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## DISCRETE MATHEMATICS

# Final Exam T1 2016

**Instruction**

- Write your name
- Read the questions carefully.
- You have 4 hours to finish the exam.
- There are 5 problems. 700 points in total. You only need to get 630 points to get full score.
- Attempt all problems, state your reasons *clearly* and *legibly*, because partial credits will be given.

Question	Full Score	Your Score
1	100	
2	120	
3	140	
4	160	
5	180	

Total:  /630

ID: \_\_\_\_\_

Name: \_\_\_\_\_

## Useful Formulas

### Sum

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2$$

$$1 + 3 + 5 + 7 + \dots + (2n-1) = n^2$$

### Euler's Formula

$$e + 2 = v + f$$

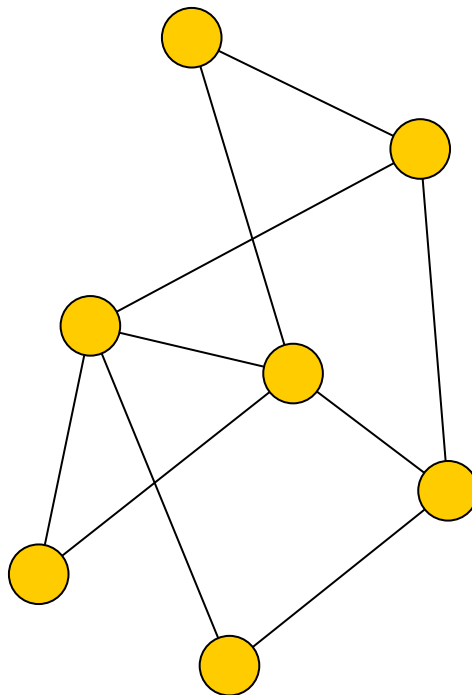
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Name: \_\_\_\_\_

1. Graph Theory(100 points. 20 each)

(a) Drawing Graph Draw a tree with exacty 5 edges and exactly 3 leaves. Please do not troll.

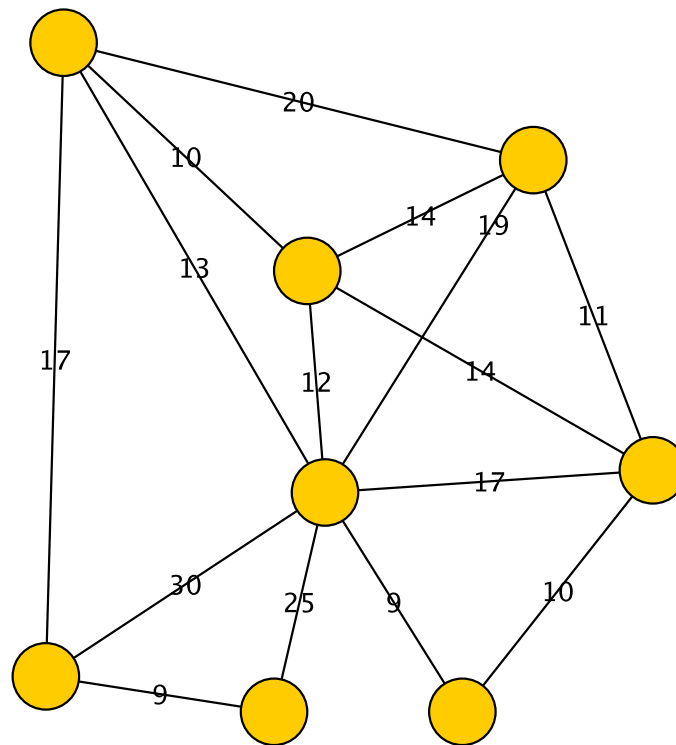
(b) Find an Euler walk for the following graph. Label the edges with *numbers* so I can follow.



ID: \_\_\_\_\_

Name: \_\_\_\_\_

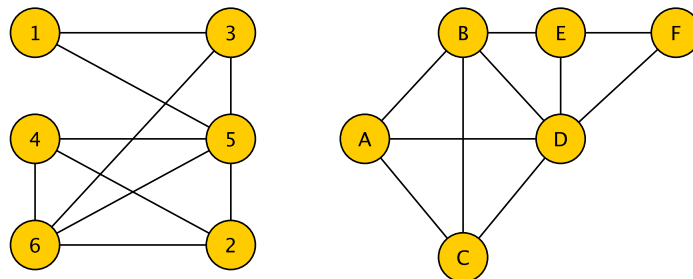
- (c) The following graph shows the cost of electric wire to connect one city to another(in millions). If your job is to connect every city to electricity grid at the lowest cost, how much money do you need?



- (d) Suppose AJ wants to do a secured take home exam. He came up with the following scheme. He first list out the list of students and find out who is friend with whom. If they are friend and AJ give out the same version of the exam to both there is a chance that they will copy each other. AJ could give out different exam for everyone but that would interfere with AJ gaming schedule too much. Use something you learn from graph theory to figure out the number of minimum different version exam paper he needs to make. (1 means friend 0 means not friend)

	A	B	C	D	E	F	G	H
A	0	0	1	0	0	1	1	0
B	0	0	0	1	0	0	1	1
C	1	0	0	1	1	0	0	1
D	0	1	1	0	0	1	0	0
E	0	0	1	0	0	0	1	1
F	1	0	0	1	0	0	1	1
G	1	1	0	0	1	1	0	1
H	0	1	1	0	1	1	1	0

- (e) Find an isomorphism between these two graphs.



2. (20 Each) Pokemon go is a game where each player go out, walk and collect pokemon. When a pokemon is caught each pokemon is assigned a Combat Power(CP) value. The higher the better.

Each pokemon even of the same kind do not come equal. Each pokemon specie has different distribution of CP.

- Charmander has mean CP of 700 with variance of 2500.
- Squirtle has mean CP of 500 with variance of 4900.

Moreover, each pokemon can evolve

- When a Charmander evolves to Charmeleon the CP of the new evolved pokemon is 2 times the old one. For example, Charmander of CP 750 after evolve will become Charmeleon of CP  $2 \times 750 = 1500$
  - When a Squirtle evolves to Wartortle the CP is multiplied by 3.
- (a) If we keep picking 2 Charmander and 3 Squirtle to form a team what would be the mean of total CP.

- (b) If we keep picking 2 Charmander and 3 Squirtle to form a team what would be the variance of total CP.

- (c) What is the expected value of CP for Charmeleon (Evolved Charmander)?

- (d) What is the variance of CP for Charmeleon (Evolved Charmander)?

ID: \_\_\_\_\_

Name: \_\_\_\_\_

(e) What is the expected value for total CP of a team of 1 Charmeleon and 1 Wartortle. (Both are evolved pokemon.)

(f) What is the variance for total CP of a team of 1 Charmeleon and 1 Wartortle. (Both are evolved pokemon.)

ID: \_\_\_\_\_

Name: \_\_\_\_\_

3. (20 Each) Random Counting Problem. In the game Overwatch, there are 4 classes for heroes to choose from.

- 7 Offensive heroes. 4 male and 3 female.
- 6 Defensive heroes 3 male, 2 female and a robot.
- 5 Tank heroes. 3 male, 2 female.
- 5 Support heroes. 3 female, 1 male and a robot.

A team is formed by choosing non identical 6 heroes. (Any permutation of the same heroes pick is count as one.)

(a) How many ways are there to form a 6 heroes team?

(b) How many ways are there to form an all female team?

(c) How many ways are there to form a 3 male, 3 female team?

(d) How many ways are there to form team with 2 offenses, 2 tanks and 2 supports?



- (e) If we want to form a team that satisfy at least one of the following
- Have exactly one robot on a team.
  - Or, have exactly one female on the team.
  - Or, have exactly one male on the team.

How many ways are there to form a team with condition shown?

- (f) To get into a competitive ranking mode, every player needs to play 10 placement matches. The result for each match could be one of the four: a win, a lose, a draw or a leave. At the end of the placement, people are into rank based on their result, eg. (4 win, 1loss, 4 draw, 1 leave),(4 win , 5 loss, 1 draw, 0 leave) regardless of the order. How many end results are there?
- (g) Leaver is a problem and it ruins everyone's game. So after third leave the player is banned from competitive match. So, the end result like 0 win, 5 lose, 1 draw and 4 leave does not exists. How many end results are there that has strictly less than three leaves?

## 4. (20 each)

Each medical test has pro and cons. Extremely specific (very few false positive) medical test is typically very expensive while a less accurate one costs less. So typically what we do is to use less accurate and less expensive one as a screening test administered to wider population.

In this problem, we are going to examine a two-step medical test for a disease that 20% of the population have the disease. **First we screen the people using less expensive test and for those whose test come out positive we are going to get them to a more expensive test. If the first test says no, then no more test is required. The person is confirmed positive if *both* tests say yes.**

Here are the information about the two tests.

### Screening Test

- Costs 100 Baht per person
- Specification
  - If you have the disease, there is a 90% chance the test says yes.
  - if you do not have the disease, there is a 30% chance the test says yes.

### Expensive Test

- The result is independent to screening test result.
- Costs 5000 Baht per person.
- Specification
  - If you have the disease there is a 95% chance the test says yes.
  - If you do not have the disease, there is a 5% chance the test says yes.

(a) If you have the disease, what is the probability that you are confirmed positive.

(b) If you do not have the disease what is the probability that you are confirmed positive.

ID: \_\_\_\_\_

Name: \_\_\_\_\_

- (c) Given that you are confirmed positive what is the probability that you have the disease?
  
  
  
  
  
  
  
  
  
  
- (d) Given that you are *not* confirmed positive what is the probability that you *do not* have the disease?
  
  
  
  
  
  
  
  
  
  
- (e) If you have the disease, what is the amount of money you are expected to pay as testing fee?
  
  
  
  
  
  
  
  
  
  
- (f) If you have the disease what is the *variance* of total medical test fee?
  
  
  
  
  
  
  
  
  
  
- (g) If you do not have the disease what is the amount of money you are expected to pay as testing fee?
  
  
  
  
  
  
  
  
  
  
- (h) If we switch the two tests, that is to have people go through expensive test first then cheaper one. Find the new false negative rate. (False negative is the probability that you have the disease but the process says no.)

5. (20 Each) There are 9 questions for this problem. A popular activity to do at new year in an office is secret santa. The goal is to get everyone in the office to buy present for someone while the receiver have no idea who bought it. To do this one needs to figure out a way to assign who buy present for whom. For this problem, we will consider the case where we have 3 people in the office:  $A$ ,  $B$  and  $C$ . We are going to consider two strategies.

**Strategy A. Random drawing.**

The first strategy for assigning the buyer and receiver is random drawing.

- Everyone write their own name on a piece of paper and put it in a bowl.
- Everyone draw one paper from the bowl. No one open the piece of paper yet.
- Every unseal the paper at the same time to reveal whom they have to buy a present for.
- If there is at least one person getting their own name, *everyone* put the paper back in the bowl and redraw.
- This keeps repeating until no one gets their own name.

Answer the following questions.

- (a) What is the probability that we need to do a redraw because at least one person draw their own name?
- (b) What is the expected number of times we need to redraw until no one gets their own name?

ID: \_\_\_\_\_

Name: \_\_\_\_\_

- (c) For each iteration, what is the expected number of people drawing out their own name?
- (d) What is the probability that  $C$  will get to buy the present for  $A$ ?
- (e) Given that we successfully end the process. Is the event that  $A$  gets to buy present for  $B$  and  $B$  gets to buy present for  $A$  independent?

**Strategy B. Drawing one at a time.**

- Everyone write their own name on a piece of paper and put it in a bowl.
  - $A$  draws a paper first and unseal it. If he gets his own name, he will keep redrawing until he doesn't get his own name.
  - Then,  $B$  draws a paper and unseal it. If he gets his own name, he will keep redrawing until he doesn't get his own name.
  - Then,  $C$  draws the last piece of paper and unseal it. If he gets his own name. Then the process fails and we repeat all over again.
- (a) What is the probability that  $C$  will have no choice but picking his own name?

ID: \_\_\_\_\_

Name: \_\_\_\_\_

(b) What is the probability that  $A$  will get to buy present for  $C$ .

(c) What is the probability that  $C$  will get to buy present for  $B$  needs to redraw?  
(Be careful)

(d) What is the probability that  $C$  will get to buy present for  $A$ ? (Be careful)