Probability & Statistics for EECS: Homework #09

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Problem 1

Problem 2

1. As for the joint PMF of X, Y, N, we have that the PMF is $P(X = x, Y = y, N = n)$, then as we have that $N = X + Y$, then only when $x + y = n$, will the PMF be non-zero. So we have that

$$P(X = x, Y = y, N = n) = P(X = x, Y = y) = (1-p)^x * p * (1-p)^y * p = (1-p)^{x+y}p^2$$

, as we have that $x + y = n$, so we get that

$$P(X = x, Y = y, N = n) = (1-p)^n p^2$$

2. As for the joint PMF os X, N, we have that the PMF is $P(X = x, N = n)$, as only when $n = x + y$ will the PMF be non-zero, so we have that

$$P(X = x, N = n) = P(X = x, Y = n - x) = (1-p)^x p(1-p)^{n-x} p = (1-p)^n p^2$$

3. As for the conditional PMF of X given $N = n$, we have that the PMF is

$$P(X = x|N = n) = \frac{P(X = x, N = n)}{P(N = n)}.$$

The numerator is the joint PMF of X and N, which is $P(X = x, N = n) = (1-p)^n p^2$, and the denominator is PMF of N, which is $P(N = n) = \sum_{x=0}^{n}(1-p)^n p^2 = (n+1)(1-p)^n p^2$, so we have that

$$P(X = x|N = n) = \frac{P(X = x, N = n)}{P(N = n)} = \frac{(1-p)^n p^2}{(n+1)(1-p)^n p^2} = \frac{1}{n+1}.$$

where x = 0, 1, 2, ..., n.
Description: The conditional PMF of X given $N = n$ is a uniform distribution, which is $P(X = x|N = n) = \frac{1}{n+1}$. The event $P(X = x)$ is a Geom distribution, while the event $N = n$ is actuallu a negative binomial distribution, which denote the fail times before the second success. So the conditional PMF of X given $N = n$ is $\frac{1}{n+1}$, which denote that the first success between the first and the second success is uniformly distributed.

Problem 3

Problem 4

Problem 5