# Instructions for Paper Submissions to AISTATS 2022: Supplementary Materials

## 1 FORMATTING INSTRUCTIONS

To prepare a supplementary pdf file, we ask the authors to use `aistats2022.sty` as a style file and to follow the same formatting instructions as in the main paper. The only difference is that the supplementary material must be in a *single-column* format. You can use `supplement.tex` in our starter pack as a starting point, or append the supplementary content to the main paper and split the final PDF into two separate files.

Note that reviewers are under no obligation to examine your supplementary material.

## 2 MISSING PROOFS

The supplementary materials may contain detailed proofs of the results that are missing in the main paper.

### 2.1 Proof of Lemma 3

*In this section, we present the detailed proof of Lemma 3 and then [ ... ]*

Given that,

The mean effect at a specific point $x_s$ is:

$$\mu(x_s) = \mathbb{E}_{\mathbf{x_c}|x_s}\left[\frac{\partial f}{\partial x_s}(x_s, \mathbf{x_c})\right] \tag{1}$$

The variance at a specific point $x_s$ is:

$$\sigma^2(x_s) = \mathrm{Var}_{\mathbf{x_c}|x_s}\left[\frac{\partial f}{\partial x_s}(x_s, \mathbf{x_c})\right] \tag{2}$$

The mean effect inside the interval $[z_1, z_2)$ is:

$$\mu(z_1, z_2) = \frac{1}{z_2 - z_1}\int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s}\left[\frac{\partial f}{\partial x_s}(x_s, \mathbf{x_c})\right] \tag{3}$$

The accumulated variance inside the interval is:

$$\sigma^2(z_1, z_2) = \int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\left(\frac{\partial f}{\partial x_s}(x_s, \mathbf{x_c}) - \mu(z_1, z_2)\right)^2\right]\partial z \tag{4}$$

The residual at a specific point $x_s$ is:

$$\rho(x_s) = \mu(x_s) - \mu(z_1, z_2) \tag{5}$$

We want to prove that:

$$\sigma^2(z_1, z_2) = \int_{z_1}^{z_2} \sigma^2(z) + \rho^2(z)\partial z = \int_{z_1}^{z_2} \sigma^2(z) + \int_{z_1}^{z_2} \rho^2(z)\partial z \tag{6}$$

Proof:

$$\sigma^2(z_1, z_2) = \int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\left(\frac{\partial f}{\partial x_s}(z, \mathbf{x_c}) - \mu(z_1, z_2)\right)^2\right]\partial z \tag{7}$$

$$= \int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\left(\frac{\partial f}{\partial x_s} - \mu(z) + \rho(z)\right)^2\right]\partial z \tag{8}$$

$$= \int_{z_1}^{z_2}\left(\mathbb{E}_{\mathbf{x_c}|x_s=z}\left[(\frac{\partial f}{\partial x_s} - \mu(z))^2\right] + \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\rho(z)^2\right] + \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[2(\frac{\partial f}{\partial x_s} - \mu(z))\rho(z)\right]\right)\partial z \tag{9}$$

$$= \int_{z_1}^{z_2}(\sigma^2(z) + \rho^2(z) + 2(\mu(z) - \mu(z))\rho(z))\partial z \tag{10}$$

$$= \int_{z_1}^{z_2} \sigma^2(z) + \rho^2(z)\partial z \tag{11}$$

$$\tag{12}$$

# 3 ADDITIONAL EXPERIMENTS

If you have additional experimental results, you may include them in the supplementary materials.

### 3.1 The Effect of Regularization Parameter

*Our algorithm depends on the regularization parameter* $\lambda$*. Figure 1 below illustrates the effect of this parameter on the performance of our algorithm. As we can see, [ ... ]*