Probability & Statistics for EECS: Homework #06

Due on Mar 26, 2023 at 23:59

Name: **Penghao Wang**

Student ID: 2021533138

Problem 1

Firstly we denote an indicator I_j, which is 1 if the j-th type toy is selected, and 0 otherwise. Then we denote that the total number of distinct toy types is X, we then have

$$X = \sum_{j=1}^{n} I_j.$$

We then find the $E(X)$, which is

$$E(X) = E(\sum_{j=1}^{n} I_j) = \sum_{j=1}^{n} E(I_j).$$

We can denote that the probability of the j-th type toy is head is p_j, then we have

$$E(I_j) = p_j.$$

So we have

$$E(X) = \sum_{j=1}^{n} p_j.$$

As $p_j = (1 - (1 - \frac{1}{n})^t)$, where t is the number that we totally collected toys. So we have

$$E(X) = \sum_{j=1}^{n} (1 - (1 - \frac{1}{n})^t) = n(1 - (1 - \frac{1}{n})^t)$$

Problem 2

We denote an indicator A_i, which is 1 if the i-th block not equal the $(i+1)$-th block, and 0 otherwise, and A is the total number of runs, then we have that

$$A = \sum_{i=1}^{n-1} A_i + 1.$$

So we have that

$$E(A) = E(\sum_{i=1}^{n-1} A_i + 1) = 1 + \sum_{i=1}^{n-1} E(A_i).$$

Then we denote event B_j is the jth block is different from $j+1th$, then $P(B_j) = p(1-p) + (1-p)p = 2p(1-p)$. So we have that $E(A) = 1 + \sum_{i=1}^{n-1} P(B_i) = 1 + 2(n-1)p(1-p)$.

Problem 3

1. Fristly we need to consider let the last one be the tagged elk, then we need to make k-1 captures. We have

$$P(X = k) = \frac{\binom{n}{m-1}\binom{N-n}{k}}{\binom{N}{m+k-1}} * \frac{n-(m-1)}{N-(m-1)-k}$$

Then the total number, we need to

Problem 4

Problem 5

To distribute the 14 balls into 5 boxes, we can use the