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Anonymous Author
Anonymous Institution

Abstract

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1 INTRODUCTION

Recently, ML has flourished in critical domains, such as healthcare and finance. In these areas, we need ML system with the capability to explain their predictions, apart from predicting with accuracy. For this reason there is an increased interest in Explainable AI (XAI), the field that provides interpretations about the behavior of complex black-box models. XAI literature distinguishes between local and global explainability techniques (Molnar et al., 2020a). Local methods explain a specific prediction, whereas global methods explain the entire model behavior. Global methods provide a universal explanation, summarizing the numerous local ones into a single interpretable outcome, usually a number or a plot. If a user wants to get a rough overview about which features are significant (feature importance) or whether a particular feature has a positive or negative effect on the output (feature effect), they should opt for a global explainability technique. On ther other hand, aggregating the individual explanations for producing a concise global one is vulnerable to misinterpretations; Under strong feature interactions, the global explanation may obfuscate heterogeneous effects (Herbinger et al., 2022) that exist under the hood; a phenomenon called aggregation bias (Mehrabi et al., 2021).

Feature effect (FE) (Grömping, 2020) is a fundamental category of global explainability methods. The objective of FE is to the isolate and visulalize the impact of a single feature on the output. [1] FE methods suffer from aggregation bias because, often, the rationale behind the average effect might be unclear. For example, a feature with zero average effect may indicate that the feature has no effect on the output or, contrarily, it has a highly positive effect in some cases and a highly negative in others. There are three widely-used FE methods; Partial Dependence Plots (PDP)(Friedman, 2001), Marignal Plots (MP)(Apley and Zhu, 2020) and Aggregated Local Effects (ALE)(Apley and Zhu, 2020). PDP and MP have been criticized for computing erroneous effects when the input features are (highly) correlated, which is a frequent scenario in many ML problems. Therefore, ALE has been established as the state-of-the-art FE method.

However, ALE faces two crucial drawbacks. First, it does not provide a way to inform the user about potential heterogeneous effects that are hidden behind the average effect. In contrast, in the case of PDP, the heterogeneous effects can be spotted by exploring the Individual Conditional Expectations (ICE)(Goldstein et al., 2015). Second, ALE requires an additional step where the axis of the feature of interest is split in K fixed-size non-overlapping intervals. So far, the user is asked to provide a value for the parameter K blindly, i.e without an indication about whether a big or a small value could be more appropriate, which often leads to unstable explanations. For convenience, in the rest of the paper we will refer to this step using the term bin-splitting problem.

In this paper, we extend ALE with a probabilistic component for measuring the uncertainty of the global explanation. The uncertainty of the global explanation expresses how certain we are that the global (averaged) explanation is valid if applied to an instance drawn at random. The probabilistic extension completes ALE, as ICE plots complement PDP, for revealing the heterogeneous effects. Furthermore, we transform the bin-splitting step into a data-driven clustering problem, i.e., we search for the optimal splitting given available instances of the training set. We, also, present a computationally-grounded algorithm for finding the optimal solution.

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[1] FE methods also isolate the effect of a pair of features to the output. Combinations of more than two features are not usual, because they encounter, among others, visualization difficulties.

Contributions. The contributions of this paper are the following:

- We introduce NAME, an extension of ALE that quantifies the uncertainty of the global explanation, i.e. the level of heterogeneous effects hidden behind the global explanation.
- We present an algorithm that automatically computes the optimal bin-splitting
- We formally prove that our method finds the optimal grouping of samples, minimizing the added uncertainty over the unavoidable heterogeneity.
- We provide empirical evaluation of the method in artificial and real datasets.

The implementation of our method and the code for reproducing all the experiments is provided in the submission and will become publicly available upon acceptance.

2 BACKGROUND AND RELATED WORK

Notation. We refer to random variables (rv) using uppercase X, whereas to simple variables with plain lowercase x. Bold denotes a vector; $\mathbf{x}$ for simple variables or $\mathbf{X}$ for rvs. Often, we partition the input vector $\mathbf{x} \in \mathbb{R}^D$ to the feature of interest $x_s \in \mathbb{R}$ and the rest of the features $\mathbf{x_c} \in \mathbb{R}^{D-1}$. For convenience we denote it as $(x_s, \mathbf{x}_c)$, but we clarify that it corresponds to the vector $(x_1, \cdots, x_s, \cdots, x_D)$. Equivalently, we denote the corresponding rv as $X = (X_s, \mathbf{X}_c)$. The black-box function is $f : \mathbb{R}^D \to \mathbb{R}$ and the FE of the s-th feature is $f^{<\texttt{method}>}(x_s)$, where $<\texttt{method}>$ is the name of the FE method.[2]

Feature Effect Methods. The three well-known feature effect methods are: PDP, MP and ALE. PDP formulates the FE of the s-th attribute as an expectation over the marginal distribution $\mathbf{X}_c$, i.e., $f^{\texttt{PDP}}(x_s) = \mathbb{E}_{\mathbf{X}_c}[f(x_s, \mathbf{X}_c)]$, whereas MP formulates it as an expectation over the conditional $\mathbf{X}_c|X_s$, i.e., $f^{\texttt{MP}}(x_s) = \mathbb{E}_{\mathbf{X}_c|X_s=x_s}[f(x_s, \mathbf{X}_c)]$. ALE computes the global effect at x_s as the accumulation of the averaged local effects:

$$f^{\texttt{ALE}}(x_s) = \int_{z_{s,min}}^{x_s} \mathbb{E}_{\mathbf{X}_c|X_s=z}\left[\frac{\partial f(z, \mathbf{X}_c)}{\partial z}\right] \partial z \qquad (1)$$

ALE has specific advantages which gain particular value in cases of corelated input features. In these cases, PDP integrates over unrealistic instances, due to the use of the marginal distribution $\mathbf{X}_c$, and MP computes aggregated effects, i.e., imputes the combined effect of sets of features to a single feature. ALE manages to resolve both issues, and is therefore the only trustable FE method in cases of correlated features.

Quantify the Heterogeneous Effects. FE methods answer the question *what is expected to happen to the output (expected effect), if the value of a specific feature is increased/decreased*. Having an answer to the question above, it comes naturally to also wonder *how certain we are about the expected change (uncertainty)*. For this reason, the quantification of the uncertainty along with the expected effect has attracted a lot of interest. The level of uncertainty is mostly quantified by measuring the existence of heterogeneous effects, i.e. whether there are local explanations that deviate from the expected global effect. ICE and d-ICE plots provide a visual understanding of the heterogeneous effects on top of PDPs. Another approach targets on grouping the heterogeneous effects, e.g., allocating ICE plots in homogeneous clusters, by dividing the input space.(Molnar et al., 2020b) Some other approaches, like H-Statistic, Greenwel, move a step behind and try to quantify the level of interaction between the input features, a possible cause of heterogeneous effects. In this case, the interpretation is indirect, since a strong interaction index is only an indicator of heterogeneous effects. The aforementioned approaches face two pathogenies; They either do not quantify the uncertainty of the FE directly or they are based on PDPs, and, therefore, they are subject to the failure modes of PDPs in cases of correlated features. To the best of our knowledge, no method so far targets on quantifying the uncertainty of ALE.

Bin-Splitting for ALE estimation. In real ML scenarios, the expected FE and the uncertainty are estimated from the limited instances of the training set. (Apley and Zhu, 2020) proposed estimating the local effects in each bin by evaluating the black box-funtion at the bin limits:

$$\hat{f}^{\texttt{ALE}}(x_s) = \sum_{k=1}^{k_x} \frac{1}{|\mathcal{S}_k|} \sum_{i:\mathbf{x}^i \in \mathcal{S}_k} \left[f(z_k, \mathbf{x_c}^i) - f(z_{k-1}, \mathbf{x_c}^i)\right] \qquad (2)$$

We denote as k_x the index of the bin that x_s belongs to, i.e. $k_x : z_{k_x-1} \leq x_s < z_{k_x}$ and $\mathcal{S}_k$ is the set of training instance that lie in the k-th bin, i.e. $\mathcal{S}_k = \{\mathbf{x}^i : z_{k-1} \leq x_s^i < z_k\}$. Afterwards, (cite) proposed the Differential ALE (DALE) that exploits differentiaton for computing the local effects on the instances training-set, instead of the bin limits:

$$\hat{f}^{\texttt{DALE}}(x_s) = \Delta x \sum_{k=1}^{k_x} \frac{1}{|\mathcal{S}_k|} \sum_{i:\mathbf{x}^i \in \mathcal{S}_k} \frac{\partial f}{\partial x_s}(\mathbf{x}^i) \qquad (3)$$

Their method has the advantages of remaining on-distribution even when bins become wider and, most im-

[2] An extensive list of all symbols used in the paper is provided in the helping material.

portantly, allows the recomputation of the accumulated effect with different bin-splitting with near-zero computational cost. However, none of the approximations above deals with the crucial problem of bin-spliting. As indicated by (Molnar, 2022), in ALE the effects are computed per interval (region) and the interpretation of the effect can only be local.

3 THE NAME METHOD

In Section 3.1 we define the component for the uncertainty quantification. In Section 3.2, we show how to estimate the average effect and the uncertainty from the limited samples of the training set and we make an important proof about the aggregated variance defined over a bin. In Section 3.3, we define and solve the problem of optimal bin-splitting. Finally, in Section 3.4, we illustrate the appropriate visualization of NAME for facilitating its interpretation by a non-expert and we discuss important aspects of the method

3.1 Uncertainty Quantification

ALE defines the local effect of the s-th feature on $f(\cdot)$ at point $(x_s, \mathbf{x_c})$ as $\frac{\partial f}{\partial x_s}(x_s, \mathbf{x_c})$. All the local explanations at x_s are, then, weigthed by the conditional distribution $p(\mathbf{x_c}|x_s)$ and are averaged, to produce the summarized effect at x_s:

$$\mu(x_s) = \mathbb{E}_{\mathbf{X}_c|x_s}\left[\frac{\partial f}{\partial x_s}(x_s, \mathbf{X}_c)\right] \tag{4}$$

The FE at x_s is the accumulation of the averaged local effects from $x_{s,min}$ until x_s, i.e. $f^{\texttt{ALE}}(x_s) = \int_{x_{s,min}}^{x_s} \mu(z)\partial z$. As described at the Introduction, limiting the explanation to the exepected value level does not shed light to possible heterogeneous effects behind the averaged explanation. Therefore, we model the uncertainty of the local effects at $\mathcal{H}(x_s)$ as the variance of the local explanations:

$$\mathcal{H}(x_s) := \sigma^2(x_s) = \mathrm{Var}_{\mathbf{X}_c|x_s}\left[\frac{\partial f}{\partial x_s}(x_s, \mathbf{X}_c)\right] \tag{5}$$

The uncertainty of the explanation emerges from the natural characteristics of the experiment, i.e., the data generating distribution and the properties of the black-box function. In Section 3.4, we propose appropriate visualizations for easier interpretation of Eq. (5). In ALE, the FE at x_s is the accumulation of the averaged local effects from x_{min} until x_s, as shown in Eq. (1). Equivalently, we define the accumulated uncertainty (variance) until x_s, as the integral of the variances of the local effects:

$$f^{\texttt{ALE}}_{\sigma^2}(x_s) = \int_{z_{s,min}}^{x_s} \sigma^2(z)\partial z \tag{6}$$

The accumulated uncertainty is not a directly interpretable quantity. It only helps us define a sensible objective for the interval spliting step, as we discuss in Section 3.3.

3.2 Interval-Based Estimation

In real scenarios, we have ignorance about the data-generating distribution $p(x_s, \mathbf{x}_c)$, so, the estimations are based on the limited instances of the training set. Estimating Eqs. (4), (5) at the granularity of a point is impossible, because the probability of observing a sample inside the interval $[x_s - h, x_s + h]$ tends to zero, when $h \to 0$. We are, therefore, obliged to split the axis of x_s into a sequence of non-overlaping intervals (bins) and estimate the mean and the variance from the samples that lie inside each bin. The mean effect at an interval $[z_1, z_2)$ is defined as the mean of the expected effects:

$$\mu(z_1, z_2) = \frac{1}{z_2 - z_1}\int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\frac{\partial f}{\partial x_s}\right]\partial z \tag{7}$$

Similarly, the accumulated variance at an interval $[z_1, z_2)$ is defined as:

$$\sigma^2(z_1, z_2) = \int_{z_1}^{z_2} \mathbb{E}_{\mathbf{x_c}|x_s=z}\left[\left(\frac{\partial f}{\partial x_s} - \mu(z_1, z_2)\right)^2\right]\partial z \tag{8}$$

Theorem 1. If we define the residual $\rho(z)$ as the difference between the expected effect at x_s and the mean expected effect at the interval, i.e $\rho(z) = \mu(z) - \mu(z_1, z_2)$, then, the accumulated variance at an interval $[z_1, z_2)$ is the accumulation of the all variances plus the accumulation of squared residuals inside the interval:

$$\sigma^2(z_1, z_2) = \int_{z_1}^{z_2} \sigma^2(z) + \rho^2(z)\partial z \tag{9}$$

The proof is in the Appendix. Theorem 1 decouples the accumulated variance at an interval into two terms. The first term $\int_{z_1}^{z_2} \sigma^2(z)\partial z$, quantifies the aggregated uncertainty due to the natural characteristics of the experiment $\mathcal{H}$ and the second term adds extra nuisance uncertainty due to the limited resolution $\mathcal{H}_n$. In other words, enforcing the computation of a single effect for all points in $[z_1, z_2)]$, burdens the estimation inside the interval with a nuisance uncertainty of:

$$\mathcal{H}_{bin}(z_1, z_2) = \int_{z_1}^{z_2} \mathcal{H}(z) + \mathcal{H}_n(z)\partial z \tag{10}$$

Eqs. (7), (8) can be directly estimated from the set $\mathcal{S}$ of the dataset instances with the s-th feature lying inside the interval, i.e., $\mathcal{S} = \{\mathbf{x}^i : z_1 \leq x_s^i < z_2\}$. The mean effect at the interval, Eq. (7) is approximated by:

$$\hat{\mu}(z_1, z_2) = \frac{1}{|\mathcal{S}|} \sum_{i:\mathbf{x}^i \in \mathcal{S}} \left[\frac{\partial f}{\partial x_s}(\mathbf{x}^i) \right] \tag{11}$$

and the accumulated variance, Eq. (8) can be approximated by

$$\hat{\sigma}^2(z_1, z_2) = \frac{z_2 - z_1}{|\mathcal{S}|} \sum_{i:\mathbf{x}^i \in \mathcal{S}} \left(\frac{\partial f}{\partial x_s}(\mathbf{x}^i) - \hat{\mu}(z_1, z_2) \right)^2 \tag{12}$$

The approximation is unbiased only if the points are uniformly distributed in $[z_1, z_2]$. Elaborate.

3.3 Interval Spliting As A Clustering Problem

We formulate the axis-splitting as an unsupervised clustering problem. We search for the optimal bin splitting, i.e., the number and size of consecutive non-overlapping intervals that minimizes the accumulated variance. The optimization problem is defined as follows:

$$\begin{aligned} \min_{\{z_0,\ldots,z_K\}} \quad & \mathcal{L} = \sum_{k=1}^{K} \hat{\sigma}^2(z_{k-1}, z_k) \\ \text{s.t.} \quad & |\mathcal{S}_k| \geq N \end{aligned} \tag{13}$$

The objective of the optimization problem is the sum of the accumulated variances in each bin, estimated by the instances of the training set. As we prove in Theorem 1, $\mathcal{L} = \sum_{k=1}^{K} \mathcal{H}_{bin}(z_{k-1}, z_k)$ is the sum of the uncertainty of the local explanations $\mathcal{H}$ due to heterogeneous effects and the added nuisance uncertainty due to bin-splitting $\mathcal{H}_n$. Given that $\mathcal{H}$ is not affected by interval splitting, optimizing $\mathcal{L}$ equals to finding the sequence of non-overlaping bins that add the minimum nuisance uncertainty. The restriction of (13) secures that each bin is populated with enough samples for a robust estimation, which is set by the parameter N. In other words, the user makes the following proposal; *find the bin sequence that adds the minimum nuisance uncertainty, given that I want at least N points per bin*.

3.3.1 Algorithm For Solving The Clustering Problem

Solving the problem of finding (a) the optimal number of bins K and (b) the optimal bin limits for each bin $[z_{k-1}, z_k] \forall k$ to minimize:

$$\mathcal{L} = \sum_{k=0}^{K} \hat{\sigma}_k(z_{k-1}, z_k) \tag{14}$$

The constraints are that all bins must include more than τ points, i.e., $|\mathcal{S}_k| \geq \tau$.

- Computational complexity

3.4 Visualization of NAME Method and Discussion

- Discuss about the meaning of ALE, to find intervals with some effect

4 SYNTHETIC EXAMPLES

5 REAL-WORLD EXAMPLES

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