Final Exam – June 17, 2020

Problem 1. (25 points) Consider the formula:

$$\neg((q \land p) \land q) \rightarrow (\neg(r \leftrightarrow p) \land r)$$

(a) Which atoms are pure in the above formula?

(b) Compute a clausal normal form C of the above formula by applying the CNF transformation algorithm with naming and optimization based on polarities of subformulas;

(c) Decide the satisfiability of the computed CNF formula C by applying the DPLL method to C. If C is satisfiable, give an interpretation which satisfies it.

Problem 2. (25 points) Consider the formula:

$$\begin{aligned} &b = c - 2 \land f(b+1) \neq b+2 \land read(A, f(c-1)) = c-2 \\ &\land (read(A, f(b+1)) = b+3 \lor read(write(A, b+2, f(c-2)), f(c-1)) = c) \end{aligned}$$

where b, c are constants, f is a unary function symbol, A is an array constant, $read$, $write$ are interpreted in the array theory, and $+, -, 1, 2, 3, \ldots$ are interpreted in the standard way over the integers.

Use the Nelson-Oppen decision procedure in conjunction with DPLL-based reasoning in the combination of the theories of arrays, uninterpreted functions, and linear integer arithmetic. Use the decision procedures for the theory of arrays and the theory of uninterpreted functions and use simple mathematical reasoning for deriving new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.

Problem 3. (25 points) Consider the KBO ordering $\succ$ generated by the precedence $g \gg a \gg f \gg b$ and the weight function w with $w(g) = 0, w(b) = 1, w(a) = 2, w(f) = 3$. Let σ be a well-behaved selection function wrt $\succ$. Consider the set S of ground formulas:

$$\begin{aligned} &f(g(a)) = a \lor f(g(b)) = a \\ &g(b) \neq g(b) \lor g(a) = a \\ &g(b) = a \\ &f(a) \neq a \end{aligned}$$

Show that S is unsatisfiable by applying saturation on S using an inference process based on the ground superposition calculus $\text{Sup}_{\succ,\sigma}$ (with the inference rules of binary resolution BR_σ included). Give details on what literals are selected and which terms are maximal.

Problem 4. (25 points) Consider the following inference:

$$\frac{P(g(a, f(g(b,d)))) \lor \neg P(h(h(a))) \lor h(h(a)) \neq g(a,a) \quad \neg P(g(a, f(y))) \lor h(h(x)) \neq g(a,x)}{\neg P(h(h(a))) \lor h(h(a)) \neq g(a,a)}$$

in the non-ground superposition inference system Sup (including the rules of the non-ground binary resolution inference system BR), where P is a predicate symbol, f, g, h are function symbols, a, b, d are constants, and x, y are variables.

(a) Prove that the above inference is a sound inference of Sup.

(b) Is the above inference a simplifying inference of Sup? Justify your answer based on conditions of clauses being redundant.