Beaver Space Systems

Mission Design: Space Telescope

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Mission Statement:

Beaver Space Systems has been tasked with the delivery of a space telescope to observe planets in the solar system and beyond. In order to optimize the performance of the telescope, the selection of its location will be determined the most effective under a few considerations. Once the ideal conditions for the telescope are determined, there are five points to choose from in the earth-sun system that will achieve a fixed position relative to earth's rotation in geosynchronous orbit. Of these five points, the location that best reduces the radiation interference from the sun will be selected for placement of the telescope. Once determined, it will be necessary to design the series of orbital maneuvers and transfers to get the space telescope to the orbital Lagrange point. Important factors to consider during these analyses of the orbit trajectory include, ΔV, Δm, cost, and of course, Lagrange point position. The necessary maneuvers will also have to be determined with respect to efficiency and existing launch vehicle capability, as the payload is rather large for these mission requirements. This leads to the selection of the launch vehicle, which must be capable of delivering the payload to the determined initial orbit and beyond, with respect to the payload's mass and overall dimensions of 10 m length by 7 m width by 5 m height and overall mass of 5000 kg. Once the launch vehicle is selected, based on mission criteria, it is then necessary to select an appropriate launch site for this mission. A suitable launch site will be determined based on the selection of the launch vehicle in order to improve the efficiency of the initial launch and potential reduction of mission cost. The launch site will also be considered based on the location of the launch vehicle manufacturing facility and required transportation to the launch pad, which has the potential to increase cost and difficulty to the setup of the mission. There are currently 18 total spaceports in the United States, that consist of a combination of FAA licensed, Exclusive Use (Non-FAA licensed), and U.S. Federal launch sites (FAA, 2021). Of these, the two most commonly used for vertical launches are the Vandenberg Space Force Base in California, and the Cape Canaveral Spaceport in Florida (Thorpe, 2021).

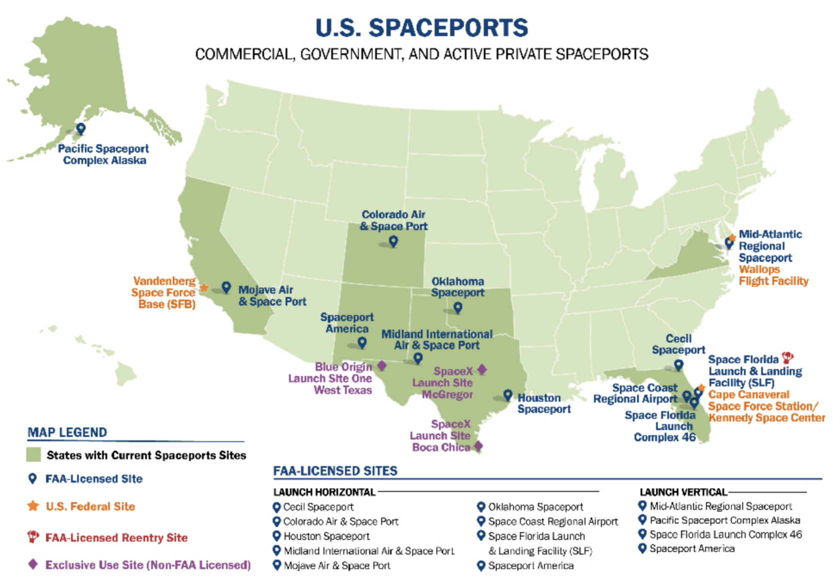

Figure 1: Source: https://www.faa.gov/space/additional_information/faq/

Orbital Determination
Lagrange Point:

In the Earth-Sun system, there are 5 known Lagrange points that hold an object at a fixed point relative to the rotation of the system. Of these five points, positions 4 and 5 are invalid selections due to the presence of gravitational forces from other large masses in space in real life conditions. Of the remaining three Lagrange points, numbers 1 and 3 are also invalid options due to the effects of the Sun's radiation. Currently Lagrange point 1 is used for direct observation of the Sun, which would be a poor selection for space observation, because satellites and instrumentation are always directly exposed to the Sun and the viewpoint is obstructed by either the Earth or the Sun. Lagrange Point 3 is a poor selection as it is much farther away from Earth than the rest of the points, and it's actually located behind the Sun, which would make connection to the satellite very difficult. Also, NASA hasn't currently found a use for point 3 in any other prior mission (NASA, 2018). This leads to the selection of point L2 in the system because it's relatively close to earth, which enables readily available communication with the satellite. This point is also ideal for the telescope, because it provides minimal interference from the Sun because it's partially shielded by the earth, and moon on occasion, which optimizes the observation capabilities of the telescope. Point L2 is also ideal for this mission, because it has also been used for previous missions by multiple space agencies such as the WMAP space probe by NASA, the Plank space probe by the ESA, and the future home of the James Webb telescope by NASA (NASA, 2018).

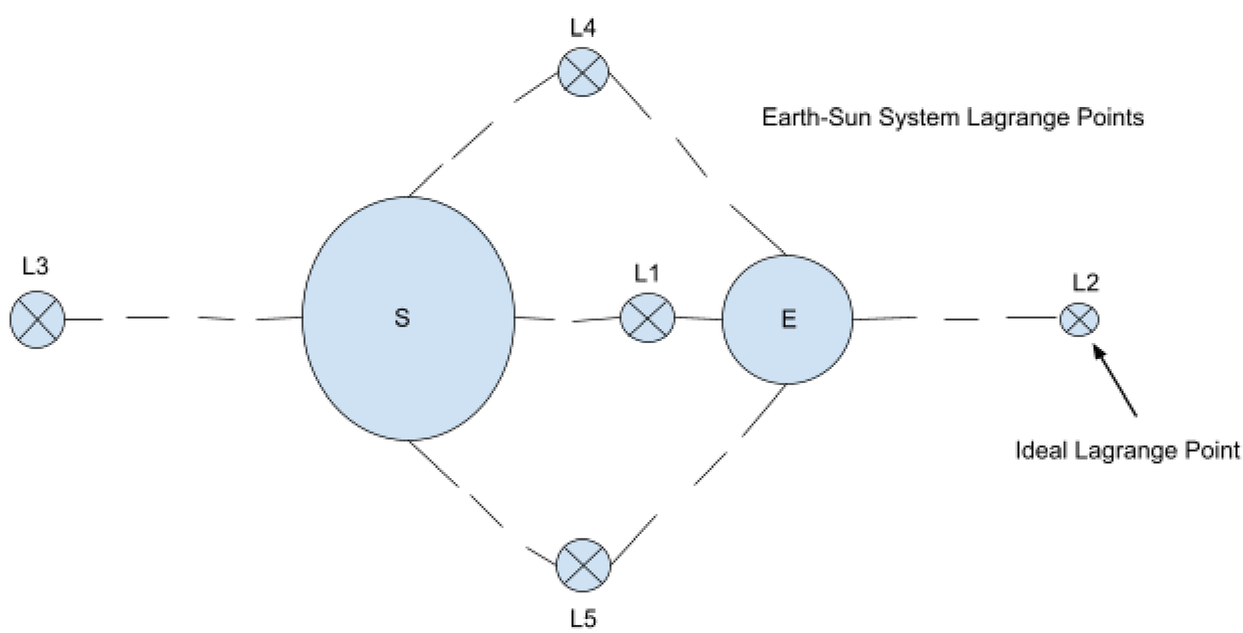

Figure 2: Earth-Sun System Lagrange Points with ideal lagrange point identified

Lagrange Point Calculations:

Altitude = 10,000km
Mass of telescope = 5000kg
Dimensions (LxWxH) = 10m x 7m x 5m
Mass of Sun (m_1) = $1.989 * 10^{30}\ kg$

Mass of Earth (m_2) = $5.97219 * 10^{24}\ kg$

Mass Ratio (Sun-Earth) (π_2)= $\frac{m_2}{m_1+m_2} = 3.0026 * 10^{-6}$

Radius from Sun to Earth $(r_{12}) = 150 * 10^{6}\ km$

Using eq 2.194 at $f(\xi) = 0{,}0 = \frac{1-\pi_2}{|\xi+\pi_2|^3}(\xi + \pi_2) + \frac{\pi_2}{|\xi+\pi_2-1|^3}(\xi + \pi_2 - 1) - \xi$

Critical points found for $\xi = -1, .99,\ 1.01\ for\ \xi_3, \xi_1, \xi_2$

Figure 3: (Critical points of function)

Lagrange point 1: $L1 = \xi_1 r_{12} = 148.5 * 10^{6}\ km$

Lagrange point 2: $L2 = \xi_2 r_{12} = 151.5 * 10^{6}\ km$

Lagrange point 3: $L3 = \xi_3 r_{12} = -150 * 10^{6}\ km$

Lagrange point 4: $L4_x = \frac{r_{12}}{2} - \pi_2 r_{12} = 7.5 * 10^{10} km$

$$L4_y = +\frac{\sqrt{3}}{2} r_{12} = 1.299 * 10^{11} km$$

$$L4 = sqrt(L4_x^{\ 2} + L4_y^{\ 2}) = 7.5 * 10^{10} km$$

Lagrange point 5: $L5_x = \frac{r_{12}}{2} - \pi_2 r_{12} = 7.5 * 10^{10} km$

$$L5_y = -\frac{\sqrt{3}}{2} r_{12} = -1.299 * 10^{11} km$$

$$L5 = sqrt(L5_x^{\ 2} + L5_y^{\ 2}) = 7.5 * 10^{10} km$$

Orbital Trajectories:

The selected Ideal Lagrange point is L2, this is because at this point the Earth will be in between the sun and the space telescope. This means that the sun will not be directly hitting the space telescope, giving it a better opportunity for high quality photos and protection from direct sun rays.

With the ideal Lagrange point selected, the Orbital path is then determined. Starting from the ground, we need to determine the required launch vehicle and the launch site. A vertical launch vehicle, with two propellant stages, is required for this mission as horizontal launch vehicles have limited payload capacities and are only suitable for small satellites (Branson, 2020). Once the launch vehicle and site are selected, the payload is put into a circular earth orbit of 275 km altitude with the burn of stage 1 (see LOE on next page for calculation work). From this orbit, the satellite will then be transferred to a second circular orbit of 10,000 km where its on board propulsion system, assumed to have a specific impulse of 440s, is then capable of achieving the required deltaV for a second Hohmann transfer that is aligned with the L2 Lagrange point.

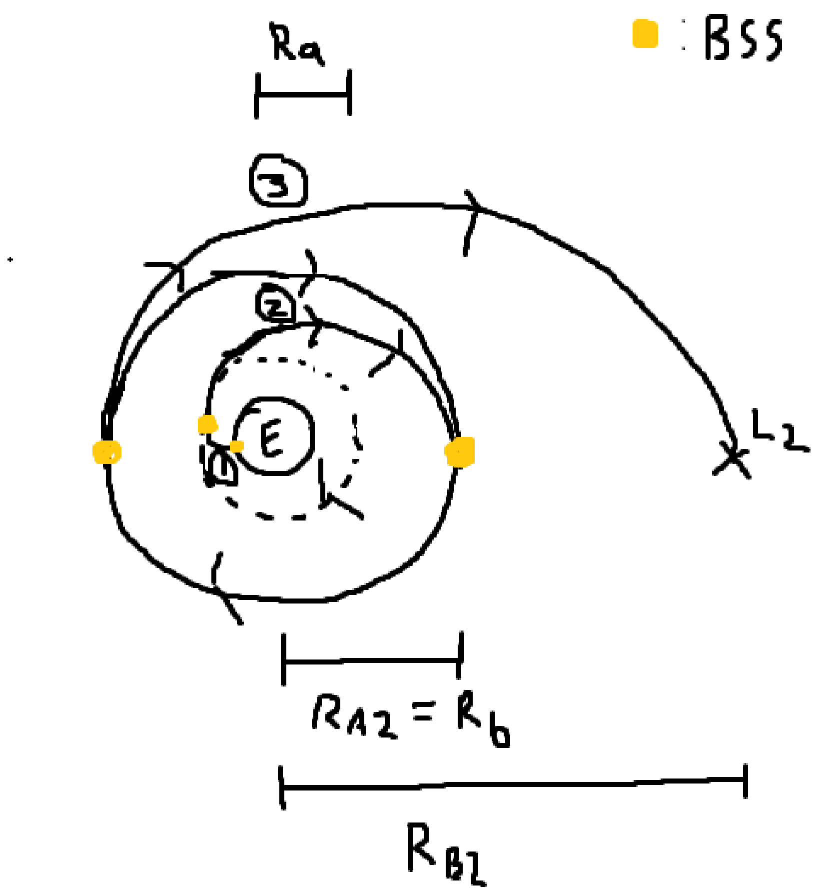

Figure 4: (Trajectory diagram of whole mission from starting point on Earth to ideal lagrange point 2)

Orbit Maneuvers and ΔV/ΔM Requirements

Low earth orbit (LOE):

```
Speed at Burnout          = 8.438331 km/s
Altitude at Burnout         = 275.589654 km
Downrange distance          = 455.786740 km
Mass of Propellent Expended = 130859.971032 kg
```

The above results were for stage 1 of the New Glenn launch vehicle at Cape Canaveral spaceport. The rocket has gained enough energy to reach low earth orbit at an altitude of 275 km while using a large amount of propellant in the process. This high burnout velocity and low atmospheric drag effects from Earth will shape how fast the vehicle will reach the parking orbit at 10,000km.

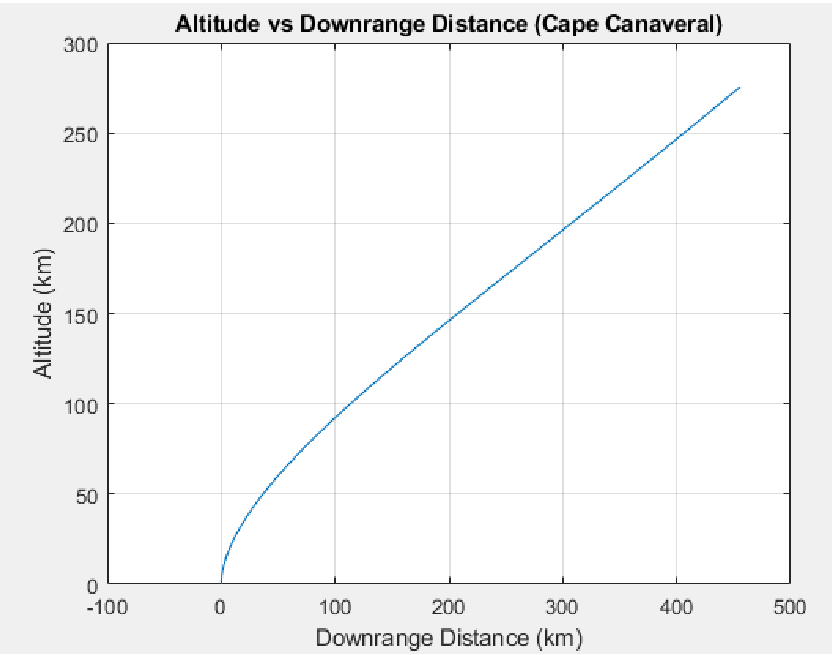

Figure 5: Plot showing the altitude vs downrange distance from Cape Canaveral of the New Glenn Launch Vehicle

First Hohmann transfer: (from low Earth orbit with altitude at burnout using stage 2 to a parking orbit at an altitude of 10,000km):
Delta v for the first Hohmann transfer is 27.066km/s from LEO to parking orbit at 10000 km. Considering how fast the rocket is going the delta v is within range for a timely transfer to our desired altitude. The time to complete the first Hohmann transfer is 102.49 seconds. The time to complete this transfer is very fast as the rocket is still riding its momentum from launch allowing high altitude orbits to be reached faster. The change in propellant (propellant expended during maneuver) of the first stage was calculated to be ΔM1 = $2.745 * 10^{6}$kg based on delta v required.

Second Hohmann transfer: (from altitude of 10,000km to Lagrange point 2 using onboard propulsion system):
Delta v for the second Hohmann transfer is 26.359 km/s from parking orbit at 10000 km to lagrange point. Time to complete the second Hohmann transfer to lagrange point 2 is approximately 38.262 days. Based on the delta v required and the considerable distance to the lagrange point 2 a time of around 38 days is within the desired range for the mission. The change in propellant (propellant expended during maneuver) of the second stage was calculated to be $\Delta M2 = 4.989 * 10^3$ kg based on the delta v required. A specific impulse of 440s was assumed for these calculations.

Total Cost of Propellant and Payload per kilogram: The cost of the New Glenn from Blue Origin is $2.5 billion. The total cost of propellant from both stages of the New Glenn launch vehicle considering the fuel ratio of 6:1 and the mixture of both oxidizer of liquid oxygen (LOX) at $0.20/kg and fuel of liquid natural gas (LNG) at $2.86/DGE gives the final cost of $880,510. The total cost of extra propellant per kilogram of the payload based on this analysis is $\$1600.5$. Based on NASA estimates, a 5000 kg telescope will cost about $110,231,100 to send to space. The final cost of the mission totals to $2.6111 Billion dollars. This Final Cost estimate does not include the cost for research and development, manufacturing, and transportation of the space telescope and other relevant materials and structures.

National Average Price Between July 1 and July 15, 2021	
Fuel	Price
Biodiesel (B20)	$3.05/gallon
Biodiesel (B99-B100)	$3.63/gallon
Electricity	$0.14/kWh
Ethanol (E85)	$2.62/gallon
Natural Gas (CNG)	$2.22/GGE
Liquefied Natural Gas	$2.86/DGE
Propane	$2.98/gallon
Gasoline	$3.09/gallon
Diesel	$3.26/gallon

Figure 6: National average Gas prices during 2021
https://afdc.energy.gov/fuels/prices.html

Launch Vehicle Determination:

After consideration of the mission criteria and required payload specifications, it was concluded that the New Glenn rocket, by Blue Origin, will be used as the launch vehicle for the Beaver Space Systems telescope. The existing selection of launch vehicles was quickly narrowed down to a few high potential candidates when considering a required payload mass of 5000 kg. One high potential choice was the Atlas V by United Launch Alliance, however this option was invalidated as the dimensions of the payload exceed the available capacity of this rocket by about 1.6 m (Kyle, 2020). As the Atlas V rocket was no longer a valid consideration, the two-stage New Glenn rocket by Blue Origin was selected as the launch vehicle for this mission. The New Glenn rocket uses a combination of liquid natural gas and liquid oxygen for both stages of the rocket (Clark, 2014). The New Glenn is a suitable launch vehicle for this mission for a number of reasons. Primarily, because it has the ability to reach the required geosynchronous orbit of 10,000 km with a payload capacity of 25 metric tons and a 7 m diameter fairing (Kyle, 2020). This rocket was also chosen because it is a three-stage rocket, which is required for the two Hohmann transfers to reach the desired geosynchronous transfer point. Also, the New Glenn is ideal because it has reusable components, which helps to greatly reduce the mission cost.

Figure 7: Source: https://www.blueorigin.com/new-glenn

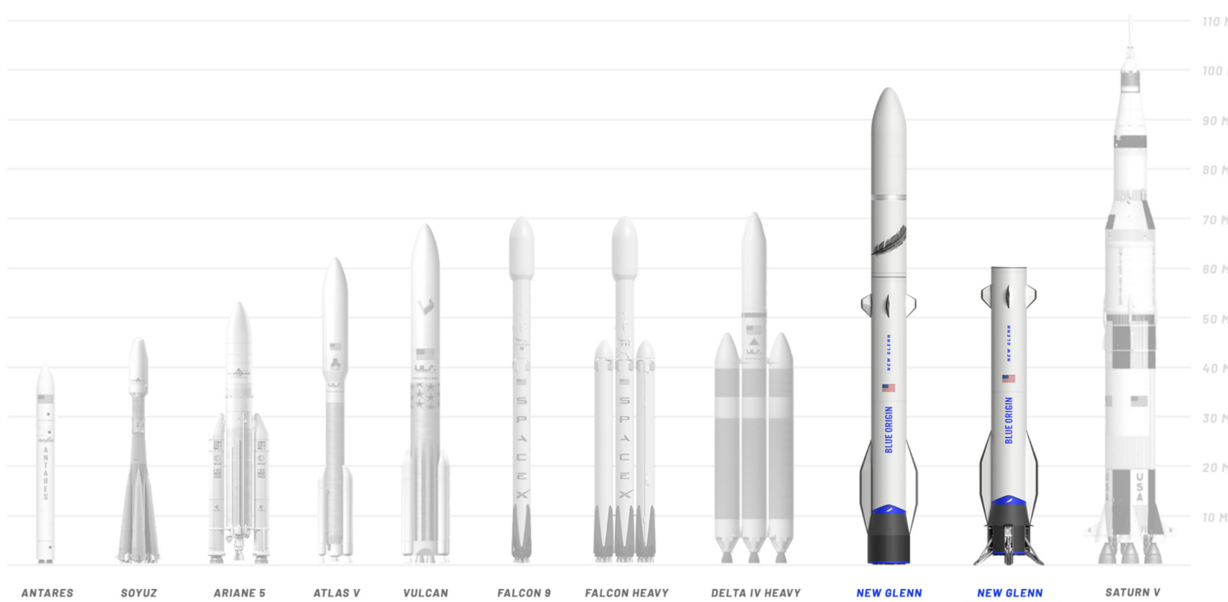

Figure 8: Source: https://www.blueorigin.com/new-glenn

New Glenn Rocket Specifications:

(used in calculations)

	LEO Payload (metric tons) 200 km x 51.6 deg	GTO (metric tons) 185 x 35,786 km x 27 deg	C3=0 km2/sec2 (metric tons)	Trans-Mars	Configuration	LIftoff Height (meters)	Liftoff Mass (metric tons) (no payload)
New Glenn 2-Stg (Superceded)	45 t	13 t	~7 t (est)	~3 t (est)	Stg 1 + Stg 2 + 7m PLF	~82 m	~1,450 t?
New Glenn 3-Stg (Superceded)	-	~30+ t (est)	~25 t (est)	~20 t (est)	Stg 1 + Stg 2 + Stg 3 + 7m PLF	~95 m	~1,450 t?
New Glen, BE-3U 2-Stg	45 t	13.6 t	>6.577 t (GEO)	~3t (est)	7xBE-4 Stg 1 + 2xBE-3U Stg 2 + 7m PLF	96 m	~1,390 t?
New Glen, BE-3U 3-Stg (Deferred)	-	-	-	-	Stg 1 + Stg 2 + Stg 3 +7m PLF	~97 m?	~1,390 t?

	Stage 1 Recoverable	Stage 2 (BE-4U) (Superceded)	Stage 2 (BE-3U)	Stage 3 (Deferred)	2-Stg Fairing (Superceded)	3-Stg Fairing
Diameter (m)	7 m	7 m	7m	7 m	5.4 m	7 m
Length (m)	57.5 m incl interstage	~19 incl engine (est)	23.4 m incl engines	~13 m (est)	~14.8 m (est)	21.9 m
Burnout or Staging Mass (tonnes)	~110 t (est)	~19 t (est)	~12 t (est)	~6 t (est)	-	-
Total Mass (tonnes)	~1,110t (est)	~270 t (est)	~120 t (est)	~55 t (est)	~4 t (est)	~4 t (est)
Engine	BE-4	BE-4U	BE-3U	BE-3U	-	-
Engine Mfgr	Blue Origin	Blue Origin	Blue Origin	Blue Origin	-	-
Fuel	LNG	LNG	LH2	LH2	-	-
Oxidizer	LOX	LOX	LOX	LOX	-	-
Thrust (SL tons)	1,746 t	-	-	-	-	-
Thrust (Vac tons)	2,041 t	290.6 t	110 t	~50 t (est)	-	-
ISP (SL sec)	~310 s (est)	-	-	-	-	-
ISP (Vac sec)	~335 s (est)	~358 (est)	~440 s (est)	~440 s (est)	-	-
Burn Time (sec)	~240 s (est)	~400 s (est)	~500 s (est)	~1,000 s (est)	-	-
No. Engines	7	1	2	1	-	-
Comments	Downrange Floating Platform Landing	-	-	-	-	-

Figure 9: Data Sheet for New Glenn [New Glenn (spacelaunchreport.com)](spacelaunchreport.com)

Launch Site Determination:

The Launch site of the New Glenn rocket will be the Cape Canaveral launch complex 36. This site is ideal because Cape Canaveral Florida is near the equator giving the added benefit of an additional push from the spinning of the Earth. The launch site also makes sense because Blue Origin has already selected this complex for the maiden flight of the New Glenn. Cape Canaveral is ideal for a number of reasons. First, being the 3000 meter runway that allows for the delivery of oversized or heavy payloads by military airplanes, which would be otherwise difficult and time consuming using ground transport (AFS, 2011). This is important in reducing the overall cost of the mission. Another reason is Blue Origin's new manufacturing facility that is 9 miles from the launch site and is also the location of mission and launch control for Blue Origin (Blue Origin, 2019). The geographical location is also an important aspect of using this launch site, because it's physically surrounded by water on three sides and borders the Atlantic Ocean. Plus, being close to the ocean allows for the use of floating launch vehicle platforms that are being used for landing sites of the reusable rocket booster of the New Glenn. Specifically the Stena Freightliner that is currently undergoing redesign to be used as a landing platform for the reusable rocket booster of the New Glenn (Little, 2018). The New Glenn's booster is advertised as being reusable for over 25 launches (Blue Origin, 2019), so a safe landing is important in considering the launch site for this mission. allowing for easy integration parameters for a safe and smooth launch of the Beaver Space System telescope.

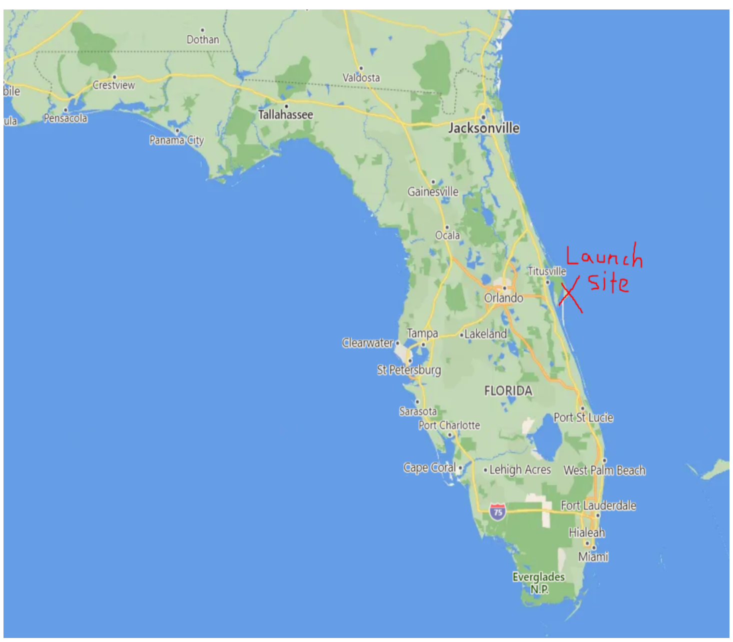

Figure 10: [google maps - Bing Maps]

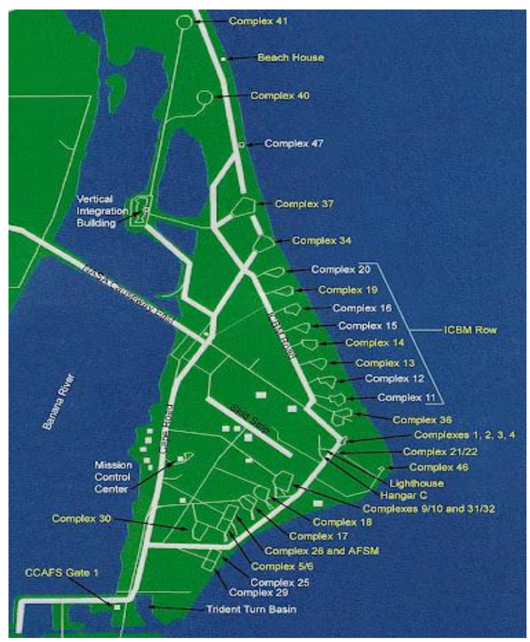

Figure 11: http://www.thousand-islands.org/launch_pads.html

Mission Debrief:

The given task from Beaver Space Systems was to deliver a space telescope to a stable Lagrange point that is selected to provide the best position for observation. Through analysis it was determined that the L2 Lagrange point (distance of $151.5 * 10^{6}\ km$ from Earth), would be the best choice as it is relatively close to earth which provides for easy communication with the satellite and shielding from the Sun's rays. This position is also ideal, as it has a history of being used for previous missions. Once the Lagrange point was selected, an orbital trajectory was then determined using two efficient Hohmann transfers after the satellite was first put in a low earth orbit with an altitude of 275 km. The first transfer is required to put the satellite in a second circular earth centered orbit of 10,000 km which the deltaV required of the first Hohmann is determined to be Δv of 27.066km/s. The second transfer, required to place the satellite in the L2 Lagrange point, has a required Δv of 26.359 km/s. The launch vehicle to be used for this mission is the New Glenn, made by Blue Origin, and it will be launched from Blue Origin's launch complex-36 at the Cape Canaveral space force station. The New Glenn is a two-stage rocket that uses a propellant mass of $2.745 * 10^{6}$ kg for the first stage. A propellant mass of $4.989 * 10^{3}$kg is used for stage two. The New Glenn uses liquid natural gas and liquid oxidizer for the propellant in both stages, which given estimates of the current cost of the fuel and oxidizer, a total cost for these propellants is $880,510. The cost of the New Glenn itself is $2.5 billion and the estimated cost of the telescope itself is about $110,231,100. In conclusion, the total cost for this mission is $2.6111 billion dollars, which is about $6 billion less than the total cost of the James Webb mission, although our analysis does not include the research costs and manufacturing of the telescope. Our mission will take about 39 days, from launch, to complete, whereas the James Webb will take around 30 days to reach the Lagrange point. These values help assure us that the calculations done in our analysis are logical.

References:

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Appendix 1: (algorithms)

Equations and method used for low earth orbit:
Using ODE45 in matlab to solve differential equations of rocket motion for stage 1. The burnout altitude of the New Glen can be determined to later calculate the first Hohmann transfer.

$$\frac{dv}{dt} = \frac{T}{m} - \frac{D}{m} - g\sin\gamma$$

$$\frac{dh}{dt} = v\sin\gamma,\ \frac{dx}{dt} = (\frac{Re}{Re+h})v\cos\gamma,\ \frac{dm}{dt} = \frac{-T}{I_{sp}g_0},\ \frac{d\gamma}{dt} = \frac{-\cos\gamma}{v} * (g - \frac{v^2}{Re+h})$$

Equations for first Hohmann transfer:

$Ra = z_1 + Re$

$Rc = Ra$

$Rb = z_2 + Re$

$h_1 = \sqrt{Ra * \mu_e * (1 + e_1 cos\theta)}$

$VA_1 = h_1/Ra$

$e_2 = (Rb - Ra)/(Rb + Ra)$

$h_2 = \sqrt{2 * us * e2}$

$VA_2 = h_2/Rb$

$\Delta VA = VA_2 - VA_1$

$VB_2 = h_2/Rb$

$VB_3 = \sqrt{\mu_e/Rb}$

$\Delta VB = VB_3 - VB_2$

$\Delta V_1 = \Delta VA + \Delta VB$

$t_1 = \pi/\sqrt{\mu_e} * ((Ra + Rb)/2)^{(3/2)}$

$\Delta m_1 = m0_2 * (1 - exp(- \Delta V1/(Isp_2 * g_e)))$

Equations for second Hohmann transfer:

$R_{a2} = z_2 + R_e$

$R_{c2} = R_{a2}$

$e\ L = L_2 - Rse$

$R_{b2} = eL + R_e$

$h_1 = \sqrt{R_{a2} * \mu_e * (1 + e_1)}$

$$V_{a1} = \frac{h_1}{R_{a2}}$$

$$e_2 = \frac{(R_{a2} - R_a)}{(R_{a2} + R_a)}$$

$$h_2 = \sqrt{2 * us * e2}$$

$$V_{a2} = \frac{h_2}{R_{a2}}$$

$$\Delta V_{a2} = V_{a2} - V_{a1}$$

$$V_{b2} = \frac{h_2}{R_{b2}}$$

$$V_{b3} = \sqrt{\frac{\mu_e}{R_{b2}}}$$

$$\Delta V_{b2} = V_{b3} - V_{b2}$$

$$t_2 = \pi / \sqrt{\mu_e} * ((Ra_2 + Rb_2)/2)^{(3/2)}$$

Cost analysis:

New Glenn cost (NG): 2.5 billion

Price for LNG fuel 1DGE = $2.86, 1DGE=2.748kg

Price for LOX oxidizer $0.20/kg

Ratio 6ox:1fuel

392,950kg of LNG = $408,970

2,357,700kg of LOX = $471,540

Total cost of propellant (P) = $880,510

Cost per kg of payload:

714.29kg LNG ⇒ $743.40

4285.71kg LOX ⇒ $857.14

Total cost of propellant per kg of payload = $1600.5

According to NASA it costs $10,000 for 1 lb payload going to space so 5000kg to lb is 11,023.11lb * 10,000 = $110,231,100 dollars for the telescope (PL).

Total cost of mission:

NG+P+PL = $2.6111billion

Appendix 2 (Matlab):

Initial to LOE:

Launch Code:

```
function CodeforProjectstage1rocket
clc
clear
close all
% givens from problem statement and first term calculations for
basic
% variables in use later in code
g0 = 9.807;
Re = 6378e3;
ho = 7500;
rho0 = 1.225;
CD = 0.5;
Ttokg = 1450*907;
mPL = 5000;
m0 = Ttokg + mPL;
TtoN = 9806.65*1746;
% T = 17122.41; %kn
T = TtoN/1000; %kn
tb0 = 240;
Isp = 310;
d = 4.6;
A = pi/4*(d)^2;
mo = 13631243.5 + mPL;
go = 9.807/1000;
deltat = tb0-0;
mf = (-T*deltat)/(Isp*go)+ mo;
n = mo/mf;
mfinal = m0/n;
% Thrust = 17122410; %n
Thrust = TtoN; %n
m_dot = (mo-mf)/tb0;
mp = m0 - mfinal;
tburn = 240;
hturn = 130;
deg = pi/180;
% Initial conditions for function
t0 = 0;
tf = tburn;
tspan = [t0,tf];
% inital vales for integration
```

```
v0 = 0;
gamma0 = deg2rad(89.85);
x0 = 0;
h0 = 0;
vD0 = 0;
vG0 = 0;
%Initial conditions vector:
Initialv = [v0; gamma0; x0; h0; vD0; vG0];
% ode45 function solves the differential equations of motion
using initial
% conditions and integrates over time until burnout
[t1,z] = ode45(@Rocketeq2flight, tspan, Initialv);
%  Rocketeq2flight is an embedded function using the
differential eqs with
% z giving the final values based on the time interval of
integration
v     =  z(:,1)/1000;
gamma =  z(:,2)/deg;
x     =  z(:,3)/1000;
h     =  z(:,4)/1000;
vD    = -z(:,5)/1000;
vG    = -z(:,6)/1000;
Answers
return
function dzdt = Rocketeq2flight(t,z)
% Function calculates rate of change in each variable for the
given
% equations of motion for gravity turn path
% initial vector is defined
dzdt = zeros(6,1);
v     = z(1);
gamma = z(2);
x     = z(3);
h     = z(4);
vD    = z(5);
vG    = z(6);
% for loop used to calculate mass over burntime and the
resulting thrust at
% each condition of launch vehicle mass
if t < tburn
   m = m0 - m_dot*t;
   T = Thrust;
else
   m = m0 - m_dot*tburn;
   T = 0;
```

```
end
% Density vs altitude, gravitational change with altitude and
drag at given
% velocity calculated for later steps
g = g0/(1 + h/Re)^2;
rho = rho0*exp(-h/ho);
D = 0.5*rho*v^2*A*CD;
% First if statement for vertical flight using differential
equations of motion up
% until altitude is above 130m. Then gravity assist begins in
else
% statement
if h <= hturn
   gamma_dot = 0;
   v_dot  = T/m - D/m - g;
   x_dot  = 0;
   h_dot  = v;
   vG_dot = -g;
else
   % when flight reaches above 130m, gravity turn begins using
differential equations of
   % motion to calculate values for the vehicle as it continues
it ascent
   v_dot = T/m - D/m - g*sin(gamma);
   gamma_dot = -1/v*(g - v^2/(Re + h))*cos(gamma);
   x_dot     = Re/(Re + h)*v*cos(gamma);
   h_dot     = v*sin(gamma);
   vG_dot    = -g*sin(gamma);
end
vD_dot = -D/m;
% Values for the equations of motion calculated stored into
matrix
dzdt(1) = v_dot;
dzdt(2) = gamma_dot;
dzdt(3) = x_dot;
dzdt(4) = h_dot;
dzdt(5) = vD_dot;
dzdt(6) = vG_dot;
end
function Answers
    % printing of important values of solution
fprintf('\n Speed at Burnout        = %f km/s',v(end))
fprintf('\n Altitude at Burnout       = %f km ',h(end))
fprintf('\n Downrange distance        = %f km ',x(end))
fprintf('\n Mass of Propellent Expended = %f kg',mp)
```

```
% plot of Altitude vs Downrange Distance
figure (1)
plot(x,h)
title('Altitude vs Downrange Distance')
xlabel('Downrange Distance (km)')
ylabel('Altitude (km)')
grid
end
End
```

Hohmann Transfers and cost analysis:

```
clc
clear
close all

Rs = 696000;
Re = 6378;
Rse = 150*10^6;
z1 = 275.59;
z2 = 10000;
L2 = 151.5*10^6;
ue = 398600;
us = 132.712*10^9;
ge = 9.81/1000;
gs = 275/1000;
mPL = 5000;
m0_1 = 13631243.5+mPL;
m0_2 = 2745862+mPL;
Isp_1 = 335;
Isp_2 = 440;
T_1 = 20015.37;
T_2 = 1078.73;
eL = L2-Rse;
e1_2 = 0;
e1 = 0;

%1st Hohmann transfer to altitude 10,000km
Ra = z1+Re;
Rc = Ra;
Rb = z2+ Re;
h1 = sqrt(Ra*ue*(1+e1));
Va1 = h1/Ra;
e2 = (Rb-Ra)/(Rb+Ra);
h2 = sqrt(2*us*e2);
```

```
Va2 = h2/Ra;
deltaVa_1 = Va2-Va1;
Vb2 = h2/Rb;
Vb3 = sqrt(ue/Rb);
deltaVb_1 = Vb3-Vb2;

deltaV_1 = deltaVa_1+deltaVb_1

t1 = pi/sqrt(ue)*((Ra+Rb)/2)^(3/2)
t1min = t1/60
deltam1 = m0_2 * (1-exp(-deltaV_1/(Isp_2*ge)));

%2nd Hohmann transfer to lagrange point
RA2 = z2+Re;
RC2 = RA2;
RB2 = eL+Re;
h1_2 = sqrt(RA2*ue*(1+e1_2));
VA1_2 = h1_2/RA2;
e2_2 = (RB2-RA2)/(RB2+RA2);
h2_2 = sqrt(2*us*e2_2);
VA2_2 = h2_2/RA2;
deltaVA2 = VA2_2-VA1_2;
VB2_2 = h2_2/RB2;
VB3_2 = sqrt(ue/RB2);
deltaVB2 = VB3_2-VB2_2;

deltaV_2 = deltaVB2+deltaVA2

t2 = pi/sqrt(ue)*((RA2+RB2)/2)^(3/2)
t2min = t2/60;
t2hrs = t2min/60;
t2days = t2hrs/24
deltam2 = mPL*(1-exp(-deltaV_2/(Isp_2*ge)));

deltam = deltam1+deltam2

%Cost analysis
NG = 2.5*10^9;
%Cost $2.86/DGE
LNG_cost = 2.86;
%Cost $0.20/kg
LOX_cost = 0.20;
%ratio for Oxidizer vs fuel is 6:1 respectively
LNG_mass = (deltam/7)*1;
LOX_mass = (deltam/7)*6;
```

```
%conversion of LNG from kg to DGE
LNG_massDGE = LNG_mass/2.748;
Fuel_cost = LNG_massDGE*LNG_cost
Ox_cost = LOX_mass*LOX_cost
Cost_prop = Fuel_cost+Ox_cost

%cost per kg of payload
LNG_mass_payload = (mPL/7)*1;
LOX_mass_payload = (mPL/7)*6;
%conversion of LNG from kg to DGE
LNG_mass_payloadDGE = LNG_mass_payload/2.748;
Fuel_cost_payload = LNG_mass_payloadDGE*LNG_cost;
Ox_cost_payload = LOX_mass_payload*LOX_cost;
cost_per_kg = Fuel_cost_payload+Ox_cost_payload
telescope = 110231100;
Total_cost = NG+Cost_prop+telescope
```