Probability & Statistics for EECS: Homework #06

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Problem 1

Firstly we denote an indicator I_j, which is 1 if the j-th type toy is selected, and 0 otherwise. Then we denote that the total number of distinct toy types is X, we then have

$$X = \sum_{j=1}^{n} I_j.$$

We then find the $E(X)$, which is

$$E(X) = E(\sum_{j=1}^{n} I_j) = \sum_{j=1}^{n} E(I_j).$$

We can denote that the probability of the j-th type toy is head is p_j, then we have

$$E(I_j) = p_j.$$

So we have

$$E(X) = \sum_{j=1}^{n} p_j.$$

As $p_j = (1 - (1 - \frac{1}{n})^t)$, where t is the number that we totally collected toys. So we have

$$E(X) = \sum_{j=1}^{n} (1 - (1 - \frac{1}{n})^t) = n(1 - (1 - \frac{1}{n})^t)$$

Problem 2

We denote an indicator A_i, which is 1 if the i-th block not equal the $(i+1)$-th block, and 0 otherwise, and A is the total number of runs, then we have that

$$A = \sum_{i=1}^{n-1} A_i + 1.$$

So we have that

$$E(A) = E(\sum_{i=1}^{n-1} A_i + 1) = 1 + \sum_{i=1}^{n-1} E(A_i).$$

Then we denote event B_j is the jth block is different from $j+1th$, then $P(B_j) = p(1-p)+(1-p)p = 2p(1-p)$. So we have that $E(A) = 1 + \sum_{i=1}^{n-1} P(B_i) = 1 + 2(n-1)p(1-p)$.

Problem 3

Problem 4

Problem 5