2023 Spring Probability & Statistics for EECS April 21, 2023

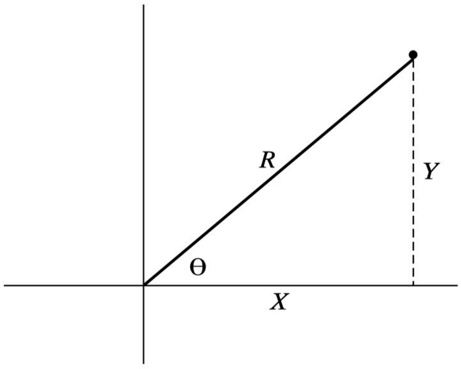
Homework 11

Professor: Ziyu Shao & Dingzhu Wen *Due: 23:59 on April 30, 2023*

1. Let X and Y be i.i.d. $\mathcal{N}(0,1)$, and let S be a random sign (1 or -1, with equal probabilities) independent of (X, Y).
   (a) Determine whether or not $(X, Y, X+Y)$ is MVN.
   (b) Determine whether or not $(X, Y, SX+SY)$ is MVN.
   (c) Determine whether or not (SX, SY) is MVN.

2. Let X and Y be i.i.d. $\mathcal{N}(0,1)$ r.v.s, $T = X+Y$, and $W = X-Y$. Show that T and W are independent using two methods: 1) properties of MVN and 2) change of variables.

3. Let (X, Y) denote a random point in the plane, and assume that the rectangular coordinates X and Y are i.i.d. $\mathcal{N}(0,1)$ r.v.s. Find the joint distribution of R and Θ (shown in the following figure). Are R and Θ independent?



4. (a) Let X and Y be i.i.d. Expo(λ), and transform them to $T = X+Y$, $W = X/Y$. Find the marginal PDFs of T and W, and the joint PDF of T and W.
   (b) Let X, Y, Z be i.i.d. Unif$(0,1)$, and $W = X+Y+Z$. Find the PDF of W using convolution.
   (c) Let X and Y be i.i.d. Expo(λ) r.v.s and $M = \max(X, Y)$. Show that M has the same distribution as $X + \frac{1}{2}Y$ using two methods: 1) properties of the Exponential and 2) convolution.

5. Programming Assignment:

(a) Use the Box-Muller Method to obtain the samples from the standard normal distribution $\mathcal{N}(0,1)$. You need to plot the pictures of both histogram and the theoretical PDF.

(b) Based on (a), generate samples from the standard bivariate Normal distribution, where the correlation is $\rho \in (-1,1)$, and the marginal PDFs are both $\mathcal{N}(0,1)$.

(c) According to the following picture format, plot the joint PDFs and the corresponding contours of standard bivariate Normal distribution with correlation $\rho = 0, 0.3, 0.5, 0.7, 0.9$.

(a) $\rho = -0.8$ (b) $\rho = -0.4$ (c) $\rho = 0$ (d) $\rho = 0.4$ (e) $\rho = 0.8$