ICMA393 Discrete Simulation: HW1

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1. Birthday Problem

The simulations ran were set to $seed = 27$ and $N = 100000$.

a)

$$p_{10} = 0.11686$$
$$CI = [0.11685, 0.11885]$$

b)

$$p_{20} = 0.41126$$
$$CI = [0.41125, 0.41430]$$

c)

$$p_{30} = 0.70595$$
$$CI = [0.70594, 0.70877]$$

d) Found n that satisfies the condition $p_n \geq 0.5$ from running the simulation on a range of n's [0, 50] and selecting the first $p_n \geq 0.5$.

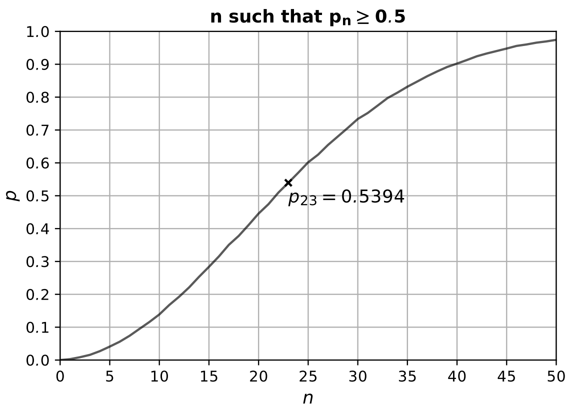

Figure 1: Closest point to $p = 0.5$ was $p_{23} = 0.5394$ which satisfies the condition.

2. Alice and Bob Play a Game

The strategy I let Alice have is the following...

Case 1: Found Exclusive Output
She presses the button recording n output values $x_1, x_2, \ldots x_n$. As she records each output she checks whether x_i is exclusive to one of the buttons output range. If x_i is exclusive she returns the corresponding button as the answer.

Case 2: No Exclusive Output
If she presses the button n times with no exclusive output appearing. She calculates $\bar{x}$ and finds the minimum difference between the mean output range of the 2 buttons. She returns the corresponding button that gets the minimum distance.

Psuedocode
Although the source code differs because loops are removed for speed, the idea is the same.

```
xs = []
for i in range(0, n):
    x_n = button_unknown.get_next_value()
    xs.append(x_n)

    # Case 1: Found exclusive output.
    if(x_n == 1)
        return 1  # it is button 1
    if(x_n == 100)
        return 2  # it is button 2

# Case 2: No exclusive output. Evaluate the
    minimum distance of means.
x_mean = sum(xs) / n
button_1_mean = (1 + 99)/2
button_2_mean = (2 + 100)/2

return argmin([abs(x_mean - button_1_mean), abs(
    x_mean - button_2_mean)]) + 1
```

Find n such that Alice is correct ≥ 0.99

Similar to problem 1 Birthday Problem, I sampled Alice's strategy over some "reasonable" range of n $[0, 500]$ and extract the n that results in probability p that Alice is correct is atleast 0.99. I set the $seed = 27$ and ran 400000 trials for all n's.

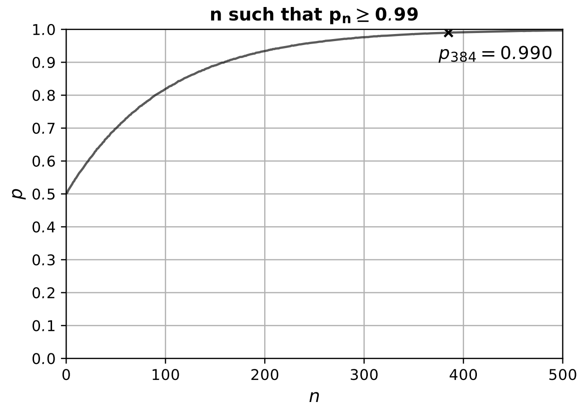

Figure 2: Closest point to p = 0.99 was p_{384} = 0.990 which satisfies the condition.

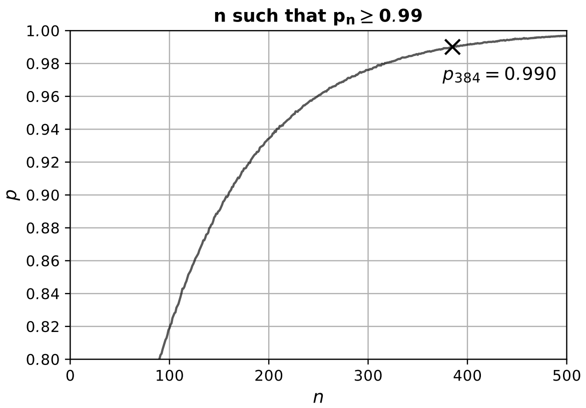

Figure 3: y-lim[0.8, 1.0] of Figure 2.

3. Practice with Uniform and Geometric Distributions

Source Code

https://github.com/AustinMaddison/discrete-simulation/tree/main/src/hw1