CS208/204: Data Structures & Abstractions
Mahidol University International College

A4 Proofs

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Contents

Task 1: Missing Tile

Theorem 1.1. *Any 2^n by 2^n grid with one painted cell can be tiled using L-shaped triominoes such that the entire grid is covered by triominoes but no triominoes overlap with each other nor the painted cell.*

1.1 Proof by induction.

1.1.1 Predicate

$P(n)$ is true when a triminoe can fit into a 2^n by 2^n grid where once cell is painted out.

1.1.2 Basis Step

$P(1)$ is true because for all the possible 2^1 by 2^1 grids with a painted out cell, triminoes can tile as shown in Figure 2.1.

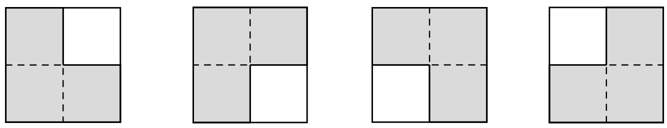


Figure 1.1: All possible triminoe placement if a single cell was painted out

1.1.3 Inductive Step

Lets assume that $P(k)$ is true for all positive interger values of k.

Want to show when $P(k)$ is true, then $P(k+1)$ is true also. Consider a $P(k+1)$ grid that is subdivided into equal quadrants vertically and horizontally. Notice that each quadrant of $P(k+1)$ is $P(k)$.Then we paint out 1 cell.

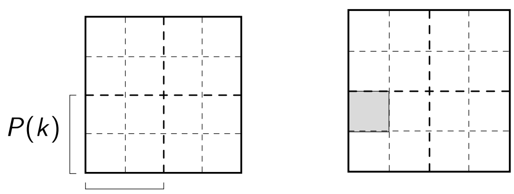


Figure 1.2: $P(k+1)$

At this stage its not apparent if the triminoes can fill the rest of the grid. We need to somehow introduce cases made in basis step into the quadrants.

Let's paint in the middle cells, only the quadrants that havn't been painted in yet. This achieves two things, firstly this achieves placing a triminoe and secondly each quadrant now has 1 cell painted out.

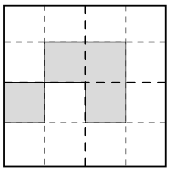


Figure 1.3: Painting out the middle cells.

Since all quadrants now satisfy the conditions of $P(k)$ which is having 1 cell painted out and we know from the basis step that every quadrant is tileable we can conclude through inuctive hypothesis that $P(k+1)$ is true too. This completes the inductive step.

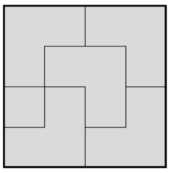


Figure 1.4: Tiling the quadrants using the basis step cases.

1.1.4 Conclusion

Since we can show that when $P(k)$ is true then $P(k+1)$ is true we can conclude by mathmatical induction that $P(n))$ is true for all positive integer values of n.

Task 4: Sum Square

Theorem 2.1. *Any 2^n by 2^n grid with one painted cell can be tiled using L-shaped triominoes such that the entire grid is covered by triominoes but no triominoes overlap with each other nor the painted cell.*

2.1 Proof by induction.

2.1.1 Predicate

$P(n)$ is true when a triminoe can fit into a 2^n by 2^n grid where once cell is painted out.

2.1.2 Basis Step

$P(1)$ is true because for all the possible 2^1 by 2^1 grids with a painted out cell, triminoes can tile as shown in Figure 2.1.

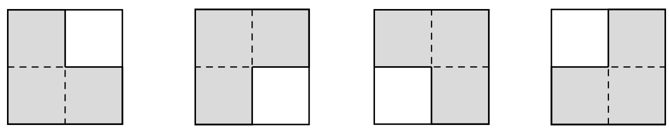


Figure 2.1: All possible triminoe placement if a single cell was painted out

2.1.3 Inductive Step

Lets assume that $P(k)$ is true for all positive interger values of k.

Want to show when $P(k)$ is true, then $P(k+1)$ is true also. Consider a $P(k+1)$ grid that is subdivided into equal quadrants vertically and horizontally. Notice that each quadrant of $P(k+1)$ is $P(k)$.Then we paint out 1 cell.

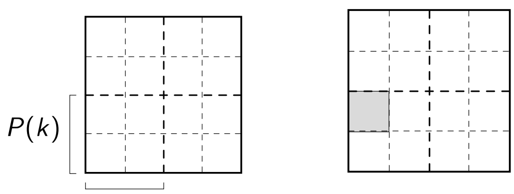
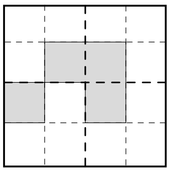


Figure 2.2: $P(k+1)$

At this stage its not apparent if the triminoes can fill the rest of the grid. We need to somehow introduce cases made in basis step into the quadrants.

Let's paint in the middle cells, only the quadrants that havn't been painted in yet. This achieves two things, firstly this achieves placing a triminoe and secondly each quadrant now has 1 cell painted out.



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Since all quadrants now satisfy the conditions of $P(k)$ which is having 1 cell painted out and we know from the basis step that every quadrant is tileable we can conclude through inuctive hypothesis that $P(k+1)$ is true too. This completes the inductive step.

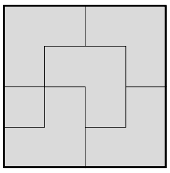

Figure 2.4: Tiling the quadrants using the basis step cases.

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Since we can show that when $P(k)$ is true then $P(k+1)$ is true we can conclude by mathmatical induction that $P(n))$ is true for all positive integer values of n.