# Diagrams and algebraic expressions at order (2,2;2) in BIMSRG

The ADG Dev Team

February 5, 2021

$C = [A, B]$ with $N_A = 2$, $N_B = 2$ and $N_C = 2$
$d_{\max} \equiv \max(d_A, d_B, d_C)$

Valid diagrams: 82
$d_{\max} = 1$ diagrams: 10
$d_{\max} = 2$ diagrams: 72

## Contents



# 1 Permutators definitions

$$
\begin{aligned}
P(k_1/k_2) &= 1 - P_{k_1k_2} \\
P(k_1/k_2k_3) &= 1 - P_{k_1k_2} - P_{k_1k_3} \\
P(k_1/k_2k_3k_4) &= 1 - P_{k_1k_2} - P_{k_1k_3} - P_{k_1k_4} \\
P(k_1k_2/k_3k_4) &= 1 - P_{k_1k_3} - P_{k_1k_4} - P_{k_2k_3} - P_{k_2k_4} + P_{k_1k_3}P_{k_2k_4} + P_{k_2k_3}P_{k_1k_4}
\end{aligned}
$$

# 2 $d_{\max} = 1$

## 2.1 $C^{00}$

**Diagram 1 ($+AB$):**

$$C^{00}(02, 20) = \frac{1}{2} \sum_{p_1p_2} A^{02}_{p_1p_2} B^{20}_{p_1p_2} \tag{1}$$

**Diagram 2** ($-BA$)**:**

$$C^{00}(20,02) = -\frac{1}{2}\sum_{p_1p_2} B^{02}_{p_1p_2} A^{20}_{p_1p_2} \tag{2}$$

## 2.2 $C^{20}$

**Diagram 3** ($+AB$)**:**

$$C^{20}(11,20) = P(k_1/k_2)\sum_{p_1} A^{11}_{k_1p_1} B^{20}_{p_1k_2} \tag{3}$$

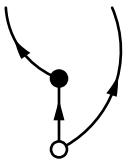
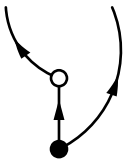


**Diagram 4** ($-BA$)**:**

$$C^{20}(20,11) = -P(k_1/k_2)\sum_{p_1} B^{11}_{k_1p_1} A^{20}_{p_1k_2} \tag{4}$$



## 2.3 $C^{11}$

**Diagram 5** ($+AB$)**:**

$$C^{11}(11,11) = \sum_{p_1} A^{11}_{k_1p_1} B^{11}_{p_1k_2} \tag{5}$$

**Diagram 6** ($+AB$)**:**

$$C^{11}(02,20) = \sum_{p_1} A^{02}_{k_2 p_1} B^{20}_{p_1 k_1} \tag{6}$$

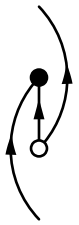
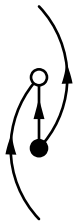


**Diagram 7** ($-BA$)**:**

$$C^{11}(11,11) = -\sum_{p_1} B^{11}_{k_1 p_1} A^{11}_{p_1 k_2} \tag{7}$$

**Diagram 8** ($-BA$)**:**

$$C^{11}(20,02) = -\sum_{p_1} B^{02}_{k_2 p_1} A^{20}_{p_1 k_1} \tag{8}$$



## 2.4 $C^{02}$

**Diagram 9** ($+AB$)**:**

$$C^{02}(02,11) = P(k_1/k_2) \sum_{p_1} A^{02}_{k_1 p_1} B^{11}_{p_1 k_2} \tag{9}$$

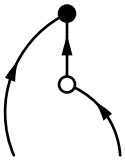
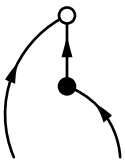


**Diagram 10** ($-BA$)**:**

$$C^{02}(11,02) = -P(k_1/k_2) \sum_{p_1} B^{02}_{k_1 p_1} A^{11}_{p_1 k_2} \tag{10}$$



# 3 $d_{\max} = 2$

## 3.1 $C^{00}$

**Diagram 11** ($+AB$)**:**

$$C^{00}(04,40) = \frac{1}{24} \sum_{p_1 p_2 p_3 p_4} A^{04}_{p_1 p_2 p_3 p_4} B^{40}_{p_1 p_2 p_3 p_4} \tag{11}$$

**Diagram 12** ($-BA$)**:**

$$C^{00}(40,04) = -\frac{1}{24} \sum_{p_1 p_2 p_3 p_4} B^{04}_{p_1 p_2 p_3 p_4} A^{40}_{p_1 p_2 p_3 p_4} \tag{12}$$

## 3.2 $C^{20}$

**Diagram 13** ($+AB$)**:**

$$C^{20}(02,40) = \frac{1}{2} \sum_{p_1 p_2} A^{02}_{p_1 p_2} B^{40}_{p_1 p_2 k_1 k_2} \tag{13}$$

**Diagram 14** ($+AB$)**:**

$$C^{20}(13,40) = P(k_1/k_2) \frac{1}{6} \sum_{p_1 p_2 p_3} A^{13}_{k_1 p_1 p_2 p_3} B^{40}_{p_1 p_2 p_3 k_2} \tag{14}$$

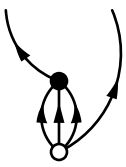


**Diagram 15** ($+AB$)**:**

$$C^{20}(22,20) = \frac{1}{2} \sum_{p_1 p_2} A^{22}_{k_1 k_2 p_1 p_2} B^{20}_{p_1 p_2} \tag{15}$$

**Diagram 16** ($-BA$)**:**

$$C^{20}(40,02) = -\frac{1}{2}\sum_{p_1p_2} B^{02}_{p_1p_2} A^{40}_{p_1p_2k_1k_2} \tag{16}$$

**Diagram 17** ($-BA$)**:**

$$C^{20}(40,13) = -P(k_1/k_2)\frac{1}{6}\sum_{p_1p_2p_3} B^{13}_{k_1p_1p_2p_3} A^{40}_{p_1p_2p_3k_2} \tag{17}$$

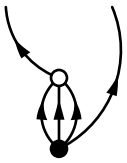


**Diagram 18** ($-BA$)**:**

$$C^{20}(20,22) = -\frac{1}{2}\sum_{p_1p_2} B^{22}_{k_1k_2p_1p_2} A^{20}_{p_1p_2} \tag{18}$$

### 3.3 $C^{40}$

**Diagram 19** ($+AB$)**:**

$$C^{40}(11,40) = P(k_1/k_2k_3k_4)\sum_{p_1} A^{11}_{k_1p_1} B^{40}_{p_1k_2k_3k_4} \tag{19}$$

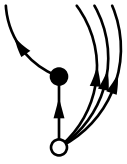


**Diagram 20** $(+AB)$**:**

$$C^{40}(22,40) = P(k_1k_2/k_3k_4)\frac{1}{2}\sum_{p_1p_2} A^{22}_{k_1k_2p_1p_2} B^{40}_{p_1p_2k_3k_4} \tag{20}$$

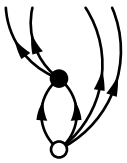


**Diagram 21** $(+AB)$**:**

$$C^{40}(31,20) = P(k_1k_2k_3/k_4)\sum_{p_1} A^{31}_{k_1k_2k_3p_1} B^{20}_{p_1k_4} \tag{21}$$

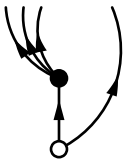


**Diagram 22** $(-BA)$**:**

$$C^{40}(40,11) = -P(k_1/k_2k_3k_4)\sum_{p_1} B^{11}_{k_1p_1} A^{40}_{p_1k_2k_3k_4} \tag{22}$$

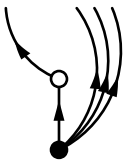


**Diagram 23** $(-BA)$**:**

$$C^{40}(40,22) = -P(k_1k_2/k_3k_4)\frac{1}{2}\sum_{p_1p_2} B^{22}_{k_1k_2p_1p_2} A^{40}_{p_1p_2k_3k_4} \tag{23}$$

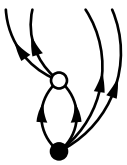
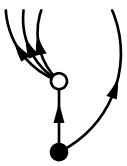


**Diagram 24** $(-BA)$**:**

$$C^{40}(20,31) = -P(k_1k_2k_3/k_4)\sum_{p_1} B^{31}_{k_1k_2k_3p_1} A^{20}_{p_1k_4} \tag{24}$$



### 3.4 $C^{11}$

**Diagram 25** $(+AB)$**:**

$$C^{11}(02,31) = \frac{1}{2}\sum_{p_1p_2} A^{02}_{p_1p_2} B^{31}_{p_1p_2k_1k_2} \tag{25}$$

**Diagram 26** $(+AB)$**:**

$$C^{11}(13,31) = \frac{1}{6}\sum_{p_1p_2p_3} A^{13}_{k_1p_1p_2p_3} B^{31}_{p_1p_2p_3k_2} \tag{26}$$

**Diagram 27** $(+AB)$**:**

$$C^{11}(04,40) = \frac{1}{6}\sum_{p_1p_2p_3} A^{04}_{k_2p_1p_2p_3} B^{40}_{p_1p_2p_3k_1} \tag{27}$$

**Diagram 28** $(+AB)$**:**

$$C^{11}(13,20) = \frac{1}{2}\sum_{p_1p_2} A^{13}_{k_1k_2p_1p_2} B^{20}_{p_1p_2} \tag{28}$$

**Diagram 29** $(-BA)$**:**

$$C^{11}(31,02) = -\frac{1}{2}\sum_{p_1p_2} B^{02}_{p_1p_2} A^{31}_{p_1p_2k_1k_2} \tag{29}$$

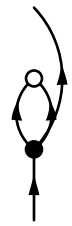


**Diagram 30** ($-BA$)**:**

$$C^{11}(31,13) = -\frac{1}{6}\sum_{p_1p_2p_3} B^{13}_{k_1p_1p_2p_3}A^{31}_{p_1p_2p_3k_2} \tag{30}$$

**Diagram 31** ($-BA$)**:**

$$C^{11}(40,04) = -\frac{1}{6}\sum_{p_1p_2p_3} B^{04}_{k_2p_1p_2p_3}A^{40}_{p_1p_2p_3k_1} \tag{31}$$

**Diagram 32** ($-BA$)**:**

$$C^{11}(20,13) = -\frac{1}{2}\sum_{p_1p_2} B^{13}_{k_1k_2p_1p_2}A^{20}_{p_1p_2} \tag{32}$$

## 3.5 $C^{31}$

**Diagram 33** ($+AB$)**:**

$$C^{31}(11,31) = P(k_1/k_2k_3)\sum_{p_1} A^{11}_{k_1p_1}B^{31}_{p_1k_2k_3k_4} \tag{33}$$

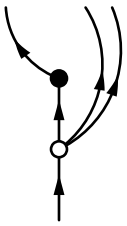


**Diagram 34** ($+AB$)**:**

$$C^{31}(22,31) = P(k_1k_2/k_3)\frac{1}{2}\sum_{p_1p_2} A^{22}_{k_1k_2p_1p_2}B^{31}_{p_1p_2k_3k_4} \tag{34}$$

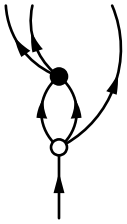


**Diagram 35** $(+AB)$**:**

$$C^{31}(31,11) = \sum_{p_1} A^{31}_{k_1 k_2 k_3 p_1} B^{11}_{p_1 k_4} \tag{35}$$

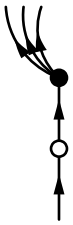


**Diagram 36** $(+AB)$**:**

$$C^{31}(02,40) = \sum_{p_1} A^{02}_{k_4 p_1} B^{40}_{p_1 k_1 k_2 k_3} \tag{36}$$

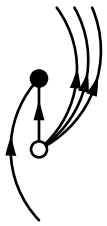


**Diagram 37** $(+AB)$**:**

$$C^{31}(13,40) = P(k_1/k_2 k_3)\frac{1}{2} \sum_{p_1 p_2} A^{13}_{k_1 k_4 p_1 p_2} B^{40}_{p_1 p_2 k_2 k_3} \tag{37}$$

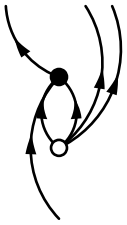


**Diagram 38** $(+AB)$**:**

$$C^{31}(22,20) = P(k_1 k_2/k_3) \sum_{p_1} A^{22}_{k_1 k_2 k_4 p_1} B^{20}_{p_1 k_3} \tag{38}$$

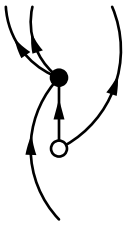


**Diagram 39** $(-BA)$**:**

$$C^{31}(31,11) = -P(k_1/k_2 k_3) \sum_{p_1} B^{11}_{k_1 p_1} A^{31}_{p_1 k_2 k_3 k_4} \tag{39}$$

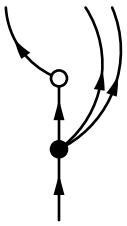


**Diagram 40** $(-BA)$**:**

$$C^{31}(31,22) = -P(k_1k_2/k_3)\frac{1}{2}\sum_{p_1p_2} B^{22}_{k_1k_2p_1p_2}A^{31}_{p_1p_2k_3k_4} \tag{40}$$

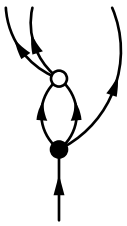


**Diagram 41** $(-BA)$**:**

$$C^{31}(11,31) = -\sum_{p_1} B^{31}_{k_1k_2k_3p_1}A^{11}_{p_1k_4} \tag{41}$$

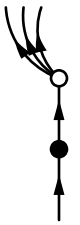


**Diagram 42** $(-BA)$**:**

$$C^{31}(40,02) = -\sum_{p_1} B^{02}_{k_4p_1}A^{40}_{p_1k_1k_2k_3} \tag{42}$$

**Diagram 43** $(-BA)$**:**

$$C^{31}(40,13) = -P(k_1/k_2k_3)\frac{1}{2}\sum_{p_1p_2} B^{13}_{k_1k_4p_1p_2}A^{40}_{p_1p_2k_2k_3} \tag{43}$$

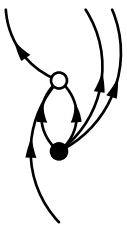
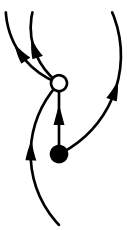


**Diagram 44** $(-BA)$**:**

$$C^{31}(20,22) = -P(k_1k_2/k_3)\sum_{p_1} B^{22}_{k_1k_2k_4p_1}A^{20}_{p_1k_3} \tag{44}$$



### 3.6 $C^{02}$

**Diagram 45** $(+AB)$**:**

$$C^{02}(02,22) = \frac{1}{2}\sum_{p_1p_2} A^{02}_{p_1p_2} B^{22}_{p_1p_2k_1k_2} \tag{45}$$

**Diagram 46** $(+AB)$**:**

$$C^{02}(04,31) = P(k_1/k_2)\frac{1}{6}\sum_{p_1p_2p_3} A^{04}_{k_1p_1p_2p_3} B^{31}_{p_1p_2p_3k_2} \tag{46}$$

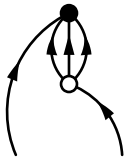


**Diagram 47** $(+AB)$**:**

$$C^{02}(04,20) = \frac{1}{2}\sum_{p_1p_2} A^{04}_{k_1k_2p_1p_2} B^{20}_{p_1p_2} \tag{47}$$

**Diagram 48** $(-BA)$**:**

$$C^{02}(22,02) = -\frac{1}{2}\sum_{p_1p_2} B^{02}_{p_1p_2} A^{22}_{p_1p_2k_1k_2} \tag{48}$$

**Diagram 49** $(-BA)$**:**

$$C^{02}(31,04) = -P(k_1/k_2)\frac{1}{6}\sum_{p_1p_2p_3} B^{04}_{k_1p_1p_2p_3} A^{31}_{p_1p_2p_3k_2} \tag{49}$$

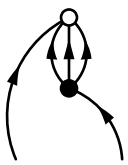


**Diagram 50** ($-BA$)**:**

$$C^{02}(20,04) = -\frac{1}{2}\sum_{p_1p_2} B^{04}_{k_1k_2p_1p_2}A^{20}_{p_1p_2} \tag{50}$$

## 3.7 $C^{22}$

**Diagram 51** ($+AB$)**:**

$$C^{22}(11,22) = P(k_1/k_2)\sum_{p_1} A^{11}_{k_1p_1}B^{22}_{p_1k_2k_3k_4} \tag{51}$$

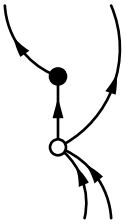


**Diagram 52** ($+AB$)**:**

$$C^{22}(22,22) = \frac{1}{2}\sum_{p_1p_2} A^{22}_{k_1k_2p_1p_2}B^{22}_{p_1p_2k_3k_4} \tag{52}$$

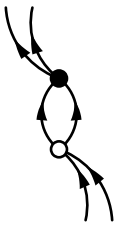


**Diagram 53** ($+AB$)**:**

$$C^{22}(02,31) = P(k_3/k_4)\sum_{p_1} A^{02}_{k_3p_1}B^{31}_{p_1k_1k_2k_4} \tag{53}$$

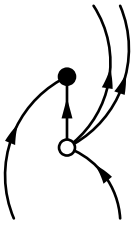


**Diagram 54** ($+AB$)**:**

$$C^{22}(13,31) = P(k_1/k_2)P(k_3/k_4)\frac{1}{2}\sum_{p_1p_2} A^{13}_{k_1k_3p_1p_2}B^{31}_{p_1p_2k_2k_4} \tag{54}$$

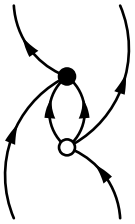


**Diagram 55** ($+AB$)**:**

$$C^{22}(22,11) = P(k_3/k_4)\sum_{p_1} A^{22}_{k_1k_2k_3p_1}B^{11}_{p_1k_4} \tag{55}$$

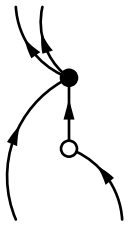


**Diagram 56** ($+AB$)**:**

$$C^{22}(04,40) = \frac{1}{2}\sum_{p_1p_2} A^{04}_{k_3k_4p_1p_2}B^{40}_{p_1p_2k_1k_2} \tag{56}$$

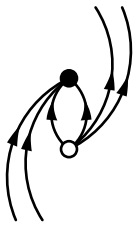


**Diagram 57** ($+AB$)**:**

$$C^{22}(13,20) = P(k_1/k_2)\sum_{p_1} A^{13}_{k_1k_3k_4p_1}B^{20}_{p_1k_2} \tag{57}$$

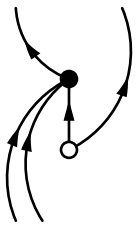


**Diagram 58** ($-BA$)**:**

$$C^{22}(22,11) = -P(k_1/k_2)\sum_{p_1} B^{11}_{k_1p_1}A^{22}_{p_1k_2k_3k_4} \tag{58}$$

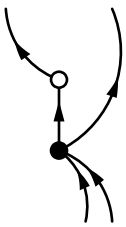


**Diagram 59** ($-BA$)**:**

$$C^{22}(22,22) = -\frac{1}{2}\sum_{p_1p_2} B^{22}_{k_1k_2p_1p_2}A^{22}_{p_1p_2k_3k_4} \tag{59}$$

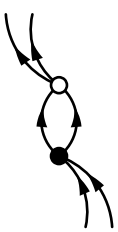


**Diagram 60** $(-BA)$**:**

$$C^{22}(31,02) = -P(k_3/k_4)\sum_{p_1} B^{02}_{k_3 p_1} A^{31}_{p_1 k_1 k_2 k_4} \tag{60}$$

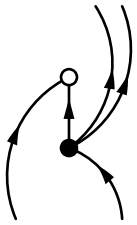


**Diagram 61** $(-BA)$**:**

$$C^{22}(31,13) = -P(k_1/k_2)P(k_3/k_4)\frac{1}{2}\sum_{p_1 p_2} B^{13}_{k_1 k_3 p_1 p_2} A^{31}_{p_1 p_2 k_2 k_4} \tag{61}$$

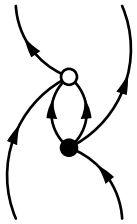


**Diagram 62** $(-BA)$**:**

$$C^{22}(11,22) = -P(k_3/k_4)\sum_{p_1} B^{22}_{k_1 k_2 k_3 p_1} A^{11}_{p_1 k_4} \tag{62}$$

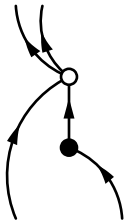


**Diagram 63** $(-BA)$**:**

$$C^{22}(40,04) = -\frac{1}{2}\sum_{p_1 p_2} B^{04}_{k_3 k_4 p_1 p_2} A^{40}_{p_1 p_2 k_1 k_2} \tag{63}$$

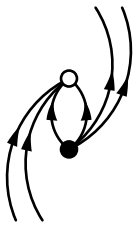
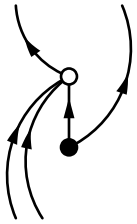


**Diagram 64** $(-BA)$**:**

$$C^{22}(20,13) = -P(k_1/k_2)\sum_{p_1} B^{13}_{k_1 k_3 k_4 p_1} A^{20}_{p_1 k_2} \tag{64}$$



### 3.8 $C^{13}$

**Diagram 65** $(+AB)$**:**

$$C^{13}(11,13) = \sum_{p_1} A^{11}_{k_1 p_1} B^{13}_{p_1 k_2 k_3 k_4} \tag{65}$$

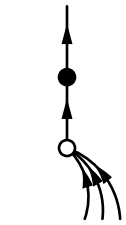


**Diagram 66** $(+AB)$**:**

$$C^{13}(02,22) = P(k_2/k_3 k_4) \sum_{p_1} A^{02}_{k_2 p_1} B^{22}_{p_1 k_1 k_3 k_4} \tag{66}$$

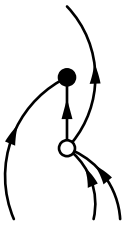


**Diagram 67** $(+AB)$**:**

$$C^{13}(13,22) = P(k_2/k_3 k_4) \frac{1}{2} \sum_{p_1 p_2} A^{13}_{k_1 k_2 p_1 p_2} B^{22}_{p_1 p_2 k_3 k_4} \tag{67}$$

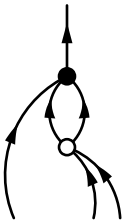


**Diagram 68** $(+AB)$**:**

$$C^{13}(04,31) = P(k_2 k_3/k_4) \frac{1}{2} \sum_{p_1 p_2} A^{04}_{k_2 k_3 p_1 p_2} B^{31}_{p_1 p_2 k_1 k_4} \tag{68}$$

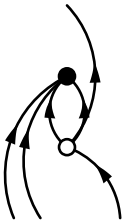


**Diagram 69** $(+AB)$**:**

$$C^{13}(13,11) = P(k_2 k_3/k_4) \sum_{p_1} A^{13}_{k_1 k_2 k_3 p_1} B^{11}_{p_1 k_4} \tag{69}$$

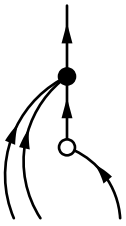


**Diagram 70** $(+AB)$**:**

$$C^{13}(04,20) = \sum_{p_1} A^{04}_{k_2k_3k_4p_1} B^{20}_{p_1k_1} \tag{70}$$

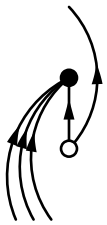


**Diagram 71** $(-BA)$**:**

$$C^{13}(13,11) = -\sum_{p_1} B^{11}_{k_1p_1} A^{13}_{p_1k_2k_3k_4} \tag{71}$$

**Diagram 72** $(-BA)$**:**

$$C^{13}(22,02) = -P(k_2/k_3k_4) \sum_{p_1} B^{02}_{k_2p_1} A^{22}_{p_1k_1k_3k_4} \tag{72}$$

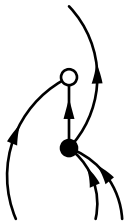


**Diagram 73** $(-BA)$**:**

$$C^{13}(22,13) = -P(k_2/k_3k_4) \frac{1}{2} \sum_{p_1p_2} B^{13}_{k_1k_2p_1p_2} A^{22}_{p_1p_2k_3k_4} \tag{73}$$

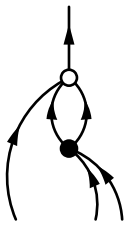


**Diagram 74** $(-BA)$**:**

$$C^{13}(31,04) = -P(k_2k_3/k_4) \frac{1}{2} \sum_{p_1p_2} B^{04}_{k_2k_3p_1p_2} A^{31}_{p_1p_2k_1k_4} \tag{74}$$

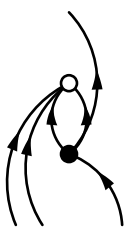


**Diagram 75** ($-BA$)**:**

$$C^{13}(11,13) = -P(k_2k_3/k_4)\sum_{p_1} B^{13}_{k_1k_2k_3p_1}A^{11}_{p_1k_4} \tag{75}$$

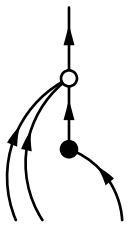
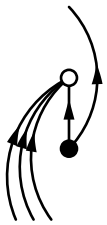


**Diagram 76** ($-BA$)**:**

$$C^{13}(20,04) = -\sum_{p_1} B^{04}_{k_2k_3k_4p_1}A^{20}_{p_1k_1} \tag{76}$$



### 3.9 $C^{04}$

**Diagram 77** ($+AB$)**:**

$$C^{04}(02,13) = P(k_1/k_2k_3k_4)\sum_{p_1} A^{02}_{k_1p_1}B^{13}_{p_1k_2k_3k_4} \tag{77}$$

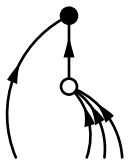


**Diagram 78** ($+AB$)**:**

$$C^{04}(04,22) = P(k_1k_2/k_3k_4)\frac{1}{2}\sum_{p_1p_2} A^{04}_{k_1k_2p_1p_2}B^{22}_{p_1p_2k_3k_4} \tag{78}$$

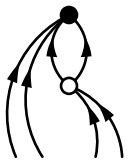


**Diagram 79** ($+AB$)**:**

$$C^{04}(04,11) = P(k_1k_2k_3/k_4)\sum_{p_1} A^{04}_{k_1k_2k_3p_1}B^{11}_{p_1k_4} \tag{79}$$

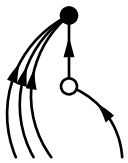


**Diagram 80 ($-BA$):**

$$C^{04}(13,02) = -P(k_1/k_2k_3k_4)\sum_{p_1} B^{02}_{k_1p_1}A^{13}_{p_1k_2k_3k_4} \tag{80}$$

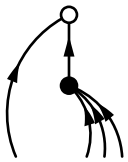


**Diagram 81 ($-BA$):**

$$C^{04}(22,04) = -P(k_1k_2/k_3k_4)\frac{1}{2}\sum_{p_1p_2} B^{04}_{k_1k_2p_1p_2}A^{22}_{p_1p_2k_3k_4} \tag{81}$$

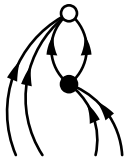


**Diagram 82 ($-BA$):**

$$C^{04}(11,04) = -P(k_1k_2k_3/k_4)\sum_{p_1} B^{04}_{k_1k_2k_3p_1}A^{11}_{p_1k_4} \tag{82}$$