Probability & Statistics for EECS: Homework #010

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Problem 1

(1) As for X discrete, Y discrete, we have that

$$P(X = x) = \sum_y P(X = x, Y = y) = \sum_y P(X = x|Y = y)P(Y = y)$$

(2) As for X continuous, Y discrete, we have that

$$P(X = x) = \sum_{-\infty}^{\infty} P(X = x|Y = y)f_Y(y)dy$$

Then we have that,

$$\lim_{\varepsilon\to 0} P(X \in (x-\varepsilon, x+\varepsilon)) = \lim_{\varepsilon\to 0} \sum_y P(X \in (x-\varepsilon, x+\varepsilon)|Y = y)P(Y = y)$$

So we have that

$$f_X(x) = \sum_y f_X(x|Y = y)P(Y = y)$$

(3) As for X discrete, Y continuous, as we have that

$$P(X = x|Y = y) = \frac{f_Y(y|X = x)P(X = x)}{f_Y(y)}$$

Then we have

$$P(X = x|Y = y)f_Y(y) = f_Y(y|X = x)P(X = x)$$

Then we integrate both sides with respect to y, we have that

$$\int_{-\infty}^{\infty} P(X = x|Y = y)f_Y(y)dy = \int_{-\infty}^{\infty} f_Y(y|X = x)P(X = x)dy$$

Then we have that

$$f_X(x) = \int_{-\infty}^{\infty} f_Y(y|X = x)P(X = x)dy = P(X = x)$$

So we get that

$$P(X = x) = \int_{-\infty}^{\infty} P(X = x|Y = y)f_Y(y)dy$$

(4) As for X continuous, Y continuous, we have

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)dy = \int_{-\infty}^{\infty} f_{Y|X}(y|x)f_X(x)dy = \int_{-\infty}^{\infty} f_{Y|X}(y|x)dy$$

So we get that

$$f_X(x) = \int_{-\infty}^{\infty} f_{Y|X}(y|x)dy$$

Problem 2

Problem 3

Problem 4

Problem 5