Anupama Goparaju[1], Riddhish Bhalodia[1], and Shireen Elhabian[1]

Scientific Computing and Imaging Institute, University of Utah, SLC, UT, USA
anupama.goparaju@utah.edu, {riddhishb,shireen}@sci.utah.edu

Abstract. Statistical shape modeling is widely used in medical image analysis. Algorithms for representing such shape models rely on reliable capture of population level variability, which in turn requires a significant number of reliable medical images. These MRI/CT images needs segmentation (manual/semi-automated) to identify the anatomicla shape of interest, thsee segmentations subjective, error-prone and easily corruptable. Due to presence of huge variability in human anatomy and lack of accurate ground truth medical data, often makes shape modeling process a challenging task. A series of manual quality checks and identification of the corrupted data (outliers) is uaually performed before building a shape model. This requires significant amount of human resource and domain specific expertise. In order to address these challenges, we propose a computational (optimization) framework to incorporate population level statistics in a relaxed shape space model, allowing for a robust shape model. As a by-product of this proposed optimization we automatically detect and identify shape based anomalies/outliers. Thereby, the framework eliminates manual and subjective categorization of samples as corrupted or inaccurate for shape analysis and enables feeding all the medical data available and acquired from different sources without additional efforts. The framework also aids in visualization of the outlier samples by highlighting the differences on the outlier samples which do not follow population level statistics, enabling clinicians to make objective decisions when the dataset comprises of normals and pathology subjects.

Keywords: Statistical shape modeling . Correspondence optimization . Robust shape modeling

1 Introduction

Statistical shape modeling(SSM) is a valuable and a powerful tool to generate detailed representation of complex anatomy that enables quantitative analysis, comparison of shapes and their variations. SSM has been used in many applications such as studying the differences between normal and pathology subjects of a human anatomy, treatment planning, discriminate species based on their specific structures.

Shape modeling, analysis and issues: Shape modeling is performed for a class or a family of shapes to find the inherent anatomical variations. A metric called shape correspondence is defined which allows comparing and enabling shape statistics in the ensemble of shapes by distributing same number of points on each shape and every point corresponds to same feature across shapes. The performance of SSM depends on accurately identifying the shape correspondence or landmark points on all the shapes by an automated means. The accuracy of correspondences is mainly dependent on the underlying optmization techniques. The techniques have been broadly classified into two types: a groupwise approach which finds the correspondence considering the variability of entire data (eg., ShapeWorks [1], Minimum Description Length - MDL [3]) and a pairwise approach (eg., SPHARM-PDM [8]) that considers mapping of each shape to predefined surface parameterization. In addition to the optimization techniques, the data obtained for the training purposes needs to be accurate, representative enough capturing the population level variability to generate a reliable shape model. As the data for the class of shapes needs to be representative enough, the data is usually gathered from different sources such as manual segmented images or semi-automated segmented images of MRI or CT scans using different segmentation algorithms resulting in corrupted, subjective and outlier images. Manual identification of these images from the cohort of training images is a time consuming, tedious and error prone process. Considering this as a common scenario of having outlier samples in the training data, the optimization technique needs to be designed to handle such outlier samples inorder to build a robust shape model. Hence this study is motivated to enhance the optmization technique of ShapeWorks to produce robust shape models considering the common inconsistency in the quality of the training data obtained.

Robust SSM: Generating a shape model when the training data consists of incomplete or outlier samples is a robust SSM process. The first method to handle this problem has been proposed by Luthi et al [6]. In their approach, during the shape modeling process, a patch based outlier detection in the training samples using a reference shape is implemented using an algorithm called PCOut, which could reject the whole patch if a specific landmark is an outlier. Another technique is proposed by Gutierrez et al [4] using Robust PCA with a reference shape in which a shape matrix is modeled as a sparse and a low rank matrix. The sparse matrix consists of the corrupted data points and the low rank matrix can be recovered as outlier free data. The issue of this technique is, the high frequency information is lost from the data which are not actually outliers. Another technique which is a combination of above 2 techniques is proposed by Honsdorf et al[5]. In their approach, every landmark point is assigned a probability and using imputation, Robust PCA and down weighting the outliers a robust shape model is developed. To the best of our knowledge there are minimal techniques covered on addressing this issue in the optmization process accurately in the shape modeling process as the different SSM tools employ hard surface constraints to distribute points on a shape when developing a shape model. We propose a computation framework incorporated in the optimization process with slack variables which would not impose hard surface constraints on the point distribution and correspondence during shape modeling and handles outliers naturally using population level statistics.

2 Methods

Statistical shape modeling provides a means to represent shapes and to perform shape statistics. Inorder to obtain shape statistics, points pertaining to the same anatomical position across shape population need to established. These points are called correspondences or landmarks in a given ensemble of shapes. Manually identifying these landmarks is a tedious and error prone process. The process of identifying landmarks is automated using a objective function. The correspondences are obtained using an optimization process which minimizes the objective function. Particle based modeling is an approach to establish point correspondences on shapes [2].

2.1 Particle based modeling

For a given ensemble of shapes, which are representative of a population, particle based modeling is applied to obtain point correspondences. In particle based modeling, sampling is performed on each shape in a consistently ordered manner to generate surface points covering the entire geometry and these are in correspondence to the points generated across shapes. If m correspondences are generated for a given shape, then each point is 3D point forming 3m shape vector representation. Each of the m 3D points form a configuration space. The 3m dimensional vector space is called a shape space which would enable statistical shape analysis [2].

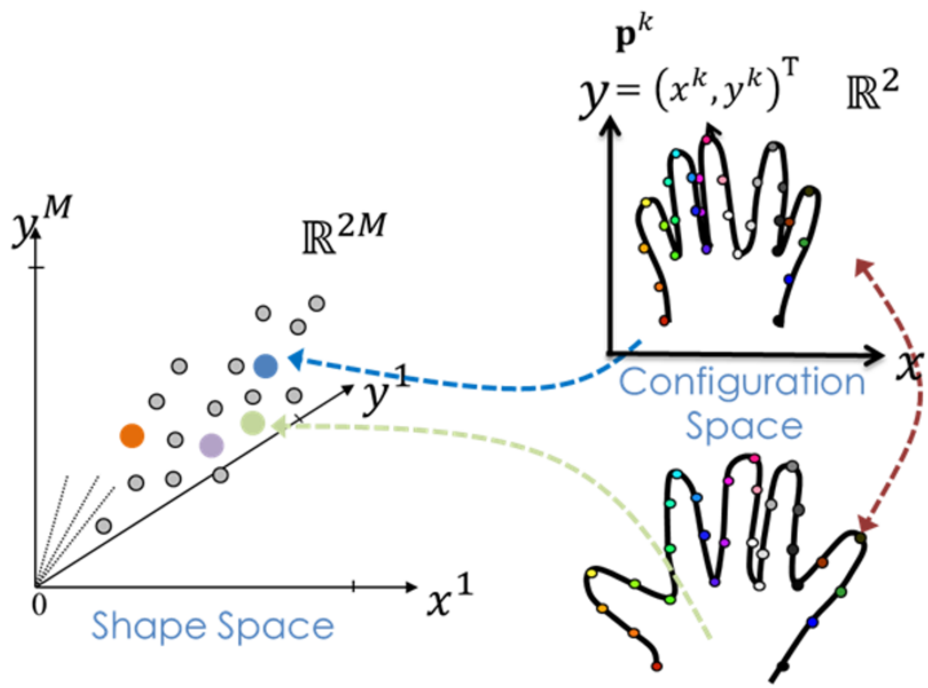

Fig. 1: Shape space and configuration space

Consider a sample of N shapes for which the surfaces representations are to be obtained. Let M represent the number of correspondences to be obtained on each surface. If d represents the dimensionality of each correspondence point, M $\times$ d represents a configuration space for each shape. Let X be a random variable representing the configuration space. A given shape is mapped from a configuration space to a d $\times$ M dimensional shape space. Let Z be the random variable representing the shape space. The correspondence positions are optimized by gradient descent on energy function that balances the negative entropy of distribution particles in a configurations space and positive entropy of distribution of shapes in the shape space.

$$Q = H(Z) - \sum_{k=1}^{N} H(X_k) \tag{1}$$

Here, H is the differential entropy. The first term generates a compact shape model in the shape space and second term produces uniform distribution of correspondence points on each shape for accurate shape representation.

2.2 Robust shape modeling

The training data fed to the shape modeling process often contains corrupted and outlier samples. An outlier could be part of a shape or an entire shape. Eliminating an entire sample when a specific part of a shape is corrupted would cause loss of information. Manual identification and elimination of individual parts or entire corrupted shapes is not only tedious but an error prone process. Hence, we enhance the optimization process described above to automatically identify the outliers point correspondences based on population level statistics and produce a robust shape model using slack or surplus variables. The differential entropy is defined as follows for a random variable X [2]:

$$H(X) = -\int_{s} p(X) \log p(X) dx \tag{2}$$

Let X denote the sampling of correspondences on a given shape based on population level statistics without any surface constraints and $\tilde{X}$ denote the sampling of correspondences on a given shape based on surface constraints. Let ΔX define the difference between X and $\tilde{X}$ which we term as an offset. We define the probability of $\tilde{X}$ to calculate the differential entropy.

$$P(\tilde{X}) = P(\tilde{X}|X) \times P(X)$$
$$P(\tilde{X}|X) \text{ is } P(\Delta X)$$
$$P(\tilde{X}) = P(\Delta X) \times P(X)$$

As X and ΔX are independent of each other, the differential entropy of $\tilde{X}$ can be defined as,

$$H(\tilde{X}) = H(X) + H(\Delta X) \tag{3}$$

Adding the changes to equation (1),

$$Q = H(Z) - \sum_{k=1}^{N} [H(\tilde{X}_k) + H(\Delta X_k)] \tag{4}$$

Particle position update Let us consider a i^{th} particle on a given shape k' as $\tilde{x_i}^{k'}$. Let the cost functions c_1 denote $H(\tilde{X}_k)$ and c_2 denote $H(\Delta X_k)$.
The update of this particle in the optimization process is derived as below:

$$\frac{\partial c_2}{\partial \tilde{x_i}^{k'}} = \frac{\partial c_2}{\partial \Delta x_i{}^{k'}} \times \frac{\partial \Delta x_i{}^{k'}}{\partial \tilde{x_i}^{k'}}$$

Using the smooth L1 norm approximation proposed by Schmidt et al[7],

$$\frac{\partial c_2}{\partial \Delta x_i{}^{k'}} = \frac{1}{(1 + exp(-\alpha \times \Delta x_i{}^{k'}))} - \frac{1}{(1 + exp(\alpha \times \Delta x_i{}^{k'}))} \tag{5}$$

Using the finite difference as an approximation to the derivative,

$$\frac{\partial \Delta x_i{}^{k'}}{\partial \tilde{x_i}^{k'}} = \begin{bmatrix} \frac{\Delta x_i{}^{k'(t)} - \Delta x_i{}^{k'(t-1)}}{\tilde{x_{i_1}}^{k'(t)} - \tilde{x_{i_1}}^{k'(t-1)}} \\ \frac{\Delta x_i{}^{k'(t)} - \Delta x_i{}^{k'(t-1)}}{\tilde{x_{i_2}}^{k'(t)} - \tilde{x_{i_2}}^{k'(t-1)}} \\ \frac{\Delta x_i{}^{k'(t)} - \Delta x_i{}^{k'(t-1)}}{\tilde{x_{i_3}}^{k'(t)} - \tilde{x_{i_3}}^{k'(t-1)}} \end{bmatrix} \tag{6}$$

$$\frac{\partial c_2}{\partial \tilde{x_i}^{k'}} = \text{equation (5)} \times \text{equation (6)} \tag{7}$$

From Cates et al[2], the sampling gradient is given by,

$$\frac{\partial c_1}{\partial \tilde{x_i}^{k'}} = \frac{1}{M} \sum_{j=1}^{M} w_{ij} (\tilde{x_i}^{k'} - \tilde{x_j}^{k'}) \tag{8}$$

From the equations (7) and (8), the particle update using gradient descent is given by below where γ is the learning rate,

$$\frac{\partial c}{\partial \tilde{x_i}^{k'}} = \text{equation (7)} + \text{equation (8)} \tag{9}$$

$$\tilde{x_i}^{k'} = \tilde{x_i}^{k'} - (\gamma \times \text{equation (9)}) \tag{10}$$

Particle offset update Let us consider a i^{th} particle on a given shape k' as $\tilde{x_i}^{k'}$ and its corresponding offset as $\Delta x_i{}^{k'}$. Let the cost functions c_1 denote $H(\tilde{X}_k)$ and c_2 denote $H(\Delta X_k)$.
The update of this particle offset in the optimization process is derived as below:

$$\frac{\partial c_1}{\partial \Delta x_i{}^{k'}} = \frac{\partial c_1}{\partial \tilde{x_i}^{k'}} \times \frac{\partial \tilde{x_i}^{k'}}{\partial \Delta x_i{}^{k'}}$$

Using the finite difference as an approximation to the derivative,

$$\frac{\partial \tilde{x_i}^{k'}}{\partial \Delta x_i^{k'}} = \begin{bmatrix} \frac{\tilde{x_{i1}}^{k'(t)} - \tilde{x_{i1}}^{k'(t-1)}}{\Delta x_i^{k'(t)} - \Delta x_i^{k'(t-1)}} \\ \frac{\tilde{x_{i2}}^{k'(t)} - \tilde{x_{i2}}^{k'(t-1)}}{\Delta x_i^{k'(t)} - \Delta x_i^{k'(t-1)}} \\ \frac{\tilde{x_{i3}}^{k'(t)} - \tilde{x_{i3}}^{k'(t-1)}}{\Delta x_i^{k'(t)} - \Delta x_i^{k'(t-1)}} \end{bmatrix} \tag{11}$$

$$\frac{\partial c_1}{\partial \Delta x_i^{k'}} = \text{equation (8)} \times \text{equation (11)} \tag{12}$$

Using the smooth L1 norm approximation proposed by Schmidt et al[7],

$$\frac{\partial c_2}{\partial \Delta x_i^{k'}} = \frac{1}{(1 + exp(-\alpha \times \Delta x_i^{k'}))} - \frac{1}{(1 + exp(\alpha \times \Delta x_i^{k'}))} \tag{13}$$

From the equations (12) and (13), the particle offset update using gradient descent is given by below where γ is the learning rate,

$$\frac{\partial c}{\partial \Delta x_i^{k'}} = \text{equation (12)} + \text{equation (13)} \tag{14}$$

$$\Delta x_i^{k'} = \Delta x_i^{k'} - (\gamma \times \text{equation (14)}) \tag{15}$$

3 Results

4 Conclusion

Acknowledgment:

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