Probability & Statistics for EECS: Homework #09

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Problem 1

Problem 2

(a) As for the joint PMF of X, Y, N, we have that the PMF is $P(X=x,Y=y,N=n)$, then as we have that $N=X+Y$, then only when $x+y=n$, will the PMF be non-zero. So we have that

$$P(X=x,Y=y,N=n)=P(X=x,Y=y)=(1-p)^x*p*(1-p)^y*p=(1-p)^{x+y}p^2$$

, as we have that $x+y=n$, so we get that

$$P(X=x,Y=y,N=n)=(1-p)^n p^2$$

(b) As for the joint PMF os X, N, we have that the PMF is $P(X=x,N=n)$, as only when $n=x+y$ will the PMF be non-zero, so we have that

$$P(X=x,N=n)=P(X=x,Y=n-x)=(1-p)^x p(1-p)^{n-x}p=(1-p)^n p^2$$

(c) As for the conditional PMF of X given $N=n$, we have that the PMF is

$$P(X=x|N=n)=\frac{P(X=x,N=n)}{P(N=n)}.$$

The numerator is the joint PMF of X and N, which is $P(X=x,N=n)=(1-p)^n p^2$, and the denominator is PMF of N, which is $P(N=n)=\sum_{x=0}^{n}(1-p)^n p^2=(n+1)(1-p)^n p^2$, so we have that

$$P(X=x|N=n)=\frac{P(X=x,N=n)}{P(N=n)}=\frac{(1-p)^n p^2}{(n+1)(1-p)^n p^2}=\frac{1}{n+1}.$$

where x = 0, 1, 2, ..., n.
Description: The conditional PMF of X given $N=n$ is a uniform distribution, which is $P(X=x|N=n)=\frac{1}{n+1}$. The event $P(X=x)$ is a Geom distribution, while the event $N=n$ is actuallu a negative binomial distribution, which denote the fail times before the second success. So the conditional PMF of X given $N=n$ is $\frac{1}{n+1}$, which denote that the first success between the first and the second success is uniformly distributed.

Problem 3

(a) To verify that the conditional distribution of X given X > c is the same as the distribution of c + X, firstly we can find the corresponding CDF of X given X > c, which is $P(X \leq x|X > c) = \frac{P(c < X \leq x)}{P(X > c)} = \frac{F(x) - F(c)}{1 - F(c)}$. As $X \sim Expo(\lambda)$, so we have $F(x) = 1 - e^{-\lambda x}$. So, the $P(X \leq x|X > c) = \frac{e^{-\lambda c} - e^{-\lambda x}}{e^{-\lambda c}} = 1 - e^{-\lambda(x-c)}$.
As for the CDF of c + X, we have that $P(c + X \leq x) = P(X \leq x - c) = 1 - e^{-\lambda(x-c)}$. So we have that $P(X \leq x|X > c) = P(c+X \leq x)$. So that the conditional CDF of X given X > c is the same as the c + X.

(b) As for the CDF of X given X < c, we have that for x < c, $P(X \leq x|X < c) = \frac{P(X \leq x, X < c)}{P(X < c)} = \frac{P(X \leq x)}{P(X < c)} = \frac{1 - e^{-\lambda x}}{1 - e^{-\lambda c}}$ As for the PDF, we have that $f(x|X < c) = (P(X \leq x|X < c))' = (\frac{P(X \leq x)}{P(X < c)})' = (\frac{1 - e^{-\lambda x}}{1 - e^{-\lambda c}})' = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda c}}$ for x < c, as for $x \geq c$, PDF is zero.

Problem 4

As we have that U_1, U_2, U_3 be i.i.d. Unif(0, 1), and let $L = min(U_1, U_2, U_3)$, $M = max(U_1, U_2, U_3)$

(a) 1. As for the marginal CDF of M is $F_M(m) = P(M \leq m) = P(U_1 \leq m, U_2 \leq m, U_3 \leq m) = P(U_1 \leq m)P(U_2 \leq m)P(U_3 \leq m) = m^3$. For $m \in [0, 1]$

2. As for the marginal PDF of M, we have that $f_M(m) = (F_M(m))' = (m^3)' = 3m^2$. For $m \in [0, 1]$

3. As for the joint CDF of M and L, firstly we consider the event $L > l, M \leq m$, which is easy to calculate, that is $P(L > l, M \leq m) = (m-l)^3$, we have that $P(L \leq l, M \leq m) = P(M \leq m) - P(L > l, M \leq m) = m^3 - (m-l)^3$ for $m \geq l$ and that $m, l \in [0, 1]$.

4. As for the joint PDF of M and L, we have that $f(l, m) = \dfrac{\partial^2 P(L \leq l, M \leq m)}{\partial l \partial m} = 6(m-l)$ for $m, l \in [0, 1]$ and that $m \geq l$.

(b) As for the conditional PDF of M given L, we firstly find the CDF, that is

Problem 5