Diagrams and algebraic expressions at order 3 in BMBPT

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Valid diagrams: 23
2N valid diagrams: 8
2N canonical diagrams for the energy: 1
2N canonical diagrams for a generic operator only: 1
2N non-canonical diagrams: 6
3N valid diagrams: 7
3N canonical diagrams for the energy: 7
3N canonical diagrams for a generic operator only: 3
3N non-canonical diagrams: 5

Contents

1 Time-structure diagrams

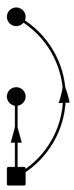
1.1 Tree diagrams

Time-structure diagram T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{1}$$

Related Feynman diagrams: 23, 22, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 6, 5, 4, 3, 2, 1.

Time-structure diagram T2:



$$\mathrm{T2} = \frac{1}{a_1 a_2} \tag{2}$$

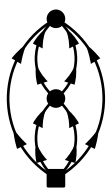
Related Feynman diagrams: 21, 7.

2 Two-body diagrams

2.1 Two-body energy canonical diagrams

Diagram 1:

$$\begin{aligned}
\mathrm{PO3.1} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^3} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{22}_{k_5k_6k_1k_2} \Omega^{04}_{k_5k_6k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_5k_6}_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}} \\
&= \frac{(-1)^2}{(2!)^3} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{22}_{k_5k_6k_1k_2} \Omega^{04}_{k_5k_6k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_3k_4k_5k_6}}
\end{aligned} \tag{3}$$

 → T1: 

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{4}$$

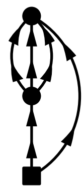
$$a_1 = \epsilon^{k_5k_6}_{k_1k_2}$$

$$a_2 = \epsilon_{k_3k_4k_5k_6}$$

2.2 Two-body canonical diagrams for a generic operator only

Diagram 2:

$$\begin{aligned}
\mathrm{PO3.2} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{31}_{k_3k_4k_5k_1} \Omega^{04}_{k_3k_4k_5k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3k_4k_5}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5}} \\
&= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{31}_{k_3k_4k_5k_1} \Omega^{04}_{k_3k_4k_5k_2}}{\epsilon_{k_1k_2}\ \epsilon_{k_2k_3k_4k_5}}
\end{aligned} \tag{5}$$



$\rightarrow$ T1:

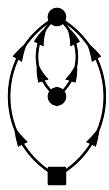
$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{6}$$

$$a_1 = \epsilon_{k_1}^{k_3 k_4 k_5}$$
$$a_2 = \epsilon_{k_2 k_3 k_4 k_5}$$

2.3 Two-body non-canonical diagrams

Diagram 3:

$$\begin{aligned} \mathrm{PO3.3} &= \lim_{\tau \to \infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} O^{20}_{k_1 k_2} \Omega^{20}_{k_3 k_4} \Omega^{04}_{k_3 k_4 k_1 k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3 k_4}} e^{-\tau_2 \epsilon_{k_1 k_2 k_3 k_4}} \\ &= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{O^{20}_{k_1 k_2} \Omega^{20}_{k_3 k_4} \Omega^{04}_{k_3 k_4 k_1 k_2}}{\epsilon_{k_1 k_2} \ \epsilon_{k_1 k_2 k_3 k_4}} \end{aligned} \tag{7}$$



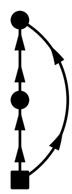
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{8}$$

$$a_1 = \epsilon^{k_3 k_4}$$
$$a_2 = \epsilon_{k_1 k_2 k_3 k_4}$$

Diagram 4:

$$\begin{aligned} \mathrm{PO3.4} &= \lim_{\tau \to \infty} (-1)^2 \sum_{k_i} O^{20}_{k_1 k_2} \Omega^{11}_{k_3 k_1} \Omega^{02}_{k_3 k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_3}} e^{-\tau_2 \epsilon_{k_2 k_3}} \\ &= (-1)^2 \sum_{k_i} \frac{O^{20}_{k_1 k_2} \Omega^{11}_{k_3 k_1} \Omega^{02}_{k_3 k_2}}{\epsilon_{k_1 k_2} \ \epsilon_{k_2 k_3}} \end{aligned} \tag{9}$$



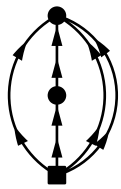
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{10}$$

$$a_1 = \epsilon_{k_1}^{k_3}$$
$$a_2 = \epsilon_{k_2 k_3}$$

Diagram 5:

$$
\begin{aligned}
\mathrm{PO3.5} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{11}_{k_5k_1} \Omega^{04}_{k_5k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1\epsilon^{k_5}_{k_1}} e^{-\tau_2\epsilon_{k_2k_3k_4k_5}} \\
&= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{11}_{k_5k_1} \Omega^{04}_{k_5k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_2k_3k_4k_5}} \qquad (11)
\end{aligned}
$$



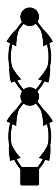
$\rightarrow$ T1:

$$
\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \qquad (12)
$$

$$
\begin{aligned}
a_1 &= \epsilon^{k_5}_{k_1} \\
a_2 &= \epsilon_{k_2k_3k_4k_5}
\end{aligned}
$$

Diagram 6:

$$
\begin{aligned}
\mathrm{PO3.6} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{22}_{k_3k_4k_1k_2} \Omega^{02}_{k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1\epsilon^{k_3k_4}_{k_1k_2}} e^{-\tau_2\epsilon_{k_3k_4}} \\
&= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{22}_{k_3k_4k_1k_2} \Omega^{02}_{k_3k_4}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4}} \qquad (13)
\end{aligned}
$$



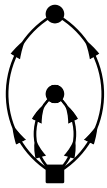
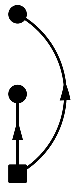
$\rightarrow$ T1:

$$
\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \qquad (14)
$$

$$
\begin{aligned}
a_1 &= \epsilon^{k_3k_4}_{k_1k_2} \\
a_2 &= \epsilon_{k_3k_4}
\end{aligned}
$$

Diagram 7:

$$
\begin{aligned}
\mathrm{PO3.7} &= \lim_{\tau\to\infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{02}_{k_1k_2} \Omega^{02}_{k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 e^{-\tau_1\epsilon_{k_1k_2}} e^{-\tau_2\epsilon_{k_3k_4}} \\
&= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{02}_{k_1k_2} \Omega^{02}_{k_3k_4}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4}} \qquad (15)
\end{aligned}
$$

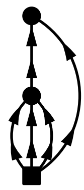
 $\rightarrow$ T2: 

$$\text{T2} = \frac{1}{a_1 a_2} \tag{16}$$

$$a_1 = \epsilon_{k_1 k_2}$$
$$a_2 = \epsilon_{k_3 k_4}$$

Diagram 8:

$$\begin{aligned} \text{PO3.8} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{13}_{k_5k_1k_2k_3} \Omega^{02}_{k_5k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5}} \\ &= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{13}_{k_5k_1k_2k_3} \Omega^{02}_{k_5k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_4k_5}} \end{aligned} \tag{17}$$

 $\rightarrow$ T1: 

$$\text{T1} = \frac{1}{(a_1 + a_2) a_2} \tag{18}$$

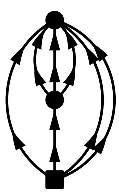
$$a_1 = \epsilon^{k_5}_{k_1 k_2 k_3}$$
$$a_2 = \epsilon_{k_4 k_5}$$

3 Three-body diagrams

3.1 Three-body energy canonical diagrams

Diagram 9:

$$\begin{aligned} \text{PO3.9} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{31}_{k_5k_6k_7k_1} \Omega^{06}_{k_5k_6k_7k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5k_6k_7}} \\ &= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{31}_{k_5k_6k_7k_1} \Omega^{06}_{k_5k_6k_7k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_2k_3k_4k_5k_6k_7}} \end{aligned} \tag{19}$$

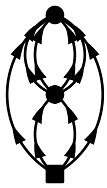
 $\rightarrow$ T1: 

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{20}$$

$$a_1 = \epsilon_{k_1}^{k_5k_6k_7}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5k_6k_7}$$

Diagram 10:

$$\begin{aligned}\mathrm{PO3.10} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{42}_{k_5k_6k_7k_8k_1k_2} \Omega^{06}_{k_5k_6k_7k_8k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon_{k_1k_2}^{k_5k_6k_7k_8}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}} \\ &= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{42}_{k_5k_6k_7k_8k_1k_2} \Omega^{06}_{k_5k_6k_7k_8k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_3k_4k_5k_6k_7k_8}}\end{aligned} \tag{21}$$



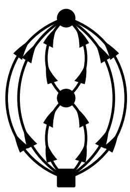
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{22}$$

$$a_1 = \epsilon_{k_1k_2}^{k_5k_6k_7k_8}$$
$$a_2 = \epsilon_{k_3k_4k_5k_6k_7k_8}$$

Diagram 11:

$$\begin{aligned}\mathrm{PO3.11} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{22}_{k_7k_8k_1k_2} \Omega^{06}_{k_7k_8k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon_{k_1k_2}^{k_7k_8}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6k_7k_8}} \\ &= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{22}_{k_7k_8k_1k_2} \Omega^{06}_{k_7k_8k_3k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_3k_4k_5k_6k_7k_8}}\end{aligned} \tag{23}$$



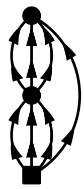
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{24}$$

$$a_1 = \epsilon_{k_1k_2}^{k_7k_8}$$
$$a_2 = \epsilon_{k_3k_4k_5k_6k_7k_8}$$

Diagram 12:

$$
\begin{aligned}
\mathrm{PO3.12} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{33}_{k_5k_6k_7k_1k_2k_3} \Omega^{04}_{k_5k_6k_7k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6k_7}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}} \\
&= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{33}_{k_5k_6k_7k_1k_2k_3} \Omega^{04}_{k_5k_6k_7k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_4k_5k_6k_7}} \qquad (25)
\end{aligned}
$$



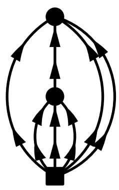
→ T1:

$$
\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \qquad (26)
$$

$$
\begin{aligned}
a_1 &= \epsilon^{k_5k_6k_7}_{k_1k_2k_3} \\
a_2 &= \epsilon_{k_4k_5k_6k_7}
\end{aligned}
$$

Diagram 13:

$$
\begin{aligned}
\mathrm{PO3.13} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{13}_{k_7k_1k_2k_3} \Omega^{04}_{k_7k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}} \\
&= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{13}_{k_7k_1k_2k_3} \Omega^{04}_{k_7k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_4k_5k_6k_7}} \qquad (27)
\end{aligned}
$$



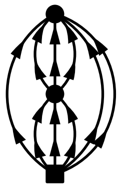
→ T1:

$$
\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \qquad (28)
$$

$$
\begin{aligned}
a_1 &= \epsilon^{k_7}_{k_1k_2k_3} \\
a_2 &= \epsilon_{k_4k_5k_6k_7}
\end{aligned}
$$

Diagram 14:

$$
\begin{aligned}
\mathrm{PO3.14} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^3} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{33}_{k_7k_8k_9k_1k_2k_3} \Omega^{06}_{k_7k_8k_9k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7k_8k_9}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}} \\
&= \frac{(-1)^2}{(3!)^3} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{33}_{k_7k_8k_9k_1k_2k_3} \Omega^{06}_{k_7k_8k_9k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_4k_5k_6k_7k_8k_9}} \qquad (29)
\end{aligned}
$$

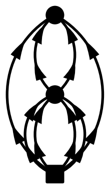
 $\rightarrow$ T1: 

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{30}$$

$$a_1 = \epsilon^{k_7k_8k_9}_{k_1k_2k_3}$$
$$a_2 = \epsilon_{k_4k_5k_6k_7k_8k_9}$$

Diagram 15:

$$\begin{aligned}
\mathrm{PO3.15} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{24}_{k_7k_8k_1k_2k_3k_4} \Omega^{04}_{k_7k_8k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_7k_8}_{k_1k_2k_3k_4}} e^{-\tau_2 \epsilon_{k_5k_6k_7k_8}} \\
&= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{24}_{k_7k_8k_1k_2k_3k_4} \Omega^{04}_{k_7k_8k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6} \ \epsilon_{k_5k_6k_7k_8}}
\end{aligned} \tag{31}$$

 $\rightarrow$ T1: 

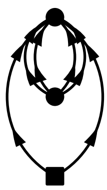
$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{32}$$

$$a_1 = \epsilon^{k_7k_8}_{k_1k_2k_3k_4}$$
$$a_2 = \epsilon_{k_5k_6k_7k_8}$$

3.2 Three-body canonical diagrams for a generic operator only

Diagram 16:

$$\begin{aligned}
\mathrm{PO3.16} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{40}_{k_3k_4k_5k_6} \Omega^{06}_{k_3k_4k_5k_6k_1k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3k_4k_5k_6}} e^{-\tau_2 \epsilon_{k_1k_2k_3k_4k_5k_6}} \\
&= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{40}_{k_3k_4k_5k_6} \Omega^{06}_{k_3k_4k_5k_6k_1k_2}}{\epsilon_{k_1k_2} \ \epsilon_{k_1k_2k_3k_4k_5k_6}}
\end{aligned} \tag{33}$$

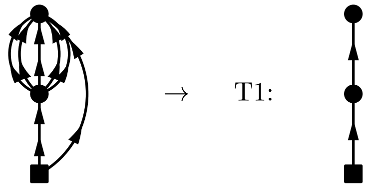
 $\rightarrow$ T1: 

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{34}$$

$$a_1 = \epsilon^{k_3k_4k_5k_6}$$
$$a_2 = \epsilon_{k_1k_2k_3k_4k_5k_6}$$

Diagram 17:

$$\begin{aligned}
\mathrm{PO3.17} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{51}_{k_3k_4k_5k_6k_7k_1} \Omega^{06}_{k_3k_4k_5k_6k_7k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3k_4k_5k_6k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5k_6k_7}} \\
&= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{51}_{k_3k_4k_5k_6k_7k_1} \Omega^{06}_{k_3k_4k_5k_6k_7k_2}}{\epsilon_{k_1k_2} \ \epsilon_{k_2k_3k_4k_5k_6k_7}}
\end{aligned} \tag{35}$$

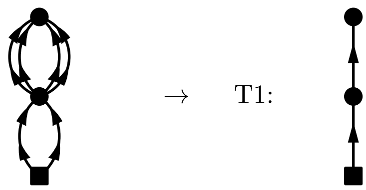


$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{36}$$

$$a_1 = \epsilon^{k_3k_4k_5k_6k_7}_{k_1}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5k_6k_7}$$

Diagram 18:

$$\begin{aligned}
\mathrm{PO3.18} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{42}_{k_3k_4k_5k_6k_1k_2} \Omega^{04}_{k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3k_4k_5k_6}_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}} \\
&= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{42}_{k_3k_4k_5k_6k_1k_2} \Omega^{04}_{k_3k_4k_5k_6}}{\epsilon_{k_1k_2} \ \epsilon_{k_3k_4k_5k_6}}
\end{aligned} \tag{37}$$



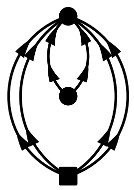
$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{38}$$

$$a_1 = \epsilon^{k_3k_4k_5k_6}_{k_1k_2}$$
$$a_2 = \epsilon_{k_3k_4k_5k_6}$$

3.3 Three-body non-canonical diagrams

Diagram 19:

$$\begin{aligned}\text{PO3.19} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{20}_{k_5k_6} \Omega^{06}_{k_5k_6k_1k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6}} e^{-\tau_2 \epsilon_{k_1k_2k_3k_4k_5k_6}} \\ &= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{20}_{k_5k_6} \Omega^{06}_{k_5k_6k_1k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_1k_2k_3k_4k_5k_6}} \end{aligned} \tag{39}$$



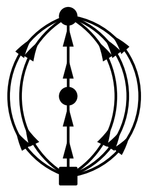
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{40}$$

$$a_1 = \epsilon^{k_5k_6}$$
$$a_2 = \epsilon_{k_1k_2k_3k_4k_5k_6}$$

Diagram 20:

$$\begin{aligned}\text{PO3.20} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{11}_{k_7k_1} \Omega^{06}_{k_7k_2k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5k_6k_7}} \\ &= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{11}_{k_7k_1} \Omega^{06}_{k_7k_2k_3k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_2k_3k_4k_5k_6k_7}} \end{aligned} \tag{41}$$



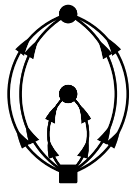
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{42}$$

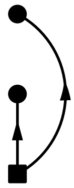
$$a_1 = \epsilon^{k_7}_{k_1}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5k_6k_7}$$

Diagram 21:

$$\begin{aligned}\text{PO3.21} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{02}_{k_1k_2} \Omega^{04}_{k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 e^{-\tau_1 \epsilon_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}} \\ &= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{02}_{k_1k_2} \Omega^{04}_{k_3k_4k_5k_6}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4k_5k_6}} \end{aligned} \tag{43}$$



$\rightarrow$ T2:

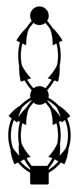


$$\mathrm{T2} = \frac{1}{a_1 a_2} \tag{44}$$

$$a_1 = \epsilon_{k_1k_2}$$
$$a_2 = \epsilon_{k_3k_4k_5k_6}$$

Diagram 22:

$$\mathrm{PO3.22} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{24}_{k_5k_6k_1k_2k_3k_4} \Omega^{02}_{k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6}_{k_1k_2k_3k_4}} e^{-\tau_2 \epsilon_{k_5k_6}}$$
$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{24}_{k_5k_6k_1k_2k_3k_4} \Omega^{02}_{k_5k_6}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_5k_6}} \tag{45}$$



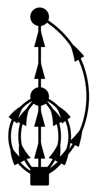
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{46}$$

$$a_1 = \epsilon^{k_5k_6}_{k_1k_2k_3k_4}$$
$$a_2 = \epsilon_{k_5k_6}$$

Diagram 23:

$$\mathrm{PO3.23} = \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{15}_{k_7k_1k_2k_3k_4k_5} \Omega^{02}_{k_7k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1k_2k_3k_4k_5}} e^{-\tau_2 \epsilon_{k_6k_7}}$$
$$= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{15}_{k_7k_1k_2k_3k_4k_5} \Omega^{02}_{k_7k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_6k_7}} \tag{47}$$



$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{48}$$

$$a_1 = \epsilon^{k_7}_{k_1k_2k_3k_4k_5}$$
$$a_2 = \epsilon_{k_6k_7}$$