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Problem 1

1.1) $\sum_{i=1}^{n}(2i+1) = 2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$

$= 2\left[\frac{n(n+1)}{2}\right] + n$

$= n(n+1) + n$

$= n^2 + n + n$

$= n^2 + 2n$

$= n(n+2)$

1.2) $\sum_{i=1}^{n}\sum_{j=1}^{m}(i+j^2) = \sum_{i=1}^{n}(i+1^2) + (i+2^2) + (i+$
$3^2) + \ldots\ldots\ldots\ldots + (i+m^2)$

$= \sum_{i=1}^{n}(i+i+i+\ldots\ldots+i) + (1^2+2^2+3^2+\ldots\ldots+m^2)$

$= \sum_{i=1}^{n}(m)i + \left(\frac{m(m+1)(2m+1)}{6}\right)$

$= \sum_{i=1}^{n}(m)i + \left(\frac{m(m+1)(2m+1)}{6}\right)$

$= \sum_{i=1}^{n}(m)i + \sum_{i=1}^{n}\left(\frac{m(m+1)(2m+1)}{6}\right)$

$= m\sum_{i=1}^{n}(i) + \left(\frac{m(m+1)(2m+1)}{6}\right)n$

$= m\frac{n(n+1)}{2} + \left(\frac{m(m+1)(2m+1)}{6}\right)n$

1.3) $\sum_{i=1}^{n}\sum_{j=1}^{m}(2^{i+2j}) = \sum_{i=1}^{n}\sum_{j=1}^{m}(2^i)(2^{2j})$

$= \sum_{i=1}^{n} 2^i[\sum_{j=1}^{m} 2^{2j}]$

$= [\sum_{i=1}^{n} 2^i][\sum_{j=1}^{m} 4^j]$

$= [2(2^n-1)][\frac{(4)(4^m-1)}{3}]$

Let $\sum_{j=1}^{m} 4^j = 4^1 + 4^2 + 4^3 + \ldots\ldots + 4^m$

$S = 4^1 + 4^2 + 4^3 + \ldots\ldots + 4^m$

$4S = 4^2 + 4^3 + 4^4 + \ldots\ldots + 4^{m+1}$

$4S = S - 4 + 4^{m+1}$

$3S = (4)(4^m - 1)$

$S = \frac{(4)(4^m-1)}{3}$

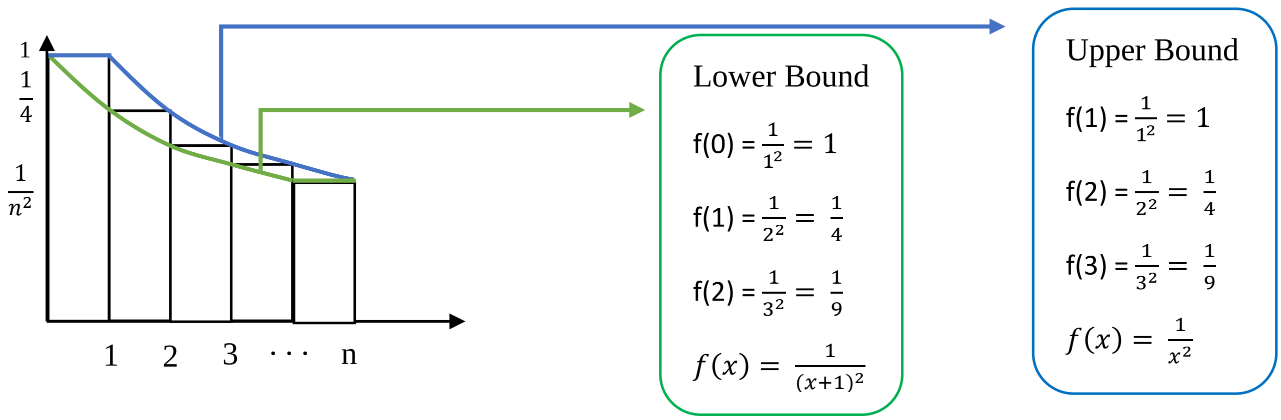
1.4) $\prod_{i=1}^{i=n}\prod_{j=1}^{m} 2^i 3^j = [\prod_{i=1}^{i=n} 2^i][\prod_{j=1}^{m} 3^j]$

$= \quad (2^1 \times 2^2 \times 2^3 \times \ldots\ldots\ldots \times 2^n)\,(3^1 \times 3^2 \times 3^3 \times \ldots\ldots\ldots \times 3^m)$

$= \quad (2^{\frac{n(n+1)}{2}})(3^{\frac{m(m+1)}{2}})$

Problem 2

2.1) $\sum_{i=1}^{n} \frac{1}{x^2} \quad = \quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots + \frac{1}{n^2}$



The area under the blue line (upper bound) $= \int_{x=1}^{n} \frac{1}{x^2}\,dx + 1$

$= \int_{x=1}^{n} x^{-2} + 1$

$= -x^{-1}|_{x=1}^{n} + 1$

$= -\frac{1}{n} - \left(-\frac{1}{1}\right) + 1$

$= -\frac{1}{n} + 2$

The area under the green line (lower bound) $= \int_{x=0}^{n-1} \frac{1}{(x+1)^2}\,dx + \frac{1}{(n)^2}$

Let u = x+1 $= \int_{u=1}^{n} \frac{1}{(u)^2}\,du + \frac{1}{(n)^2}$

$= \int_{u=1}^{n} u^{-2}du + \frac{1}{(n)^2}$

$= (-u)^{-1}|_{u=1}^{n} + \frac{1}{(n)^2}$

$= -\frac{1}{n} - \left(-\frac{1}{1}\right) + \frac{1}{(n)^2}$

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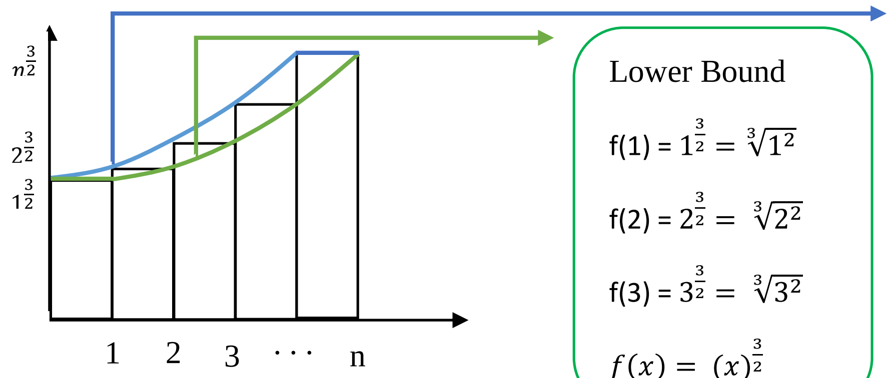
$$= \quad -\frac{1}{n} + \frac{1}{(n)^2} + 1$$

$$-\frac{1}{n} + \frac{1}{(n)^2} + 1 < \sum_{i=1}^{n} \frac{1}{x^2} < -\frac{1}{n} + 2$$

Problem 2.2)

$$\sum_{i=1}^{n} x^{\frac{3}{2}} \quad = \quad 1^{\frac{3}{2}} + 2^{\frac{3}{2}} + 3^{\frac{3}{2}} + \ldots\ldots\ldots + n^{\frac{3}{2}}$$



Lower Bound

f(1) = $1^{\frac{3}{2}} = \sqrt[3]{1^2}$

f(2) = $2^{\frac{3}{2}} = \sqrt[3]{2^2}$

f(3) = $3^{\frac{3}{2}} = \sqrt[3]{3^2}$

$f(x) = (x)^{\frac{3}{2}}$

Upper Bound

f(0) = $1^{\frac{3}{2}} = \sqrt[3]{1^2}$

f(1) = $2^{\frac{3}{2}} = \sqrt[3]{2^2}$

f(2) = $3^{\frac{3}{2}} = \sqrt[3]{3^2}$

$f(x) = (x+1)^{\frac{3}{2}}$

The area under the blue line (upper bound) $= \int_{x=0}^{n-1} (x+1)^{\frac{3}{2}}\, dx + (n)^{\frac{3}{2}}$

Let u = x+1 $= \int_{x=0}^{n-1} (u)^{\frac{3}{2}}\, du + (n)^{\frac{3}{2}}$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}}\Big|_{x=0}^{n-1} + (n)^{\frac{3}{2}}$$

$$= \frac{2}{5}(n)^{\frac{5}{2}} + (n)^{\frac{3}{2}} - \frac{2}{5}$$

The area under the green line (lower bound) $= \int_{x=1}^{n} (x)^{\frac{3}{2}}\, dx + (1)^{\frac{3}{2}}$

$$= \frac{2}{5}(x)^{\frac{5}{2}}\Big|_{x=1}^{n} + 1$$

$$= \frac{2}{5}(n)^{\frac{5}{2}} - \frac{2}{5}(1)^{\frac{5}{2}} + 1$$

$$= (n)^{\frac{5}{2}} - \frac{2}{5} + 1$$

$$= (n)^{\frac{5}{2}} + \frac{3}{5}$$

$$(n)^{\frac{5}{2}} + \frac{3}{5} < \sum_{i=1}^{n} x^{\frac{3}{2}} < \frac{2}{5}(n)^{\frac{5}{2}} + (n)^{\frac{3}{2}} - \frac{2}{5}$$

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Problem 3

$Total\ cards\ of\ a\ n\ stories\ tall\ house\ of\ card = \sum_{i=1}^{n} 2i + \sum_{j=1}^{n-1} j$

$n = 10;\quad Total\ number\ of\ cards \quad = \sum_{i=1}^{10} 2i + \sum_{j=1}^{10-1} j$

$= 2\sum_{i=1}^{10} i + \sum_{j=1}^{9} j$

$= 2\frac{10(11)}{2} + \frac{9(10)}{2}$

$= 10(11) + 9(5)$

$= 155\ cards$

Problem 4

4.1) T(n) = T(n − 1) + n where T(1) = 1.

$T(5) = T(4) + 5$

$= [T(3) + 4] + 5$

$= [T(2) + 3] + 4 + 5$

$= [T(1) + 2] + 3 + 4 + 5$

$= 1 + 2 + 3 + 4 + 5$

$T(n) = 1 + 2 + 3 + \ .\ .\ .\ .\ .\ .\ .\ . + n$

$= \frac{n(n+1)}{2}$

We guess that from $T(n) = \frac{n(n+1)}{2}$

$T_{guess}(n) = \frac{n(n+1)}{2}$

Theorem: $T(n)\ and\ T_{guess}(n)\ are\ the\ same\ thing.$

$T(n) = T_{guess}(n) \ ;\ \forall n \geq 1$

Proof: Base Case: $T(1) = T_{guess}(1)$

By T(n) definition $1 = \frac{1(2)}{2}$

$1 = 1$

Predicate P(S): = $T(S) = T_{guess}(S)$

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Inductive Steps: Assume $T(i) = T_{guess}(i)\ for\ i = 1,2,3, ... , k$

We want to show $T(k+1) =?\ T_{guess}(k+1)$

$$
\begin{aligned}
\text{LHS} &= T(k+1) && \text{By T(n) definition} \\
&= T(k) + (k+1) && \text{By IH} \\
&= T_{guess}(k) + (k+1) && \text{By } T_{guess}\text{(n) definition} \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= (k+1)[\frac{k}{2} + 1] && \text{By } T_{guess}\text{(n) definition} \\
&= \frac{k+1(k+2)}{2} = T_{guess}(k+1) = \frac{k+1(k+2)}{2}
\end{aligned}
$$

4.2) T(n) = T(n − 1) + n^3 where T(1) = 1.

$$
\begin{aligned}
T(5) &= T(4) + 5^3 \\
&= [T(3) + 4^3] + 5^3 \\
&= [T(2) + 3^3] + 4^3 + 5^3 \\
&= [T(1) + 2^3] + 3^3 + 4^3 + 5^3 \\
&= 1 + 2^3 + 3^3 + 4^3 + 5^3
\end{aligned}
$$

$$
\begin{aligned}
T(n) &= 1 + 2^3 + 3^3 + ... + n^3 \\
&= [\frac{n(n+1)}{2}]^2
\end{aligned}
$$

4.3) $\text{T(n)} = \text{T}\left(\frac{\text{n}}{2}\right) + 1$ where $\text{T}(1) = 1$.

$$
\begin{aligned}
T(32) &= T(16) + 1 \\
&= [T(8) + 1] + 1 \\
&= [T(4) + 1] + 1 + 1 \\
&= [T(2) + 1] + 1 + 1 + 1 \\
&= [T(1) + 1] + 1 + 1 + 1 + 1 \\
&= 1 + 1 + 1 + 1 + 1 + 1
\end{aligned}
$$

$$
\text{T(n)} = \log_2 n \quad , \quad where\ n = 2^k
$$

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4.4) $T(n) = T\left(\frac{n}{2}\right) + n$ where $T(1) = 1.$

$$
\begin{aligned}
T(32) &= T(16) + 32 \\
&= [T(8)+16] + 32 \\
&= [T(4) + 8]+16+32 \\
&= [T(2) + 4] + 8+16+32 \\
&= [T(1) + 2] + 4 + 8+16+32 \\
&= 1 + 2 + 4 + 8+16+32 \\
&= 1 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5
\end{aligned}
$$

$$
\begin{aligned}
S &= 1 + 2^1 + 2^2 + 2^3 + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,. + 2^n \\
2S &= \quad 2^1 + 2^2 + 2^3 + \,.\,.\,.\,.\,.\,.\,.\,.\,.\,. + 2^n + 2^{n+1} \\
2S &= S - 1 + 2^{n+1} \\
S &= 2^{n+1} - 1 \\
T(n) &= 2^{n+1} - 1 \\
&= 2(n) - 1 \qquad ;\text{ since } 2^n = n
\end{aligned}
$$

4.5) $T(n) = 2 \times T\left(\frac{n}{2}\right) + n$ where $T(1) = 1.$

$$
\begin{aligned}
T(2^5) &= 2 \times T(2^4) + 2^5 \\
&= 2 \times [2 \times T(2^3) + 2^4] + 2^5 \\
&= 2^2 \times T(2^3) + 2^5 + 2^5 \\
&= 2^2 \times [2 \times T(2^2) + 2^3] + 2^5 + 2^5 \\
&= 2^3 \times T(2^2) + 2^5 + 2^5 + 2^5 \\
&= 2^3 \times [2 \times T(2^1) + 2^2] + 2^5 + 2^5 + 2^5 \\
&= 2^4 \times T(2^1) + 2^5 + 2^5 + 2^5 + 2^5 \\
&= 2^4 \times [2 \times T(2^0) + 2^1] + 2^5 + 2^5 + 2^5 + 2^5 \\
&= 2^5 \times T(2^0) + 2^5 + 2^5 + 2^5 + 2^5 + 2^5 \\
&= 2^5 + 2^5 + 2^5 + 2^5 + 2^5 + 2^5 \\
T(n) &= (\log_2 n + 1)\mathrm{n} \qquad , \qquad where\ n = 2^k
\end{aligned}
$$

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4.6) $\mathrm{T(n)} = 2 \times \mathrm{T}\left(\frac{\mathrm{n}}{2}\right) + 1$ where $\mathrm{T}(1) = 1$.

$$T(2^5) = 2 \times T(2^4) + 1$$
$$= 2 \times [2 \times T(2^3) + 1] + 1$$
$$= 2^2 \times T(2^3) + 2 + 1$$
$$= 2^2 \times [2 \times T(2^2) + 1] + 2^1 + 1$$
$$= 2^3 \times T(2^2) + 2^2 + 2^1 + 1$$
$$= 2^3 \times [2 \times T(2^1) + 1] + 2^2 + 2^1 + 1$$
$$= 2^4 \times T(2^1) + 2^3 + 2^2 + 2^1 + 1$$
$$= 2^4 \times [2 \times T(2^0) + 1] + 2^3 + 2^2 + 2^1 + 1$$
$$= 2^5 \times T(2^0) + 2^4 + 2^3 + 2^2 + 2^1 + 1$$
$$= 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 1$$

$$\mathrm{S} = 2^0 + 2^1 + 2^2 + \ldots\ldots\ldots\ldots + 2^n$$
$$2\mathrm{S} = 2^1 + 2^2 + 2^3 + \ldots\ldots\ldots\ldots + 2^{n+1}$$
$$2\mathrm{S} = S - 1 + 2^{n+1}$$
$$\mathrm{T(n)} = 2^{n+1} - 1$$
$$= 2(n) - 1$$

4.7) $\mathrm{T(n)} = 2 \times \mathrm{T}\left(\frac{\mathrm{n}}{2}\right) + \log_2 n$ where $\mathrm{T}(1) = 1$.

$$T(2^5) = 2 \times T(2^4) + \log_2 2^5$$
$$= 2 \times T(2^4) + 5$$
$$= 2 \times [2 \times T(2^3) + \log_2 2^4] + 5$$
$$= 2^2 \times T(2^3) + 2(4) + 5$$
$$= 2^2 \times [2 \times T(2^2) + \log_2 2^3] + 2(4) + 5$$
$$= 2^3 \times T(2^2) + 2^2(3) + 2(4) + 5$$
$$= 2^3 \times [2 \times T(2^1) + \log_2 2^2] + 2^2(3) + 2(4) + 5$$
$$= 2^4 \times T(2^1) + 2^3(2) + 2^2(3) + 2(4) + 5$$
$$= 2^4 \times [2 \times T(2^1) + \log_2 2^1] + 2^3(2) + 2^2(3) + 2(4) + 5$$

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$$= \quad 2^5 \times T(2^1) + 2^4(1) + 2^3(2) + 2^2(3) + 2(4) + 5$$

$$= \quad 2^5 + 2^4(1) + 2^3(2) + 2^2(3) + 2^1(4) + 2^0(5)$$

$$T(n) \quad = \quad \sum_{i=0}^{n} 2^i(n-i)$$

$$= \quad \sum_{i=0}^{n} 2^i(n) - 2^i(i)$$

$$= \quad \sum_{i=0}^{n} 2^i(n) - \sum_{i=0}^{n} 2^i(i)$$

$$= \quad (n)\sum_{i=0}^{n} 2^i - \sum_{i=0}^{n} 2^i(i)$$

$$= \quad (n)(2^{n+1} - 1) - \sum_{i=0}^{n} 2^i(i)$$

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Problem 5

5.1) f(n) = 30n + 900 log n, g(n) = n

<u>Sol</u> $\lim_{n\to\infty} \frac{30\text{n} + 900\log\text{n}}{n}$ = 30

1) $f \sim ?\, g$ ×
2) $f \in ?\, O(g)$ ✓
3) $f \in ?\, o(g)$ ×
4) $f \in ?\, \omega(g)$ ×
5) $f \in ?\, \Omega(g)$ ✓
6) $f \in ?\, \Theta(g)$ ✓

5.2) f(n) = log n, g(n) = n

<u>Sol</u> $\lim_{n\to\infty} \frac{\log \text{n}}{n}$ = 0

1) $f \sim ?\, g$ ×
2) $f \in ?\, O(g)$ ✓
3) $f \in ?\, o(g)$ ✓
4) $f \in ?\, \omega(g)$ ×
5) $f \in ?\, \Omega(g)$ ×
6) $f \in ?\, \Theta(g)$ ×

5.3) f(n) = $\log_2 n$, g(n) = n

<u>Sol</u> $\lim_{n\to\infty} \frac{\log_2 n}{n}$ = 0

1) $f \sim ?\, g$ ×
2) $f \in ?\, O(g)$ ✓
3) $f \in ?\, o(g)$ ✓
4) $f \in ?\, \omega(g)$ ×
5) $f \in ?\, \Omega(g)$ ×
6) $f \in ?\, \Theta(g)$ ×

5.4) f(n) =$\sqrt{n}$, g(n) = 99log n^9

<u>Sol</u> $\lim_{n\to\infty} \frac{\sqrt{n}}{99\log n^9}$ = ∞

1) $f \sim ?\, g$ ×
2) $f \in ?\, O(g)$ ×
3) $f \in ?\, o(g)$ ×
4) $f \in ?\, \omega(g)$ ✓
5) $f \in ?\, \Omega(g)$ ✓
6) $f \in ?\, \Theta(g)$ ×

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5.5) f(n) = $n2^n$, g(n) = n

Sol $\lim_{n\to\infty} \frac{n2^n}{n}$ = ∞

1) $f \sim ?\, g$ ✗
2) $f \in ?\, O(g)$ ✗
3) $f \in ?\, o(g)$ ✗
4) $f \in ?\, \omega(g)$ ✓
5) $f \in ?\, \Omega(g)$ ✓
6) $f \in ?\, \Theta(g)$ ✗

5.6) f(n) = n^2 , g(n) = 1.0000000001^n

Sol $\lim_{n\to\infty} \frac{n^2}{1.0000000001^n}$ = ∞

1) $f \sim ?\, g$ ✗
2) $f \in ?\, O(g)$ ✗
3) $f \in ?\, o(g)$ ✗
4) $f \in ?\, \omega(g)$ ✓
5) $f \in ?\, \Omega(g)$ ✓
6) $f \in ?\, \Theta(g)$ ✗

5.7) f(n) = 200000, g(n) = 1

Sol $\lim_{n\to\infty} \frac{200000}{1}$ = 200000

1) $f \sim ?\, g$ ✗
2) $f \in ?\, O(g)$ ✓
3) $f \in ?\, o(g)$ ✗
4) $f \in ?\, \omega(g)$ ✗
5) $f \in ?\, \Omega(g)$ ✓
6) $f \in ?\, \Theta(g)$ ✓

5.8) f(n) = 2^n , g(n) = 10^n

Sol $\lim_{n\to\infty} \frac{2^n}{10^n}$ = 0

1) $f \sim ?\, g$ ✗
2) $f \in ?\, O(g)$ ✓
3) $f \in ?\, o(g)$ ✓
4) $f \in ?\, \omega(g)$ ✗
5) $f \in ?\, \Omega(g)$ ✗
6) $f \in ?\, \Theta(g)$ ✗

Problem 6

6.1) $T_{i+1} = 5T_i - 6T_{i-1};\ T_0 = 8, T_1 = 17$

Guess: a^n

$T_{guess}(n)$ = a^n

then; a^n = $5(a^{n-1}) - 6(a^{n-2})$

$\times a^2$; a^{n+2} = $5(a^{n+1}) - 6(a^n)$

a^2 = $5(a^1) - 6(1)$

$a^2 - 5a + 6$ = 0

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$(a-3)(a-2) = 0$

$a = 3$

b = 2

Then we have

1. $T_{guess}(n) = 3^n$

2. $T_{guess}(n) = 2^n$

Try 1. first:

$3^n =? 5(3^{n-1}) - 6(3^{n-2})$

$=? 3^n[\left(\frac{5}{3}\right) - \frac{6}{9}]$

$=? 3^n[\left(\frac{5}{3}\right) - \frac{2}{3}]$

$=? 3^n$ ✓

Try 2. first:

$2^n =? 5(2^{n-1}) - 6(2^{n-2})$

$=? 2^n[\left(\frac{5}{2}\right) - \frac{6}{4}]$

$=? 2^n[\left(\frac{5}{2}\right) - \frac{3}{2}]$

$=? 2^n$ ✓

$T_{last\_guess}(n) = A3^n + B2^n$

$T_{last\_guess}(0) = 8 = A3^0 + B2^0 = A + B \quad -①$

$T_{last\_guess}(1) = 17 = A(3^1) + B(2^1) = 3A + 2B \quad -②$

①×3; $3A + 3B = 24 \quad -③$

③-②; $B = 7$, $A = 1$

$T_{best\_guess}(n) = 3^n + 7(2^n)$

$T_{best\_guess}(0) = 3^0 + 7(2^0) = 8$ ✓

$T_{best\_guess}(1) = 3^1 + 7(2^1) = 17$ ✓

$T_{best\_guess}(2) = 3^2 + 7(2^2) = 37$ ✓

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6.2) $T_{i+1} = -5T_i + 12T_{i-1};\ T_0 = 0, T_1 = -7$

$Guess: \quad a^n$

$T_{guess}(n) = \ a^n$

Then; $a^n = -5a^{n-1} + 12a^{n-2}$

$\times a^2;\quad a^{n+2} = -5a^{n+1} + 12a^n$

$a^2 = -5a^1 + 12$

$a^2 + 5a - 12 = 0$

$$a = \frac{-5 \pm \sqrt{25-4(1)(-12)}}{2}$$

$$a = \frac{-5 \pm \sqrt{73}}{2}, \quad a = \frac{-5+\sqrt{73}}{2}$$

$$b = \frac{-5-\sqrt{73}}{2}$$

Then we have,

$$a^n + 5a^{n-1} - 12a^{n-2} = 0$$

$$\left[\frac{-5+\sqrt{73}}{2}\right]^n + 5\left[\frac{-5+\sqrt{73}}{2}\right]^{n-1} - 12\left[\frac{-5+\sqrt{73}}{2}\right]^{n-2} = 0 \quad , \quad a^n = \left[\frac{-5+\sqrt{73}}{2}\right]^n$$

$$\left[\frac{-5-\sqrt{73}}{2}\right]^n + 5\left[\frac{-5-\sqrt{73}}{2}\right]^{n-1} - 12\left[\frac{-5-\sqrt{73}}{2}\right]^{n-2} = 0 \quad , \quad b^n = \left[\frac{-5-\sqrt{73}}{2}\right]^n$$

$T_a(0) = 0$ ✓ (By calculator), $\quad T_a(1) = 0$ ✗ (By calculator)

$T_b(0) = 0$ ✓ (By calculator), $\quad T_b(1) = 0$ ✗ (By calculator)

We know

$T_a = a^n \ solves$

$T_a(n+1) + 5[T_a(n)] - 12[T_a(n-1)] = 0$

$T_b = b^n \ solves$

$T_b(n+1) + T_b[T(n)] - 12[T_b(n-1)] = 0$

$$T_a + T_b = A(T_a) + B(T_b)$$

$$= T_{a+b}(n+1) + 5[T_{a+b}(n)] - 12[T_{a+b}(n-1)]$$

$= A[T_a(n+1)] + B[T_b(n+1)] + 5A[T_a(n)] + 5B[T_b(n) - 12A[T_a(n-1)] - 12B[T_b(n-1)]]$

$= 0$

Guess:

$$T(n) = A\left[\frac{-5+\sqrt{73}}{2}\right]^n + B\left[\frac{-5-\sqrt{73}}{2}\right]^n$$

$$T(0) = 0 = A + B \quad -①$$

$$T(1) = -7 = A\left[\frac{-5+\sqrt{73}}{2}\right]^1 + B\left[\frac{-5-\sqrt{73}}{2}\right]^1 \quad -②$$

$$①\times\left[\frac{-5+\sqrt{73}}{2}\right]; \quad 0 = A\left[\frac{-5+\sqrt{73}}{2}\right] + B\left[\frac{-5+\sqrt{73}}{2}\right] \quad -③$$

$$②-③; \quad -7 = B\left[\left[\frac{-5-\sqrt{73}}{2}\right] + \left[\frac{5-\sqrt{73}}{2}\right]\right]$$

$$-7 = B(-\sqrt{73})$$

$$B = \frac{7}{\sqrt{73}}$$

$$A = -\frac{7}{\sqrt{73}} \quad \text{From ①; } A + B = 0$$

$$\text{Then; } T(n) = -\frac{7}{\sqrt{73}}\left[\frac{-5+\sqrt{73}}{2}\right]^n + \frac{7}{\sqrt{73}}\left[\frac{-5-\sqrt{73}}{2}\right]^n$$

$T(0) = 0$ ✓ (By calculator)

$T(1) = -7$ ✓ (By calculator)