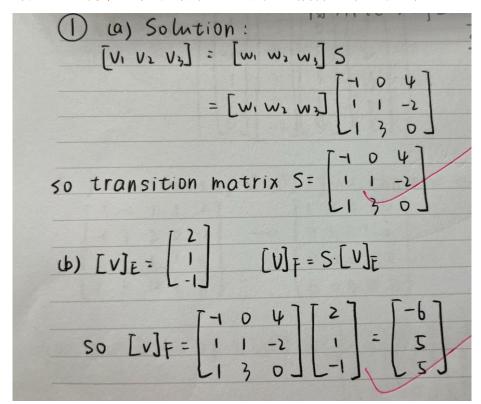
# HW3 参考答案及常见问题

#### 第一题:

这题错的人非常多,首先注意基底 v1,v2,v3,w1,w2,w3 都是列向量;其次要注意到底哪个基底是 new 哪个基底是 old,题目要求是 old 到 new,还是 new 到 old,要注意审题;还有相当大一部分人矩阵从上面搬到下面就长得不一样了,计算坐标时简单四则运算错的乱七八糟,还请各位同学注意细节。



### 第二题:

2. Solution: (0) By doing elementary row transformation, we can get the reduced row echelon form of A:

$$A = \begin{bmatrix} 1 & -2 & 1 & 1 & 3 \\ -1 & 3 & 2 & 0 & -2 \\ 0 & 1 & 3 & 1 & 4 \\ 1 & 2 & 13 & 5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 7 & 3 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So rank A = 3 = dim ((A), Let A=[a c c; a c; then c; 7c+3c+0c; c4=3c+c2+0cs

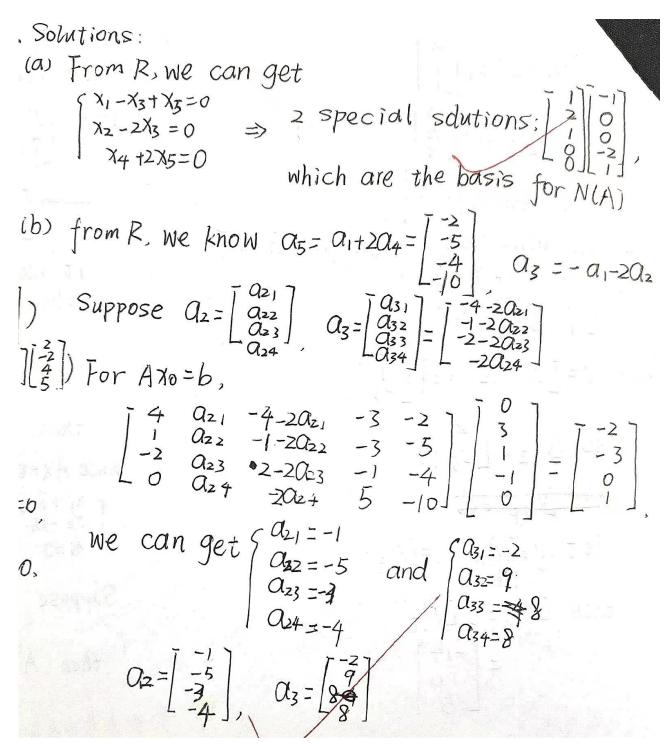
So 
$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 3 & -2 \\ 0 & 1 & 4 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 7 & 3 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
A basis of  $C(A)$  is  $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 4 \\ 5 \end{bmatrix} \right\}$ , a basis of  $C(A^T)$  is  $\left\{ \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ 

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Let Ax=0, then \begin{bmatrix} 10730\\ 01310\\ 00001 \end{bmatrix} x=0.
     So we have \begin{cases} x_1 + 7 x_3 + 3 x_4 = 0 \\ x_2 + 3 x_3 + 4 x_4 = 0 \end{cases}
           Do elementary row transformation on AT:
Then ATX=0 equals to
    So a basis of N(A^T) is \begin{bmatrix} 17\\ 13\\ -4 \end{bmatrix}
  (b) Port 1: For an man matrix A whose rank is r,
                                                                 dim C(A) = dim C(AT) = r,
                                               dim NCA) = n-r, dim N(AT) = m-r.
       Verification: from (a) we know that m=4, h=5, r=rank A=3
                                  dim C(A) = dim C(AT) = 3 = r,
                                din N(A) = 2 = n-r, din N(A) = 1 = M-r.
     Part 2: N(A) = C(A) 1, N(AT) = C(A)1
        Verification: for each basis for N(A) and C(AT).
                    \begin{bmatrix} 7 \\ 3 \\ -0 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 \\ 9 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 \\ 0 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -1 \\ 0 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 \\ 0 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -1 \\ 0 \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ -1 \\ 0 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\end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = 
             So it can be inferred that N(A) and C(AT) are orthogonal.
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Similarly, for the bases for  $N(A^{T})$  and C(A):  $\begin{bmatrix} 17\\13\\3\\-4 \end{bmatrix} \begin{bmatrix} 1\\1\\3\\-4 \end{bmatrix} \begin{bmatrix} 1\\1\\3\\-4 \end{bmatrix} \begin{bmatrix} 1\\3\\2\\-4 \end{bmatrix} \begin{bmatrix} 1\\3\\3\\-4 \end{bmatrix} \begin{bmatrix} 2\\4\\3 \end{bmatrix} = 0.$ 

So N(AT) and C(A) are orthogonal, too.

### 第三题:



## 第四题:

该题第一小问要从两个方向加以证明

```
(a) to suppose mES, then m=a, x+azy
  suppose M E N(A) then AM=0
: ( * * xy<sup>T</sup> + y x<sup>T</sup>) M=0
0 = T(Txy+Tyx) TM:
 :.MT(yx+xyT)=0
0 = x( Tyx+ Txy) TM :.
\therefore M^{\mathsf{T}}(y \times^{\mathsf{T}} x + x y^{\mathsf{T}} x) = M^{\mathsf{T}}(y \times 1 + 0) = 0
 0= p TM ..
: MT (yxT+xyT)y=MT(yxTy+xyTy)=MT(0+x141)=0
: MT x = 0
: MT · m = a, MT x + a2MTy = 0
: N(A) ES1
@ suppose NES1, then NT. m=0
\therefore \alpha_1 N^T X + \alpha_2 N^T Y = 0
"NTX=0, NTy=0
\frac{1}{N^{T}(yx^{T}+xy^{T})}=N^{T}yx^{T}+N^{T}xy
==N(Txy+Tyx):-
: SIENA)
., N(A) = \s^1
B: dim S= 2. SER
.. dim 5 = n-2
: N(A)= S1
.'. dim N(A)= n-z= n-R(A)
∴R(A)=2
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