2023 Spring Probability & Statistics for EECS	April 9, 2023

Homework 09

Professor: Ziyu Shao & Dingzhu Wen — *Due: 23:59 on April 16, 2023*

1. Show the proof of general Bayes' Rule (four cases).

	Y **discrete**	Y **continuous**
X **discrete**	$P(Y=y\|X=x)=\frac{P(X=x\|Y=y)P(Y=y)}{P(X=x)}$	$f_Y(y\|X=x)=\frac{P(X=x\|Y=y)f_Y(y)}{P(X=x)}$
X **continuous**	$P(Y=y\|X=x)=\frac{f_X(x\|Y=y)P(Y=y)}{f_X(x)}$	$f_{Y\|X}(y\|x)=\frac{f_{X\|Y}(x\|y)f_Y(y)}{f_X(x)}$

2. Let X and Y be i.i.d. Geom(p), and $N = X + Y$.
   (a) Find the joint PMF of X, Y, N.
   (b) Find the joint PMF of X and N.
   (c) Find the conditional PMF of X given $N = n$, and give a simple description in words of what the result says.

3. Let $X \sim$ Expo(λ), and let c be a positive constant.
   (a) If you remember the memoryless property, you already know that the conditional distribution of X given $X > c$ is the same as the distribution of $c + X$ (think of waiting c minutes for a "success" and then having a fresh Expo(λ) additional waiting time). Derive this in another way, by finding the conditional CDF of X given $X > c$ and the conditional PDF of X given $X > c$.
   (b) Find the conditional CDF and conditional PDF of X given $X < c$.

4. Let U_1, U_2, U_3 be i.i.d. Unif(0, 1), and let $L = \min(U_1, U_2, U_3)$, $M = \max(U_1, U_2, U_3)$.
   (a) Find the marginal CDF and marginal PDF of M, and the joint CDF and joint PDF of L, M.
   (b) Find the conditional PDF of M given L.

5. Let X and Y be i.i.d. Geom(p), $L = \min(X, Y)$, and $M = \max(X, Y)$.
   (a) Find the joint PMF of L and M. Are they independent?

(b) Find the marginal distribution of L in two ways: using the joint PMF, and using a story.

(c) Find $E[M]$.

(d) Find the joint PMF of L and $M - L$. Are they independent?