

ICMA393 Discrete Simulation: HW1

Mahidol University International College
Austin Jetrin Maddison
September 28, 2024

1. Birthday Problem

The simulations ran were set to *seed* = 27
and $N = 400000$.

a)

$$p_{10} = 0.11686$$

$$CI = [0.11685, 0.11885]$$

b)

$$p_{20} = 0.41126$$

$$CI = [0.41125, 0.41430]$$

c)

$$p_{30} = 0.70595$$

$$CI = [0.70594, 0.70877]$$

d) Found n that satisfies the condition $p_n \geq 0.5$ from running the simulation on a range of n 's $[0, 50]$ and selecting the first $p_n \geq 0.5$.

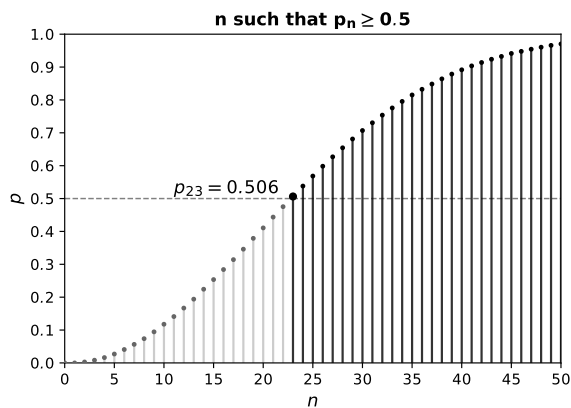


Figure 1: The first point $\geq p = 0.5$ was $p_{23} = 0.506$ which satisfies the condition.

e) We can model p_n using a product function. We do this by getting the product of probabilities where we always pick somebody with a unique birthday. Then we invert the result by minus 1 to get the probability of picking somebody with a non-unique birthday.

$$p(n) = 1 - \prod_{i=0}^{n-1} \frac{365 - i}{365}$$

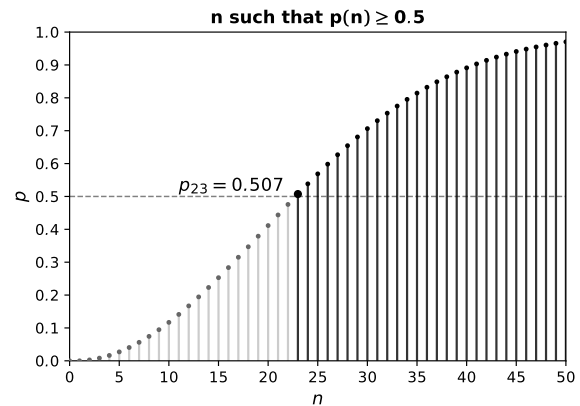


Figure 2: $p(n)$ results are similar to the simulation. The difference between simulation p_n and $p(n)$ is $MSE = 2.013 \times 10^{-7}$.

For all intensive purposes $p(n)$ models the results in simulation p_n .

2. Alice and Bob Play a Game

The strategy I let Alice have is the following...

Case 1: Found Exclusive Output

She presses the button recording n output values x_1, x_2, \dots, x_n . As she records each output she checks whether x_i is exclusive to one of the buttons output range. If x_i is exclusive she returns the corresponding button as the answer.

Case 2: No Exclusive Output

If she presses the button n times with no exclusive output appearing. She calculates \bar{x} and finds the minimum difference between the mean output range of the 2 buttons. She returns the corresponding button that gets the minimum distance.

Find n such that Alice is correct ≥ 0.99 I sampled Alice's strategy varying n between $[0, 500]$ and extracted the first n that results in probability atleast 0.99. I set the *seed* = 27 and ran 400000 trials for all n 's.

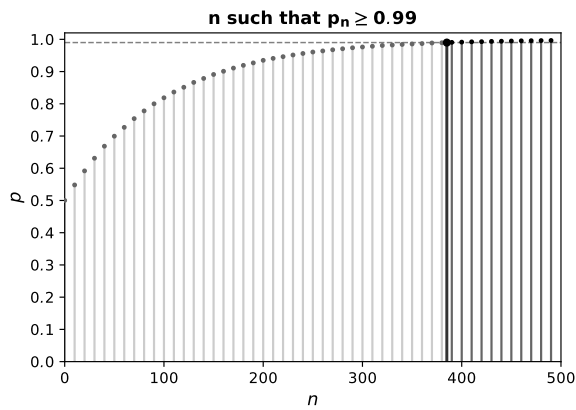


Figure 3: The first point to $\geq p = 0.99$ was $p_{384} = 0.990$ which satisfies the condition.

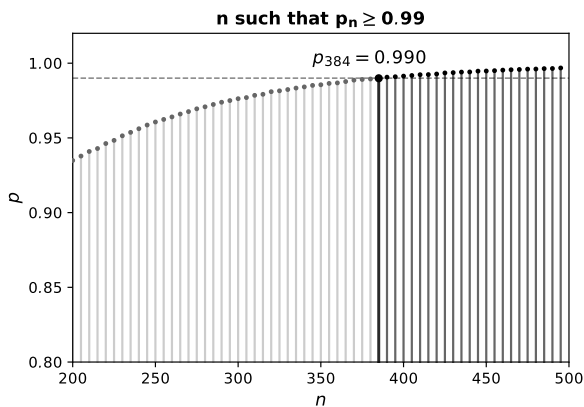


Figure 4: Crop of Figure 3.

Psuedocode

Although the source code differs because loops are removed for speed, the idea is the same.

```
xs = []
for i in range(0, n):
    x_n = button_unknown.get_next_value()
    xs.append(x_n)

    # Case 1: Found exclusive output.
    if(x_n == 1)
        return 1 # it is button 1
    if(x_n == 100)
        return 2 # it is button 2

# Case 2: No exclusive output. Evaluate the
# minimum distance of means.
x_mean = sum(xs) / n
button_1_mean = (1 + 99)/2
button_2_mean = (2 + 100)/2

return argmin([abs(x_mean - button_1_mean), abs(
    x_mean - button_2_mean)]) + 1
```

3. Practice with Uniform and Geometric Distributions

First find $E[X]$ and $V[X]$ of the geometric distribution. Since X is a fair dice $p\{X = 5\} = 1/6$.

$$E[X] = \frac{1}{1/6} = 6$$

$$V[X] = \frac{1 - 1/6}{(1/6)^2} = 30$$

Then I do the same for uniform distribution of Y while equating the corresponding known values from X .

$$E[Y] = \frac{n-1}{2} = 6$$

$$V[Y] = \frac{(n^2 - 1)}{12} = 30$$

Since n is determined by the setting of a and b we can rewrite it like this.

$$E[Y] = \frac{(b-a+1)-1}{2} = 6$$

$$V[Y] = \frac{(b-a+1)^2 - 1}{12} = 30$$

Now it's easy to find a and b by solving the system of equations. However I am not going to do that because that is not how I found it at first.

What I did is rearranged $E[Y]$ such that $a+b = 12$. Then I did a search of pairs of a, b st, $a+b = 12$ and $a < b$ until I got $V[Y] = 30$.

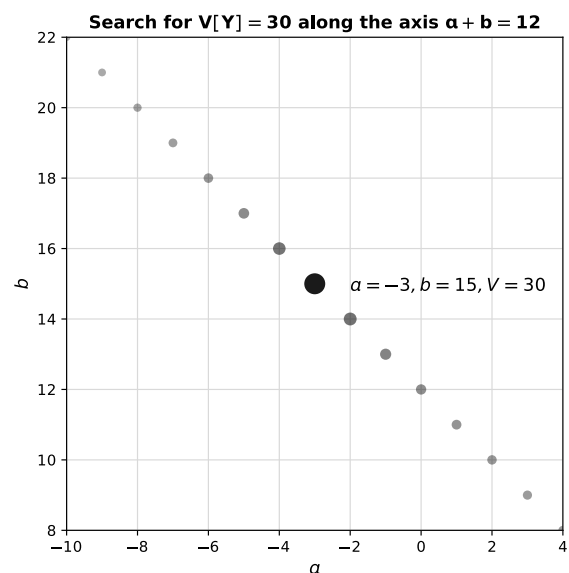


Figure 5: The size and opacity of the discs is the inverse distance away from $V[Y] = 30$.

For $E[X] = E[Y]$ and $V[X] = V[Y]$,
 $a = -3, b = 15$

Source Code

<https://github.com/AustinMaddison/discrete-simulation/tree/main/hw1/source>