(%i3)
```
ResetToA0():=[a0:0,a3:0,x0:'x0,x1:'x1,x3:'x3,v0:'v0,v1:'v1,v3:'v3,a1:'a1,j1:'j1,Jf:'Jf,t1:'t1,t2:'t2,t3:'t3,tf:t1+t2+t3,DeltaX:'DeltaX]$
/·ResetToBase():=[xm1:'xm1,x0:'x0,x1:'x1,x2:'x2,x3:'x3,x4:'x4,vm1:'vm1,v0:'v0,v1:'v1,v2:'v2,v3:'v3,v4:'v4,a0:'a0,a1:'a1,a2:'a2,a3:'a3,a4='a4,j1:'j1,Jf:'Jf,t0:'t0,t1:'t1,t2:'t2,t3
ResetToBase():= remvalue(xm1,x0,x1,x2,x3,x4,vm1,v0,v1,v2,v3,v4,a0,a1,a2,a3,a4,j1,Jf,t0,t1,t2,t3,t4,tf,DeltaX,t,delta,alpha);
maxima_userdir;
```

(%o2) ResetToBase (*Bug: Missing contents*) :=
remvalue $(xm1,x0,x1,x2,x3,x4,vm1,v0,v1,v2,v3,v4,a0,a1,a2,a3,a4,j1,Jf,t0,t1,t2,t3,t4,tf,DeltaX,t,\delta,\alpha)$

(%o3) *C:/Users/afsharn/maxima*

RSCN PVT Scan-Skip formulation

1 PVT kinematics formulation

In this section, some useful combinations of the Postion Velocity Time motion are formulated. The equations are in a general form, contain a mid PVT move which is constant acceleration at maximum acceleration.
S-Curved acceleration is a special case of this solution with t1=t2

1.1 PVT as a linear acceleration equation

(%i6)
```
define(f_a1(t),a0+(a1−a0)/t1·t);
define(f_v1(t),v0+integrate(f_a1(t), t));
define(f_x1(t),x0+integrate(f_v1(t), t));
```

(%o4) $$f_{a1}(t):=\frac{(a1-a0)\,t}{t1}+a0$$

(%o5) $$f_{v1}(t):=v0+\frac{(a1-a0)\,t^2}{2\,t1}+a0\,t$$

(%o6) $$f_{x1}(t):=x0+t\,v0+\frac{(a1-a0)\,t^3}{6\,t1}+\frac{a0\,t^2}{2}$$

1.2 Solving single PVT with known initial and final points

(%i17)
```
ResetToBase()$
eqx1: x1=f_x1(t1);
eqv1: v1=f_v1(t1);
Sln1: solve([eqx1,eqv1],[t1,a1]);
define(s_t1_1(x0,v0,x1,v1,a0),ev(t1,Sln1[1][1]));
define(s_a1_1(x0,v0,x1,v1,a0),ev(a1,Sln1[1][2]));
define(s_t1_2(x0,v0,x1,v1,a0),ev(t1,Sln1[2][1]));
define(s_a1_2(x0,v0,x1,v1,a0),ev(a1,Sln1[2][2]));
Sln2: solve([eqx1,eqv1],[a0,a1]);
define(p_a0_1(x0,v0,x1,v1,t1),ev(a0,Sln2[1]));
define(p_a1_1(x0,v0,x1,v1,t1),ev(a1,Sln2[1]));
```

(eqx1) $$x1=x0+t1\,v0+\frac{(a1-a0)\,t1^2}{6}+\frac{a0\,t1^2}{2}$$

(eqv1) $$v1=v0+\frac{(a1-a0)\,t1}{2}+a0\,t1$$

(Sln1) $$\left[\left[t1=\frac{6\,x1-6\,x0}{-\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v1+2\,v0},a1=-\frac{v1\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v0\left(-\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}-v1\right)+3\,a0\,x1-3\,a0\,x0-v1^2+2\,v0^2}{3\,x1-3\,x0}\right],\left[t1=\frac{6\,x1-6\,x0}{\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v1+2\,v0},a1=-\frac{v0\left(\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}-v1\right)-v1\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+3\,a0\,x1-3\,a0\,x0-v1^2+2\,v0^2}{3\,x1-3\,x0}\right]\right]$$

(%o11) $$s\_t1_1(x0,v0,x1,v1,a0):=\frac{6\,x1-6\,x0}{-\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v1+2\,v0}$$

(%o12) $$s\_a1_1(x0,v0,x1,v1,a0):=-\frac{v1\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v0\left(-\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}-v1\right)+3\,a0\,x1-3\,a0\,x0-v1^2+2\,v0^2}{3\,x1-3\,x0}$$

(%o13) $$s\_t1_2(x0,v0,x1,v1,a0):=\frac{6\,x1-6\,x0}{\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+v1+2\,v0}$$

(%o14) $$s\_a1_2(x0,v0,x1,v1,a0):=-\frac{v0\left(\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}-v1\right)-v1\sqrt{6\,a0\,x1-6\,a0\,x0+v1^2+4\,v0\,v1+4\,v0^2}+3\,a0\,x1-3\,a0\,x0-v1^2+2\,v0^2}{3\,x1-3\,x0}$$

(Sln2) $$\left[\left[a0=\frac{6\,x1-6\,x0-2\,t1\,v1-4\,t1\,v0}{t1^2},a1=-\frac{6\,x1-6\,x0-4\,t1\,v1-2\,t1\,v0}{t1^2}\right]\right]$$

(%o16) $$p\_a0_1(x0,v0,x1,v1,t1):=\frac{6\,x1-6\,x0-2\,t1\,v1-4\,t1\,v0}{t1^2}$$

(%o17) $$p\_a1_1(x0,v0,x1,v1,t1):=-\frac{6\,x1-6\,x0-4\,t1\,v1-2\,t1\,v0}{t1^2}$$

1.2.1 Symmetric 2 segment turn around

We only need to solve it for one segment. The 2nd segment would be the oposite move: v21=-v10, t21=t11

(%i24)
```
ResetToBase()$
eq11: 0=p_a0_1(0,v0,x1,0,t1);
eq12: a1=p_a1_1(0,v0,x1,0,t1);
eqSym2SegRet: solve([eq11,eq12],[t1,x1]);
second(eqSym2SegRet[1][1]);
define(tOutFn(v0,a1),second(eqSym2SegRet[1][1]));
define(d1OutFn(v0,a1),second(eqSym2SegRet[1][2]));
```

(eq11) $0=\frac{6\,x1-4\,t1\,v0}{t1^2}$

(eq12) $a1=-\frac{6\,x1-2\,t1\,v0}{t1^2}$

(eqSym2SegRet) $[[t1=-\frac{2\,v0}{a1},x1=-\frac{4\,v0^2}{3\,a1}],[t1=0,x1=0]]$

(%o22) $-\frac{2\,v0}{a1}$

(%o23) $\mathrm{tOutFn}(v0,a1):=-\frac{2\,v0}{a1}$

(%o24) $\mathrm{d1OutFn}(v0,a1):=-\frac{4\,v0^2}{3\,a1}$

1.3 Single PVT with known initial and final points, for Opt-jerk

→
```
ResetToBase()$
eq11: ai=p_a0_1(x0,v0,x1,v1,t1)−Delta;
eq12: af=p_a1_1(x0,v0,x1,v1,t1)+Delta;
eqSngMinJrk: solve([eq11,eq12],[t1,Delta]);
```

1.4 Single PVT accelerating with constant acceleration

→
```
ResetToBase()$
eq11: af=p_a0_1(x0,v0,x0+dX,v0+dV,t1),factor;
eq12: af=p_a1_1(x0,v0,x0+dX,v0+dV,t1),factor;
eqSngAcc: solve([eq11,eq12],[af,dX]),factor;
```

2 *Symmetric 4 segment turn around*

Move is a sequence of 4 moves:
point(-1) to point(0): t0, X constant speed=XVel, Y increasing accel to a0,
point(0) to point(1): t1, X decel to speed=0, Y decreasing accel to 0,
point(1) to point(2): t2=0, not used
point(2) to point(3): t3, X accel to speed=XVel, Y decreasing accel to a3=-a0,
point(3) to point(4): t4, X constant speed=XVel, Y decreasing accel to 0,

When position crosses the grid X boundary, Y is at ax2Step·(K/2) and returns at ax2Step·(1-K/2). 0<K<1

Solving the equations in general form shows that only a symmetric solution with a0=-a3, v0=v3 and t0=t4 can be valid when Y pre and post moves are equal

2.1 Initial formulation and analysis

2.2 (OBS) Symmetric 4 segment turn around formulation

2.3 symmetric 4+1 segment move, with constant velocity segment a2=0, v2=v3, t2=given

Trick is to have the same set of equations as 4 segment, with additional assumption that the total travel is now DeltaX + t2·v2, where DeltaX is the 4 segment travel equivalent, t2 is given and v2 can be written in terms of t0,t3 and a0

(%i37)
```
ResetToBase()$K:'K$
[xm1:0,vm1:0,v4:0,x0:xm1+DeltaXTot·(K/2),x3:xm1+DeltaXTot·(1−K/2),x4:xm1+DeltaXTot,v4:0,a3:−a0,a4:0,v3:v0,t4:t0,t1:t3];
DeltaXTot: DeltaX −t2·(t0+t3)·a0/2;
eqm10: 0=p_a0_1(xm1,vm1,x0,v0,t0)$
eqm11: a0=p_a1_1(xm1,vm1,x0,v0,t0)$
eq00: a0=p_a0_1(x0,v0,x3,v3,t1+t3)$
eq01: a3=p_a1_1(x0,v0,x3,v3,t1+t3)$
eq30: a3=p_a0_1(x3,v3,x4,v4,t4)$
eq31: a4=p_a1_1(x3,v3,x4,v4,t4)$
eqSet:[eqm10,eqm11,eq00,eq01,eq30,eq31];
s5S2:solve(eqSet,[K,v0,t0]);
s5S3:solve(eqSet,[K,v0,t3]);
```

(%o27) $$[0,0,0,\frac{DeltaXTot\ K}{2},DeltaXTot\left(1-\frac{K}{2}\right),DeltaXTot,0,-a0,0,v0,t0,t3]$$

(DeltaXTot) $$DeltaX-\frac{a0\ t2\ (t3+t0)}{2}$$

(eqSet) $$[0=\frac{3\ DeltaXTot\ K-2\ t0\ v0}{t0^2},a0=-\frac{3\ DeltaXTot\ K-4\ t0\ v0}{t0^2},a0=\frac{-12\ t3\ v0-3\ DeltaXTot\ K+6\ DeltaXTot\left(1-\frac{K}{2}\right)}{4\ t3^2},-a0=-$$
$$\frac{-12\ t3\ v0-3\ DeltaXTot\ K+6\ DeltaXTot\left(1-\frac{K}{2}\right)}{4\ t3^2},-a0=\frac{-4\ t0\ v0-6\ DeltaXTot\left(1-\frac{K}{2}\right)+6\ DeltaXTot}{t0^2},0=-\frac{-2\ t0\ v0-6\ DeltaXTot\left(1-\frac{K}{2}\right)+6\ DeltaXTot}{t0^2}]$$

(s5S2) $$[[K=0,v0=-\frac{4\ a0\ t3^2+3\ a0\ t2\ t3-6\ DeltaX}{12\ t3},t0=0],[K=(t3$$
$$(-21\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+81\ a0^2\ t2^2+144\ DeltaX\ a0)+t3^2$$
$$(72\ a0^2\ t2-10\ a0\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0})-9\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-12\ DeltaX$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+12\ a0^2\ t3^3+27\ a0^2\ t2^3+108\ DeltaX\ a0\ t2)/(t3$$
$$(27\ a0^2\ t2^2-3\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0})-9\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-12\ DeltaX$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+6\ a0^2\ t2\ t3^2+27\ a0^2\ t2^3+108\ DeltaX\ a0\ t2),v0=-(t3(-81\ a0^2\ t2^3$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-168\ DeltaX\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+297\ a0^3\ t2^4+1224\ DeltaX\ a0^2\ t2^2+$$
$$576\ DeltaX^2\ a0)+t3^2$$
$$(-72\ a0^2\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-40\ DeltaX\ a0\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+378\ a0^3\ t2^3+696\ DeltaX\ a0^2\ t2)$$
$$+t3^3(-16\ a0^2\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+192\ a0^3\ t2^2+48\ DeltaX\ a0^2)-27\ a0^2\ t2^4$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-108\ DeltaX\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-48\ DeltaX^2$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+32\ a0^3\ t2\ t3^4+81\ a0^3\ t2^5+540\ DeltaX\ a0^2\ t2^3+720\ DeltaX^2\ a0\ t2)/(t3$$
$$(-60\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}-48\ DeltaX\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+252\ a0^2\ t2^3+624\ DeltaX\ a0\ t2)$$
$$+t3^2(-16\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+168\ a0^2\ t2^2+32\ DeltaX\ a0)-36\ a0\ t2^3\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}$$
$$-96\ DeltaX\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+32\ a0^2\ t2\ t3^3+108\ a0^2\ t2^4+576\ DeltaX\ a0\ t2^2+384\ DeltaX^2),t0=-$$
$$\frac{-\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+6\ a0\ t3+3\ a0\ t2}{4\ a0}],[K=(t3$$
$$(21\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+81\ a0^2\ t2^2+144\ DeltaX\ a0)+t3^2$$
$$(10\ a0\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+72\ a0^2\ t2)+9\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+12\ DeltaX$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+12\ a0^2\ t3^3+27\ a0^2\ t2^3+108\ DeltaX\ a0\ t2)/(t3$$
$$(3\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+27\ a0^2\ t2^2)+9\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+12\ DeltaX$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+6\ a0^2\ t2\ t3^2+27\ a0^2\ t2^3+108\ DeltaX\ a0\ t2),v0=-(t3(81\ a0^2\ t2^3$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+168\ DeltaX\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+297\ a0^3\ t2^4+1224\ DeltaX\ a0^2\ t2^2+$$
$$576\ DeltaX^2\ a0)+t3^2$$
$$(72\ a0^2\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+40\ DeltaX\ a0\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+378\ a0^3\ t2^3+696\ DeltaX\ a0^2\ t2)$$
$$+t3^3(16\ a0^2\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+192\ a0^3\ t2^2+48\ DeltaX\ a0^2)+27\ a0^2\ t2^4$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+108\ DeltaX\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+48\ DeltaX^2$$
$$\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+32\ a0^3\ t2\ t3^4+81\ a0^3\ t2^5+540\ DeltaX\ a0^2\ t2^3+720\ DeltaX^2\ a0\ t2)/(t3$$
$$(60\ a0\ t2^2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+48\ DeltaX\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+252\ a0^2\ t2^3+624\ DeltaX\ a0\ t2)$$
$$+t3^2(16\ a0\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+168\ a0^2\ t2^2+32\ DeltaX\ a0)+36\ a0\ t2^3\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+$$
$$96\ DeltaX\ t2\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+32\ a0^2\ t2\ t3^3+108\ a0^2\ t2^4+576\ DeltaX\ a0\ t2^2+384\ DeltaX^2),t0=-$$
$$\frac{\sqrt{4\ a0^2\ t3^2+12\ a0^2\ t2\ t3+9\ a0^2\ t2^2+48\ DeltaX\ a0}+6\ a0\ t3+3\ a0\ t2}{4\ a0}]]$$

(s5S3) $$[[K=(-24\ a0\ t0^2\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+72\ a0^2\ t0^2\ t2+16\ a0^2\ t0^3)/(t2$$
$$(6\ a0\ t0\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+12\ a0^2\ t0^2+648\ DeltaX\ a0)+t2^2$$
$$(-27\ a0\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}-72\ a0^2\ t0)-72\ DeltaX\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+81\ a0^2\ t2^3+48\ DeltaX$$
$$a0\ t0),v0=\frac{a0\ t0}{2},t3=-\frac{-\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+3\ a0\ t2+6\ a0\ t0}{8\ a0}],[K=(24\ a0\ t0^2\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+$$
$$72\ a0^2\ t0^2\ t2+16\ a0^2\ t0^3)/(t2(-6\ a0\ t0\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+12\ a0^2\ t0^2+648\ DeltaX\ a0)+t2^2$$
$$(27\ a0\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}-72\ a0^2\ t0)+72\ DeltaX\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+81\ a0^2\ t2^3+48\ DeltaX$$
$$a0\ t0),v0=\frac{a0\ t0}{2},t3=-\frac{\sqrt{9\ a0^2\ t2^2-12\ a0^2\ t0\ t2+4\ a0^2\ t0^2+96\ DeltaX\ a0}+3\ a0\ t2+6\ a0\ t0}{8\ a0}]]$$

2.3.1 RSCN substitution of symmetric 4+1 segment turnaround, with constant velocity segment a2=0, v2=v3, t2=tEdgeTotal

(%i59)
```
forget(a0>0);
_simpSub:[DeltaX=delta·a0,t2=−(2·t3−4·tau)/3];
_simpSln:solve(_simpSub,[tau,delta]);
/·print("subst into")$·/
s5S2[2][3];
slnPlus:ev(%,_simpSub),factor;
assume(a0>0);
subs0:[t0=tIn/1,v0=v2In,t2=tEdgeTotal,t3=tOut/1,DeltaX=d2Step,a0=a2Max]$
Pre4CSegSubs: subst(subs0,_simpSln[1]);
Pre4CSegPlus:subst(subs0,slnPlus),expand;
s5S2[3][3]$
slnMinus:ev(%,_simpSub),factor$
Pre4CSegMinus:subst(subs0,slnMinus),expand;
define(tInFn(tOut,tEdgeTotal,a2Max,d2Step),expand(ev(second(Pre4CSegPlus),Pre4CSegSubs)));
forget(a0>0);
d2InSet: '[x0=0,v0=0,a0=0,a1=a2Max,t1=tIn]$
d2InEq: d2In=ev(f_x1(t1),d2InSet),factor;
v2InEq: v2In=ev(f_v1(t1),d2InSet);
d2OutSet: '[x0=0,v0=second(v2InEq),a0=a2Max,a1=0,t1=tOut]$
d2OutEq: d2Out=ev(f_x1(t1),d2OutSet),factor;
v2OutEq: v2Out=ev(f_v1(t1),d2OutSet),factor;
a2MaxMAX: ev(a2Max,subst(append(subs0),a0MAX));
subst(append(subs0,[tOut=2·XVel/a1Max·1]),a0MAX);
```

(%o38) $[a0>0]$

(_simpSub) $[DeltaX=a0\,\delta,t2=\frac{4\,\tau-2\,t3}{3}]$

(_simpSln) $[[\tau=\frac{2\,t3+3\,t2}{4},\delta=\frac{DeltaX}{a0}]]$

(%o41) $t0=-\frac{-\sqrt{4\,a0^2\,t3^2+12\,a0^2\,t2\,t3+9\,a0^2\,t2^2+48\,DeltaX\,a0}+6\,a0\,t3+3\,a0\,t2}{4\,a0}$

(slnPlus) $t0=\frac{|a0|\sqrt{\tau^2+3\,\delta}-a0\,\tau-a0\,t3}{a0}$

(%o43) $[a0>0]$

(Pre4CSegSubs) $[\tau=\frac{2\,tOut+3\,tEdgeTotal}{4},\delta=\frac{d2Step}{a2Max}]$

(Pre4CSegPlus) $tIn=\sqrt{\tau^2+3\,\delta}-\tau-tOut$

(Pre4CSegMinus) $tIn=-\sqrt{\tau^2+3\,\delta}-\tau-tOut$

(%o50) $\mathrm{tInFn}(tOut,tEdgeTotal,a2Max,d2Step):=\sqrt{\frac{tOut^2}{4}+\frac{3\,tEdgeTotal\,tOut}{4}+\frac{9\,tEdgeTotal^2}{16}+\frac{3\,d2Step}{a2Max}}-\frac{3\,tOut}{2}-\frac{3\,tEdgeTotal}{4}$

(%o51) $[a0>0]$

(d2InEq) $d2In=\frac{a2Max\,tIn^2}{6}$

(v2InEq) $v2In=\frac{a2Max\,tIn}{2}$

(d2OutEq) $d2Out=\frac{a2Max\,tOut\,(2\,tOut+3\,tIn)}{6}$

(v2OutEq) $v2Out=\frac{a2Max\,(tOut+tIn)}{2}$

(a2MaxMAX) $a2Max$

(%o59) $a0MAX$

(%i75)
```
forget(a0>0);
_simpSub:[DeltaX=delta·a0,t2=−(2·t3−4·tau)/3];
_simpSln:solve(_simpSub,[tau,delta]);
/·print("subst into")$·/
s5S2[2][3];
slnPlus:ev(%,_simpSub),factor;
assume(a0>0);
tmp: t3=(tOut+(tEdge/2));
subs0:[t0=tIn/1,v0=ax2VlSkip,t2=tEFSkip,DeltaX=d2Step,a0=a2Max]$
/·
subs0:append(tmp,subs0)$
·/
tEdgeTotal=(tEdge/2)+tEFSkip;
Pre4CSegSubs: subst(append([tmp],subs0),_simpSln[1]);

tmpt3: solve(_simpSub[2],[t3]);
subs0:append(tmpt3,subs0)$

Pre4CSegPlus:subst(subs0,slnPlus),expand;
s5S2[3][3]$
slnMinus:ev(%,_simpSub),factor$
Pre4CSegMinus:subst(subs0,slnMinus),expand;
```

(%o60) $[a0>0]$

(_simpSub) $[DeltaX=a0\,\delta, t2=\frac{4\,\tau-2\,t3}{3}]$

(_simpSln) $[[\tau=\frac{2\,t3+3\,t2}{4}, \delta=\frac{DeltaX}{a0}]]$

(%o63) $t0=-\frac{-\sqrt{4\,a0^2\,t3^2+12\,a0^2\,t2\,t3+9\,a0^2\,t2^2+48\,DeltaX\,a0}+6\,a0\,t3+3\,a0\,t2}{4\,a0}$

(slnPlus) $t0=\frac{|a0|\sqrt{\tau^2+3\,\delta}-a0\,\tau-a0\,t3}{a0}$

(%o65) $[a0>0]$

(tmp) $t3=tOut+\frac{tEdge}{2}$

(%o68) $tEdgeTotal=\frac{tEdge}{2}+tEFSkip$

(Pre4CSegSubs) $[\tau=\frac{2\left(tOut+\frac{tEdge}{2}\right)+3\,tEFSkip}{4}, \delta=\frac{d2Step}{a2Max}]$

(tmpt3) $[t3=\frac{4\,\tau-3\,t2}{2}]$

(Pre4CSegPlus) $tIn=\sqrt{\tau^2+3\,\delta}-3\,\tau+\frac{3\,tEFSkip}{2}$

(Pre4CSegMinus) $tIn=-\sqrt{\tau^2+3\,\delta}-3\,\tau+\frac{3\,tEFSkip}{2}$

(%i76)
```
a2Max;
```

(%o76) $a2Max$

(%i91)
```
/* Now calculate (tEdge/2) for ax2 moves */
forget(a2Max>0);
forget(d2In>0);

rscn_tIn_ax2: solve(d2InEq,[tIn])[2];

tmp: Pre4CSegPlus-tIn;

%-third(second(%));
tmp: %^2;
solve(tmp,[tau]),expand$
tauEq: %[2];
deltaEq: solve(tmp,[delta])[1];
tmp: solve(Pre4CSegSubs[1],[tEdge]);

incase: [tEFSkip=0];
rscn_tau_ax2: subst(incase,tauEq),radcan;
/*subst([Pre4CSegSubs[2]],%);*/
subst(incase,tmp),ratsimp;
rscn_tEdge_ax2: subst(rscn_tau_ax2,%);

rscn_delta_ax2: subst(incase,deltaEq),ratsimp;
```

(%o77) $[a2Max>0]$

(%o78) $[d2In>0]$

(rscn_tIn_ax2) $tIn=\sqrt{6}\sqrt{\frac{d2In}{a2Max}}$

(tmp) $0=\sqrt{\tau^2+3\,\delta}-3\,\tau-tIn+\frac{3\,tEFSkip}{2}$

(%o81) $tIn=\sqrt{\tau^2+3\,\delta}-3\,\tau+\frac{3\,tEFSkip}{2}$

(tmp) $tIn^2=\left(\sqrt{\tau^2+3\,\delta}-3\,\tau+\frac{3\,tEFSkip}{2}\right)^2$

(tauEq) $\tau=\frac{\sqrt{-12\,tEFSkip\,\sqrt{\tau^2+3\,\delta}+36\,\tau^2+40\,tIn^2-9\,tEFSkip^2-12\,\delta}}{20}+\frac{3\,\sqrt{\tau^2+3\,\delta}}{10}+\frac{9\,tEFSkip}{20}$

(deltaEq) $\delta=-\frac{(12\,tEFSkip-24\,\tau)\,\sqrt{\tau^2+3\,\delta}+40\,\tau^2-36\,tEFSkip\,\tau-4\,tIn^2+9\,tEFSkip^2}{12}$

(tmp) $[tEdge=4\,\tau-2\,tOut-3\,tEFSkip]$

(incase) $[tEFSkip=0]$

(rscn_tau_ax2) $\tau=\frac{\sqrt{9\,\tau^2+10\,tIn^2-3\,\delta}+3\,\sqrt{\tau^2+3\,\delta}}{10}$

(%o89) $[tEdge=4\,\tau-2\,tOut]$

(rscn_tEdge_ax2) $[tEdge=\frac{2\,(\sqrt{9\,\tau^2+10\,tIn^2-3\,\delta}+3\,\sqrt{\tau^2+3\,\delta})}{5}-2\,tOut]$

(rscn_delta_ax2) $\delta=\frac{6\,\tau\,\sqrt{\tau^2+3\,\delta}-10\,\tau^2+tIn^2}{3}$

(%i92)
```
solve(rscn_tIn_ax2^2,[a2Max]);
```

(%o92) $[a2Max=\frac{6\,d2In}{tIn^2}]$

(%i96)
```
subst([delta=d2Step/a2Max],rscn_delta_ax2);
solve(%,[a2Max]);
diff(second(%[1]),tau,1),ratsimp;
subst([tau=(tEdge/2)/2+tOut/2],1/%),factor;
```

(%o93) $$\frac{d2Step}{a2Max}=\frac{6\,\tau\sqrt{\tau^2+\frac{3\,d2Step}{a2Max}}-10\,\tau^2+tIn^2}{3}$$

(%o94) $$\left[a2Max=\frac{3\,d2Step}{6\,\tau\sqrt{\frac{a2Max\,\tau^2+3\,d2Step}{a2Max}}-10\,\tau^2+tIn^2}\right]$$

(%o95) $$\frac{60\,a2Max\,d2Step\,\tau\sqrt{\frac{a2Max\,\tau^2+3\,d2Step}{a2Max}}-36\,a2Max\,d2Step\,\tau^2-54\,d2Step^2}{36\,a2Max\,\tau^2\left(\frac{a2Max\,\tau^2+3\,d2Step}{a2Max}\right)^{3/2}-120\,a2Max\,\tau^5+\sqrt{\frac{a2Max\,\tau^2+3\,d2Step}{a2Max}}\,(100\,a2Max\,\tau^4-20\,a2Max\,tIn^2\,\tau^2+a2Max\,tIn^4)+(12\,a2Max\,tIn^2-360\,d2Step)\,\tau^3+36\,d2Step\,tIn^2\,\tau}$$

(%o96) $$\begin{aligned}&(36\,a2Max\,tOut^2\left(\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}\right)^{3/2}+36\,a2Max\,tEdge\,tOut\\&\left(\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}\right)^{3/2}+9\,a2Max\,tEdge^2\left(\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}\right)^{3/2}+400\\&a2Max\,tOut^4\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}+800\,a2Max\,tEdge\,tOut^3\\&\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}-320\,a2Max\,tIn^2\,tOut^2\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}+600\\&a2Max\,tEdge^2\,tOut^2\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}-320\,a2Max\,tEdge\,tIn^2\,tOut\\&\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}+200\,a2Max\,tEdge^3\,tOut\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}+64\\&a2Max\,tIn^4\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}-80\,a2Max\,tEdge^2\,tIn^2\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}\\&+25\,a2Max\,tEdge^4\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}-960\,a2Max\,tOut^5-2400\,a2Max\,tEdge\,tOut^4+384\,a2Max\,tIn^2\,tOut^3-2400\\&a2Max\,tEdge^2\,tOut^3-11520\,d2Step\,tOut^3+576\,a2Max\,tEdge\,tIn^2\,tOut^2-1200\,a2Max\,tEdge^3\,tOut^2-17280\,d2Step\,tEdge\,tOut^2+288\,a2Max\,tEdge^2\,tIn^2\,tOut+\\&4608\,d2Step\,tIn^2\,tOut-300\,a2Max\,tEdge^4\,tOut-8640\,d2Step\,tEdge^2\,tOut+48\,a2Max\,tEdge^3\,tIn^2+2304\,d2Step\,tEdge\,tIn^2-30\,a2Max\,tEdge^5-1440\,d2Step\\&tEdge^3)/(192\,d2Step\,(10\,a2Max\,tOut\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}+5\,a2Max\,tEdge\\&\sqrt{\frac{4\,a2Max\,tOut^2+4\,a2Max\,tEdge\,tOut+a2Max\,tEdge^2+48\,d2Step}{a2Max}}-12\,a2Max\,tOut^2-12\,a2Max\,tEdge\,tOut-3\,a2Max\,tEdge^2-72\,d2Step))\end{aligned}$$

(%i105)
```
assume((tOut+tau+tInLLM)>0);
assume(a2Max>0);
tmp:subst([tIn=tInLLM],Pre4CSegPlus);
assume((2·tInLLM−3·tEFSkip+6·tau)>0);
dumeq:solve(tmp, [delta]);
subst(Pre4CSegSubs,dumeq),factor;

assume((2·tInLLM−3·tEFSkip+6·tau)<0);
dumeq:solve(tmp, [delta]);
subst(Pre4CSegSubs,dumeq),factor;
```

(%o97) $[\tau+tOut+tInLLM>0]$

(%o98) $[a2Max>0]$

(tmp) $$tInLLM=\sqrt{\tau^2+3\,\delta}-3\,\tau+\frac{3\,tEFSkip}{2}$$

(%o100) $[6\,\tau+2\,tInLLM-3\,tEFSkip>0]$

(dumeq) $$\left[\delta=\frac{32\,\tau^2+(24\,tInLLM-36\,tEFSkip)\,\tau+4\,tInLLM^2-12\,tEFSkip\,tInLLM+9\,tEFSkip^2}{12}\right]$$

(%o102) $$\left[\frac{d2Step}{a2Max}=\frac{(2\,tOut+2\,tInLLM+tEdge)\,(4\,tOut+2\,tInLLM+2\,tEdge+3\,tEFSkip)}{12}\right]$$

(%o103) $[inconsistent]$

(dumeq) $$\left[\delta=\frac{32\,\tau^2+(24\,tInLLM-36\,tEFSkip)\,\tau+4\,tInLLM^2-12\,tEFSkip\,tInLLM+9\,tEFSkip^2}{12}\right]$$

(%o105) $$\left[\frac{d2Step}{a2Max}=\frac{(2\,tOut+2\,tInLLM+tEdge)\,(4\,tOut+2\,tInLLM+2\,tEdge+3\,tEFSkip)}{12}\right]$$

(%i109)
```
s5S3[2][1];
subst(_simpSub,%);
subst(subs0,%);
subst([a2Max=a^2],%);
```

(%o106) $K=\left(24\,a0\,t0^2\sqrt{9\,a0^2\,t2^2-12\,a0^2\,t0\,t2+4\,a0^2\,t0^2+96\,DeltaX\,a0}+72\,a0^2\,t0^2\,t2+16\,a0^2\,t0^3\right)/\left(t2\left(-6\,a0\,t0\sqrt{9\,a0^2\,t2^2-12\,a0^2\,t0\,t2+4\,a0^2\,t0^2+96\,DeltaX\,a0}+12\,a0^2\,t0^2+648\,DeltaX\,a0\right)+t2^2\left(27\,a0\sqrt{9\,a0^2\,t2^2-12\,a0^2\,t0\,t2+4\,a0^2\,t0^2+96\,DeltaX\,a0}-72\,a0^2\,t0\right)+72\,DeltaX\sqrt{9\,a0^2\,t2^2-12\,a0^2\,t0\,t2+4\,a0^2\,t0^2+96\,DeltaX\,a0}+81\,a0^2\,t2^3+48\,DeltaX\,a0\,t0\right)$

(%o107) $K=\left(24\,a0\,t0^2\sqrt{a0^2\,(4\tau-2\,t3)^2-4\,a0^2\,t0\,(4\tau-2\,t3)+4\,a0^2\,t0^2+96\,a0^2\,\delta}+24\,a0^2\,t0^2\,(4\tau-2\,t3)+16\,a0^2\,t0^3\right)/\left(\frac{(4\tau-2\,t3)\left(-6\,a0\,t0\sqrt{a0^2\,(4\tau-2\,t3)^2-4\,a0^2\,t0\,(4\tau-2\,t3)+4\,a0^2\,t0^2+96\,a0^2\,\delta}+12\,a0^2\,t0^2+648\,a0^2\,\delta\right)}{3}+\frac{(4\tau-2\,t3)^2\left(27\,a0\sqrt{a0^2\,(4\tau-2\,t3)^2-4\,a0^2\,t0\,(4\tau-2\,t3)+4\,a0^2\,t0^2+96\,a0^2\,\delta}-72\,a0^2\,t0\right)}{9}+72\,a0\,\delta\sqrt{a0^2\,(4\tau-2\,t3)^2-4\,a0^2\,t0\,(4\tau-2\,t3)+4\,a0^2\,t0^2+96\,a0^2\,\delta}+3\,a0^2\,(4\tau-2\,t3)^3+48\,a0^2\,\delta\,t0\right)$

(%o108) $K=\left(24\,a2Max\,tIn^2\sqrt{4\,a2Max^2\,tIn^2-12\,a2Max^2\,tEFSkip\,tIn+9\,a2Max^2\,tEFSkip^2+96\,a2Max^2\,\delta}+16\,a2Max^2\,tIn^3+72\,a2Max^2\,tEFSkip\,tIn^2\right)/\left(tEFSkip\left(-6\,a2Max\,tIn\sqrt{4\,a2Max^2\,tIn^2-12\,a2Max^2\,tEFSkip\,tIn+9\,a2Max^2\,tEFSkip^2+96\,a2Max^2\,\delta}+12\,a2Max^2\,tIn^2+648\,a2Max^2\,\delta\right)+tEFSkip^2\left(27\,a2Max\sqrt{4\,a2Max^2\,tIn^2-12\,a2Max^2\,tEFSkip\,tIn+9\,a2Max^2\,tEFSkip^2+96\,a2Max^2\,\delta}-72\,a2Max^2\,tIn\right)+72\,a2Max\,\delta\sqrt{4\,a2Max^2\,tIn^2-12\,a2Max^2\,tEFSkip\,tIn+9\,a2Max^2\,tEFSkip^2+96\,a2Max^2\,\delta}+48\,a2Max^2\,\delta\,tIn+81\,a2Max^2\,tEFSkip^3\right)$

(%o109) $K=\left(24\,a^2\,tIn^2\sqrt{4\,a^4\,tIn^2-12\,a^4\,tEFSkip\,tIn+9\,a^4\,tEFSkip^2+96\,a^4\,\delta}+16\,a^4\,tIn^3+72\,a^4\,tEFSkip\,tIn^2\right)/\left(tEFSkip\left(-6\,a^2\,tIn\sqrt{4\,a^4\,tIn^2-12\,a^4\,tEFSkip\,tIn+9\,a^4\,tEFSkip^2+96\,a^4\,\delta}+12\,a^4\,tIn^2+648\,a^4\,\delta\right)+tEFSkip^2\left(27\,a^2\sqrt{4\,a^4\,tIn^2-12\,a^4\,tEFSkip\,tIn+9\,a^4\,tEFSkip^2+96\,a^4\,\delta}-72\,a^4\,tIn\right)+72\,a^2\,\delta\sqrt{4\,a^4\,tIn^2-12\,a^4\,tEFSkip\,tIn+9\,a^4\,tEFSkip^2+96\,a^4\,\delta}+48\,a^4\,\delta\,tIn+81\,a^4\,tEFSkip^3\right)$

→
```
;
```

(%i116)
```
Dummy: 'Dummy;
CalcCase: [v0=0.5,a1=−500,a2Max=2,ax2Step=0.002];
ev(eqSym2SegRet[1], CalcCase);
subst(%,[tOut=t3,tEdgeTotal=0.03−t3]);
append(%,ev(CalcCase,%));
append(%,ev(Pre4CSegSubs,%));
ev(Pre4CSegPlus,%),float;
```

(Dummy) $Dummy$

(CalcCase) $[v0=0.5,a1=-500,a2Max=2,ax2Step=0.002]$

(%o112) $[t3=0.002,x1=6.666666666666666\;10^{-4}]$

(%o113) $[tOut=0.002,tEdgeTotal=0.028]$

(%o114) $[tOut=0.002,tEdgeTotal=0.028,v0=0.5,a1=-500,a2Max=2,ax2Step=0.002]$

(%o115) $\left[tOut=0.002,tEdgeTotal=0.028,v0=0.5,a1=-500,a2Max=2,ax2Step=0.002,\tau=\frac{2\left(\frac{tEdge}{2}+0.002\right)+3\,tEFSkip}{4},\delta=\frac{d2Step}{2}\right]$

(%o116) $tIn=-0.75\,(2\,(0.5\,tEdge+0.002)+3\,tEFSkip)+\left(0.0625\,(2\,(0.5\,tEdge+0.002)+3\,tEFSkip)^2+1.5\,d2Step\right)^{0.5}+1.5\,tEFSkip$

2.3.2 Rascan time and range analysis

2.4 symmetric 4+1 segment move, with constant velocity segment a2=0, v2=v3, K=given, t0=given

3 symmetric 5 segment move

General solution for symmetric 5 segment. DeltaX and t1 are given

→
```
ResetToBase()$
Ki:'Ki,Ko:'Ko$
[xm1:0,vm1:0,am1:0,x0:DeltaX·(Ki/2),x1:DeltaX·(Ko/2),x2:DeltaX·(1−Ko/2),v2:v1,a2:−a1,t3:t1,x3:DeltaX·(1−Ki/2),v3:v0,a3:−a0,t4:t0,x4:DeltaX,v4:0,a4:0];
/·DeltaXTot: DeltaX +x2;·/

eqm10: am1=p_a0_1(xm1,vm1,x0,v0,t0)$
eqm11: a0=p_a1_1(xm1,vm1,x0,v0,t0)$
eq00: a0=p_a0_1(x0,v0,x1,v1,t1)$
eq01: a1=p_a1_1(x0,v0,x1,v1,t1)$
eq10: a1=p_a0_1(x1,v1,x2,v2,t2)$
eq11: a2=p_a1_1(x1,v1,x2,v2,t2)$
eq20: a2=p_a0_1(x2,v2,x3,v3,t3)$
eq21: a3=p_a1_1(x2,v2,x3,v3,t3)$
eq30: a3=p_a0_1(x3,v3,x4,v4,t4)$
eq31: a4=p_a1_1(x3,v3,x4,v4,t4)$
assume(t0>0,t2>0,Ki<Ko and 0<Ki,Ko<1 and 0<Ko);
eqSet:[eqm10,eqm11,eq00,eq01,eq10],factor;
solveradcan:false$
```

→
```
s5ST:solve(eqSet,[t0,Ko,t2,v0,v1]);
```

→
```
expand(s5ST[1][2]);
```

```
→	forget(t0>0,t1>0,t2>0,DeltaX>0,Ki>0,Ko>0);

→	s5ST[1][5];
	ev(%,[Ko=1]);
	solve(%,Ki);

→	solveradcan:true;
	solvedecomposes:true;
	last(s5ST[1][1]);
	/·expand(ev(s5ST[1][1],[t2=0]));·/
	solve(%,[t0]);
```

4 Asymmetric 5-1 segment move

General solution for 5 segment. DeltaX and t1 are given
For applications like moving between the pixels, it makes sense to try a non-symmetric solution with peak acceleration and velocity towards begining of the move.
Idea is to shift the acceleration and Jerk towards the begining of the step move, improving settling performance.
This could be done by solving a 4 segment move into the first segments of a 5 segment move and leave the last move with 0 acceleration/Velocity.
However this would need an active t2.

Another approach is to put peak acceleration at a0, decelerate with minimum jerk from there.

,
-a0 =< a1 < 0
So we may keep the exit and entry points symmetric

Genaral equations for 5 segment move:

```
(%i131)	ResetToBase()$
	Ki:'Ki,Ko:'Ko$
	specSet:[]$

	forget(t0,t1,t2,a3)$
	eqm10: am1=p_a0_1(xm1,vm1,x0,v0,t0),factor$
	eqm11: a00=p_a1_1(xm1,vm1,x0,v0,t0),factor$

	eq00: a0=p_a0_1(x0,v0,x1,v1,t1),factor$
	eq01: a1=p_a1_1(x0,v0,x1,v1,t1),factor$

	/·
	eq10: a1=p_a0_1(x1,v1,x2,v2,t2),factor$
	eq11: a2=p_a1_1(x1,v1,x2,v2,t2),factor$
	·/

	eq20: a2=p_a0_1(x2,v2,x3,v3,t3),factor$
	eq21: a3=p_a1_1(x2,v2,x3,v3,t3),factor$
	eq30: a3=p_a0_1(x3,v3,x4,v4,t4),factor$
	eq31: a4=p_a1_1(x3,v3,x4,v4,t4),factor$

	eqSet_4SegFull:ev([eqm10,eqm11,eq00,eq01,eq20,eq21,eq30,eq31]);
	[xm1:0,vm1:0,am1:0,v4:0,a4:0]$
	eqSet_4Seg:ev([eq00,eq01,eq20,eq21,eq30,eq31]);
```

(eqSet_4SegFull) $$\left[am1=-\frac{2\,(3\,xm1-3\,x0+2\,t0\,vm1+t0\,v0)}{t0^2}, a00=\frac{2\,(3\,xm1-3\,x0+t0\,vm1+2\,t0\,v0)}{t0^2}, a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2}, a1=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2}, a2=\frac{2\,(3\,x3-3\,x2-t3\,v3-2\,t3\,v2)}{t3^2}, a3=-\frac{2\,(3\,x3-3\,x2-2\,t3\,v3-t3\,v2)}{t3^2}, a3=\frac{2\,(3\,x4-3\,x3-t4\,v4-2\,t4\,v3)}{t4^2}, a4=-\frac{2\,(3\,x4-3\,x3-2\,t4\,v4-t4\,v3)}{t4^2} \right]$$

(eqSet_4Seg) $$\left[a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2}, a1=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2}, a2=\frac{2\,(3\,x3-3\,x2-t3\,v3-2\,t3\,v2)}{t3^2}, a3=-\frac{2\,(3\,x3-3\,x2-2\,t3\,v3-t3\,v2)}{t3^2}, a3=\frac{2\,(3\,x4-3\,x3-2\,t4\,v3)}{t4^2}, 0=-\frac{2\,(3\,x4-3\,x3-t4\,v3)}{t4^2} \right]$$

4.1 Rascan solutions

(%i137)
```
/* Special RSCN condition as t1=t3=tOut: [t3:t1]$*/
ax2_specSet:append([t3=t1],specSet)$

/* Eliminate secion 2 (t2=0) for now: [x2:x1,v2:v1,v00:v0]$*/
ax2_specSet:append([x2=x1,v2=v1,v00=v0],ax2_specSet)$
/* Trajectory design: allowing acceleration discontinuity at t0 which means a00<>a0, but v00=v0
THIS is the essential design of the motion

from worst (higher accels) to best

[a1:0,a2:−a0]$
[a1:−a0/2,a2:−a0]$
[a1:−a0,a2:−a0]$

*/
ax2_specSet:append([a1=−a0,a2=−a0],ax2_specSet)$

/* Given departure and arrival points at t0 and t3: [x0:x4*(Ki/2),x3:x4*(1−Ki/2)]$ */
[Ki:1/2]$
ax2_specSetQuarter:append([x0=x4*(Ki/2),x3=x4*(1−Ki/2)],ax2_specSet);
rascanSubs:[t1=tOut,t4=tIESkip,x0=d2In,x1=d2Out,x3=ax2Entry,x4=d2Step,v1=v2In,v3=v2Out,a0=a2Max];
```

(ax2_specSetQuarter) $[x0=\frac{x4}{4},x3=\frac{3\,x4}{4},a1=-a0,a2=-a0,x2=x1,v2=v1,v00=v0,t3=t1]$

(rascanSubs) $[t1=tOut,t4=tIESkip,x0=d2In,x1=d2Out,x3=ax2Entry,x4=d2Step,v1=v2In,v3=v2Out,a0=a2Max]$

4.1.1 AX2 For given x0/x3 posits and settling acceleration a3

(%i141)
```
eqSet1:subst(ax2_specSetQuarter,eqSet_4Seg);
/*
Solving for a positive overall travel (step), which implies negative a3,
we can substitute x4 with delta^2 and a3 with −alpha^2 for simplicity:
a3:−alpha^2$
x4:delta^2$
x0:ev(x0)$
x3:ev(x3)$
*/
ax2_specSet1:append([a3=−alpha^2,x4=delta^2],ax2_specSetQuarter);
a4V:solve(subst(ax2_specSet1,eqSet1),[v0,a0,x1,v1,v3,t4]);
/* Chose the solution so that t4 is positive:
might be better to replace t4 with tau^2
*/
if ev(t4,ev(a4V[1][6],[delta=1,alpha=−1])) > 0 then
   a4V1:a4V[1]
else if ev(t4,ev(a4V[2][6],[delta=1,alpha=−1])) > 0 then
      a4V1:a4V[2]
   else
      a4V1:'a4V1;
```

(eqSet1) $$[a0=\frac{2\left(-\frac{3\,x4}{4}+3\,x1-t1\,v1-2\,t1\,v0\right)}{t1^2},-a0=-\frac{2\left(-\frac{3\,x4}{4}+3\,x1-2\,t1\,v1-t1\,v0\right)}{t1^2},-a0=\frac{2\left(\frac{9\,x4}{4}-3\,x1-t1\,v3-2\,t1\,v1\right)}{t1^2},a3=-\frac{2\left(\frac{9\,x4}{4}-3\,x1-2\,t1\,v3-t1\,v1\right)}{t1^2},a3=\frac{2\left(\frac{3\,x4}{4}-2\,t4\,v3\right)}{t4^2},0=-\frac{2\left(\frac{3\,x4}{4}-t4\,v3\right)}{t4^2}]$$

(ax2_specSet1) $[a3=-\alpha^2,x4=\delta^2,x0=\frac{x4}{4},x3=\frac{3\,x4}{4},a1=-a0,a2=-a0,x2=x1,v2=v1,v00=v0,t3=t1]$

(a4V) $$[[v0=-\frac{\sqrt{6}\,\alpha\,\delta\,t1-6\,\delta^2}{20\,t1},a0=-\frac{5\,\alpha^2\,t1^2+3\sqrt{6}\,\alpha\,\delta\,t1-3\,\delta^2}{5\,t1^2},x1=-\frac{10\,\alpha^2\,t1^2+9\sqrt{6}\,\alpha\,\delta\,t1-39\,\delta^2}{60},v1=-\frac{\sqrt{6}\,\alpha\,\delta\,t1-6\,\delta^2}{20\,t1},v3=\frac{\sqrt{6}\,\alpha\,\delta}{4},t4=\frac{\sqrt{6}\,\delta}{2\,\alpha}],[v0=\frac{\sqrt{6}\,\alpha\,\delta\,t1+6\,\delta^2}{20\,t1},a0=-\frac{5\,\alpha^2\,t1^2-3\sqrt{6}\,\alpha\,\delta\,t1-3\,\delta^2}{5\,t1^2},x1=-\frac{10\,\alpha^2\,t1^2-9\sqrt{6}\,\alpha\,\delta\,t1-39\,\delta^2}{60},v1=\frac{\sqrt{6}\,\alpha\,\delta\,t1+6\,\delta^2}{20\,t1},v3=-\frac{\sqrt{6}\,\alpha\,\delta}{4},t4=-\frac{\sqrt{6}\,\delta}{2\,\alpha}]]$$

(%o141) $$[v0=\frac{\sqrt{6}\,\alpha\,\delta\,t1+6\,\delta^2}{20\,t1},a0=-\frac{5\,\alpha^2\,t1^2-3\sqrt{6}\,\alpha\,\delta\,t1-3\,\delta^2}{5\,t1^2},x1=-\frac{10\,\alpha^2\,t1^2-9\sqrt{6}\,\alpha\,\delta\,t1-39\,\delta^2}{60},v1=\frac{\sqrt{6}\,\alpha\,\delta\,t1+6\,\delta^2}{20\,t1},v3=-\frac{\sqrt{6}\,\alpha\,\delta}{4},t4=-\frac{\sqrt{6}\,\delta}{2\,\alpha}]$$

(%i145)
```
/*
Note the discontinuity in acceleration at t0 means we have two a0 values
*/
[eqm10,eqm11];
a40s: solve(%,[t0,a00]);
t0F:a40s[1][1]; a0F:a40s[1][2];
```

(%o142) $[am1=-\frac{2\,(3\,xm1-3\,x0+2\,t0\,vm1+t0\,v0)}{t0^2},a00=\frac{2\,(3\,xm1-3\,x0+t0\,vm1+2\,t0\,v0)}{t0^2}]$

(a40s) $[[t0=\frac{3\,x0}{v0},a00=\frac{2\,v0^2}{3\,x0}]]$

(t0F) $t0=\frac{3\,x0}{v0}$

(a0F) $a00=\frac{2\,v0^2}{3\,x0}$

4.1.2 test case

```
tmp : ev([x0,a4V1],specSetQuarter)$
testCase :[t1=0.01,delta=sqrt(0.001),alpha=−sqrt(0.1)]$
display(ev([t1,delta^2,alpha^2],testCase));


tmp:a4V1$

ev(ev([0,x0,x0,x1,x2,x3,x4],tmp),eval,specSet1)$
posits:ev(%,testCase),float$
PosNrm : %/lmax(abs(%))$

ev(ev([0,v00,v0,v1,v2,v3,v4],tmp),eval,specSet1)$
velos:ev(%,testCase),float$
VelNrm : %/lmax(abs(%))$

ev(ev([0,ev(a00,a0F),a0,a1,a2,a3,0],tmp),eval,specSet1)$
accels:ev(%,testCase),float$
AccNrm : %/lmax(abs(%))$
ev(ev([ev(−t0,a40s),0,0,t1,t1,2·t1,2·t1+t4],tmp),eval,specSet1)$
times:ev(%,testCase),float$
display(times);
display(posits);
display(velos);
display(accels);
```

```
ptemp:[['discrete, times, PosNrm],['discrete, times, VelNrm],['discrete, times,AccNrm]]$
wxplot2d(ptemp,[title,"Nrm Acceleration, and point position and velocicy"],[grid2d,true],[style,lines],[legend,"Pos","Vel","Acc"])$
```

```
/·ResetToBase()·/
$i:'i$ptemp:[]$

push((posits[7]−posits[1])/4,ptemp)$
push((posits[7]−posits[1])·3/4,ptemp)$

for i in [1,3,5,6] do (

  ftemp : ev(f_x1(t),[x0=posits[i],
      v0=velos[i],
           a0=accels[i],
           a1=accels[i+1],
 t1=times[i+1]−times[i],
           t=t−times[i]]),

   P_x1(t) := if (t > times[i]) and (t <= times[i+1]) then ftemp,

  push(P_x1(t),ptemp)
   )$
wxplot2d(ptemp,[t,times[1],times[7]],[title,"Asymetric 4 segment optimized step move"],[xlabel,"t[s]"],[ylabel,"Position[mm]"],[grid2d,true],[legend,"ISkip2","OSkip2","OSki
```

```
/·ResetToBase()$·/
i:'i$ptemp:[]$
for i in [1,3,5,6] do (

     ftemp : ev(f_v1(t),[x0=posits[i],
           v0=velos[i],
           a0=accels[i],
           a1=accels[i+1],
           t1=times[i+1]−times[i],
           t=t−times[i]]),

   P_v1(t) := if (t > times[i]) and (t <= times[i+1]) then ftemp,

   push(P_v1(t),ptemp)
   )$
wxplot2d(ptemp,[t,times[1],times[7]],[title,"Asymetric 4 segment optimized step move"],[xlabel,"t[s]"],[ylabel,"Velocity[mm/s]"],[grid2d,true],[legend,"ISkip2","OSkip2","OSk
```

4.2 Optimal solution for limited Jerk Step move: Given x0 and x3, continuos acceleration

A4V1 solution for t1=0.015, x4=0.001 and a3=-0.25 leads to an almost continuous acceleration
 which suggests that by adding the condition as a1=a2, and relaxing a3 as a avriable,
the set can be solved for that solution. It means that the arrivinng acceleration will be dictated
by the solution. So for any given tOut and Step size, there will be a solution. Maximum Acceleration
can be compared to maximu allowed to validate the solution.

(%i151)
```
Ki:'Ki$Ko:'Ko$
X: matrix([Ki/2,kx1,kx2,(1−Ko/2),1,kx5])·x4;
T: matrix([kt0,1,kt2,kt3,kt4,kt5])·t1;
V: matrix([kv0,kv1,kv2,kv3,kv4,kv5])·x4/t1;
A: matrix([ka0,ka1,ka2,−ka3,ka4,ka5])·x4/t1^2;
```

(X) $$\begin{bmatrix} \frac{Ki\ x4}{2} & kx1\ x4 & kx2\ x4 & \left(1-\frac{Ko}{2}\right)\ x4 & x4 & kx5\ x4 \end{bmatrix}$$

(T) $$\begin{bmatrix} kt0\ t1 & t1 & kt2\ t1 & kt3\ t1 & kt4\ t1 & kt5\ t1 \end{bmatrix}$$

(V) $$\begin{bmatrix} \frac{kv0\ x4}{t1} & \frac{kv1\ x4}{t1} & \frac{kv2\ x4}{t1} & \frac{kv3\ x4}{t1} & \frac{kv4\ x4}{t1} & \frac{kv5\ x4}{t1} \end{bmatrix}$$

(A) $$\begin{bmatrix} \frac{ka0\ x4}{t1^2} & \frac{ka1\ x4}{t1^2} & \frac{ka2\ x4}{t1^2} & -\frac{ka3\ x4}{t1^2} & \frac{ka4\ x4}{t1^2} & \frac{ka5\ x4}{t1^2} \end{bmatrix}$$

4.3 General case solution for Step move with given exit and entry points

(%i159)
```
assume(t1>0,x4>0);
linax2_specSet: [t4=T[1,5],x1=X[1,2],v0=V[1,1],v1=V[1,2],v3=V[1,4],a0=A[1,1],a3=A[1,4],x0=X[1,1],x3=X[1,4]];
tmp:subst(a00=a0,a0F)$
tmp:append([tmp],eqSet_4Seg)$
tmp:subst(ax2_specSet,tmp)$
lineqSet:subst(linax2_specSet,tmp)$
a4VO2:solve(lineqSet,[kt4,kx1,kv0,kv1,kv3,ka0,ka3])$
a4VOsubd: subst(a4VO2[2],linax2_specSet);
```

(%o152) $[t1>0, x4>0]$

(linax2_specSet) $$\left[t4=kt4\ t1, x1=kx1\ x4, v0=\frac{kv0\ x4}{t1}, v1=\frac{kv1\ x4}{t1}, v3=\frac{kv3\ x4}{t1}, a0=\frac{ka0\ x4}{t1^2}, a3=-\frac{ka3\ x4}{t1^2}, x0=\frac{Ki\ x4}{2}, x3=\left(1-\frac{Ko}{2}\right)\ x4\right]$$

<< Expression longer than allowed by the configuration setting! >>

→
```
/·Simplifying the formulas::: not succsessful do far!!!!
_simpSub1 :kappa^2=3·Ki·Ko+2·Ko−6·Ki^2+14·Ki−4;
_simpSub2 :gamma=Ko+25·Ki;
_simpSub: [_simpSub1,_simpSub2];
_simpSln: solve(_simpSub,[Ki,Ko]);
tmp:subst(_simpSln[1],a4VOsubd[1]);
expand(tmp);
subst(_simpSln[1],tmp);
tmp:solve([(25·Ki+Ko)=gamma]],Ki)[1];
second(ratsimp(a4VOsubd)[2]);
factor(subst(tmp,%));
%·sqrt(Ko)·gamma^2/x4;
factor(%);
/·2·Ko^(5/2)+100·Ki·Ko^(3/2)+1250·Ki^2·sqrt(Ko)=delta,sqrt((3·Ki+2)·Ko−6·Ki^2+14·Ki−4)=alpha·/;
```

4.3.1 test case numerical solution (OBS)

4.4 Special case: asymetric move in symeteric time frame: t4=t0

```
(%i167) subst(a00=a0,a0F)$
tmp: append([%],eqSet_4Seg);
ev(t4=t0, a40s[1][1]);
tmp:append([%],tmp);
tmp:subst(ax2_specSet,tmp);
lineqSet2:subst(linax2_specSet,tmp);
a4VO22:solve(lineqSet2,[kt4,kx1,kv0,kv1,kv3,ka0,ka3,Ko]);
a4VOsubd2: append([a4VO22[1][8]],subst(a4VO22[1],linax2_specSet));
```

(tmp) $\left[a0=\frac{2\,v0^2}{3\,x0},a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2},a1=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2},a2=\frac{2\,(3\,x3-3\,x2-t3\,v3-2\,t3\,v2)}{t3^2},a3=-\frac{2\,(3\,x3-3\,x2-2\,t3\,v3-t3\,v2)}{t3^2},a3=\frac{2\,(3\,x4-3\,x3-2\,t4\,v3)}{t4^2},0=-\frac{2\,(3\,x4-3\,x3-t4\,v3)}{t4^2}\right]$

(%o162) $t4=\frac{3\,x0}{v0}$

(tmp) $\left[t4=\frac{3\,x0}{v0},a0=\frac{2\,v0^2}{3\,x0},a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2},a1=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2},a2=\frac{2\,(3\,x3-3\,x2-t3\,v3-2\,t3\,v2)}{t3^2},a3=-\frac{2\,(3\,x3-3\,x2-2\,t3\,v3-t3\,v2)}{t3^2},a3=\frac{2\,(3\,x4-3\,x3-2\,t4\,v3)}{t4^2},0=-\frac{2\,(3\,x4-3\,x3-t4\,v3)}{t4^2}\right]$

(tmp) $\left[t4=\frac{3\,x0}{v0},a0=\frac{2\,v0^2}{3\,x0},a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2},-a0=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2},-a0=\frac{2\,(3\,x3-3\,x1-t1\,v3-2\,t1\,v1)}{t1^2},a3=-\frac{2\,(3\,x3-3\,x1-2\,t1\,v3-t1\,v1)}{t1^2},a3=\frac{2\,(3\,x4-3\,x3-2\,t4\,v3)}{t4^2},0=-\frac{2\,(3\,x4-3\,x3-t4\,v3)}{t4^2}\right]$

(lineqSet2) $\left[kt4\,t1=\frac{3\,Ki\,t1}{2\,kv0},\frac{ka0\,x4}{t1^2}=\frac{4\,kv0^2\,x4}{3\,Ki\,t1^2},\frac{ka0\,x4}{t1^2}=\frac{2\left(3\,kx1\,x4-kv1\,x4-2\,kv0\,x4-\frac{3\,Ki\,x4}{2}\right)}{t1^2},-\frac{ka0\,x4}{t1^2}=-\frac{2\left(3\,kx1\,x4-2\,kv1\,x4-kv0\,x4-\frac{3\,Ki\,x4}{2}\right)}{t1^2},-\frac{ka0\,x4}{t1^2}=\frac{2\left(-3\,kx1\,x4-kv3\,x4-2\,kv1\,x4+3\left(1-\frac{Ko}{2}\right)x4\right)}{t1^2},-\frac{ka3\,x4}{t1^2}=-\frac{2\left(-3\,kx1\,x4-2\,kv3\,x4-kv1\,x4+3\left(1-\frac{Ko}{2}\right)x4\right)}{t1^2},-\frac{ka3\,x4}{t1^2}=\frac{2\left(-2\,kt4\,kv3\,x4-3\left(1-\frac{Ko}{2}\right)x4+3\,x4\right)}{kt4^2\,t1^2},0=-\frac{2\left(-kt4\,kv3\,x4-3\left(1-\frac{Ko}{2}\right)x4+3\,x4\right)}{kt4^2\,t1^2}\right]$

(a4VO22) $\left[\left[kt4=-\frac{2\,Ki}{Ki-1},kx1=-\frac{Ki^2-4\,Ki-1}{8\,Ki},kv0=-\frac{3\,Ki-3}{4},kv1=-\frac{3\,Ki-3}{4},kv3=-\frac{9\,Ki^2-12\,Ki+3}{4\,Ki+4},ka0=\frac{3\,Ki^2-6\,Ki+3}{4\,Ki},ka3=\frac{9\,Ki^3-21\,Ki^2+15\,Ki-3}{4\,Ki^2+4\,Ki},Ko=\frac{3\,Ki^2-Ki}{Ki+1}\right]\right]$

(a4VOsubd2) $\left[Ko=\frac{3\,Ki^2-Ki}{Ki+1},t4=-\frac{2\,Ki\,t1}{Ki-1},x1=-\frac{(Ki^2-4\,Ki-1)\,x4}{8\,Ki},v0=-\frac{(3\,Ki-3)\,x4}{4\,t1},v1=-\frac{(3\,Ki-3)\,x4}{4\,t1},v3=-\frac{(9\,Ki^2-12\,Ki+3)\,x4}{(4\,Ki+4)\,t1},a0=\frac{(3\,Ki^2-6\,Ki+3)\,x4}{4\,Ki\,t1^2},a3=-\frac{(9\,Ki^3-21\,Ki^2+15\,Ki-3)\,x4}{(4\,Ki^2+4\,Ki)\,t1^2},x0=\frac{Ki\,x4}{2},x3=\left(1-\frac{3\,Ki^2-Ki}{2\,(Ki+1)}\right)x4\right]$

4.4.1 Rascan formulation

Rascan for

```
(%i168) ax2_A4VT:subst(append(rascanSubs,ax2_specSet),a4VOsubd2),float;
```

(ax2_A4VT) $\left[Ko=\frac{3\,Ki^2-Ki}{Ki+1},tIESkip=-\frac{2\,Ki\,tOut}{Ki-1},d2Out=-\frac{0.125\,(Ki^2-4\,Ki-1)\,d2Step}{Ki},v0=-\frac{0.25\,(3\,Ki-3)\,d2Step}{tOut},v2In=-\frac{0.25\,(3\,Ki-3)\,d2Step}{tOut},v2Out=-\frac{(9\,Ki^2-12\,Ki+3)\,d2Step}{(4\,Ki+4)\,tOut},a2Max=\frac{0.25\,(3\,Ki^2-6\,Ki+3)\,d2Step}{Ki\,tOut^2},a3=-\frac{(9\,Ki^3-21\,Ki^2+15\,Ki-3)\,d2Step}{(4\,Ki^2+4\,Ki)\,tOut^2},d2In=0.5\,Ki\,d2Step,ax2Entry=\left(1-\frac{0.5\,(3\,Ki^2-Ki)}{Ki+1}\right)d2Step\right]$

a Two Segment ax1 solution (uniform a1HLM) can't be matched with an ax2 A4VT, because ax2 demands tIn>tOut and ax1 needs tiSkip < tOut:

```
(%i173) ax1_S2VT: subst([t1=tOut,x1=d1Out,v0=v1Out,a1=-a1HLM],eqSym2SegRet[1]);
ax1_S2VT_Add: subst([t1=tOut+tIn,x1=d1Out+d1In,v0=v1Out+v1Diff,a1=-a1HLM],eqSym2SegRet[1]);
ax1_S2VT_All: factor(-ax1_S2VT+ax1_S2VT_Add);
ax2_tIn : solve(ax2_A4VT[2],[Ki]);
Ki_c : factor(ev(ev(ev(ax2_tIn,[tIESkip=tIn]),ax1_S2VT),ax1_S2VT_All));
```

(ax1_S2VT) $\left[tOut=\frac{2\,v1Out}{a1HLM},d1Out=\frac{4\,v1Out^2}{3\,a1HLM}\right]$

(ax1_S2VT_Add) $\left[tOut+tIn=\frac{2\,(v1Out+v1Diff)}{a1HLM},d1Out+d1In=\frac{4\,(v1Out+v1Diff)^2}{3\,a1HLM}\right]$

(ax1_S2VT_All) $\left[tIn=\frac{2\,v1Diff}{a1HLM},d1In=\frac{4\,v1Diff\,(2\,v1Out+v1Diff)}{3\,a1HLM}\right]$

(ax2_tIn) $\left[Ki=\frac{tIESkip}{2\,tOut+tIESkip}\right]$

(Ki_c) $\left[Ki=\frac{v1Diff}{2\,v1Out+v1Diff}\right]$

4.5 Case solutions for given Ki and Ko, or Ki

4.6 Turn-around symmetric 4 segment solution

(%i179)
```
eqSet_4SegTurn:[eqSet_4SegFull[1],eqSet_4SegFull[2],eqSet_4SegFull[3],eqSet_4SegFull[4]];
[vm1:'vm1, am1:'am1, xm1:'xm1,v4:'v4,a4:'a4];
ax1_specSet:[am1=0,xm1=0,a00=a0]$
/* Eliminate secion 2 (t2=0) for now: [x2:x1,v2:v1,v00:v0]$*/
/*ax1_specSet:append([x2=x1,v2=v1,a2=a1],ax1_specSet);
/*symmetric move */
/*ax1_specSet: append([t3=t1,t4=t0],ax1_specSet);
/* turnaround */
ax1_specSet:append([v1=0],ax1_specSet)$
/* velocity tolerance */
ax1_specSet:append([v0=vm1-delta],ax1_specSet)$
ax1_eqSet1:subst(ax1_specSet,eqSet_4SegTurn);
```

(eqSet_4SegTurn) $[am1=-\frac{2\,(3\,xm1-3\,x0+2\,t0\,vm1+t0\,v0)}{t0^2},a00=\frac{2\,(3\,xm1-3\,x0+t0\,vm1+2\,t0\,v0)}{t0^2},a0=\frac{2\,(3\,x1-3\,x0-t1\,v1-2\,t1\,v0)}{t1^2},a1=-\frac{2\,(3\,x1-3\,x0-2\,t1\,v1-t1\,v0)}{t1^2}]$

(%o175) $[vm1,am1,xm1,v4,a4]$

(ax1_eqSet1) $[0=-\frac{2\,(-3\,x0+t0\,(vm1-\delta)+2\,t0\,vm1)}{t0^2},a0=\frac{2\,(-3\,x0+2\,t0\,(vm1-\delta)+t0\,vm1)}{t0^2},a0=\frac{2\,(3\,x1-3\,x0-2\,t1\,(vm1-\delta))}{t1^2},a1=-\frac{2\,(3\,x1-3\,x0-t1\,(vm1-\delta))}{t1^2}]$

(%i180)
```
s4V_turnAround :solve(ax1_eqSet1,[t1,x0,x1,a0]);
```

(s4V_turnAround) $[[t1=0,x0=\frac{3\,t0\,vm1-\delta\,t0}{3},x1=\frac{3\,t0\,vm1-\delta\,t0}{3},a0=-\frac{2\,\delta}{t0}],[t1=-\frac{2\,t0\,vm1-2\,\delta\,t0}{a1\,t0-2\,\delta},x0=\frac{3\,t0\,vm1-\delta\,t0}{3},x1=-\frac{(4\,a1\,t0^2-4\,\delta\,t0)\,vm1^2+(-3\,a1^2\,t0^3+4\,a1\,\delta\,t0^2-4\,\delta^2\,t0)\,vm1+a1^2\,\delta\,t0^3}{3\,a1^2\,t0^2-12\,a1\,\delta\,t0+12\,\delta^2},a0=-\frac{2\,\delta}{t0}]]$

(%i187)
```
ax1_rascanSubs:[t0=tIn,t1=tOut,x0=d1In,x1=d1Out+d1In,vm1=v1Scan,v0=ax1VISkip,delta=v1Diff,a0=ax1IAcc,a1=-a1Max];
rscn_ax1_Tol_sol: subst(ax1_rascanSubs,s4V_turnAround[2]),factor;
rscn_ax1_Tol_sol[3]:factor(%[3]-%[2])$
rscn_d1Out:%;
sub_alpha:alpha=a1Max*tIn+2*v1Diff;
/*a1Max=(alpha-2*v1Diff)/tIn;*/
solve(%,[a1Max]);
rscn_ax1_frml :subst(%,rscn_ax1_Tol_sol);
```

(ax1_rascanSubs) $[t0=tIn,t1=tOut,x0=d1In,x1=d1Out+d1In,vm1=v1Scan,v0=ax1VISkip,\delta=v1Diff,a0=ax1IAcc,a1=-a1Max]$

(rscn_ax1_Tol_sol) $[tOut=\frac{2\,tIn\,(v1Scan-v1Diff)}{2\,v1Diff+a1Max\,tIn},d1In=\frac{tIn\,(3\,v1Scan-v1Diff)}{3},d1Out+d1In=\frac{tIn\,(4\,v1Diff\,v1Scan^2+4\,a1Max\,tIn\,v1Scan^2+4\,v1Diff^2\,v1Scan+4\,a1Max\,tIn\,v1Diff\,v1Scan+3\,a1Max^2\,tIn^2\,v1Scan-a1Max^2\,tIn^2\,v1Diff)}{3\,(2\,v1Diff+a1Max\,tIn)^2},ax1IAcc=-\frac{2\,v1Diff}{tIn}]$

(rscn_d1Out) $d1Out=\frac{4\,tIn\,(v1Diff+a1Max\,tIn)\,(v1Scan-v1Diff)^2}{3\,(2\,v1Diff+a1Max\,tIn)^2}$

(sub_alpha) $\alpha=2\,v1Diff+a1Max\,tIn$

(%o186) $[a1Max=-\frac{2\,v1Diff-\alpha}{tIn}]$

(rscn_ax1_frml) $[tOut=\frac{2\,tIn\,(v1Scan-v1Diff)}{\alpha},d1In=\frac{tIn\,(3\,v1Scan-v1Diff)}{3},d1Out=\frac{4\,tIn\,(\alpha-v1Diff)\,(v1Scan-v1Diff)^2}{3\,\alpha^2},ax1IAcc=-\frac{2\,v1Diff}{tIn}]$

(%i188)
```
solve(rscn_ax1_frml[2],[tIn]);
```

(%o188) $[tIn=\frac{3\,d1In}{3\,v1Scan-v1Diff}]$

(%i192)
```
tmp: rscn_ax1_Tol_sol[1];
[v1Out=v1Scan-v1Diff];
tmp:subst(%,tmp);
/*
takeouttEdgeTotal: tOut=tOut-tEdgeTotal;
subst(%,tmp);
*/
rscn_VAdd_ax1:solve(tmp,[v1Diff]),factor;
```

(tmp) $tOut=\frac{2\,tIn\,(v1Scan-v1Diff)}{2\,v1Diff+a1Max\,tIn}$

(%o190) $[v1Out=v1Scan-v1Diff]$

(tmp) $tOut=\frac{2\,tIn\,(v1Scan-v1Diff)}{2\,v1Diff+a1Max\,tIn}$

(rscn_VAdd_ax1) $[v1Diff=\frac{tIn\,(2\,v1Scan-a1Max\,tOut)}{2\,(tOut+tIn)}]$

(%i198) /* VAdd will slow down ax1 which means there ios additional time in tIn */
tAdd=tIn-d1In/v1Scan;
rscn_tAdd:subst(rscn_ax1_frml[2],%),factor;
diff(%,v1Diff,1);
/* to check if in each incremental step, itr worth increasing VAdd to eat up -tEdgeTotal/2: */
subst([tAdd=t+deltat,v1Diff=v1Diff+deltaVAdd],rscn_tAdd);
solve(%,[deltat]),expand;
rscn_VAdd_ax1;

(%o193) $tAdd=tIn-\frac{d1In}{v1Scan}$

(rscn_tAdd) $tAdd=\frac{tIn\ v1Diff}{3\ v1Scan}$

(%o195) $0=\frac{tIn}{3\ v1Scan}$

(%o196) $t+deltat=\frac{tIn\ (v1Diff+deltaVAdd)}{3\ v1Scan}$

(%o197) $[deltat=\frac{tIn\ v1Diff}{3\ v1Scan}+\frac{deltaVAdd\ tIn}{3\ v1Scan}-t]$

(%o198) $[v1Diff=\frac{tIn\ (2\ v1Scan-a1Max\ tOut)}{2\ (tOut+tIn)}]$

(%i206) tOvrHd_basic: tOvrHd=tAdd+tOut+(tEdge/2);
subst(rscn_tAdd,%),factor;
subst([tEdge=0],%);
subst([solve(rscn_ax1_Tol_sol[1],[tIn])],%),expand;
diff(%,v1Diff,1),expand;
solve(%,[v1Diff]);
subst([tOut=tOut+deltaT],%);
solve(%[1],[deltaT]);

(tOvrHd_basic) $tOvrHd=tOut+\frac{tEdge}{2}+tAdd$

(%o200) $tOvrHd=\frac{6\ tOut\ v1Scan+3\ tEdge\ v1Scan+2\ tIn\ v1Diff}{6\ v1Scan}$

(%o201) $tOvrHd=\frac{6\ tOut\ v1Scan+2\ tIn\ v1Diff}{6\ v1Scan}$

(%o202) $tOvrHd=tOut-\frac{2\ tOut\ v1Diff^2}{-6\ v1Scan^2+6\ v1Diff\ v1Scan+3\ a1Max\ tOut\ v1Scan}$

(%o203) $0=\frac{12\ tOut\ v1Diff^2\ v1Scan}{36\ v1Scan^4-72\ v1Diff\ v1Scan^3-36\ a1Max\ tOut\ v1Scan^3+36\ v1Diff^2\ v1Scan^2+36\ a1Max\ tOut\ v1Diff\ v1Scan^2+9\ a1Max^2\ tOut^2\ v1Scan^2}-\frac{4\ tOut\ v1Diff}{-6\ v1Scan^2+6\ v1Diff\ v1Scan+3\ a1Max\ tOut\ v1Scan}$

(%o204) $[v1Diff=2\ v1Scan-a1Max\ tOut,v1Diff=0]$

(%o205) $[v1Diff=2\ v1Scan-a1Max\ (tOut+deltaT),v1Diff=0]$

(%o206) $[deltaT=\frac{2\ v1Scan-v1Diff-a1Max\ tOut}{a1Max}]$

Double optimisation: Fly and Step axes

(%i209) /* tOvrHd_basic: tOvrHd=tAdd+tOut+(tEdge/2); */
rscn_tEdge_ax2;
subst(rscn_tEdge_ax2,tOvrHd_basic);

rscn_tOv:subst(rscn_tAdd,%),expand;
/* Overhead time is monotonic to VAdd
But non-monotonic to tIn*/;

(%o207) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-2\ tOut]$

(%o208) $tOvrHd=\frac{\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-2\ tOut}{2}+tOut+tAdd$

(rscn_tOv) $tOvrHd=\frac{tIn\ v1Diff}{3\ v1Scan}+\frac{\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}{5}+\frac{3\sqrt{\tau^2+3\ \delta}}{5}$

(%i212) rscn_tEdge_ax2;
subst(rscn_ax1_frml[1],%);
subst(sub_alpha,%);

(%o210) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-2\ tOut]$

(%o211) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-\frac{4\ tIn\ (v1Scan-v1Diff)}{\alpha}]$

(%o212) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-\frac{4\ tIn\ (v1Scan-v1Diff)}{2\ v1Diff+a1Max\ tIn}]$

1 Optimisation formulation: simplify the optimisation problem

1.1 d1Out

(%i214) subst([delta=d2Step/a2Max],rscn_tOv);
rscn_d1Out;

(%o213) $tOvrHd=\frac{tIn\ v1Diff}{3\ v1Scan}+\frac{\sqrt{9\ \tau^2+10\ tIn^2-\frac{3\ d2Step}{a2Max}}}{5}+\frac{3\sqrt{\tau^2+\frac{3\ d2Step}{a2Max}}}{5}$

(%o214) $d1Out=\frac{4\ tIn\ (v1Diff+a1Max\ tIn)\ (v1Scan-v1Diff)^2}{3\ (2\ v1Diff+a1Max\ tIn)^2}$

1.2 Variation of tOvrHd for tIn and v1Diff

(%i221) rscn_tOv;
tOv_diff_tIn: diff(rscn_tOv,tIn,1),expand;

solve(tOv_diff_tIn,[tIn]);
(sqrt(tIn^2+24·delta)/%)^2;
rscn_Opt_tIn: solve(%,[tIn]),radcan;
solve(tOv_diff_tIn,[v1Diff]);
diff(rscn_tOv,v1Diff,1);

(%o215) $tOvrHd=\frac{tIn\ v1Diff}{3\ v1Scan}+\frac{\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}{5}+\frac{3\sqrt{\tau^2+3\ \delta}}{5}$

(tOv_diff_tIn) $0=\frac{v1Diff}{3\ v1Scan}+\frac{2\ tIn}{\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}$

(%o217) $[tIn=-\frac{\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}\ v1Diff}{6\ v1Scan}]$

(%o218) $[\frac{tIn^2+24\ \delta}{tIn^2}=\frac{36\ (tIn^2+24\ \delta)\ v1Scan^2}{(9\ \tau^2+10\ tIn^2-3\ \delta)\ v1Diff^2}]$

(rscn_Opt_tIn) $[tIn=-\frac{\sqrt{3}\sqrt{3\ \tau^2-\delta}\ v1Diff}{\sqrt{2}\sqrt{18\ v1Scan^2-5\ v1Diff^2}},tIn=\frac{\sqrt{3}\sqrt{3\ \tau^2-\delta}\ v1Diff}{\sqrt{2}\sqrt{18\ v1Scan^2-5\ v1Diff^2}},tIn=-2^{3/2}\sqrt{3}\sqrt{-\delta},tIn=2^{3/2}\sqrt{3}\sqrt{-\delta}]$

(%o220) $[v1Diff=-\frac{6\ tIn\ v1Scan}{\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}]$

(%o221) $0=\frac{tIn}{3\ v1Scan}$

tOvrHd is monotonically increasing for v1Diff. All the roots are outside.

(%i226) /· tIn − VAdd variation of tOv ·/
solve([3^(5/2)·v1Scan−4·sqrt(3)·v1Diff], [v1Diff]);
solve([9·v1Scan^2−9·v1Diff·v1Scan+2·v1Diff^2], [v1Diff]);
rscn_WellCond:solve([28·v1Delta−9·v1Scan], [v1Delta]);
solve(diff(9·v1Scan^2−9·v1Diff·v1Scan+2·v1Diff^2,v1Diff,1),[v1Diff]);
subst([v1Diff=v1Delta],%);

(%o222) $[v1Diff=\frac{9\ v1Scan}{4}]$

(%o223) $[v1Diff=\frac{3\ v1Scan}{2},v1Diff=3\ v1Scan]$

(rscn_WellCond) $[v1Delta=\frac{9\ v1Scan}{28}]$

(%o225) $[v1Diff=\frac{9\ v1Scan}{4}]$

(%o226) $[v1Delta=\frac{9\ v1Scan}{4}]$

1.2.1 Optimal v1Diff for tEdge=0 for given tIn

tOvrHd is monotonically increasing for v1Diff. So for overhead time optimization, v1Diff shall be minimized.
In a region which tOvrHd is limited by ax2 (tEdge>0), increasing v1Diff MAY decrease d1Out without adding to overall tOvrHd.

(%i232) rscn_tEdge_ax2;
subst(rscn_ax1_Tol_sol[1],%);
tmp: subst([tEdge=0],%),expand;
(sqrt(tIn^2+24·delta)/4−%)^2;
rscn_v1Diff_tE0: solve(tmp,[v1Diff]);
rscn_tIn_tE0: solve(tmp,[tIn]);

(%o227) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-2\ tOut]$

(%o228) $[tEdge=\frac{2\ (\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\sqrt{\tau^2+3\ \delta})}{5}-\frac{4\ tIn\ (v1Scan-v1Diff)}{2\ v1Diff+a1Max\ tIn}]$

(tmp) $[0=-\frac{4\ tIn\ v1Scan}{2\ v1Diff+a1Max\ tIn}+\frac{4\ tIn\ v1Diff}{2\ v1Diff+a1Max\ tIn}+\frac{2\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}{5}+\frac{6\sqrt{\tau^2+3\ \delta}}{5}]$

(%o230) $[\frac{tIn^2+24\ \delta}{16}=\left(\frac{4\ tIn\ v1Scan}{2\ v1Diff+a1Max\ tIn}-\frac{4\ tIn\ v1Diff}{2\ v1Diff+a1Max\ tIn}-\frac{2\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}}{5}-\frac{6\sqrt{\tau^2+3\ \delta}}{5}+\frac{\sqrt{tIn^2+24\ \delta}}{4}\right)^2]$

(rscn_v1Diff_tE0) $[v1Diff=\frac{10\ tIn\ v1Scan-a1Max\ tIn\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}-3\ a1Max\ tIn\sqrt{\tau^2+3\ \delta}}{2\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+6\sqrt{\tau^2+3\ \delta}+10\ tIn}]$

(rscn_tIn_tE0) $[tIn=-\frac{2\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}\ v1Diff+6\sqrt{\tau^2+3\ \delta}\ v1Diff}{-10\ v1Scan+10\ v1Diff+a1Max\sqrt{9\ \tau^2+10\ tIn^2-3\ \delta}+3\ a1Max\sqrt{\tau^2+3\ \delta}}]$

(%i238)
```
subst([tEdge=0],tOvrHd_basic);
subst(rscn_tAdd,%),factor;
subst([solve(rscn_ax1_Tol_sol[1],[tOut])],%),expand;
diff(%,tIn,1);
solve(%,[tIn]);
subst([v1Diff=v1Scan·Kv],%),radcan;
```

(%o233) $tOvrHd = tOut + tAdd$

(%o234) $tOvrHd = \frac{3\, tOut\, v1Scan + tIn\, v1Diff}{3\, v1Scan}$

(%o235) $tOvrHd = \frac{2\, tIn\, v1Scan}{2\, v1Diff + a1Max\, tIn} + \frac{tIn\, v1Diff}{3\, v1Scan} - \frac{2\, tIn\, v1Diff}{2\, v1Diff + a1Max\, tIn}$

(%o236) $0 = \frac{2\, v1Scan}{2\, v1Diff + a1Max\, tIn} - \frac{2\, a1Max\, tIn\, v1Scan}{(2\, v1Diff + a1Max\, tIn)^2} + \frac{v1Diff}{3\, v1Scan} - \frac{2\, v1Diff}{2\, v1Diff + a1Max\, tIn} + \frac{2\, a1Max\, tIn\, v1Diff}{(2\, v1Diff + a1Max\, tIn)^2}$

(%o237) $[tIn = -\frac{2\sqrt{3}\sqrt{v1Diff\, v1Scan - v1Scan^2} + 2\, v1Diff}{a1Max}, tIn = \frac{2\sqrt{3}\sqrt{v1Diff\, v1Scan - v1Scan^2} - 2\, v1Diff}{a1Max}]$

(%o238) $[tIn = -\frac{2\sqrt{3}\sqrt{Kv - 1}\,|v1Scan| + 2\, Kv\, v1Scan}{a1Max}, tIn = \frac{2\sqrt{3}\sqrt{Kv - 1}\,|v1Scan| - 2\, Kv\, v1Scan}{a1Max}]$

1.3 Variation of d1Out for tIn and v1Diff

(%i242)
```
rscn_d1Out;
diff(rscn_d1Out,v1Diff,1);
solve(%,[v1Diff]),radcan;
solve((−63·a1Max^2·tIn^2−36·a1Max·v1Scan·tIn+4·v1Scan^2),[tIn]);
```

(%o239) $d1Out = \frac{4\, tIn\, (v1Diff + a1Max\, tIn)\, (v1Scan - v1Diff)^2}{3\, (2\, v1Diff + a1Max\, tIn)^2}$

(%o240) $0 = \frac{4\, tIn\, (v1Scan - v1Diff)^2}{3\, (2\, v1Diff + a1Max\, tIn)^2} - \frac{16\, tIn\, (v1Diff + a1Max\, tIn)\, (v1Scan - v1Diff)^2}{3\, (2\, v1Diff + a1Max\, tIn)^3} - \frac{8\, tIn\, (v1Diff + a1Max\, tIn)\, (v1Scan - v1Diff)}{3\, (2\, v1Diff + a1Max\, tIn)^2}$

(%o241) $[v1Diff = -\frac{\sqrt{2\, v1Scan - 7\, a1Max\, tIn}\sqrt{2\, v1Scan + a1Max\, tIn} + 2\, v1Scan + 3\, a1Max\, tIn}{4}, v1Diff = \frac{\sqrt{2\, v1Scan - 7\, a1Max\, tIn}\sqrt{2\, v1Scan + a1Max\, tIn} - 2\, v1Scan - 3\, a1Max\, tIn}{4}, v1Diff = v1Scan]$

(%o242) $[tIn = -\frac{2\, v1Scan}{3\, a1Max}, tIn = \frac{2\, v1Scan}{21\, a1Max}]$

2 *Generalised formulation: solve the two systems together, then formulate the optimisation problem*

(%i251) /· forming a 2x eq set to solve for tIn and tOut: ·/

```
tmp:[rscn_ax1_Tol_sol[1],tIn=tInFn(tOut,tEdgeTotal,a2Max,ax2Step)];
assuming:[tEdgeTotal=0];
tmp:subst(%,tmp);
substituting:[ax2Step=delta·a2Max,tOut=t[o]·4/2−3/2·0,tIn=t[i],v1Scan=v+3/2·Delta,a1Max=a,v1Diff=Delta/2];
tmp: subst(substituting,tmp);
ax1_tIO_eq: solve(tmp[1],[t[o]])[1];
ax2_tIO_eq : solve(tmp[2],[t[i]])[1];
assume((3·tOut)/2+tIn>0);
rscn_tIO_eq : [ax1_tIO_eq,ax2_tIO_eq];
```

(tmp) $[tOut = \frac{2\, tIn\, (v1Scan - v1Diff)}{2\, v1Diff + a1Max\, tIn}, tIn = \sqrt{\frac{tOut^2}{4} + \frac{3\, tEdgeTotal\, tOut}{4} + \frac{9\, tEdgeTotal^2}{16} + \frac{3\, ax2Step}{a2Max}} - \frac{3\, tOut}{2} - \frac{3\, tEdgeTotal}{4}]$

(assuming) $[tEdgeTotal = 0]$

(tmp) $[tOut = \frac{2\, tIn\, (v1Scan - v1Diff)}{2\, v1Diff + a1Max\, tIn}, tIn = \sqrt{\frac{tOut^2}{4} + \frac{3\, ax2Step}{a2Max}} - \frac{3\, tOut}{2}]$

(substituting) $[ax2Step = a2Max\, \delta, tOut = 2\, t_o, tIn = t_i, v1Scan = v + \frac{3\,\Delta}{2}, a1Max = a, v1Diff = \frac{\Delta}{2}]$

(tmp) $[2\, t_o = \frac{2\, t_i\, (v + \Delta)}{a\, t_i + \Delta}, t_i = \sqrt{t_o^2 + 3\,\delta} - 3\, t_o]$

(ax1_tIO_eq) $t_o = \frac{t_i\, v + \Delta\, t_i}{a\, t_i + \Delta}$

(ax2_tIO_eq) $t_i = \sqrt{t_o^2 + 3\,\delta} - 3\, t_o$

(%o250) $[\frac{3\, tOut}{2} + tIn > 0]$

(rscn_tIO_eq) $[t_o = \frac{t_i\, v + \Delta\, t_i}{a\, t_i + \Delta}, t_i = \sqrt{t_o^2 + 3\,\delta} - 3\, t_o]$

```
(%i256) subst(ax1_tIO_eq,ax2_tIO_eq);
tmp2 : %-second(second(%));
/*subst(theta=S/a-2*Delta/a,%)*/;
%^2;
tmp: ratnumer(%-second(%));
t[i] < second(solve(first(tmp2),t[i])[1]);
```

(%o252) $$t_i=\sqrt{\frac{(t_i\,v+\Delta\,t_i)^2}{(a\,t_i+\Delta)^2}+3\,\delta}-\frac{3\,(t_i\,v+\Delta\,t_i)}{a\,t_i+\Delta}$$

(tmp2) $$\frac{3\,(t_i\,v+\Delta\,t_i)}{a\,t_i+\Delta}+t_i=\sqrt{\frac{(t_i\,v+\Delta\,t_i)^2}{(a\,t_i+\Delta)^2}+3\,\delta}$$

(%o254) $$\left(\frac{3\,(t_i\,v+\Delta\,t_i)}{a\,t_i+\Delta}+t_i\right)^2=\frac{(t_i\,v+\Delta\,t_i)^2}{(a\,t_i+\Delta)^2}+3\,\delta$$

(tmp) $$8\,t_i^2\,v^2+(6\,a\,t_i^3+22\,\Delta\,t_i^2)\,v+a^2\,t_i^4+8\,\Delta\,a\,t_i^3+(-3\,a^2\,\delta+15\,\Delta^2)\,t_i^2-6\,\Delta\,a\,\delta\,t_i-3\,\Delta^2\,\delta=0$$

(%o256) $$t_i<-\frac{3\,v+4\,\Delta}{a}$$

```
(%i258) 1/ax1_tIO_eq;
expand(%);
```

(%o257) $$\frac{1}{t_o}=\frac{a\,t_i+\Delta}{t_i\,v+\Delta\,t_i}$$

(%o258) $$\frac{1}{t_o}=\frac{a\,t_i}{t_i\,v+\Delta\,t_i}+\frac{\Delta}{t_i\,v+\Delta\,t_i}$$

```
(%i259) solve(tmp,[t[i]]);
```

<< Expression longer than allowed by the configuration setting! >>

(%i260) %[1];

(%o260)

$$t_i=-\mathit{sqrt}(-(3a^2\left(-\frac{(8\Delta+6v)^3}{a^3}+\frac{4(8\Delta+6v)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^2a}+\frac{8(6\delta\Delta)}{a}\right))/(4\,\mathit{sqrt}((9a^4((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6+4224$$

$$\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v$$

$$+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(2/3)+(33a^2v^2+84\Delta a^2v+18a^4\delta+54\Delta^2a^2)((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta$$

$$(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+$$

$$(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)+64v^4+352\Delta v^3+(724\Delta^2-48a^2\delta)v^2+(660\Delta^3-24\Delta a^2\delta)v$$

$$+9a^4\delta^2+18\Delta^2a^2\delta+225\Delta^4)/((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+$$

$$(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+$$

$$\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)))-((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6$$

$$+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+$$

$$(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)+(\frac{(-1)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^2}{9a^4}-\frac{36\delta\Delta^2+36v\delta\Delta}{3a^2})/((2$$

$$\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+$$

$$(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)+\frac{(8\Delta+6v)^2}{2a^2}-\frac{4(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{3a^2})/2-\mathit{sqrt}((9a^4($$

$$(2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+$$

$$(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(2/3)+(33a^2v^2+84\Delta a^2v+18a^4\delta+54\Delta^2a^2)((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta$$

$$(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+$$

$$(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+$$

$$\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)+64v^4+352\Delta v^3+(724\Delta^2-48a^2\delta)v^2+(660\Delta^3-24\Delta a^2\delta)v$$

$$+9a^4\delta^2+18\Delta^2a^2\delta+225\Delta^4)/((2\Delta(v+\Delta)\,\mathit{sqrt}(-\delta(512v^6+4224\Delta v^5+(1152a^2\delta+14496\Delta^2)v^4+(7632\Delta a^2\delta+26488\Delta^3)v^3+$$

$$(-1080a^4\delta^2+17811\Delta^2a^2\delta+27180\Delta^4)v^2+(432\Delta a^4\delta^2+17712\Delta^3a^2\delta+14850\Delta^5)v+216a^6\delta^3+1377\Delta^2a^4\delta^2+6399\Delta^4a^2\delta+3375\Delta^6)))/(3a^5)+$$

$$\frac{(15\Delta^2+22v\Delta-3\delta a^2+8v^2)^3}{27a^6}+\frac{\frac{(36\delta\Delta^2+36v\delta\Delta)(15\Delta^2+22v\Delta-3\delta a^2+8v^2)}{a^4}-\frac{3(12\delta\Delta^4+24v\delta\Delta^3+12v^2\delta\Delta^2)}{a^4}}{6})^\wedge(1/3)))/(6a^2)+\frac{(-1)(8\Delta+6v)}{4a}$$

(%i262) assume([a>0,x>0]);
solve(a·x^4+b·x^3+c·x^2−d=0,[x]);

(%o261) *[meaningless]*

(%o262)
$$[x=-\mathrm{sqrt}(-(3\,a\left(\frac{4\,b\,c}{a\,a}-\frac{b^3}{a^3}\right))/(2\,\mathrm{sqrt}((36\,a^2$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{2/3}+(9\,b^2-24\,a\,c)$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}-48\,a\,d+4\,c^2)/$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}))-$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}+$$
$$\frac{\frac{(-1)\,c^2}{9\,a^2}+\frac{4\,d}{3\,a}}{\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}}-\frac{4\,c}{3\,a}+\frac{b^2}{2\,a^2})/2-\mathrm{sqrt}((36\,a^2$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{2/3}+(9\,b^2-24\,a\,c)$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}-48\,a\,d+4\,c^2)/$$

$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3})/(12\,a)+\frac{(-1)\,b}{4\,a},x=\mathrm{sqrt}(-(3\,a\left(\frac{4\,b\,c}{a\,a}-\frac{b^3}{a^3}\right)$$
$$)/(2\,\mathrm{sqrt}((36\,a^2\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{2/3}+(9\,b^2-24\,a\,c)$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}-48\,a\,d+4\,c^2)/$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}))-$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}+$$
$$\frac{\frac{(-1)\,c^2}{9\,a^2}+\frac{4\,d}{3\,a}}{\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}}-\frac{4\,c}{3\,a}+\frac{b^2}{2\,a^2})/2-\mathrm{sqrt}((36\,a^2$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{2/3}+(9\,b^2-24\,a\,c)$$
$$\left(\frac{\sqrt{d\,(256\,a^3\,d^2+(128\,a^2\,c^2-144\,a\,b^2\,c+27\,b^4)\,d+16\,a\,c^4-4\,b^2\,c^3)}}{2\,3^{3/2}\,a^3}+\frac{c^3}{27\,a^3}+\frac{\frac{3\,(4\,d\,c\,a-d\,b^2)}{a^3}-\frac{c\,(4\,d)}{a\,a}}{6}\right)^{1/3}-48\,a\,d+4\,c^2)/$$

3 Asymmetric axis1 return path

In case of tEdge>0, the symmetric double optimization solution leads to acceleration discontinuity at start and end of the edgr move, as axis 1 needs to stop for axis 2 to catch up.
This issue can be addressed by finding a different solution for acceleration of axis 1, while keeping maximum decceleration ti minimise the outrun distance, d1Out.
This part of the trajectory can be overriden without invalidating the rest of the solution, using the known times (tOut and tEdge)
Problem can be defined as below:
Find d1Edge1, v1Edge1, d1Edge2, v1Edge2, d1Out2 so that: ax1 accelerates throgh Edge1, Edge2 and Out2 phases to v1Out at potition 0
d1Out2 + d1Edge1 + d1Edge2 = d1Out

As it turns out that with tEdge1=tEdge2=tEdge/2 the solution is symetric, making ax1 move back and forth with a1Max acceleration which is not acceptable.
On the other hand, the Asymmetric tEdge dividing doesnt lead to a viable solution.

(%i276)
```
ResetToBase()$
specSet:[];

[x0:0, v0:0, a0:a1Max]$

[x1:d1Edge1, v1:v1Edge1, t1:tEdge1]$

eq00: a0=p_a0_1(x0,v0,x1,v1,t1),factor$
eq01: a1=p_a1_1(x0,v0,x1,v1,t1),factor$

[x2: x1+d1Edge2, v2: v1Edge2, t2: tEdge2]$

eq10: a1=p_a0_1(x1,v1,x2,v2,t2),factor$
eq11: a2=p_a1_1(x1,v1,x2,v2,t2),factor$

[x3: d1Out, t3: tOut]$

eq20: a2=p_a0_1(x2,v2,x3,v3,t3),factor$
eq21: a3=p_a1_1(x2,v2,x3,v3,t3),factor$

[a1: −a0,tEdge2:tEdge−tEdge1]$

eqSet_Ax1Asym:ev([eq00,eq01,eq10,eq11,eq20,eq21]);
/·
[xm1:0,vm1:0,am1:0,v4:0,a4:0]$
eqSet_4Seg:ev([eq00,eq01,eq20,eq21,eq30,eq31]);
·/;
```

(specSet) $[\,]$

(eqSet_Ax1Asym) $$\left[a1Max=-\frac{2\,(tEdge1\;v1Edge1-3\;d1Edge1)}{tEdge1^2},-a1Max=\frac{2\,(2\;tEdge1\;v1Edge1-3\;d1Edge1)}{tEdge1^2},-a1Max=-\frac{2\,((tEdge-tEdge1)\;v1Edge2+2\,(tEdge-tEdge1)\;v1Edge1-3\;d1Edge2)}{(tEdge-tEdge1)^2},a2=\frac{2\,(2\,(tEdge-tEdge1)\;v1Edge2+(tEdge-tEdge1)\;v1Edge1-3\;d1Edge2)}{(tEdge-tEdge1)^2},a2=-\frac{2\,(tOut\;v3+2\;tOut\;v1Edge2-3\;d1Out+3\;d1Edge2+3\;d1Edge1)}{tOut^2},a3=\frac{2\,(2\;tOut\;v3+tOut\;v1Edge2-3\;d1Out+3\;d1Edge2+3\;d1Edge1)}{tOut^2}\right]$$

(%i277)
```
sln_Ax1Asym: solve(eqSet_Ax1Asym,[a2,tEdge1,d1Edge1,v1Edge1,d1Edge2,v1Edge2]);
```

(sln_Ax1Asym)

$$[[a2=\frac{tEdge\,(6\,v3-3\,a3\,tOut)+6\,tOut\,v3-2\,a3\,tOut^2+a1Max\,tEdge^2-6\,d1Out}{tOut^2+3\,tEdge\,tOut+2\,tEdge^2},tEdge1=0,d1Edge1=0,v1Edge1=\frac{-4\,tOut\,v3+tEdge\,(a3\,tOut-2\,v3)+a3\,tOut^2+a1Max\,(tEdge\,tOut+tEdge^2)+6\,d1Out}{2\,tOut+4\,tEdge},$$

$$d1Edge2=\frac{tEdge\,(-12\,tOut^2\,v3+3\,a3\,tOut^3+18\,d1Out\,tOut)+tEdge^2\,(-12\,tOut\,v3+4\,a3\,tOut^2+12\,d1Out)+a1Max\,tEdge^2\,tOut^2}{6\,tOut^2+18\,tEdge\,tOut+12\,tEdge^2},$$

$$v1Edge2=-\frac{4\,tOut^2\,v3+tEdge^2\,(2\,a3\,tOut-4\,v3)-a3\,tOut^3+a1Max\,tEdge^2\,tOut-6\,d1Out\,tOut}{2\,tOut^2+6\,tEdge\,tOut+4\,tEdge^2}],$$

$$[a2=\frac{6\,tOut\,v3-2\,a3\,tOut^2+a1Max\,tEdge^2-6\,d1Out}{tOut^2},tEdge1=tEdge,d1Edge1=\frac{a1Max\,tEdge^2}{6},v1Edge1=0,d1Edge2=0,v1Edge2=-\frac{4\,tOut\,v3-a3\,tOut^2+a1Max\,tEdge^2-6\,d1Out}{2\,tOut}],$$

$$[a2=\frac{-4\,v3^2+a1Max\,(tEdge\,(4\,v3-2\,a3\,tOut)+6\,tOut\,v3-2\,a3\,tOut^2-6\,d1Out)+4\,a3\,tOut\,v3-a3^2\,tOut^2+a1Max^2\,tEdge^2}{2\,tOut\,v3+a1Max\,(tOut^2+2\,tEdge\,tOut+tEdge^2)-6\,d1Out},$$

$$tEdge1=\frac{-4\,tOut\,v3+tEdge\,(a3\,tOut-2\,v3)+a3\,tOut^2+a1Max\,(tEdge\,tOut+tEdge^2)+6\,d1Out}{-2\,v3+a1Max\,(tOut+2\,tEdge)+a3\,tOut},$$

$$\begin{aligned}d1Edge1=(&a1Max\,(tEdge\,(16\,tOut\,v3^2+(-12\,a3\,tOut^2-24\,d1Out)\,v3+2\,a3^2\,tOut^3+12\,a3\,d1Out\,tOut)+tEdge^2\,(4\,v3^2-4\,a3\,tOut\,v3+a3^2\,tOut^2)+16\,tOut^2\,v3^2\\&+(-8\,a3\,tOut^3-48\,d1Out\,tOut)\,v3+a3^2\,tOut^4+12\,a3\,d1Out\,tOut^2+36\,d1Out^2)+a1Max^2\\&(tEdge\,(-8\,tOut^2\,v3+2\,a3\,tOut^3+12\,d1Out\,tOut)+tEdge^2\,(-12\,tOut\,v3+4\,a3\,tOut^2+12\,d1Out)+tEdge^3\,(2\,a3\,tOut-4\,v3))+a1Max^3\\&(tEdge^2\,tOut^2+2\,tEdge^3\,tOut+tEdge^4))/(24\,v3^2+a1Max\,(-24\,tOut\,v3+tEdge\,(24\,a3\,tOut-48\,v3)+12\,a3\,tOut^2)-24\,a3\,tOut\,v3+a1Max^2\\&(6\,tOut^2+24\,tEdge\,tOut+24\,tEdge^2)+6\,a3^2\,tOut^2),v1Edge1=0,\end{aligned}$$

$$\begin{aligned}d1Edge2=-(&64\,tOut^2\,v3^4+a1Max\,(tEdge\\&(-64\,tOut^2\,v3^3+(64\,a3\,tOut^3+192\,d1Out\,tOut)\,v3^2+(-20\,a3^2\,tOut^4-144\,a3\,d1Out\,tOut^2-144\,d1Out^2)\,v3+2\,a3^3\,tOut^5+24\,a3^2\,d1Out\,tOut^3+72\,a3\,d1Out^2\,tOut)\\&+tEdge^2\,(32\,tOut\,v3^3+(-40\,a3\,tOut^2-48\,d1Out)\,v3^2+(16\,a3^2\,tOut^3+48\,a3\,d1Out\,tOut)\,v3-2\,a3^3\,tOut^4-12\,a3^2\,d1Out\,tOut^2)-32\,tOut^3\,v3^3+48\,a3\\&tOut^4\,v3^2+(-18\,a3^2\,tOut^5-72\,a3\,d1Out\,tOut^3+216\,d1Out^2\,tOut)\,v3+2\,a3^3\,tOut^6+18\,a3^2\,d1Out\,tOut^4-216\,d1Out^3)+(-96\,a3\,tOut^3-192\,d1Out\,tOut)\\&v3^3+a1Max^2\,(tEdge\,(64\,tOut^3\,v3^2+(-32\,a3\,tOut^4-192\,d1Out\,tOut^2)\,v3+4\,a3^2\,tOut^5+48\,a3\,d1Out\,tOut^3+144\,d1Out^2\,tOut)+tEdge^3\\&(-32\,tOut\,v3^2+(24\,a3\,tOut^2+48\,d1Out)\,v3-4\,a3^2\,tOut^3-24\,a3\,d1Out\,tOut)+tEdge^4\,(4\,v3^2-4\,a3\,tOut\,v3+a3^2\,tOut^2)+32\,tOut^4\,v3^2+tEdge^2\\&((12\,a3\,tOut^3-72\,d1Out\,tOut)\,v3-3\,a3^2\,tOut^4+108\,d1Out^2)+(-16\,a3\,tOut^5-96\,d1Out\,tOut^3)\,v3+2\,a3^2\,tOut^6+24\,a3\,d1Out\,tOut^4+72\,d1Out^2\,tOut^2)+\\&(52\,a3^2\,tOut^4+240\,a3\,d1Out\,tOut^2+144\,d1Out^2)\,v3^2+a1Max^3\,(tEdge^2\,(16\,tOut^3\,v3-4\,a3\,tOut^4-24\,d1Out\,tOut^2)+tEdge^3\\&(32\,tOut^2\,v3-8\,a3\,tOut^3-48\,d1Out\,tOut)+tEdge^4\,(6\,tOut\,v3-18\,d1Out)+tEdge^5\,(2\,a3\,tOut-4\,v3))+\\&(-12\,a3^3\,tOut^5-96\,a3^2\,d1Out\,tOut^3-144\,a3\,d1Out^2\,tOut)\,v3+a3^4\,tOut^6+12\,a3^3\,d1Out\,tOut^4+a1Max^4\,(2\,tEdge^4\,tOut^2+4\,tEdge^5\,tOut+tEdge^6)+36\,a3^2\\&d1Out^2\,tOut^2)/(48\,tOut\,v3^3+a1Max\,(tEdge\,(-48\,tOut\,v3^2+288\,d1Out\,v3+12\,a3^2\,tOut^3-144\,a3\,d1Out\,tOut)+tEdge^2\\&(24\,v3^2-24\,a3\,tOut\,v3+6\,a3^2\,tOut^2)-24\,tOut^2\,v3^2+144\,d1Out\,tOut\,v3+6\,a3^2\,tOut^4-72\,a3\,d1Out\,tOut^2)+(-48\,a3\,tOut^2-144\,d1Out)\,v3^2+a1Max^2\,(\\&tEdge\,(-48\,tOut^2\,v3+48\,a3\,tOut^3-144\,d1Out\,tOut)+tEdge^2\,(-72\,tOut\,v3+60\,a3\,tOut^2-144\,d1Out)-12\,tOut^3\,v3+tEdge^3\,(24\,a3\,tOut-48\,v3)+12\,a3\\&tOut^4-36\,d1Out\,tOut^2)+(12\,a3^2\,tOut^3+144\,a3\,d1Out\,tOut)\,v3+a1Max^3\,(6\,tOut^4+36\,tEdge\,tOut^3+78\,tEdge^2\,tOut^2+72\,tEdge^3\,tOut+24\,tEdge^4)-36\,a3^2\\&d1Out\,tOut^2),\end{aligned}$$

$$v1Edge2=-\frac{-8\,tOut\,v3^2+a1Max\,(4\,tOut^2\,v3+tEdge^2\,(a3\,tOut-2\,v3)-a3\,tOut^3-6\,d1Out\,tOut)+(6\,a3\,tOut^2+12\,d1Out)\,v3-a3^2\,tOut^3+a1Max^2\,tEdge^2\,tOut-6\,a3\,d1Out\,tOut}{4\,tOut\,v3+a1Max\,(2\,tOut^2+4\,tEdge\,tOut+2\,tEdge^2)-12\,d1Out}]$$

$$]$$

(%i284)
```
sln_N:2;
factor(sln_Ax1Asym[sln_N][1]);
factor(sln_Ax1Asym[sln_N][2]);
fm_d1Edge1: factor(sln_Ax1Asym[sln_N][3]);
fm_v1Edge1: factor(sln_Ax1Asym[sln_N][4]);
fm_d1Edge2: factor(sln_Ax1Asym[sln_N][5]);
fm_v1Edge2: factor(sln_Ax1Asym[sln_N][6]);
```

(sln_N) 2

(%o279) $a2=\frac{6\,tOut\,v3-2\,a3\,tOut^2+a1Max\,tEdge^2-6\,d1Out}{tOut^2}$

(%o280) $tEdge1=tEdge$

(fm_d1Edge1) $d1Edge1=\frac{a1Max\,tEdge^2}{6}$

(fm_v1Edge1) $v1Edge1=0$

(fm_d1Edge2) $d1Edge2=0$

(fm_v1Edge2) $v1Edge2=-\frac{4\,tOut\,v3-a3\,tOut^2+a1Max\,tEdge^2-6\,d1Out}{2\,tOut}$