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① Solve the following recurrence relation.

a) $x(n) = x(n-1) + 5$ for $n > 1$ with $x(1) = 0$.

1) write down the first two terms & identify the pattern.

$x(1) = 0$.
$x(2) = x(1) + 5 = 0$.
$x(3) = x(2) + 8 = 10$.
$x(4) = x(3) + 5 = 15$.

2) Identify the pattern ~~for the~~ (or) the general term

→ The first term $x(1) = 0$
The common difference $d = 5$
The general formula for the nth term of an AP is $x(n) = 0 + (n-1)5 = 5(n-1)$.
The solution is
$x(n) = 5(n-1)$.

b) $x(n) = 3x(n-1)$ for $n > 1$ with $x(1) = 11$

1) write down the first two terms to identify

$x(1) = 4$.
$x(2) = 3x(1) = 3.4 = 12$ $x(4) = 3x(3) = 108$
$x(3) = 3x(2) = 36$.

2) identify the general term.

→ The first term $x(1) = 4$.
→ The common ratio $= 3$.
The general formula for the $n^{th}$ term of Gp is $x(n) = x(1) \cdot r^{n-1}$.

Substituting the given values.

$x(n) = 4 \cdot 3^{n-1}$.

The solution is

$x(n) = 4 \cdot 3^{n-1}$.

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (Solve for $n = 2k$)

For $n = 2k$ we can write recurrence in term of $k$.

1) Substitute $n = 2k$ in the recurrence.

$x(2k) = x(2k-1) + 2k$.

2) Write down the first few terms to identify the pattern.

$x(1) = 1$.

$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$.

$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$.

$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$.

3) Identify the general term by finding the pattern. We observe that.

$x(2^k) = x(2^{k-1}) + 2^k$.

since $x(1) = 1$.

$x(2^k) = 2^k + 2^{k-1} + 2^{k-2}$.

The geometric series with the term $d = 2$ and the last term $2^k$ Except for the additional $+1$ term.

The sum of a geometric series $S$ with ratio $r = 2$ is given by

$$S = a\frac{r^n - 1}{r - 1}$$

where $a = 2$, $r = 2$ and $n = k$.

2) Evaluate the following recurrences completely.

1) $T(n) = T(n/2) + 1$, where $n = 2k$ for all $1/20$.

The recurrence relation can be solved using algorithm method.

1) substitute $n = 2k$.

2) Iterate the recurrence.

for $k = 0$: $T(20) = T(1) = T(1)$.

$k = 1$ : $T(2) = T(1) + 1 =$

$k = 2$ : $T(22) = T(4) = T(n) + 1 = (T(1) + 2) + 1$.

$= T(1) + 2$

$k = 3$ : $T(23) = T(8) = T(n1) + 1 = T(1) + 2 + 1$

$= T(1) + 3$

3) generate the pattern.

$T(2^k) = T(1) + k$.

Since $n = 2^k$, $k = \log_2 n$.

$T(n) = T(2^k) = T(1) + \log_2 n$.

4) Assume $T(1)$ is constrate.

$T(n) = c + \log_2 n$.

The solution.

ii) $T(n) = T(n/3) + T(2n/3) + n$, where $k$ is constru $n$ for divide and conque.

$T(n) = aT(n/2) + F(n)$.

where $a = 2$, $b = 3$ and $F(n) = n$.

Lets determine the value of $\log a$:

$\log_b a = \log_3 2$.

using the properties of logarithms.

$$\log_3 2 = \frac{\log 2}{\log 3}.$$

Now we compare $F(n) = Cn$ with $n^{\log_2}$.

$$F(n) = O(n).$$

$$n = n^1$$

since $\log_3 2$ we are in the third case of the theorem.

$$F(n) = O(n^2) \text{ with } c > \log_b a.$$

The solution is:

with $T(n) = O(F(n)) = O((n) = O(n)$

d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$

(save for $n = 3^k$).

For $n = 3^k$.

1) Substitute $n = 3^k$ in the recurser.

$$x(3^k) = x(3^{k-1}) + 1.$$

2) write down the first few terms.

$$x(1) = 1.$$

$$x(3) = x(3) = x(1) + 1 = 1 + 1 = 2.$$

$$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3.$$

$$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4.$$

3) identify the generical term.

we observe thee.

$$x(3^k) = x(3^k) + 1$$

summing up the series.

3 Consider the following recurrence algorithm.

min [A [0 . . . n-2]].

if n = 1 Return A[0].

Else temp = min [A[0 . . . n-2].

If temp <= A(n-1) return temp.

Else.

return A(n-1)

a). What does this algorithm compute?

The given Algorithm min [A(0 . . n-1)]

Computes the minimum value in the array "A" from Index "0" for "n-1" It does this Recursively finding the minimum value.

b) Set up recurrence relation for the algorithm basic operation count and Solve it.

The solution is

$T(n) = n$.

This means the algorithm performs n basic parameters for an input array sing "n".

4) Analyse the order of growth.

i) $F(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega g(n)$ notation.

To analyze the order of growth and use the $\Omega$ notation, we need to compare the given function.

$F(n) = c \cdot g(n)$

→ Ignore the lower order returns for longer

$2n^2 \geq 7cn$.

→ Divide both sides by $n$.

$2n \geq 7c$.

→ Solve for $n$:

$n \geq 7c/2$

4) Choose the constants.

Let $c = 1$.

$n \geq \frac{7 \cdot 1}{2} = 3.5$.

for $n \geq n$ the inequality notes:

$2n^2 + 5 \geq 7n$ for all $n \geq n$.

we have shown that there exist constants $c = 1$ and $n_0 = n$ such that for all $n \geq n_0$.

$2n^2 + 5 \geq 7n$.

Thus we can conclude that:-

$F(n) = 2n^2 5 = \Omega(7m)$.

in $\Omega$ notation the dominant term $2n^2$ in $F(n)$ closely grows faster.

$F(n) = \Omega(n^2)$.