omega(g)$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} \to \infty$	$f \underbrace{>}_{x\to\infty} g$
Big Omega	$f \in \Omega(g)$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} > 0$	$f \underbrace{\geq}_{x\to\infty} g$
Theta	$f \in \Theta(g)$	iff	$\lim_{x\to\infty} \frac{f(x)}{g(x)} = c, c \neq 0$	$f \underbrace{=}_{x\to\infty} g$

Sum

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Integral

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} \qquad \text{if } n \neq -1$$
$$\int \frac{1}{x} \, dx = \ln(x)$$

Quadratic

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. (100 points) Easy part.
   (a) (10 points) Determine which of the following are propositions.
   You do **NOT** need to explain your answer.
      i. **(Yes/No)** Everyone in Thailand got Covid-19 vaccines.
      ii. **(Yes/No)** Will Bitcoin ever reach $65,000 per coin?
      iii. **(Yes/No)** $c^2 = a^2 + b^2$ (Without quantifier nor definition of a, b, c)
      iv. **(Yes/No)** Every positive integer can be written as a sum of two primes.
   (b) (10 points) Give at least two mathematical propositions not shown in this exam paper.
      i. Proposition 1
      ii. Proposition 2
      iii. More proposition (if case you want to give more answers)
   (c) (20 points) Draw truth table for

$$Q \vee (P \to \sim R)$$

(d) (20 points) Find the closed form of the following sum

$$\sum_{i=0}^{n}((i+1)^3 - x^i)$$

(e) (20 points) Show that $3n^2 + 2n\ln(n)$ is $O(n^2)$ but not $o(n^2)$.

(f) (20 points) **Disprove** the following theorem.

All digits of a product of two even integers are even.
Ex: $24 \times 2 = 48$. Both 4 and 8 are even

2. (100 points) Easy proof

(a) (50 points) You are playing an online game with lucky box system. Each lucky box opening, you will receive one item from 30 selected items. However, if you open a lucky box and receive the item that you already have in your inventory box, then you will receive 50 credits.

Prove that you will receive at least 50 credits if you purchase and open 31 lucky boxes.

(b) (50 points) The formal notation of division remainder is 5 mod 3. This is also known as 5%3. It is defined as

$$n \bmod m = r \text{ if and only if } n = x \times m + r \ \exists x \in I.$$

For example, 17 mod 3 = 2 since $\underbrace{17}_{n} = \underbrace{5}_{x} \times \underbrace{3}_{m} + \underbrace{2}_{r}$

Prove the following

$$[(a \bmod 3) + (b \bmod 3)] \bmod 3 = (a + b) \bmod 3 \quad \forall a, b \in I$$

3. (100 points) Prove that

$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$$

whenever n is a positive integer.

4. (100 points) AJ Wasakorn and AJ Piti really wants to take a break. The two made the following statement on Day number 1.

   - AJ Piti said that Day d is a holiday if d is divisible by 4 or 4 days ago was a holiday(of either of AJ).
   - AJ Wasakorn said that Day d is a holiday if d is divisible by 11 or 11 days ago was a holiday(of either of AJ).

   Prove that after a certain date D, everyday is a holiday. Find D.

   Ex: Day 8 is a holiday since it's divisible by 4. Day 19 is also a holidy since 11 days ago is a holiday.

   Note: Make sure you include all the necessary base cases and specify clearly where you use the inductive hypothesis.

5. (100 points) In year 2030, Covid has evolved into a new strain called Covid Epsilon. Here are the characteristics.

   - If a person is sick, one should choose to **cure oneself** by **infecting the virus to 5 healthy people**. No more. No less.
   - If a person is healthy, one can altruistically choose to **infect oneself** with virus from exactly 7 sick people. The **7 sick people are then cured**.
   - You may not need this but for those of you who tends to overthink. There is no other cure for the virus, no other way to get infected and assume that people never die and new born child are healthy. Plus, virus don't mutate etc.

   Today, there are 7 people infected with Covid Epsilon. Prove that there is no way to eradicate the virus (make the infection go down to 0).

6. (100 points) Recurrence Solve the following recurrences.

   (a) (30 points) $T(n) = 2T(n-1) + n$. with $T(0) = 1$

   (b) (30 points) $T(n) = T\left(\frac{n}{2}\right) + 2n$ with $T(1) = 1$

(c) (40 points) $T(n) = -2T(n-1) + 15T(n-2)$ with $T(0) = 1, T(1) = 1$