Big Omega Notation : Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$.

$$g(n) = c \cdot n^3$$

$$g(n) = n^3 + 2n^2 + 4n.$$

For Finding constants c and n_0.

$$n^3 + 2n^2 + 4n \geq c \cdot n^3.$$

Divide both sides with n^3.

$$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$$

$$1 + 2/n + 4/n^2 \geq c.$$

Hence $2/n$ and $4/n^2$ approaches 0.

$$1 + 2/n + 4/n^2 \approx 1.$$

Example $c = 1/2$ $(1 \geq 1/2, n \geq 1)$.

$$1 + 2/n + 4/n^2 \geq 1/2.$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1.$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1/2$$

Thus $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$.

Big theta Notation : Determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

$$c_1 n^2 \leq h(n) \leq c_2 n^2.$$

In Upper bound $h(n)$ is $O(n^2)$.

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n$.

$h(n) \leq c_2 n^2$.

$4n^2 + 3n \leq c_2 n^2 \Rightarrow 4n^2 + 3n \leq 5n^2$

Let's. $c_2 = 5$.

Divide both sides by n^2.

$4 + 3/n \leq 5$.

$h(n) = 4n^2 + 3n$ is $O(n^2)$ ($c_2 = 5, n_0 = 1$).

Lower bound:-

$h(n) = 4n^2 + 3n$.

$h(n) \geq c_1 4n^2$.

$4n^2 + 3n \geq c_1 n^2$.

Lets both sides by n^2.

$4 + 3/n \geq 4$.

$h(n) = 4n^2 + 3n$ ($c_1 = 4$, $n_0 = 1$).

$h(n) = 4n^2 + 3n$.

Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $f(n) = \Omega(g(n))$ is true or False and Justify your answer.

$F(n) \geq (g(n))$.

Substuting $F(n)$ and $g(n)$ into this inEquality we get.

$n^3 - 2n^2 + n \geq c \cdot (-n^2)$.

Find c and no holds $n \geq n_0$.

$n^3 - 2n^2 + n \geq -cn^2$.

$n^3 - 2n^2 + n + cn^2 \geq 0$.

$n^3 + (c-2)n^2 + n \geq 0$.

$n^3 + (c-2)n^2 + n \geq 0$.

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$

$F(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

statement $F(n) = \Omega(g(n))$ is

true

mine whether $h(n) = n\log n$ is $\Theta(n\log n)$ prove a rigoros proof for your conclusion.

$$C_1 n\log n \le h(n) \le C_2 n\log n.$$

Upperbound :

$$h(n) \le C_2 n\log n.$$

$$h(n) = n\log n + n.$$

$$n\log n + n \le C_2 n\log n.$$

Divide both sides by $n\log n$.

$$1 + \frac{n}{n\log n} \le 2.$$

$$1 + \frac{1}{\log n} \le C_2. \quad (\text{simply } (C_2 = 2).$$

$$1 + \frac{1}{\log n} \le 2.$$

Then $h(n)$ is $O(n\log n)$. $(C_2 = 2, n_0 = 2)$.

Lower bound:

$$h(n) \ge c_1 n\log n.$$

$$h(n) = n\log n + n.$$

$$n\log n \ge C_1 n\log n.$$

Divide both sides by $n\log n$.

$$1 + \frac{n}{n\log n} \ge c_1.$$

$$1 + \frac{1}{\log n} \ge c_1.$$

$$1 + \frac{1}{\log n} \ge 1 \quad (c_1 = 1)$$

$$\frac{1}{\log n} \ge 0. \quad (c_1 = 1), n_0 = 1$$

$h(n)$ is $\Omega(n \log n)$

$h(n) = n\log n + n$ is $\Theta(n \log n)$.

Solve the following recurrence relations and find the order of growth of solutions.

$T(n) = 4T(n/2) + n^2, T(1) = 1$.

$T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$.

$a = 4, b = 2, f(n) = n^2$.

Applying master theorem.

$T(n) = aT(n/b) + f(n)$.

$f(n) = O(n^{\log_b a - \epsilon})$.

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, then $T(n) = \Theta$

Calculating $\log_b a$.

$\log_b a = \log_2 4 = 2$.

$f(n) = n^2 = \Theta(n^2)$.

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$,

$T(n) = 4T(n/2) + n^2$.

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$.

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$.

is $\Theta(n^2 \log n)$.