Homework 6

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> There is no need to get numerical value for the answer. But you must put a short note on where each term comes from.

> It is extremely important that you try this **alone** first before working it with your friends.

Problem 1 Poker hands. Let us count how many poker hands there are. This is an excellent exercise. It is important that you do this alone first before checking it with others.

A deck of card consists of 52 cards. There are 13 ranks $\{2, 3, 4, \ldots, 9, 10, J, Q, K, A\}$ and each of them comes with 4 suits: Spade(♠), Heart(♥), Diamond(♦) and club(♣).

The game is played by dealing 5 cards to a player.

1.1) How many 5 cards hands are there?

1.2) A *royal straight flush* happens when the player have $10, J, Q, K, A$ all of the same suit. How many royal straight flush are there?
Ex: $10♥, J♥, Q♥, K♥, A♥$

1.3) 4-of-a-kind happens when 4 of the 5 cards are of the same rank. How many 4-of-a-kind are there?
Ex: $3♠, 3♥, 3♥, 3♣, 7♥$

1.4) Full house happens when 3 of the 5 cards form 3 of a kind and the other 2 form a pair. How many full house hands are there?
Ex: $J♥, J♠, J♦, 9♣, 9♥$

1.5) Straight Flush happens when you have 5 consecutive cards of the same suit that is not a Royal Straight Flush. (Note that $A, 2, 3, 4, 5$ is not considered consecutive.) How many straight flush hands are there?
Ex: $2♥, 3♥, 4♥, 5♥, 6♥$

1.6) A flush happens when the 5 cards you get are of the same suit but not staright flush or a royal straight flush. How many of a flush hands are there?
Ex: $5♦, 3♦, 8♦, 10♦, K♦$

1.7) A straight happens when all 5 cards are of consecutive rank but not a straight flush or royal straight flush. How many straight hands are there?
Ex: $7♣, 8♣, 9♥, 10♠, J♥$

1.8) 3-of-a-kind happens when exactly 3 of 5 cards are of the same rank but not a full house. How many 3-of-a-kind hands are there?
Ex: $A♠, A♥, A♦, 5♠, 7♥$

1.9) 2-pair happens when you have exactly 2 pairs in the hand. How many 2 pairs hand are there? Be careful about double counting this one.
Ex: $3♠, 3♥, J♠, J♦, A♠$

1.10) 1-pair hands happens when you have exactly 1 pair in your hand but not a full house.

Ex: $5\clubsuit, 5\heartsuit, A\heartsuit, 2\diamondsuit, 10\clubsuit$

1.11) High card happens when you do not have anything special(all of the above) in your hand. How many how many high card hands are there? There is an easy way to do this and there is a cool way to do this? Try figure out both ways.

Ex: $2\clubsuit, 3\clubsuit, 10\spadesuit, J\spadesuit, 5\heartsuit$

Problem 2 How many numbers in $1, 2, 3, \ldots, 1000$ are there that is divisible by 2 or 5 or 7?

Problem 3 Read up the lecture notes on injective, surjective, and bijective function and decide whether the following functions $A \to B$ is injective, surjective, or surjective.

3.1) $A = \{a, b\}$, $B = \{1, 2\}$, $f(a) = 1$, $f(b) = 1$.

3.2) $A = \{a, b\}$, $B = \{1, 2, 3\}$, $g(a) = 1$, $g(b) = 2$.

3.3) $A = \{a, b, c\}$, $B = \{1, 2\}$, $h(a) = 1$, $h(b) = 2$, $h(c) = 1$.

3.4) $A = \{a, b\}$, $B = \{1, 2\}$, $q(a) = 1$, $q(b) = 2$.

Problem 4 Combinatoric proof. Find a combinatoric proof of the following identity.

$$\sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i} = \binom{2n}{n}$$

If you have problem with this, read up lecture notes. This is very similar.

Problem 5 Best, Boss, Tow+ and Majeed love to play a card game called Jubmoo so much. Ask them for how to play. The games begins with dealing cards to 4 players. Each player will get exactly 13 cards.

5.1) How many ways are there to deal 52 cards to 4 players such that each player get exactly 13 cards?

However, if it happens that at least 1 person has no $\heartsuit$ in his/her hand the card will have to be redealt.

5.2) How many ways are there such that Tow+ gets no $\heartsuit$? (The other three may get no $\heartsuit$ too.)

5.3) How many ways are there such that Tow+ and Boss gets no $\heartsuit$? (The other two may get no $\heartsuit$ too.)

5.4) How many ways are there such that Tow+, Boss and Majeed gets no $\heartsuit$?

5.5) How many ways are there such that four people has no $\heartsuit$? (Trick question.)

5.6) How many ways are there such that *at least* one person has no $\heartsuit$? (Hint: Inclusion-Exclusion)

5.7) What is the probability that they have to redeal the cards? Give me numerical answer. Use computer or something to do it. (I got around 5%.)

Problem 6 Mixed up deck. There are three decks of card in 1409 they got all mixed up so we have a pile of (156 cards). How many ways are there to arrange them? Assume that the card of the same rank and same suit from the three decks looks identical.

Problem 7 Our computer concept students are doing a new robot competition. Grid walking robot. The robot must walk along the grid lines. The robot can *only go up or right.* The robot starts at (0,0). The treasure is placed at (20,10).
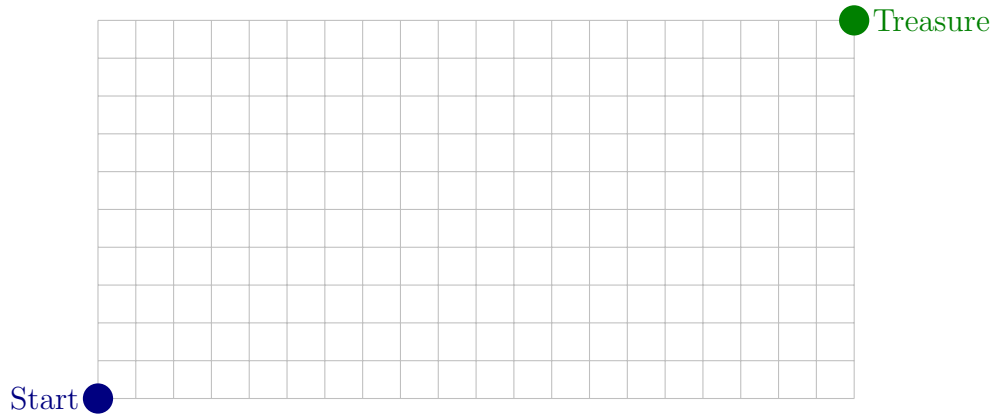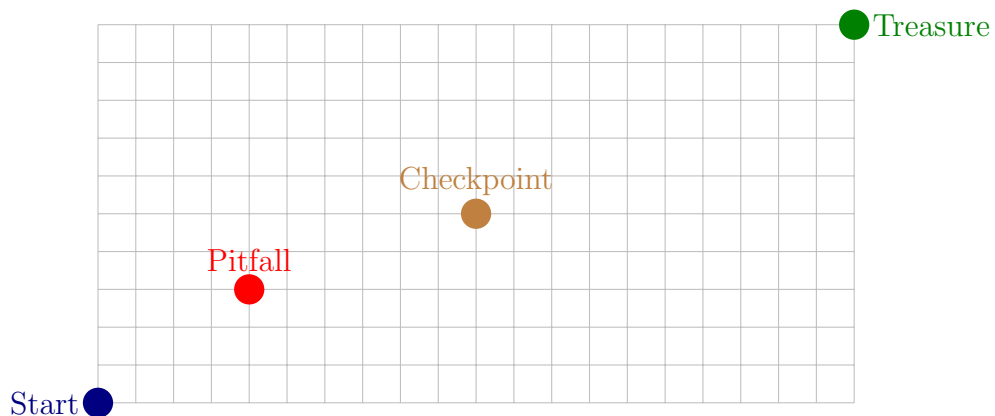

7.1) How many ways are there for the robot to reach the goal?

Since the game seems to be too hard. AJ Boonyanit decide to place a checkpoint at (10,5)
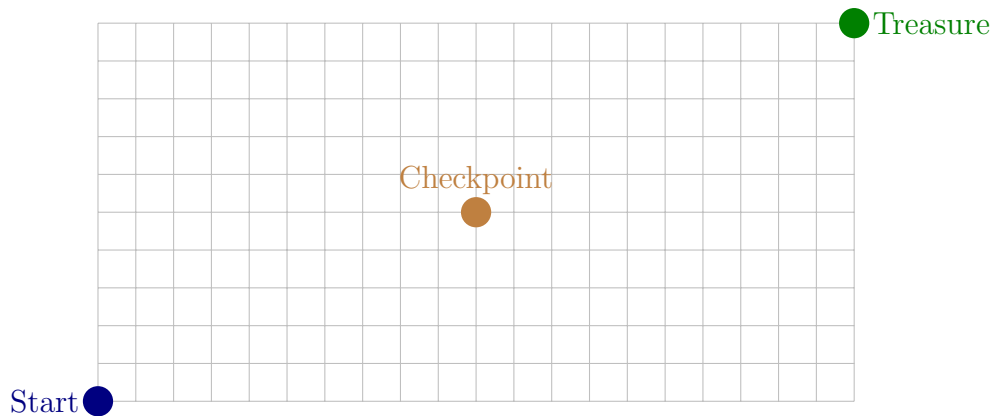
7.2) How many ways are there for the robot to reach the checkpoint then the goal?

AJ Piti feels that the competition is too easy so he places a pitfall at (4,3).

7.3) How many ways are there for the robot to reach the checkpoint then the goal while avoiding the pitfall?

Problem 8 Consider Tower Hanoi with 3 poles and 10 disks of distinct sizes. A disk can only be place on the bottom level or on the bigger disk.

8.1) How many valid configurations are there if the three poles are distinct?

8.2) If the three poles are identical, how many valid configurations are there? Be careful on this one and make sure your answer is integer.

Problem 9 How many ways can n books be placed on k distinguishable shelves

9.1) if the books are indistinguishable copies of the same title?

9.2) if no two books are the same, and the positions of the books on the shelves matter?