

ME 332 Project 1

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1. Problem statement and objective:

Our objective is to determine the heating and cooling requirements for a shipping container over the course of a year with both insulation and without insulation. In this analysis we must account for radiation, convection, conduction and applied flux. We then must do a cost analysis of our system to determine the annual energy cost. Finally we can determine the cost of insulation so that we can directly compare it to the daily energy cost without insulation through payback period.

Givens:

$$\epsilon_w = 0.925, \epsilon_R = 0.22, k_{\text{window}} = 1.4 \text{ W/mK}, k_{\text{in}} = 0.042 \text{ W/mK}, k_{ER} = 14.9 \text{ W/mK}, k_{\text{wall}} = 54 \text{ W/mK},$$

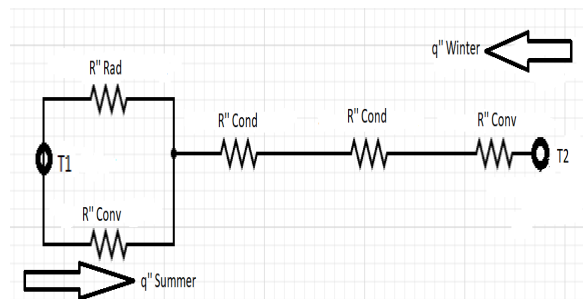
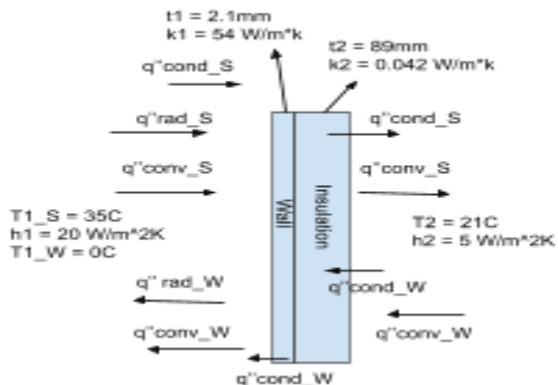
$$t_{ER} = 0.002 \text{ m}, t_{\text{wall}} = 0.0021 \text{ m}, t_{\text{in}} = 0.089 \text{ m}, t_{\text{window}} = 0.005 \text{ m}, t_{\text{gap}} = 0.025 \text{ m}, \sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \text{K}^4)$$

$$T_{1,S} = 35^\circ\text{C}, T_{1,W} = 0^\circ\text{C}, T_2 = 21^\circ\text{C}, h_1 = 20 \text{ W/m}^2\text{K}, h_2 = 5 \text{ W/m}^2\text{K}, \eta = 50\%, \eta_H = 90\%,$$

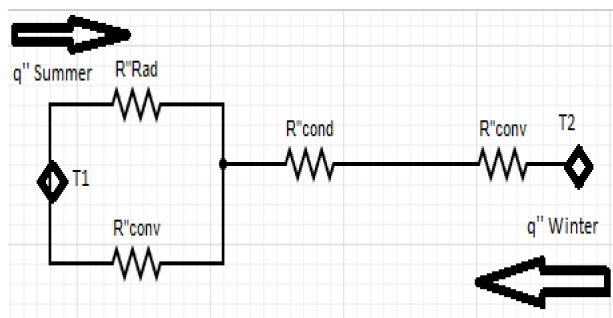
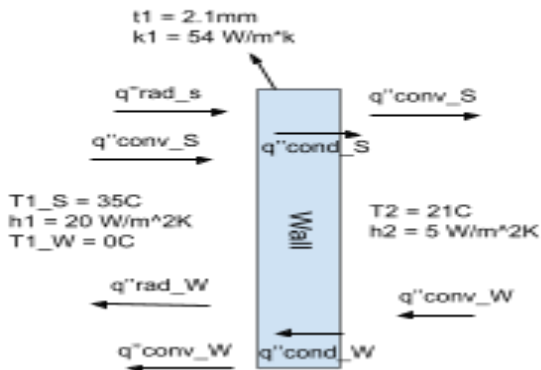
$$L_{\text{wall}} = 12.2 \text{ m}, W_{\text{wall}} = 2.4 \text{ m}, h_{\text{wall}} = 2.7 \text{ m}, W_{\text{win}} = 0.5 \text{ m}, h_{\text{win}} = 1.0 \text{ m}$$

2. Diagrams:

Summer and Winter Wall Heat Flux Diagram



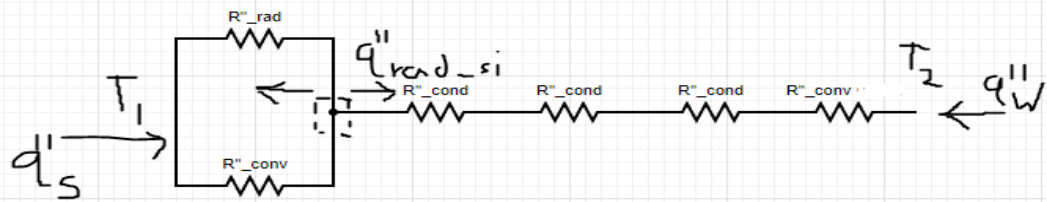
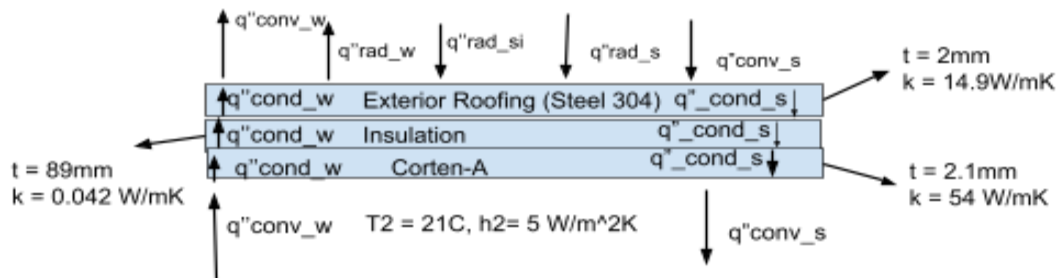
Summer and Winter Wall Heat Flux Diagram With No Insulation



Roof of Container with Insulation

$$T_{1,w} = 0^\circ\text{C}, h_1 = 20 \text{ W/m}^2\text{K}$$

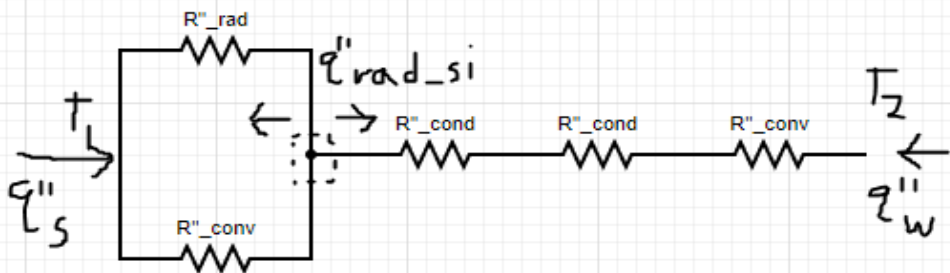
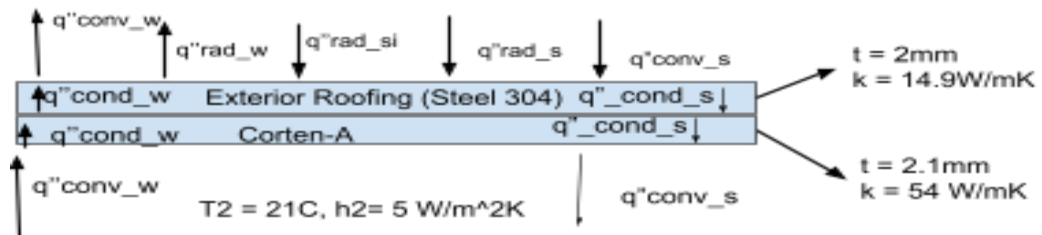
$$T_{1,s} = 35^\circ\text{C}$$

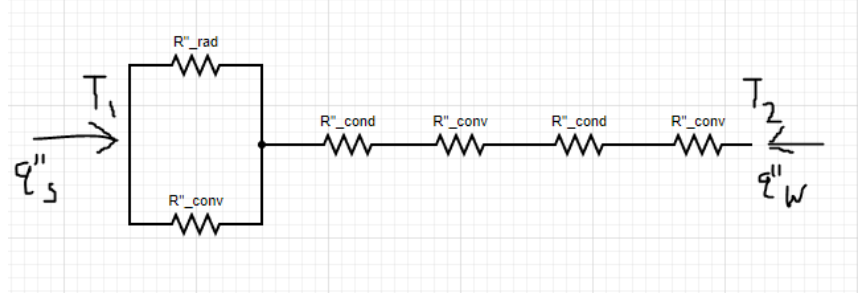
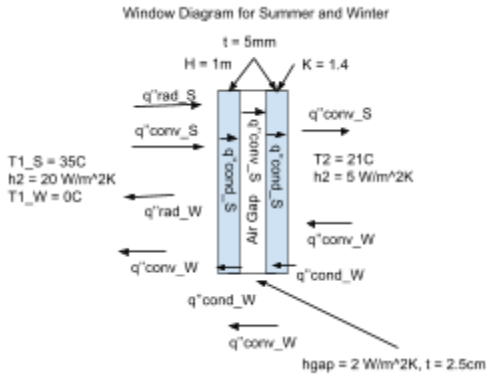


Roof of Container Without Insulation

$$T_{1,w} = 0^\circ\text{C}, h_1 = 20 \text{ W/m}^2\text{K}$$

$$T_{1,s} = 35^\circ\text{C}$$





3. Approach:

Assumptions: Steady-state, 1-D, neglect contact resistance between surfaces, all given properties at 300K, CORTEN-A treated as clean stainless steel (for emissivity on walls), roof has the same dimensions as floor (no roof overhang), no generation aside from applied flux.

4. Energy balance, energy cost equations, and other relevant analyses:

Evaluation of Walls and Flooring:

$$R''_{rad,S} = 1/h_{rad,S}$$

$$R''_{rad,W} = 1/h_{rad,W}$$

$$h_{rad,S} = 4 * \sigma * \epsilon_R * T_{avg,S}^4$$

$$h_{rad,W} = 4 * \sigma * \epsilon_R * T_{avg,W}^4$$

$$T_{avg,S} = (T_{1,S} + T_2)/2$$

$$T_{avg,W} = (T_{1,W} + T_2)/2$$

$$R''_{convL} = 1/h_1$$

$$R''_{convR} = 1/h_2$$

$$R''_{condw} = t_{wall}/k_{wall}$$

$$R''_{condin} = t_{in}/k_{in}$$

$$\Sigma R''_{wall, insulation} = 1/(1/R''_{rad,S/W} + 1/R''_{convL}) + R''_{condw} + R''_{condin} + R''_{convR}$$

$$q''_{wall, insulation, summer} = (T_{1,S} - T_2)/\Sigma R''_{wall, insulation}$$

$$q''_{wall, insulation, winter} = (T_{1,W} - T_2)/\Sigma R''_{wall, insulation}$$

$$\Sigma R''_{wall, no insulation} = 1/(1/R''_{rad,S/W} + 1/R''_{convL}) + R''_{cond} + R''_{convR}$$

$$q''_{wall, no insulation, summer} = (T_{1,S} - T_2)/\Sigma R''_{wall, no insulation}$$

$$q''_{wall, no insulation, winter} = (T_{1,W} - T_2)/\Sigma R''_{wall, no insulation}$$

Evaluation of Windows:

$$R''_{conv\ gap} = 1/h_{gap}$$

$$R''_{condwindow} = t_{window}/k_{window}$$

$$R''_{radw,S/W} = 1/h_{rad,S/W}$$

$$h_{rad,S} = 4 * \sigma * \epsilon_W * T_{avg,S}^4$$

$$h_{rad,W} = 4 * \sigma * \epsilon_W * T_{avg,W}^4$$

$$\Sigma R''_{window, no\ insulation} = 1/(1/R''_{radw,S/W} + 1/R''_{convL}) + R''_{cond\ window} + R''_{conv\ gap} + R''_{cond\ window} + R''_{convR}$$

$$q''_{window, summer} = (T_{1,S} - T_2)/\Sigma R''_{window, no\ insulation}$$

$$q''_{window, winter} = (T_2 - T_{1,W})/\Sigma R''_{window, no\ insulation}$$

Evaluation of Roof (Day):

$$R''_{condER} = t_{ER}/k_{ER}$$

$$q''_{roof, in, day, S/W} = q''_{R, in, day, S/W} + q''_{L, in, day, S/W}$$

$$q''_{L, in, day, S/W} = (T_{1,S/W} - T_{O,S/W})/(1/(1/R''_{rad,S/W} + 1/R''_{convL}))$$

$$q''_{R, in, day, S/W} = (T_{O,S/W} - T_2)/(R''_{condER} + R''_{condin} + R''_{condw} + R''_{convR})$$

$$q''_{roof, non-in, day, S/W} = q''_{R, non-in, day, S/W} + q''_{L, non-in, day, S/W}$$

$$q''_{L, non-in, day, S/W} = (T_{1,S/W} - T_{O,S/W})/(1/(1/R''_{rad,S/W} + 1/R''_{convL}))$$

$$q''_{R, non-in, day, S/W} = (T_{O,S/W} - T_2)/(R''_{condER} + R''_{condw} + R''_{convR})$$

Using these equations of q''_R and q''_L we can find $T_{O,S/W}$ which is the outside surface temperature

of our wall. We then use those values to calculate the values for q''_R and q''_L which we use to calculate the other temperature values for the roof (equation shown under temperature evaluation).

Evaluation of Roof (Night):

$$\Sigma R''_{roof, in, night, S/W} = 1/(1/R''_{rad,S/W} + 1/R''_{convL}) + R''_{condER} + R''_{condin} + R''_{condw} + R''_{convR}$$

$$q''_{roof, in, night, S} = (T_{1,S} - T_2)/\Sigma R''_{roof, in, night, S}$$

$$q''_{roof, in, night, W} = (T_2 - T_{1,W})/\Sigma R''_{roof, in, night, W}$$

$$\Sigma R''_{roof, non-in, night, S/W} = 1/(1/R''_{rad,S/W} + 1/R''_{convL}) + R''_{condER} + R''_{condw} + R''_{convR}$$

$$q''_{roof, non-in, night, S} = (T_{1,S} - T_2)/\Sigma R''_{roof, non-in, night, S}$$

$$q''_{roof, non-in, night, W} = (T_2 - T_{1,W})/\Sigma R''_{roof, non-in, night, W}$$

Equation for temperature evaluation:

$$q'' = (T_H - T_C) / \Sigma R''$$

We use this base equation to solve for all inner and outer wall temperatures, in summer T_H (hot temperature) comes from outside, in winter T_H comes from inside. $\Sigma R''$ only contains the resistance over the portion of the circuit we are looking at. For example:

$$q''_{Wall,S} = (T_{i_{Wall,S}} - T_2) / R''_{convR} \Rightarrow T_{i_{Wall,S}} = (q''_{Wall,S} * R''_{convR}) + T_2$$

Totals:

$$q'_{Window, W/S, in/non-in} = 4 * (q''_{window, in/non-in, S/W} * (W_{win} * h_{win}))$$

Multiply by 4 because there are 4 windows

$$q_{Wall, W/S, in/non-in} = 2 * (q''_{wall, in/non-in, S/W} * (L_{wall} * h_{wall} - W_{win} * h_{win})) + 2 * (q''_{wall, in/non-in, S/W} * (W_{wall} * h_{wall} - W_{win} * h_{win})) + q''_{wall, in/non-in, S/W} (L_{wall} * W_{wall})$$

Two walls have different lengths, subtract window area from wall area, add floor at the end because $q''_{wall} = q''_{floor}$

$$q_{Roof(Day), W/S, in} = q''_{roof, day, in, S/W} (L_{wall} * W_{wall})$$

$$\text{Daytime total: } q_{day, S/W, in/non-in} = q_{Window, W/S, in/non-in} + q_{Wall, W/S, in/non-in} + q_{Roof(Day), W/S, in/non-in}$$

$$q_{Roof(night), W/S, in} = q''_{roof, night, in, S/W} (L_{wall} * W_{wall})$$

$$\text{Nighttime total: } q_{night, S/W, in/non-in} = q_{Window, W/S, in/non-in} + q_{Wall, W/S, in/non-in} + q_{Roof(Night), W/S, in/non-in}$$

$$\text{Daily total heat transfer: } DTHT_{in/no in} = 28800s * q_{S/W, in/no in} + 57600s * q_{S/W, in/no in}$$

Multiplying by 28800 seconds because the day lasts 28800 seconds and the night lasts 57600 seconds.

$$\text{Single day cooling energy: } SDCE_{in/no in} = ((8hr * q_{S/W, in/no in}) / \eta_c) / 1000$$

Multiplying by 8 hr because the day lasts 8 hrs.

$$\text{Single night cooling energy: } SNCE_{in/no in} = ((16hr * q_{night, S/W, in/no in}) / \eta_c) / 1000$$

Multiplying by 16 hr because the night lasts 16 hrs.

$$\text{Total daily cooling energy: } TDCE_{in/no in} = ((8hr * q_{S/W, in/no in}) / \eta_c) / 1000 + ((16hr * q_{night, S/W, in/no in}) / \eta_c) / 1000$$

$$\text{Single day heating energy: } SDHE_{in/no in} = ((8hr * DTHT_{in/no in}) / \eta_H) / 1000$$

$$\text{Single night heating energy: } SNHE_{in/no in} = ((16hr * DTHT_{in/no in}) / \eta_H) / 1000$$

$$\text{Total daily heating energy: } TDHE_{in/no in} = (SDHE_{in/no in} + SNHE_{in/no in})$$

$$\text{Total annual energy: } TAE_{in/no in} = 180days * TDCE_{in/no in} + 180days * SNHE_{in/no in}$$

$$\text{Total annual energy cost: } TAEC_{in, no in} = \$0.13 * 180days * TDCE_{in/no in} + \$0.03 * 180days * SNHE_{in/no in}$$

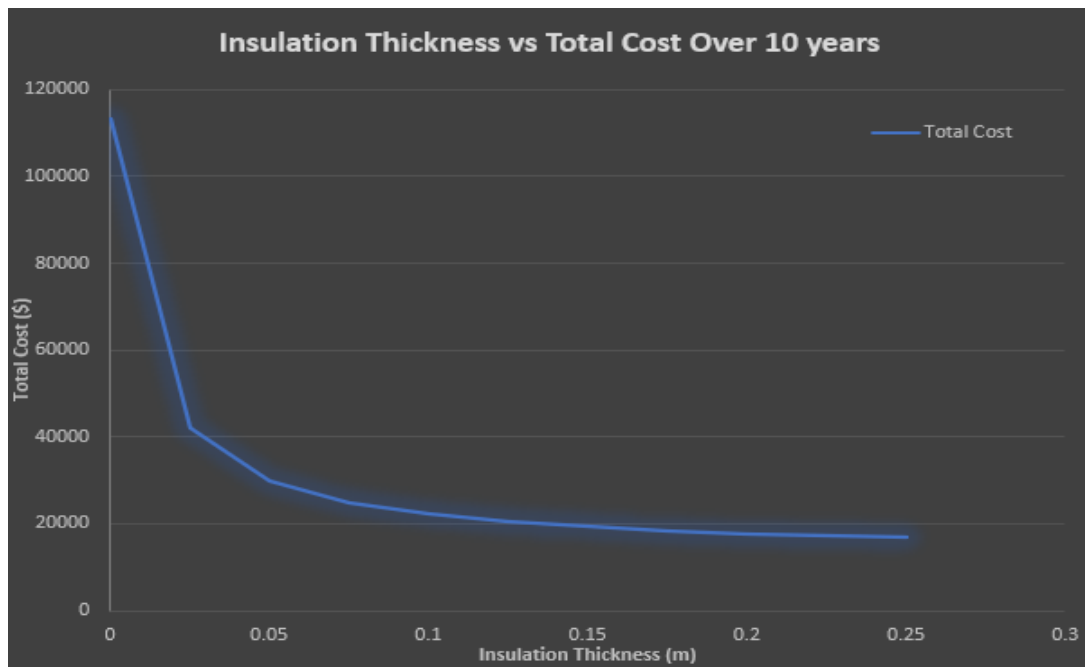
$$\text{Cost to insulate trailer: } CTIT = \$5.00((L_{wall} * h_{wall} - W_{win} * h_{win}) * t_{wall} + (L_{wall} * h_{wall} - W_{win} * h_{win}) * t_{wall} + 2 * (L_{wall} * W_{wall} * t_{wall}))$$

$$\text{Payback period: } PP = (CTIT / (TAEC_{non-in} - TAEC_{in})) * 360days$$

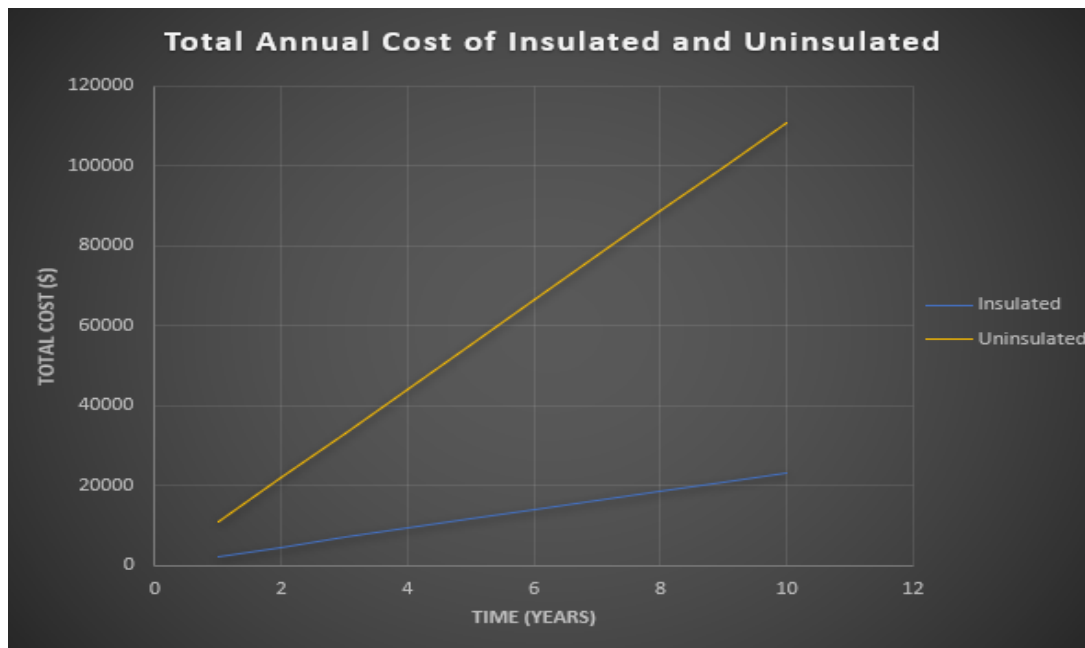
This payback period will show us how many days it will take to be in profit from buying insulation rather than having no insulation.

5. Results:

Plot 1:



Plot 2:



6. Conclusions:

a.

The total cost for insulation is extremely low, because of this it is completely undebatable that buying the insulation is the best option. Because of the high cost of energy when there is no insulation, the payback period is less than one day when you buy insulation. In plot 1 the optimal thickness for energy-saving features such as insulation was determined by iteration and can be asymptotically found based on the developed plot to be around 0.2m and 0.25m. This range of thickness is optimal because it requires less annual energy which reduces the total cost. In plot 2 the total annual cost of insulated and uninsulated is represented over a 10 year period. This shows the the immense cost of uninsulated vs the relatively lower cost associated with insulation for the container.

b.

To reduce the energy usage more insulative materials could be used in the container, this would increase the resistance through the walls/floor/roof. This would decrease the amount of heat that is able to transfer through the walls, requiring less heating and cooling from inside the container. They also could keep the inside temperature of the container at a slightly higher temperature in Summer ($21 < T_i < 35$), this way there would be less energy required to keep the container at said temperature. You can also apply this in winter, you would need to keep the temperature inside the container at a lower temperature though ($0 < T_i < 21$). One way to improve the efficiency of the window would be to simply increase the thickness.

c.

If the price of energy increases then our total cost will also increase. If the system was moved to a more temperate area where the outside temperature is lower in summer and higher in winter then the total energy cost would decrease. If the system were moved to a more extreme environment where temperature is higher in summer and lower in winter, the total energy cost would increase because it would take more energy to keep the inside temperature of the container at 21 degrees Celsius.

d. Extended learning:

Assuming we cover the roof in solar panels ($A=30m^2$) the initial cost will be \$4008.33 and they produce 7800kWh/yr. This means they produce 21.67kWh/day which we factor into our cost analysis.

Annual Energy Cost (insulated):

$$(\$0.13 * 180days * (TDCE - 21.67)) + (\$0.03 * 180days * (TDHE - 21.67)) \\ \Rightarrow \$1700.32$$

There is also a one time \$4008.33 installation fee.

Payback Period:

$$\$4008.33 / (2324.42\$/yr - 1700.32\$/yr) = 6.42 \text{ years}$$

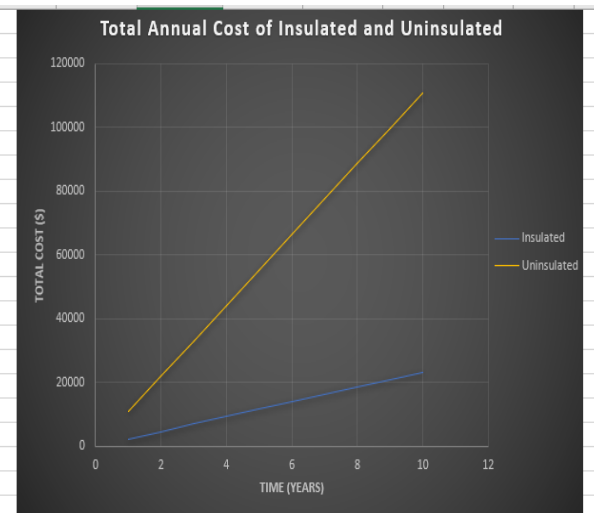
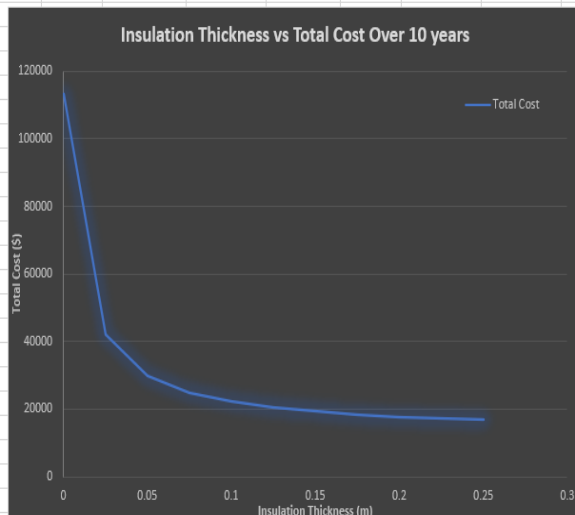
\$4008.33 is the initial investment for solar panels, and \$2324.42-\$1700.32 is how much money you save annually with solar panels vs without solar panels. According to our equation, it will take 6.42 years to break even, after that you are saving money with the panels.

7. Appendix:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1				h values			k values								
2		Temps		h1	20		k1	54		etac	0.5		q" _S	550	
3	T1_W	0		h2	5		k2	0.042		etah	0.9		q" _W	175	
4	T1_S	35	35	hgap	2		k3	14.9						ep_W	0.925
5	T2_in	21	21	hrad_S	1.3627442					CEC	0.13				
6	T_avg_S	28	301.15	hrad_W	1.1387125					CGH	0.03				
7	T_avg2_W	10.5	283.65	W_hrad_S	5.73										
8				W_hrad_W	4.79										
9															
10															
11															
12				Wall		Total Thermal Resistances				Window			Wall		
13				Summer	Winter					Summer	Winter		q_s	q_w	
14		Insulated		2.3659	2.3664					0.746008	0.747485335	Insulated	5.917417435	8.874265306	
15		Non Insulated		0.25	0.2473455							Non Insulated	56.71475233	84.9014978	
16				Wall (Insulated)			Wall (Non-Insulated)			Window					
17				Summer	Winter		Summer	Winter		Summer	Winter				
18		R"rad		0.73	0.88	R"rad	0.7338	0.878184779	R"rad	0.174529	0.208865569				
19		R"conv		0.05	0.05	R"conv	0.05	0.05	R"conv	0.05	0.05			Roof Insulation (DAY)	
20		R"cond	3.88889E-05	3.889E-05	R"cond	3.88889E-05	3.88889E-05	R"cond	0.003571	0.003571429			q_s	q_w	
21		R"cond	2.119047619	2.1190476	R"conv	0.2	0.2	R"conv	0.5	0.5				112.7874912	29.98214964
22		R"conv		0.2	0.2			R"cond	0.003571	0.003571429					
23								R"conv	0.2	0.2					
24															
25															
26															
27			Roof Non-Insulated (NIGHT)			Roof Insulated (NIGHT)				Roof Insulated (DAY)			Roof Non-Insulated (DAY)		
28			Summer	Winter		Summer	Winter			Summer	Winter		Summer	Winter	
29		R"rad	0.733813448	0.878184779	R"rad	0.733813448	0.878184779	R"rad	0.733813448	0.878184779	R"rad		0.733813448	0.878184779	
30		R"conv	0.05	0.05	R"conv	0.05	0.05	R"conv	0.05	0.05	R"conv		0.05	0.05	
31		R"cond	0.000134228	0.000134228	R"cond	0.000134228	0.000134228	R"cond	0.000134228	0.000134228	R"cond		0.000134228	0.000134228	
32		R"cond	3.88889E-05	3.88889E-05	R"cond	2.119047619	2.119047619	R"cond	2.119047619	2.119047619	R"cond				
33		R"conv	0.2	0.2	R"cond	3.88889E-05	3.88889E-05	R"cond	3.88889E-05	3.88889E-05	R"cond		3.88889E-05	3.88889E-05	
34					R"conv	0.2	0.2	R"conv	0.2	0.2	R"conv		0.2	0.2	

		Annual				Insulation		
		Insulated	Non-Insulated			CTIT	60.253	
	TAE	23284.81	128601.4826			PP	2.480721257	
	TAEC	2324.417	11068.27685					
		12.91343						
				TOTALS for q'				
	Windows			Walls			Roof (DAY)	
	Insulated	Non-Insulated		Insulated	Non-Insulated	Insulated	Non-Insulated	
Summer	37.533089	37.53309	Summer	627.9563382	6018.56952	Summer	3302.417741	3547.810324
Winter	56.188393	56.18839	Winter	941.7370343	9009.74695	Winter	877.8773415	3145.0332
							Roof (NIGHT)	
							Insulated	Non-Insulated
						Summer	173.2521531	1659.705458
						Winter	259.8237503	2484.567543

		Summer	173.2521531	1659.705458		
		Winter	259.8237503	2484.567543		
Plot 1		plot 2		plot 2		
Thickness	Total cost 10 year	1	2384.669593	1	11068.27684	
0	113218.3345	2	4709.086185	2	22136.55368	
0.025	42080.16018	3	7033.502778	3	33204.83052	
0.05	29962.25749	4	9357.91937	4	44273.10736	
0.075	24948.86825	5	11682.33596	5	55341.3842	
0.1	22208.37465	6	14006.75256	6	66409.66104	
0.125	20480.71879	7	16331.16915	7	77477.93788	
0.15	19291.98274	8	18655.58574	8	88546.21472	
0.175	18424.02499	9	20980.00233	9	99614.49156	
0.2	17762.4391	10	23304.41893	10	110682.7684	
0.225	17241.44262					
0.25	16820.526					



Solar panels Cost	area
4008.333333	30m^2
Solar panel energy	
7800 kWh/yr	
Annual Energy	
15484.80594	
Annual Energy Cost	
5708.653926	
1700.320593	
PP	
6.422623015	