

# ICMA393 Discrete Simulation: HW1

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## 1. Birthday Problem

The simulations ran were set to *seed* = 27  
and  $N = 400000$ .

a)

$$p_{10} = 0.11686$$

$$CI = [0.11685, 0.11885]$$

b)

$$p_{20} = 0.41126$$

$$CI = [0.41125, 0.41430]$$

c)

$$p_{30} = 0.70595$$

$$CI = [0.70594, 0.70877]$$

d) Found  $n$  that satisfies the condition  $p_n \geq 0.5$  from running the simulation on a range of  $n$ 's  $[0, 50]$  and selecting the first  $p_n \geq 0.5$ .

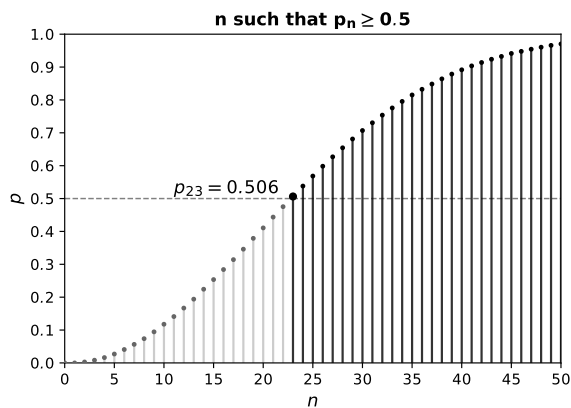


Figure 1: The first point  $\geq p = 0.5$  was  $p_{23} = 0.506$  which satisfies the condition.

e) We can model  $p_n$  using a product function. We do this by getting the product of probabilities where we always pick somebody with a unique birthday. Then we invert the result by minus 1 to get the probability of picking somebody with a non-unique birthday.

$$p(n) = 1 - \prod_{i=0}^{n-1} \frac{365 - i}{365}$$

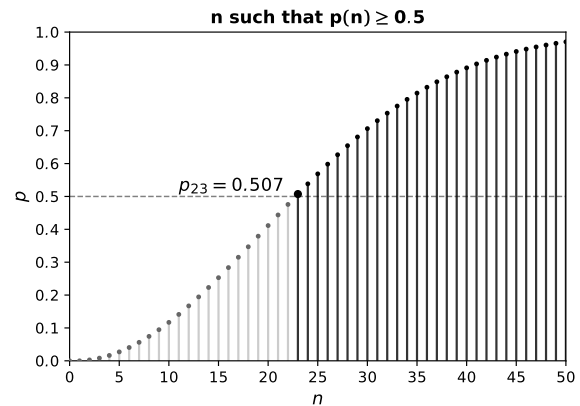


Figure 2:  $p(n)$  results are similar to the simulation. The difference between simulation  $p_n$  and  $p(n)$  is  $MSE = 2.013 \times 10^{-7}$ .

For all intensive purposes  $p(n)$  models the results in simulation  $p_n$ .

## 2. Alice and Bob Play a Game

The strategy I let Alice have is the following...

### Case 1: Found Exclusive Output

She presses the button recording  $n$  output values  $x_1, x_2, \dots, x_n$ . As she records each output she checks whether  $x_i$  is exclusive to one of the buttons output range. If  $x_i$  is exclusive she returns the corresponding button as the answer.

### Case 2: No Exclusive Output

If she presses the button  $n$  times with no exclusive output appearing. She calculates  $\bar{x}$  and finds the minimum difference between the mean output range of the 2 buttons. She returns the corresponding button that gets the minimum distance.

**Find  $n$  such that Alice is correct  $\geq 0.99$**  I sampled Alice's strategy varying  $n$  between  $[0, 500]$  and extracted the first  $n$  that results in probability atleast 0.99. I set the *seed* = 27 and ran 400000 trials for all  $n$ 's.

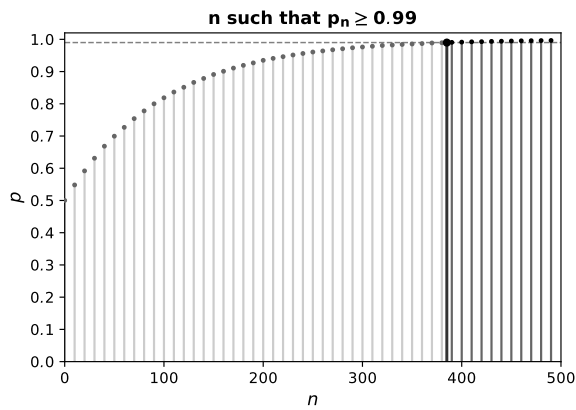


Figure 3: The first point to  $\geq p = 0.99$  was  $p_{384} = 0.990$  which satisfies the condition.

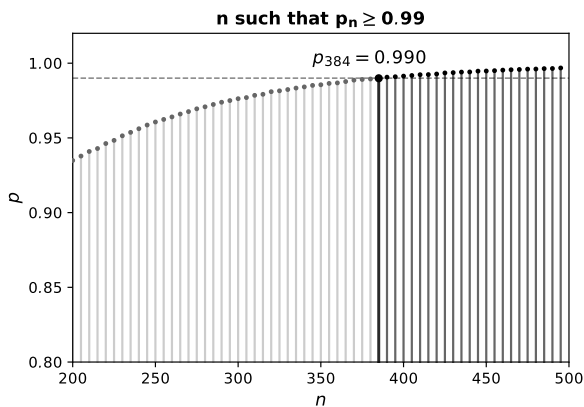


Figure 4: Crop of Figure 3.

### Pseudocode

Although the source code differs because loops are removed for speed, the idea is the same.

```
xs = []
for i in range(0, n):
    x_n = button_unknown.get_next_value()
    xs.append(x_n)

    # Case 1: Found exclusive output.
    if(x_n == 1)
        return 1 # it is button 1
    if(x_n == 100)
        return 2 # it is button 2

# Case 2: No exclusive output. Evaluate the
# minimum distance of means.
x_mean = sum(xs) / n
button_1_mean = (1 + 99)/2
button_2_mean = (2 + 100)/2

return argmin([abs(x_mean - button_1_mean), abs(
    x_mean - button_2_mean)]) + 1
```

### 3. Practice with Uniform and Geometric Distributions

First find  $E[X]$  and  $V[X]$  of the geometric distribution. Since  $X$  is a fair dice  $p\{X = 5\} = 1/6$ .

$$E[X] = \frac{1}{1/6} = 6$$

$$V[X] = \frac{1 - 1/6}{(1/6)^2} = 30$$

Then I do the same for uniform distribution of  $Y$  while equating the corresponding known values from  $X$ .

$$E[Y] = \frac{n-1}{2} = 6$$

$$V[Y] = \frac{(n^2 - 1)}{12} = 30$$

Since  $n$  is determined by the setting of  $a$  and  $b$  we can rewrite it like this.

$$E[Y] = \frac{(b-a+1)-1}{2} = 6$$

$$V[Y] = \frac{(b-a+1)^2 - 1}{12} = 30$$

Now it's easy to find  $a$  and  $b$  by solving the system of equations. However I am not going to do that because that is not how I found it at first.

What I did is rearranged  $E[Y]$  such that  $a+b = 12$ . Then I did a search of pairs of  $a, b$  st,  $a+b = 12$  and  $a < b$  until I got  $V[Y] = 30$ .

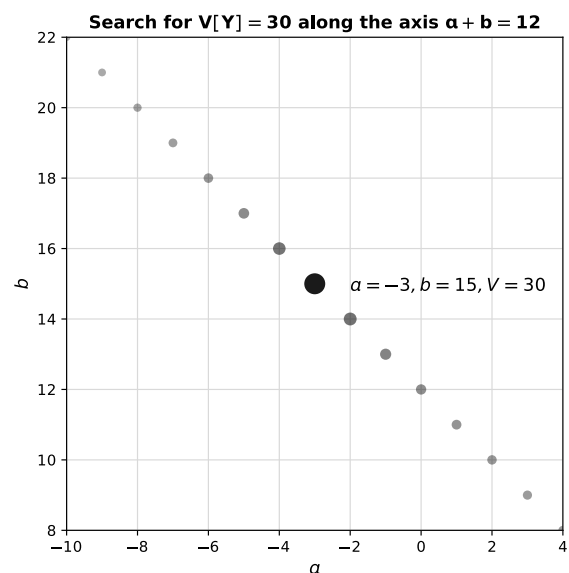


Figure 5: The size and opacity of the discs is the inverse distance away from  $V[Y] = 30$ .

For  $E[X] = E[Y]$  and  $V[X] = V[Y]$ ,  
 $a = -3, b = 15$

## Source Code

<https://github.com/AustinMaddison/discrete-simulation/tree/main/src/hw1>