Thm. $n^3 \leq 3^n$....

Predicate: $P(s) = s^3 \leq 3^s$.

Base case: $k=3$. $3^3 \leq 3^5$ ✓

Inductive Step: Let us assume that $\cancel{\forall}$ $k^3 \leq 3^k$. $\exists k \geq 3$.

we want to show that $(k+1)^3 \leq 3^{k+1}$

Consider $(k+1)^3 \underset{\text{Alg}}{=} k^3 + 3k^2 + 3k + 1 \underset{\text{by IH}}{\leq} 3^k + 3k^2 + 3k + 1$ — ①

(A) consider $3k^2$

we know that $3 \leq k$, $\cancel{k \geq 3}$ $\Rightarrow 3k^2 \overset{\text{mult } k \text{ on both sides.}}{\leq} k^3 \underset{\text{by IH.}}{\leq} 3^k$

(B) consider $3k+1$

we know that $3k+1 \underset{\substack{\text{since} \\ k \geq 1}}{\leq} 3k+k \underset{\text{Alg.}}{\leq} 4k \underset{k>0}{\leq} 9k \underset{k \geq 3.}{\leq} k^3 \underset{\text{by IH.}}{\leq} 3^k$

① $(k+1)^3 \leq 3^k + 3k^2 + 3k + 1$

$\leq 3^k + 3^k + 3k + 1$ by (A)

$\leq 3^k + 3^k + 3^k$ by (B)

$\leq 3 \cdot 3^k$

$\leq 3^{k+1}$

$\therefore (k+1)^3 \leq 3^{k+1}$

Thus, by mathematical induction, $n^3 \leq 3^n$ for $n \geq 3$.

So.. fill in $1 \leq 3^1$ and $2 \leq 3^2$ $\therefore$ $n^3 \leq 3^n$ for $n \geq 1$.