Lazy Caterer's Sequence

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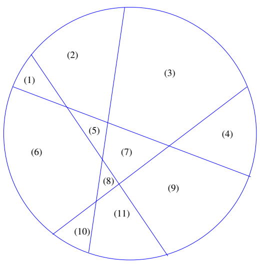
1 Illustration of structures

1.1 Pancake Structure

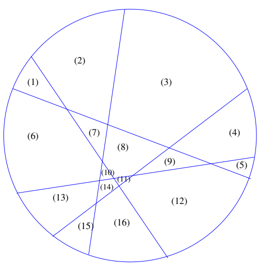
1. $n = 4$

   (a) Pancake structure



2. $n = 5$

   (a) Pancake structure



1.2 Binary Words

When $n > 3$ the sequence gives the number of binary words where there are at least two ones and at most two two's. In these formulas n represents the length of a binary word, so $a(n)$ gives the number of possible binary words formed with these constraints. The following are two illustrations of this structure. The first is when $n = 4$ and the second is when $n = 5$.

1. $n = 4$

   (a) Binary words structure illustration

   $$\boxed{0011, 0101, 0110, 0111, 1001, 1010, 1011, 1100, 1101, 1110, 1111}$$

2. $n = 5$

   (a) Binary words structure illustration

   $$\boxed{\begin{gathered}11111, 01111, 10111, 11011, 11101, 11110, 00111, 10011, 10101, 10110,\\ 01011, 01101, 01110, 11100, 11001, 11010\end{gathered}}$$

2 Formula Demonstrations

2.1 Binomial

1. $a(n) = Binomial(n + 2, 1) - 2 \times Binomial(n + 1, 1) + Binomial(n + 2, 2)$

   $\binom{n+2}{1} - 2 \times \binom{n+1}{1} + \binom{n+2}{2}$

   (a) $n = 4$

   $$\boxed{\begin{gathered}a(4) = \binom{4+2}{1} - 2 \times \binom{4+1}{1} + \binom{4+2}{2}\\[2ex] a(4) = \binom{6}{1} - 2 \times \binom{5}{1} + \binom{6}{2}\\[2ex] a(4) = 6 - 10 + 15\\[2ex] a(4) = 11\end{gathered}}$$

(b) $n = 5$

$$\boxed{\begin{gathered} a(5) = \binom{5+2}{1} - 2 \times \binom{5+1}{1} + \binom{5+2}{2} \\ a(5) = \binom{7}{1} - 2 \times \binom{6}{1} + \binom{7}{2} \\ a(5) = 7 - 12 + 21 \\ a(4) = 16 \end{gathered}}$$

2.2 Generating Function

1. $G.f : A(x) = (1 - x + x^2)/(1 - x)^3$

   (a) $n = 4$

   $$\boxed{\begin{gathered} [\frac{d^4}{dx^4}(1 - x + x^2)/(1 - x)^3]_{x=0} = 264 \\ And\ \frac{264}{4!} = 11 \end{gathered}}$$

   (b) $n = 5$

   $$\boxed{\begin{gathered} [\frac{d^5}{dx^5}(1 - x + x^2)/(1 - x)^3]_{x=0} = 1920 \\ And\ \frac{1920}{5!} = 16 \end{gathered}}$$

2.3 Exponential Generating Function

1. $E.g.f : A(x) = e^x + xe^x + \frac{e^x x^2}{2}$

   (a) $n = 4$

   $$\boxed{[\frac{d^4}{dx^4}(e^x + xe^x + \frac{e^x x^2}{2})]_{x=0} = 11}$$

   (b) $n = 5$

   $$\boxed{[\frac{d^5}{dx^5}(e^x + xe^x + \frac{e^x x^2}{2})]_{x=0} = 16}$$