Probability & Statistics for EECS: Homework #09

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Problem 1

Problem 2

(a) As for the joint PMF of X, Y, N, we have that the PMF is $P(X = x, Y = y, N = n)$, then as we have that $N = X + Y$, then only when $x + y = n$, will the PMF be non-zero. So we have that

$$P(X = x, Y = y, N = n) = P(X = x, Y = y) = (1 - p)^x * p * (1 - p)^y * p = (1 - p)^{x+y} p^2$$

, as we have that $x + y = n$, so we get that

$$P(X = x, Y = y, N = n) = (1 - p)^n p^2$$

(b) As for the joint PMF os X, N, we have that the PMF is $P(X = x, N = n)$, as only when $n = x + y$ will the PMF be non-zero, so we have that

$$P(X = x, N = n) = P(X = x, Y = n - x) = (1 - p)^x p(1 - p)^{n-x} p = (1 - p)^n p^2$$

(c) As for the conditional PMF of X given $N = n$, we have that the PMF is

$$P(X = x|N = n) = \frac{P(X = x, N = n)}{P(N = n)}.$$

The numerator is the joint PMF of X and N, which is $P(X = x, N = n) = (1 - p)^n p^2$, and the denominator is PMF of N, which is $P(N = n) = \sum_{x=0}^{n} (1 - p)^n p^2 = (n + 1)(1 - p)^n p^2$, so we have that

$$P(X = x|N = n) = \frac{P(X = x, N = n)}{P(N = n)} = \frac{(1 - p)^n p^2}{(n + 1)(1 - p)^n p^2} = \frac{1}{n + 1}.$$

where x = 0, 1, 2, ..., n.
Description: The conditional PMF of X given $N = n$ is a uniform distribution, which is $P(X = x|N = n) = \frac{1}{n+1}$. The event $P(X = x)$ is a Geom distribution, while the event $N = n$ is actuallu a negative binomial distribution, which denote the fail times before the second success. So the conditional PMF of X given $N = n$ is $\frac{1}{n+1}$, which denote that the first success between the first and the second success is uniformly distributed.

Problem 3

(a) To verify that the conditional distribution of X given X > c is the same as the distribution of c + X, firstly we can find the corresponding CDF of X given X > c, which is $F_X(x|X > c) = \dfrac{P(c < X \leq x)}{P(X > c)} = \dfrac{F_X(x) - F_X(c)}{1 - F_X(c)}$. Then we can find the corresponding CDF of X given X > c, which is $F_X(x|X > c) = \dfrac{F_X(x) - F_X(c)}{1 - F_X(c)}$, as $X \sim Expo(\lambda)$, so we have $F_X(x) = 1 - e^{-\lambda x}$. So, the $F_X(x|X > c) = \dfrac{e^{-\lambda c} - e^{-\lambda x}}{e^{-\lambda c}} = 1 - e^{-\lambda(x-c)}$.
As for the CDF of c + X, we have that $F_{c+X}(x) = F_X(x - c) = 1 - e^{-\lambda(x-c)}$. So we have that $F_{c+X}(x) = F_X(x|X > c)$. So that the conditional CDF of X given X > c is the same as the distribution of c + X.

(b) As for the CDF of X given X < c, we have that for x < c, $P(X \leq x|X < c) = \dfrac{P(X \leq x, X < c)}{P(X < c)} = \dfrac{P(X \leq x)}{P(X < c)} = \dfrac{1 - e^{-\lambda x}}{1 - e^{-\lambda c}}$ As for the PDF, we have that $f_X(x|X < c) = (P(X \leq x|X < c))' = (\dfrac{P(X \leq x)}{P(X < c)})' = (\dfrac{1 - e^{-\lambda x}}{1 - e^{-\lambda c}})' = \dfrac{\lambda e^{-\lambda x}}{1 - e^{-\lambda c}}$ for x < c, as for $x \geq c$, PDF is zero.

Problem 4

As we have that U_1, U_2, U_3 be i.i.d. Unif(0, 1), and let $L = min(U_1, U_2, U_3)$, $M = max(U_1, U_2, U_3)$

(a) 1. As for the marginal CDF of M is $F_M(m) = P(M \leq m) = P(U_1 \leq m, U_2 \leq m, U_3 \leq m) = P(U_1 \leq m)P(U_2 \leq m)P(U_3 \leq m) = m^3$.

2. As for the marginal PDF of M, we have that $f_M(m) = (F_M(m))' = (m^3)' = 3m^2$.

3. As for the joint CDF of M and L, firstly we consider the event $L \geq l, M \leq m$, which is easy to calculate, that is $P(L \geq l, M \leq m) = (m - l)^3$we have that $F_{M,L}(m, l) = P(M \leq m, L \leq l) =$.

4. As for the joint PDF of M and L, we have that $f_{M,L}(m, l) = (F_{M,L}(m, l))'$.

(b) As for the conditional PDF of M given L, we firstly find the CDF, that is

Problem 5