Solution 0

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Problem 1 Basic Stuff.

1.1) Write down the set of integers that is less than 8 and greater than 2.

Answer: $\{3, 4, 5, 6, 7, 8\}$

1.2) Write down the members of $\{x \in I | x^2 < 10\}$.

Answer: $\{-3, -2, -1, 0, 1, 2, 3\}$

1.3) Write down the members of

$$\{x \in I^+ | x^2 < 10\} \cup \{x \in I^+ | 2 < x < 8\}.$$

Answer: $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

1.4) Write down the members of

$$\{x \in I^+ | x^2 < 10\} \cap \{x \in I^+ | 2 < x < 8\}.$$

Answer: $\{3\}$

Problem 2 Notation Exercise.

- Let M be the set of all MUIC students.
- Let $P(x)$ be the predicate for student x takes Discrete Math class this term.
- Let $Q(x)$ be the predicate for student x taks Data Structure this term.
- Let $F(x, y)$ be the predicate for student x and student y are friends on facebook.
- Let $\mathbb{A} = \{x \in M | P(x)\}$
- Let $\mathbb{B} = \{x \in M | Q(x)\}$

Write the following propositions in mathematical form using $\cap$, $\cup$, $\rightarrow$, $\forall$, $\exists$ and all those set notations.

2.1) There are some students at MUIC who take Discrete Math is term. (Ex: $\exists x \in M$ such that $P(x)$)

Answer: $\exists x \in M$ such that $P(x)$

2.2) There are some students who take both Discrete Math and Data Structure this term.

Answer: $\exists x \in M$ such that $P(x) \wedge Q(x)$

2.3) There exists some students who take Discrete math but not Data Structure.

Answer: $\exists x \in M$ such that $P(x) \wedge \neg Q(x)$

2.4) Everyone takes Discrete Math.

Answer: $\forall x \in M, P(x)$

2.5) No one takes Data Structure.

Answer: $\forall x \in M, \neg Q(x)$

2.6) All students who takes discrete math this term also take Data structure. (Ex: $\forall x \in \mathbb{A}, Q(x)$ or you can use $\forall x \in M, P(x) \rightarrow Q(x)$)

Answer: $\forall x \in \mathbb{A}, Q(x)$

2.7) There exists some students who take Data Structure but not Discrete Math.

Answer: $\exists x \in \mathbb{B}$ such that $\neg P(x)$

2.8) There exists a student in Discrete Math who is friend to every one in Data structure.

Answer: $\exists x \in \mathbb{A}$ such that $F(x, y) \forall y \in \mathbb{B}$

2.9) Everyone in Data Structure has at least one friend in Discrete Math.

Answer: $\forall x \in \mathbb{B} \exists y \in \mathbb{A} F(x, y)$

2.10) There exists a student in Discrete Math who is friend to no one in Data Structure.

Answer: $\exists x \in \mathbb{A}$ such that $\forall y \in \mathbb{B} \neg F(x, y)$

Problem 3 Write down truth table for the following statements:

3.1) $P \Longrightarrow (\neg Q \vee P)$

3.2) $P \Longrightarrow (P \wedge Q)$

Answer: Too easy. Do it yourself.

Problem 4 Fun With Quantifiers.

4.1) Are the following two propositions equivalent?

A) $\forall x \in I, \exists y \in I$ such that $x + y = 23$

B) $\exists y \in I$ such that $x + y = 23$ $\forall x \in I$

Answer: No. See lecture notes.

4.2) Let X be the set of all boys in the class and Y be the set of all girl in teh class $P(a, b)$ be a predicate that is true only if a secretly likes b. Are the follwing propositions true for all predicate P.

Translate these confusing symbols in to plain english and determine whether follwing propositions true. Explain/Prove/Disprove it.

A) $\forall x \in X, \exists y \in Y$ such that $P(x, y) \Longrightarrow \exists y \in Y$ such that $P(x, y)\ \forall x \in X$
If for all boy in class there exists a girl that he likes, then there exists a girl in class that all boy likes.

B) $\exists y \in Y$ such that $P(x, y) \forall x \in X \Longrightarrow \forall x \in X, \exists y \in Y$ such that $P(x, y)$
If there exists a girl in which all boy likes, then every boy has a girl he likes. Of course, it's that girl.