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Discrete Mathematics

Final Exam T3 2019

Instruction

- Write your name
- Read the questions carefully.
- There are 4 problems. 440 points in total. You only need to get $0.9 \times 440 = 396$ points to get full score.
- Attempt all problems, state your reasons *clearly* and *legibly*, because partial credits will be given.

Question	Full Score	Your Score
1	100	
2	100	
3	100	
4	140	
Bonus	30	

Total: /486

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Useful Formulas

Sum

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$
$$1 + 3 + 5 + 7 + \ldots + (2n-1) = n^2$$

Euler's Formula

$$e + 2 = v + f$$

Tree

$$e = v - 1$$

Bayes' Formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Combinatoric

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

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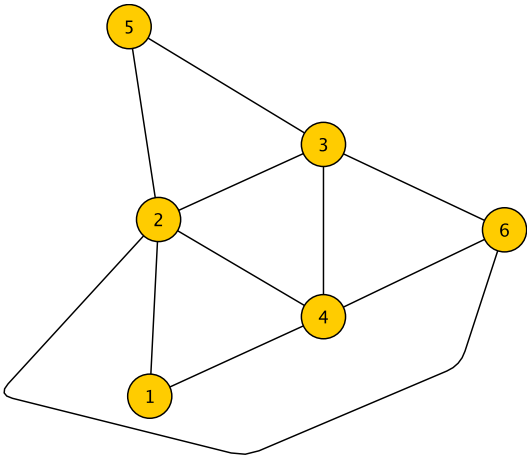
1. Graph Theory(20 each)

   (a) Draw a **tree** that has exactly 5 vertex which exactly one vertex of degree 3.

   (b) Find an Euler walk for the following graph. Label the edges with *numbers* so I can follow.



   (c) Consider 30 random points on a piece of paper. What is the maximum number of partitions(not counting the outside) can we make if we can draw only 60 non intersecting lines? (Yes, it's a planar graph)

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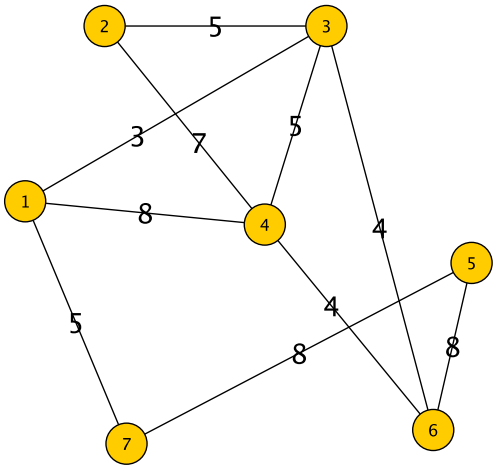
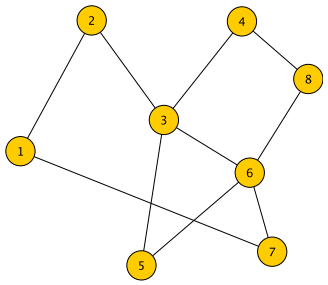
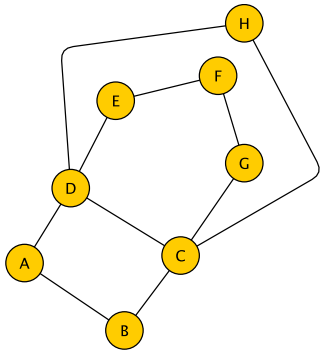
(d) Find a minimum spanning tree from the following graph.



(e) Find an isomorphism between these two graphs.





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2. (20 each) Consider 6 bowls each has different color. There are 20 identical chocolate chips. And, there are 4 students.

   (a) How many ways are there to line up 4 students?

   (b) How many ways there to give 1 bowl to each students?

   (c) How many ways are there to give all 6 bowls to each student such that every one get at least 1 bowl?

   (d) How many different ways are there to put *all* chocolate chips into 6 bowls such that each bowl has at least 1 chocolate chips?

   (e) How many different ways are there to put *all* chocolate chips into 6 bowls such that each bowl has at least 2 chocolate chips? (Hint: try to reduce the problem to the former one)

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3. (20 each) This is a made up number don't take these number as medical advice. Consider a COVID-19 test. The performance metric of the test is as follows:

   - if the patient do have the COVID-19 there is a 90% chance the test will come up positive.
   - if the patient *do not* have COVID-19 there is a 10% chance the test will say positive(wrongly).

   The fraction of population that has COVID-19 disease(prevalance) is 10%.

   (a) Are the event that someone actually has the disease and the event that the test comes out positive independent?

   (b) If we pick a random person from the street, what is the probability that the test comes out positive?

   (c) If you take the test once and the test come up positive what is the probability that you actually have the disease?

   (d) If you take the test twice and both test results comes up positive what is the proabability that you actually have the disease? Let us assume that the two tests are independent.

(e) Let us supposed that you are over paranoid and you goal is to take the test until the test comeup positive. If you do not actually have the disease, what is the expected number of test you need to take until the result show up postive?

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4. (20 each) Consider the experiments of betting on the result of a fair independent coin tosses. If the coin come up head, the player get the original bet back with the profit equal to the original bet. If the coin come up tail, the player loses the original bet.

   - Mr. A bet 100 Baht on 1 coin toss.
   - Mr. B bet 1 Baht on 100 coin toss.

   (a) What is the probability that Mr. A will lose 100 Baht?

   (b) What is the probability that Mr. B will lose 100 Baht?

   (c) Find the expected profit of Mr. A.

   (d) Find the expected profit of Mr. B.

   (e) Find the standard deviation of Mr. A profit.

(f) Find the standard deviation Mr. B profit.

(g) What is the probability that Mr. A's profit will be greater than or equal to Mr. B's?

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5. Bonus. (Require some thought.)In Pokemon Sword and Shield, players can obtain berries from shaking berry tree. Once the player shake the berry tree on of the two things can happen

   (a) 3 Berries falls down and the player is given a choice whether to shake the berry tree again

   (b) A squirrel falls down eat all the berries user collected so far. The berry tree then becomes unusable.

   The player could also stop shaking the tree and collect all the berries accumulated so far.

   The more the berry tree get shaken the more likely a squirrel will falls down. Specifically at turn n the probability that the user will get 3 berries falls down is given by (first turn is turn $n = 1$).

   $$P_n = (1 - n/100)$$

   The question is find out at which turn should the player decide to stop shaking the tree to get the highest expected value of number of berries?

   Note: ($P_i = 0$ if $i > 100$) This means at turn 100 there is no point of shaking the tree. I didn't put it up there so you don't get confused.