Homework 9

Last updated: Wednesday 15th July, 2015 16:09

Problem 1 Mine Sweeper Theorem.

Mine Sweeper is game that is played on a rectangular grid the bombs randomly place on grid. The goal is to uncover all the cell that has no bomb. You can uncover one cell at a time. If you uncover a cell and found a bomb behind it then it is game over. Every time you uncover a space with no bomb a number indicating how many bombs are there in the adjacent cells (there are 8 of them including the diagonal one).

We are not trying to solve mine sweeper here. We are interested in a very interesting property of the grid.

Let us consider the following 5×5 grid.

1	1	2	2	💣
3	💣	4	💣	2
💣	💣	💣	3	2
2	3	3	💣	2
0	0	1	2	💣

The sum of the hints is

$$3 \times 1 + 7 \times 2 + 4 \times 3 + 1 \times 4 = 33.$$

Now, let us consider the complement of this grid where we replace bomb cell to all non-bomb cell and all non-bomb cell to bomb cell. The hint numbers are recalculated accordingly.

💣	💣	💣	💣	2
💣	5	💣	6	💣
3	5	4	💣	💣
💣	💣	💣	6	💣
💣	💣	💣	💣	2

The sum of the hints on this complement board is

$$2 \times 2 + 1 \times 3 + 1 \times 4 + 2 \times 5 + 2 \times 6 = 33$$

Magic!. Your job to make it less magical. Show that this trick always works: the sum of the hints of any mine sweeper board and its complement are the same.

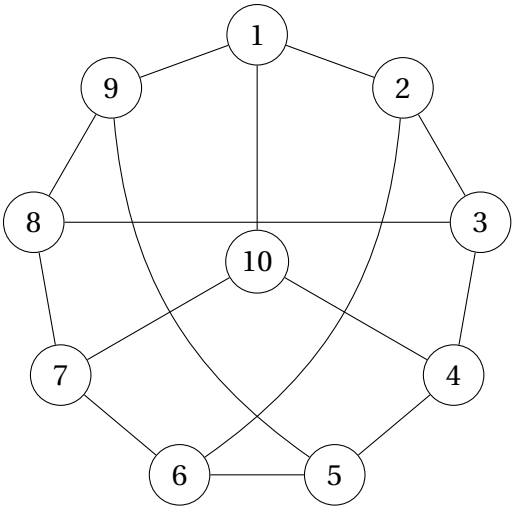
Hint: This is similar to facebook problem we discussed.

Answer: If we draw edges between non bomb cell and adjacent bomb cell. The number on the grid is the degree of the non bomb node. The sum is then the number of edge.

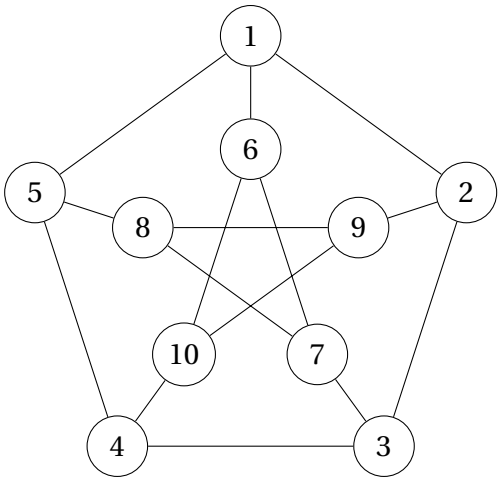
When we invert the grid all the edge stays the same. The number is the degree for each bomb cell. The sum is still the same number of edges.

Problem 2 Isomorphism. (Stolen from MIT's Note).

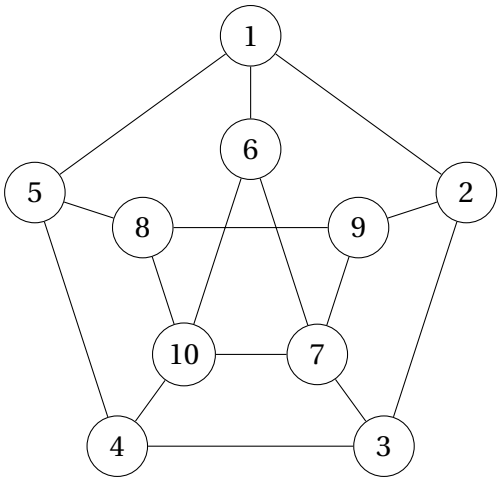
Which pair of the follwing graphs are isomorphic? If so, find an isomorphism between the two. If not, explain briefly why.



G_1



G_2



G_3

Answer: G_1 and G_2 are isomorphic. The isomorphism $G_1 \to G_2$ is given by $1 \to 1, 2 \to 23 \to 3, 4 \to 7, 10 \to 69 \to 5, 8 \to 4, 7 \to 10, 5 \to 8$

Problem 3 Suppose we want to hold couple meetings among executives.

Meeting	Members
A	{Smith, Jones, Brown, Green}
B	{Jones, Wagner, Chase}
C	{Harris, Oliver}
D	{Harris, Jones, Mason}
E	{Oliver, Cummings, Larson}

We want to schedule the meeting such that there is no time conflict for any member. Each member can attend one meeting at a time and each meeting lasts 1 hour. We want to get all these meeting done as soon as possible. How many hours do we need and how should we schedule the meetings?

Answer:3 hours. Use graph coloring. Each meeting is a node and there is an edge between the two if the two meeting require the same person to attend.

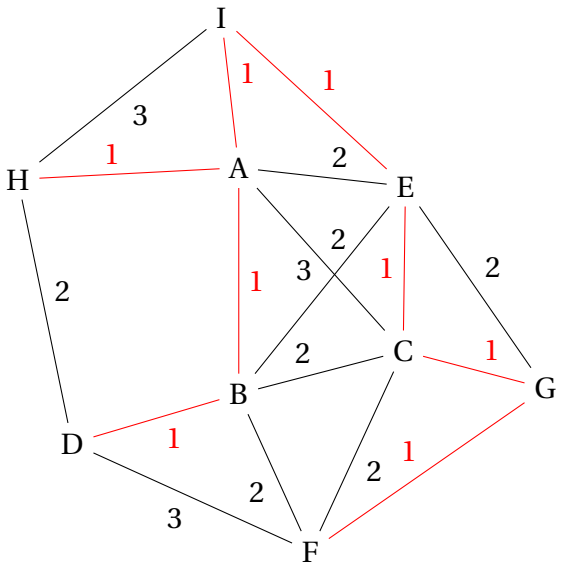
Problem 4 DNA Sequence. Suppose that you perform shotgun sequencing and found that you got the following subsequence

$$\{CAT, TCA, AAG, GCT, GCA, ATC, CAA, AGC\}$$

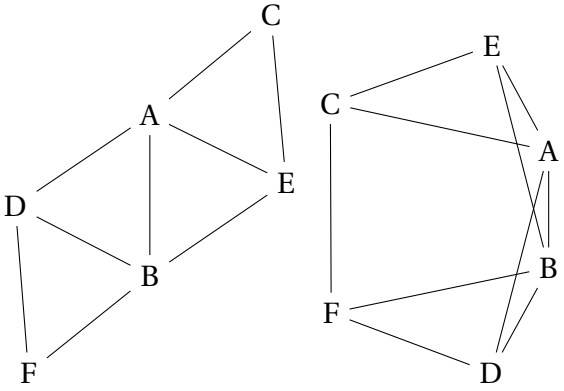
What is the original sequence?

Answer: See lecture notes. GCATCAAGCT.

Problem 5 Yoyo Distribution Network. Imagine you are now the CEO of Yoyo company. You you want to distribute Yoyo to all distribution center at the cheapest cost. Here is what your distribution network look like along with the cost for transfering on each route. Find the optimal way to distribute the Yoyo. Note AC is of cost 2 and AD is of cost while EB is of cost 3.



Problem 6 One Stroke Game. For each of the following graph show how to draw each one of them with one stroke. If it is not possible state also why it is impossible.



Answer: For the left one start at D end at E. The right one is not possible since there are more than 2 vertex of odd degree.