

Lazy Caterer's Sequence

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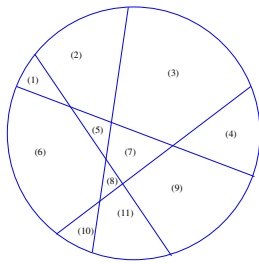
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1 Illustration of structures

1.1 Pancake Structure

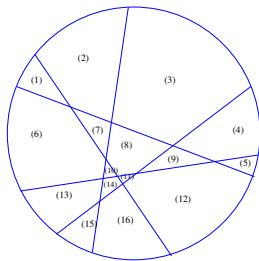
1. $n = 4$

(a) Pancake structure



2. $n = 5$

(a) Pancake structure



1.2 Binary Words

When $n > 3$ the sequence gives the number of binary words where there are at least two ones and at most two two's. In these formulas n represents the length of a binary word, so $a(n)$ gives the number of possible binary words formed with these constraints. The following are two illustrations of this structure. The first is when $n = 4$ and the second is when $n = 5$.

1. $n = 4$

(a) Binary words structure illustration

0011, 0101, 0110, 0111, 1001, 1010, 1011, 1100, 1101, 1110, 1111

2. $n = 5$

(a) Binary words structure illustration

11111, 01111, 10111, 11011, 11101, 11110, 00111, 10011, 10101, 10110,
01011, 01101, 01110, 11100, 11001, 11010

2 Formula Demonstrations

2.1 Binomial

1. $a(n) = \text{Binomial}(n+2, 1) - 2 \times \text{Binomial}(n+1, 1) + \text{Binomial}(n+2, 2)$

$$\binom{n+2}{1} - 2 \times \binom{n+1}{1} + \binom{n+2}{2}$$

(a) $n = 4$

$$\begin{aligned} a(4) &= \binom{4+2}{1} - 2 \times \binom{4+1}{1} + \binom{4+2}{2} \\ a(4) &= \binom{6}{1} - 2 \times \binom{5}{1} + \binom{6}{2} \\ a(4) &= 6 - 10 + 15 \\ a(4) &= 11 \end{aligned}$$

(b) $n = 5$

$$\begin{aligned}a(5) &= \binom{5+2}{1} - 2 \times \binom{5+1}{1} + \binom{5+2}{2} \\a(5) &= \binom{7}{1} - 2 \times \binom{6}{1} + \binom{7}{2} \\a(5) &= 7 - 12 + 21 \\a(4) &= 16\end{aligned}$$

2.2 Generating Function

1. $G.f : A(x) = (1 - x + x^2)/(1 - x)^3$

(a) $n = 4$

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(b) $n = 5$

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