

Lazy Caterer's Sequence

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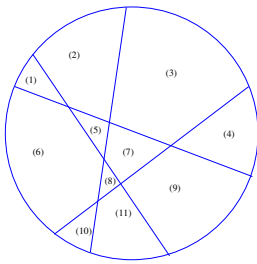
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1 Illustration of structures

1.1 Pancake Structure

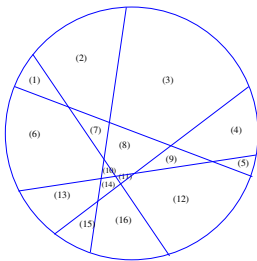
1. $n = 4$

(a) Pancake structure



2. $n = 5$

(a) Pancake structure



1.2 Binary Words

When $n > 3$ the sequence gives the number of binary words where there are at least two ones and at most two two's. In these formulas n represents the length of a binary word, so $a(n)$ gives the number of possible binary words formed with these constraints. The following are two illustrations of this structure. The first is when $n = 4$ and the second is when $n = 5$.

1. $n = 4$

(a) Binary words structure illustration

0011, 0101, 0110, 0111, 1001, 1010, 1011, 1100, 1101, 1110, 1111

2. $n = 5$

(a) Binary words structure illustration

11111, 01111, 10111, 11011, 11101, 11110, 00111, 10011, 10101, 10110,
01011, 01101, 01110, 11100, 11001, 11010

2 Formula Demonstrations

2.1 Binomial

1. $a(n) = \text{Binomial}(n+2, 1) - 2 \times \text{Binomial}(n+1, 1) + \text{Binomial}(n+2, 2)$

$$\binom{n+2}{1} - 2 \times \binom{n+1}{1} + \binom{n+2}{2}$$

(a) $n = 4$

$$\begin{aligned} a(4) &= \binom{4+2}{1} - 2 \times \binom{4+1}{1} + \binom{4+2}{2} \\ a(4) &= \binom{6}{1} - 2 \times \binom{5}{1} + \binom{6}{2} \\ a(4) &= 6 - 10 + 15 \\ a(4) &= 11 \end{aligned}$$

(b) $n = 5$

$$a(5) = \binom{5+2}{1} - 2 \times \binom{5+1}{1} + \binom{5+2}{2}$$

$$a(5) = \binom{7}{1} - 2 \times \binom{6}{1} + \binom{7}{2}$$

$$a(5) = 7 - 12 + 21$$

$$a(4) = 16$$

2.2 Generating Function

1. $G.f : A(x) = (1 - x + x^2)/(1 - x)^3$

(a) $n = 4$

$$\left[\frac{d^4}{dx^4} (1 - x + x^2)/(1 - x)^3 \right]_{x=0} = 264$$

$$\text{And } \frac{264}{4!} = 11$$

(b) $n = 5$

$$\left[\frac{d^5}{dx^5} (1 - x + x^2)/(1 - x)^3 \right]_{x=0} = 1920$$

$$\text{And } \frac{1920}{5!} = 16$$

2.3 Exponential Generating Function

1. $E.g.f : A(x) = e^x + xe^x + \frac{e^x x^2}{2}$

(a) $n = 4$

$$\left[\frac{d^4}{dx^4} (e^x + xe^x + \frac{e^x x^2}{2}) \right]_{x=0} = 11$$

(b) $n = 5$

$$\left[\frac{d^5}{dx^5} (e^x + xe^x + \frac{e^x x^2}{2}) \right]_{x=0} = 16$$

2.4 Recursive Formula

1. $a(n+3) = 3 * a(n+2) - 3 * a(n+1) + a(n), a(0) = 1, a(1) = 2, a(2) = 4, a(3) = 7, a(4) = 11, a(5) = 16, a(6) = 22, a(7) = 29, a(8) = 37$

(a) $n = 6$

$$a(6+3) = a(9) = 3 * a(6+2) - 3 * a(6+1) + a(6)$$

$$a(9) = 3 * a(8) - 3 * a(7) + a(6)$$

$$a(9) = 3 * 37 - 3 * 29 + 22$$

$$a(9) = 111 - 87 + 22 = 46$$

(b) $n = 7$

$$a(7+3) = a(10) = 3 * a(7+2) - 3 * a(7+1) + a(7)$$

$$a(10) = 3 * a(9) - 3 * a(8) + a(7)$$

$$a(10) = 3 * 46 - 3 * 37 + 29$$

$$a(10) = 138 - 111 + 29 = 56$$

(c) $n = 8$

$$a(8+3) = a(11) = 3 * a(8+2) - 3 * a(8+1) + a(8)$$

$$a(11) = 3 * a(10) - 3 * a(9) + a(8)$$

$$a(11) = 3 * 56 - 3 * 46 + 37$$

$$a(11) = 168 - 138 + 37 = 67$$