

CSC 211: Computer Programming

Recursion

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Recursion

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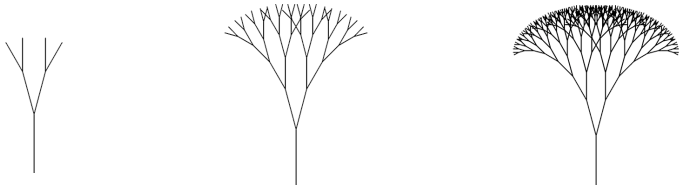
- Problem solving technique in which we solve a task by reducing it to smaller tasks (of the same kind)
 - ✓ then use same approach to solve the smaller tasks
- Technically, a recursive function is one that **calls itself**
- General form:
 - ✓ **base case**
 - solution for a **trivial case**
 - it can be used to stop the recursion (prevents “stack overflow”)
 - every recursive algorithm needs at least one base case
 - ✓ **recursive call(s)**
 - divide problem into **smaller instance(s)** of the **same structure**

General form

```
function() {  
    if (this is the base case) {  
        calculate trivial solution  
    } else {  
        break task into subtasks  
        solve each task recursively  
        combine solutions if necessary  
    }  
}
```

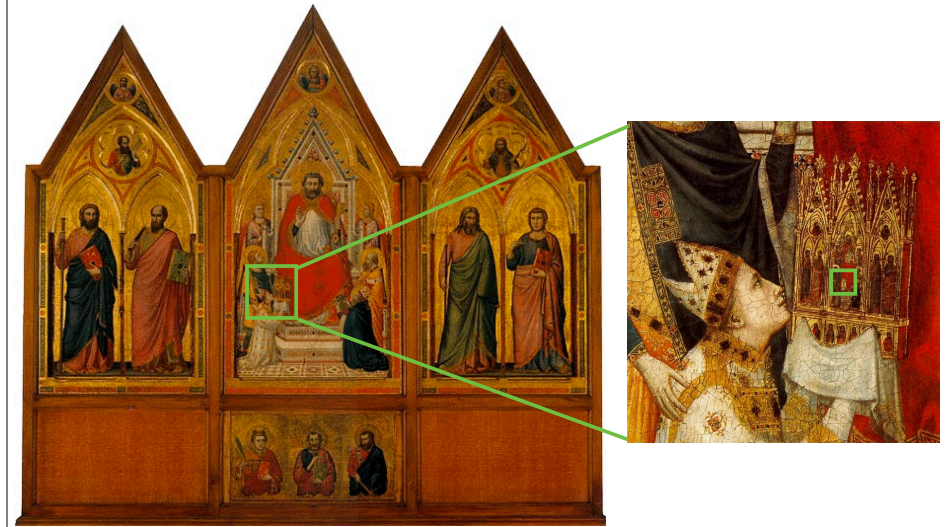
Why recursion?

- Can we live without it?
 - ✓ yes, you can write “any program” with arrays, loops, and conditionals
- However ...
 - ✓ some formulas are explicitly recursive
 - ✓ some problems exhibit a natural recursive solution



<https://courses.cs.washington.edu/courses/cse120/17sp/labs/11/tree.html>

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The Stefaneschi Altarpiece is a triptych by the Italian medieval painter Giotto, commissioned by Cardinal Giacomo Stefaneschi to serve as an altarpiece for one of the altars of Old St. Peter's Basilica in Rome. It is now at the Pinacoteca Vaticana, Rome. Circa 1320.

https://en.wikipedia.org/wiki/Stefaneschi_Triptych

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Example: factorial

$$n! = 1 \cdot 2 \cdot \dots \cdot n = \prod_{k=1}^n k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n-1)! \cdot n & \text{if } n > 0 \end{cases}$$

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Example: factorial

- Apply the recursive definition of factorial to calculate:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n-1)! \cdot n & \text{if } n > 0 \end{cases}$$

▸ 3!

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General form

```
function() {  
    if (this is the base case) {  
        calculate trivial solution  
    } else {  
        break task into subtasks  
        solve each task recursively  
        combine solutions if necessary  
    }  
}
```

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Example: factorial

```
int fact(int n) {  
  
    // base case  
    if (n == 0) {  
        return 1;  
    }  
  
    // recursive call  
    return fact(n-1) * n;  
}
```

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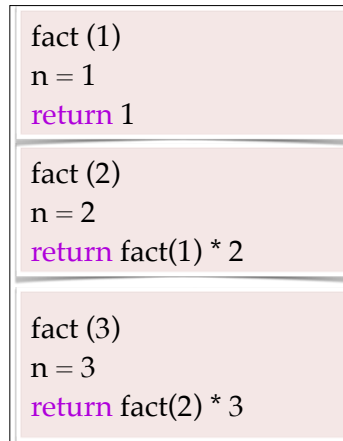
Example: factorial

```
int fact(int n) {  
  
    // base case  
    if (n < 2) {  
        return 1;  
    }  
  
    // recursive call  
    return fact(n-1) * n;  
}
```

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Recursion call stack

fact(3) == 6



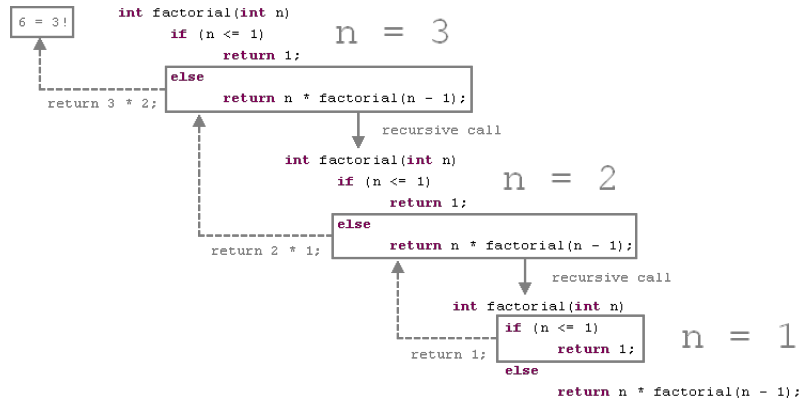
Stack

```
int fact(int n) {  
    if (n < 2) {  
        return 1;  
    }  
    return fact(n-1) * n;  
}
```

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Example

Factorial



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Question

- Given $f(n) = f(n - 1) + 2n - 1$, what is the value of $f(3)$?

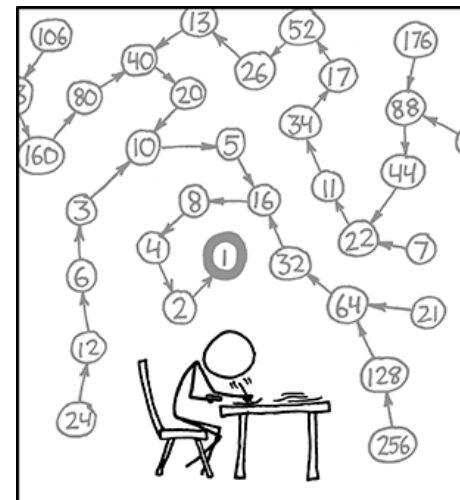
Must have base case and make progress towards base case

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Rules of the game

- Your code must have **at least one base case** for a trivial solution
 - that is, for a non-recursive solution
- Recursive calls should **make progress** towards the base case
- Your code must break a larger problem into smaller problems
 - each smaller problem should be of the same '**nature**' as the larger problem

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THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

No one knows whether or not this function terminates for all values of N

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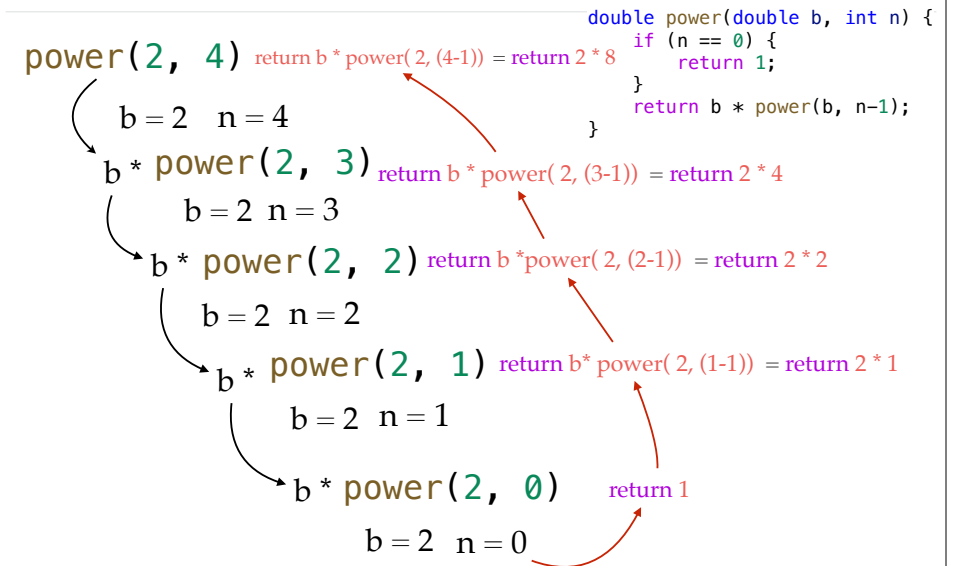
Example: power of a number

$$b^n = \underbrace{b \cdot b \cdot \dots \cdot b}_{n \text{ times}}$$

base case?
recursive case?

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Recursion call tree (tracing recursion)



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What is the output of foo(1234)?

```
int foo(int n) {  
    if (n < 10) {  
        return n;  
    }  
    int b = n % 10;  
    return b + foo(n/10);  
}
```

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What is the output of foo(1234)?

```
int foo(int n) {  
    if (n < 10) {  
        return n;  
    }  
    int b = n % 10;  
    return b + foo(n/10);  
}
```

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What is the output of mystery(7)?

```
void mystery(unsigned int n) {
    if (n < 2) {
        std::cout << n;
    } else {
        mystery(n/2);
        std::cout << n % 2;
    }
}
```

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What is the output of mystery(7)?

```
void mystery(unsigned int n) {
    if (n < 2) {
        std::cout << n;
    } else {
        mystery(n/2);
        std::cout << n % 2;
    }
}
```

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Indirect Recursion

```
void f2(int n);

void f1(int n) {
    if (n > 1) {
        std::cout << "1";
        f2(n - 1);
    }
}

void f2(int n) {
    std::cout << "0";
    f1(n - 1);
}
```

f1(1) ?
f1(2) ?
f1(4) ?
f1(7) ?
f1(10) ?

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Indirect Recursion

```
void f2(int n);

void f1(int n) {
    if (n > 1) {
        std::cout << "1";
        f2(n - 1);
    }
}

void f2(int n) {
    std::cout << "0";
    f1(n - 1);
}
```

f1(1) =
f1(2) =
10
f1(4) =
1010
f1(7) =
101010
f1(10) =
1010101010

Some considerations and summary

- Recursion is a powerful technique that solves problems by **breaking them down into smaller subproblems** of the same form, and applying the same strategy to solve the subproblems
- One can **always write an iterative solution** to a problem solved recursively
 - recursive code is often simpler to read, write, and maintain
- Not always an efficient** solution (iterative counterparts are faster)
 - why not?
 - overhead

Overhead is any combination of excess or indirect computation time, memory, bandwidth, or other resources that are required to perform a specific task.

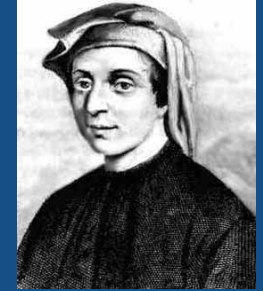
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Lets Try It - Fibonacci sequence

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$



0 1 1 2 3 5 8 13 21 34 ...

The **Fibonacci sequence** first appears in the book **Liber Abaci** (1202) by Fibonacci, using it to calculate the growth of rabbit populations. The sequence had been described by Indian mathematicians as early as the **sixth century**.

from: wikipedia

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Fib we've seen before

- Write a program to print the first 50 terms of the Fibonacci sequence (pick your favorite loop)

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

```
int num = 50;
int x = 0, y = 1, z = 0;
for (int i = 0; i < num; i++) {
    std::cout << x << " ";
    z = x + y;
    x = y;
    y = z;
}
```

0 1 1 2 3 5 8 13 21 34 ...

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