

Diagrams and algebraic expressions at order 2 in PBMBPT

The ADG Dev Team

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Valid diagrams: 33
 2N valid diagrams: 33
 2N canonical diagrams for the energy: 7
 2N canonical diagrams for a generic operator only: 11
 2N non-canonical diagrams: 15


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1 Time-structure diagrams

1.1 Tree diagrams

Time-structure diagram T1:



$$T1 = \frac{1}{a_1 a_2} \quad (1)$$

Resummation power: 2
 Number of related Feynman diagrams: 12.
 Related Feynman diagrams: 4.3, 5.3, 6.2, 2.4, 7.4, 8.3, 2.3, 4.2, 6.3, 6.1, 8.4, 8.2.

Time-structure diagram T2:



$$T2 = \frac{1}{(a_1 + a_2) a_2} \quad (2)$$

Resummation power: 1
 Number of related Feynman diagrams: 21.
 Related Feynman diagrams: 3.4, 4.4, 5.2, 5.1, 1.6, 1.5, 2.6, 2.5, 7.3, 7.2, 7.1, 8.6, 1.2, 1.1, 2.2, 2.1, 3.1, 4.1, 8.7, 8.5, 8.1.

2 Two-body diagrams

2.1 Two-body energy canonical diagrams

Diagram 3.4:

$$\begin{aligned} \text{PO2.3.4} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_3}^{13} \Omega_{k_5 k_4 k_6 k_7}^{04} R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2 k_3}^{k_5}} e^{-\tau_2 \epsilon_{k_4 k_5 k_7 k_6}} \\ &= \frac{(-1)^2}{2(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_3}^{13} \Omega_{k_5 k_4 k_6 k_7}^{04} R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_3 k_4 k_7 k_6} \epsilon_{k_4 k_5 k_7 k_6}} \end{aligned} \quad (3)$$



→ T2:



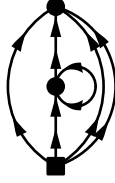
$$\text{T2} = \frac{1}{(a_1 + a_2)a_2} \quad (4)$$

$$a_1 = \epsilon_{k_1 k_2 k_3}^{k_5}$$

$$a_2 = \epsilon_{k_4 k_5 k_7 k_6}$$

Diagram 4.4:

$$\begin{aligned} \text{PO2.4.4} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_6 k_7}^{13} \Omega_{k_5 k_2 k_3 k_4}^{04} R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_7 k_6}^{k_5}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_5}} \\ &= \frac{(-1)^2}{2(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_6 k_7}^{13} \Omega_{k_5 k_2 k_3 k_4}^{04} R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_3 k_4 k_7 k_6} \epsilon_{k_2 k_3 k_4 k_5}} \end{aligned} \quad (5)$$



→ T2:



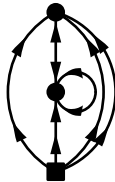
$$\text{T2} = \frac{1}{(a_1 + a_2)a_2} \quad (6)$$

$$a_1 = \epsilon_{k_1 k_7 k_6}^{k_5}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_5}$$

Diagram 4.3:

$$\begin{aligned} \text{PO2.4.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_7 k_8 k_5}^{04} \Omega_{k_6 k_2 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_8 k_5 k_7}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_6}} \\ &= \frac{(-1)^2}{2(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_7 k_8 k_5}^{04} \Omega_{k_6 k_2 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_1 k_8 k_7 k_5} \epsilon_{k_2 k_3 k_4 k_6}} \end{aligned} \quad (7)$$



→ T1:



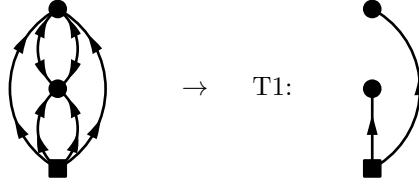
$$T1 = \frac{1}{a_1 a_2} \quad (8)$$

$$a_1 = \epsilon_{k_1 k_8 k_5 k_7}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_6}$$

Diagram 5.3:

$$\begin{aligned} \text{PO2.5.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^3} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2 k_5 k_7}^{04} \Omega_{k_8 k_6 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_2 k_5 k_7}} e^{-\tau_2 \epsilon_{k_3 k_4 k_6 k_8}} \\ &= \frac{(-1)^2}{2(2!)^3} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2 k_5 k_7}^{04} \Omega_{k_8 k_6 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_7} \epsilon_{k_3 k_4 k_6 k_8}} \end{aligned} \quad (9)$$



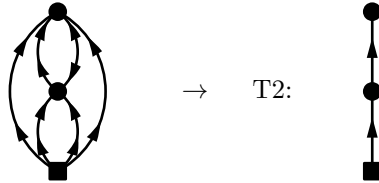
$$T1 = \frac{1}{a_1 a_2} \quad (10)$$

$$a_1 = \epsilon_{k_1 k_2 k_5 k_7}$$

$$a_2 = \epsilon_{k_3 k_4 k_6 k_8}$$

Diagram 5.2:

$$\begin{aligned} \text{PO2.5.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_6}^{13} \Omega_{k_5 k_7 k_3 k_4}^{04} R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2 k_6}^{k_5}} e^{-\tau_2 \epsilon_{k_3 k_4 k_5 k_7}} \\ &= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_6}^{13} \Omega_{k_5 k_7 k_3 k_4}^{04} R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_3 k_4 k_7 k_6} \epsilon_{k_3 k_4 k_5 k_7}} \end{aligned} \quad (11)$$



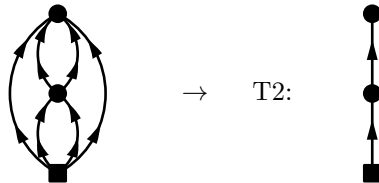
$$T2 = \frac{1}{(a_1 + a_2) a_2} \quad (12)$$

$$a_1 = \epsilon_{k_1 k_2 k_6}^{k_5}$$

$$a_2 = \epsilon_{k_3 k_4 k_5 k_7}$$

Diagram 5.1:

$$\begin{aligned} \text{PO2.5.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)^3} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_6 k_1 k_2}^{22} \Omega_{k_5 k_6 k_3 k_4}^{04} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2}^{k_5 k_6}} e^{-\tau_2 \epsilon_{k_3 k_4 k_5 k_6}} \\ &= \frac{(-1)^2}{(2!)^3} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_6 k_1 k_2}^{22} \Omega_{k_5 k_6 k_3 k_4}^{04}}{\epsilon_{k_1 k_2 k_3 k_4} \epsilon_{k_3 k_4 k_5 k_6}} \end{aligned} \quad (13)$$



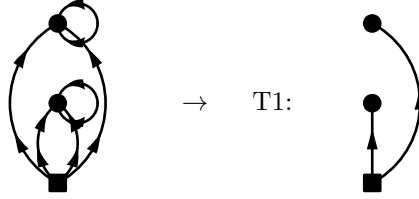
$$T2 = \frac{1}{(a_1 + a_2)a_2} \quad (14)$$

$$a_1 = \epsilon_{k_1 k_2}^{k_5 k_6}$$

$$a_2 = \epsilon_{k_3 k_4 k_5 k_6}$$

Diagram 6.2:

$$\begin{aligned} \text{PO2.6.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{8(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2 k_5 k_6}^{04}(\varphi) \Omega_{k_3 k_4 k_7 k_8}^{04}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_2 k_6 k_5}} e^{-\tau_2 \epsilon_{k_3 k_4 k_8 k_7}} \\ &= \frac{(-1)^2}{8(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2 k_5 k_6}^{04}(\varphi) \Omega_{k_3 k_4 k_7 k_8}^{04}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_6 k_5} \epsilon_{k_3 k_4 k_8 k_7}} \end{aligned} \quad (15)$$



$$T1 = \frac{1}{a_1 a_2} \quad (16)$$

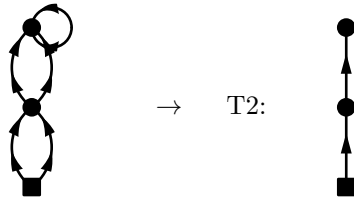
$$a_1 = \epsilon_{k_1 k_2 k_6 k_5}$$

$$a_2 = \epsilon_{k_3 k_4 k_8 k_7}$$

2.2 Two-body canonical diagrams for a generic operator only

Diagram 1.6:

$$\begin{aligned} \text{PO2.1.6} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_2}^{22}(\varphi) \Omega_{k_3 k_4 k_5 k_6}^{04}(\varphi) R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2}^{k_3 k_4}} e^{-\tau_2 \epsilon_{k_3 k_4 k_6 k_5}} \\ &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_2}^{22}(\varphi) \Omega_{k_3 k_4 k_5 k_6}^{04}(\varphi) R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_6 k_5} \epsilon_{k_3 k_4 k_6 k_5}} \end{aligned} \quad (17)$$



$$T2 = \frac{1}{(a_1 + a_2)a_2} \quad (18)$$

$$a_1 = \epsilon_{k_1 k_2}^{k_3 k_4}$$

$$a_2 = \epsilon_{k_3 k_4 k_6 k_5}$$

Diagram 1.5:

$$\begin{aligned} \text{PO2.1.5} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_2 k_4}^{13}(\varphi) \Omega_{k_3 k_5 k_6 k_7}^{04}(\varphi) R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2 k_4}^{k_3}} e^{-\tau_2 \epsilon_{k_3 k_5 k_7 k_6}} \\ &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_2 k_4}^{13}(\varphi) \Omega_{k_3 k_5 k_6 k_7}^{04}(\varphi) R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4 k_7 k_6} \epsilon_{k_3 k_7 k_6 k_5}} \end{aligned} \quad (19)$$



→ T2:



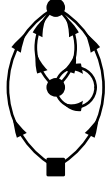
$$T2 = \frac{1}{(a_1 + a_2)a_2} \quad (20)$$

$$a_1 = \epsilon_{k_1 k_2 k_4}^{k_3}$$

$$a_2 = \epsilon_{k_3 k_5 k_7 k_6}$$

Diagram 2.6:

$$\begin{aligned} \text{PO2.2.6} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_5 k_6}^{22} \Omega_{k_3 k_4 k_1 k_2}^{04} R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_6 k_5}^{k_3 k_4}} e^{-\tau_2 \epsilon_{k_1 k_2 k_3 k_4}} \\ &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_5 k_6}^{22} \Omega_{k_3 k_4 k_1 k_2}^{04} R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_6 k_5} \epsilon_{k_1 k_2 k_3 k_4}} \end{aligned} \quad (21)$$



→ T2:



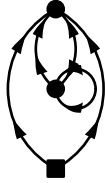
$$T2 = \frac{1}{(a_1 + a_2)a_2} \quad (22)$$

$$a_1 = \epsilon_{k_6 k_5}^{k_3 k_4}$$

$$a_2 = \epsilon_{k_1 k_2 k_3 k_4}$$

Diagram 2.5:

$$\begin{aligned} \text{PO2.2.5} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_6 k_7 k_4}^{13} \Omega_{k_3 k_5 k_1 k_2}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_7 k_4 k_6}^{k_3}} e^{-\tau_2 \epsilon_{k_1 k_2 k_3 k_5}} \\ &= \frac{(-1)^2}{2(2!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_6 k_7 k_4}^{13} \Omega_{k_3 k_5 k_1 k_2}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_7 k_6 k_5 k_4} \epsilon_{k_1 k_2 k_3 k_5}} \end{aligned} \quad (23)$$



→ T2:



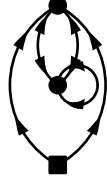
$$T2 = \frac{1}{(a_1 + a_2)a_2} \quad (24)$$

$$a_1 = \epsilon_{k_7 k_4 k_6}^{k_3}$$

$$a_2 = \epsilon_{k_1 k_2 k_3 k_5}$$

Diagram 2.4:

$$\begin{aligned}
 \text{PO2.2.4} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_7 k_8 k_3 k_5}^{04} \Omega_{k_6 k_4 k_1 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_8 k_3 k_5 k_7}} e^{-\tau_2 \epsilon_{k_1 k_2 k_4 k_6}} \\
 &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_7 k_8 k_3 k_5}^{04} \Omega_{k_6 k_4 k_1 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_8 k_7 k_3 k_5} \epsilon_{k_1 k_2 k_4 k_6}}
 \end{aligned} \tag{25}$$



→ T1:



$$T1 = \frac{1}{a_1 a_2}$$

(26)

$$a_1 = \epsilon_{k_8 k_3 k_5 k_7}$$

$$a_2 = \epsilon_{k_1 k_2 k_4 k_6}$$

Diagram 7.4:

$$\begin{aligned}
 \text{PO2.7.4} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3 k_5 k_7}^{04} \Omega_{k_8 k_6 k_4 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_3 k_5 k_7}} e^{-\tau_2 \epsilon_{k_2 k_4 k_6 k_8}} \\
 &= \frac{(-1)^2}{2(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3 k_5 k_7}^{04} \Omega_{k_8 k_6 k_4 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_1 k_3 k_5 k_7} \epsilon_{k_2 k_4 k_6 k_8}}
 \end{aligned} \tag{27}$$



→ T1:



$$T1 = \frac{1}{a_1 a_2}$$

(28)

$$a_1 = \epsilon_{k_1 k_3 k_5 k_7}$$

$$a_2 = \epsilon_{k_2 k_4 k_6 k_8}$$

Diagram 7.3:

$$\begin{aligned}
 \text{PO2.7.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_6}^{13} \Omega_{k_3 k_7 k_5 k_2}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_4 k_6}^{k_3}} e^{-\tau_2 \epsilon_{k_2 k_3 k_5 k_7}} \\
 &= \frac{(-1)^2}{(2!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_6}^{13} \Omega_{k_3 k_7 k_5 k_2}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4 k_7 k_6} \epsilon_{k_2 k_3 k_5 k_7}}
 \end{aligned} \tag{29}$$



→ T2:



$$T2 = \frac{1}{(a_1 + a_2) a_2}$$

(30)

$$a_1 = \epsilon_{k_1 k_4 k_6}^{k_3}$$

$$a_2 = \epsilon_{k_2 k_3 k_5 k_7}$$

Diagram 7.2:

$$\begin{aligned}
 \text{PO2.7.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_5}^{22} \Omega_{k_3 k_4 k_6 k_2}^{04} R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_5}^{k_3 k_4}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_6}} \\
 &= \frac{(-1)^2}{(2!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_5}^{22} \Omega_{k_3 k_4 k_6 k_2}^{04} R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_6 k_5} \epsilon_{k_2 k_3 k_4 k_6}}
 \end{aligned} \tag{31}$$



→ T2:



$$T2 = \frac{1}{(a_1 + a_2)a_2} \tag{32}$$

$$a_1 = \epsilon_{k_1 k_5}^{k_3 k_4}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_6}$$

Diagram 7.1:

$$\begin{aligned}
 \text{PO2.7.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_5 k_1}^{31} \Omega_{k_3 k_4 k_5 k_2}^{04} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_3 k_4 k_5}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_5}} \\
 &= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_5 k_1}^{31} \Omega_{k_3 k_4 k_5 k_2}^{04}}{\epsilon_{k_1 k_2} \epsilon_{k_2 k_3 k_4 k_5}}
 \end{aligned} \tag{33}$$



→ T2:



$$T2 = \frac{1}{(a_1 + a_2)a_2} \tag{34}$$

$$a_1 = \epsilon_{k_1}^{k_3 k_4 k_5}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_5}$$

Diagram 8.6:

$$\begin{aligned}
 \text{PO2.8.6} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{4} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_5}^{13} \Omega_{k_3 k_2 k_6 k_7}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_5 k_4}^{k_3}} e^{-\tau_2 \epsilon_{k_2 k_3 k_7 k_6}} \\
 &= \frac{(-1)^2}{4} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_5}^{13} \Omega_{k_3 k_2 k_6 k_7}^{04} R_{k_5 k_4}^{--}(\varphi) R_{k_7 k_6}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4 k_7 k_6} \epsilon_{k_2 k_3 k_7 k_6}}
 \end{aligned} \tag{35}$$



→ T2:



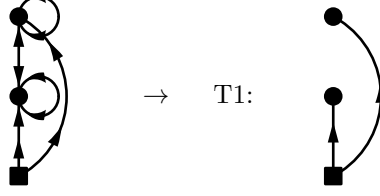
$$T2 = \frac{1}{(a_1 + a_2)a_2} \tag{36}$$

$$a_1 = \epsilon_{k_1 k_5 k_4}^{k_3}$$

$$a_2 = \epsilon_{k_2 k_3 k_7 k_6}$$

Diagram 8.3:

$$\begin{aligned}
 \text{PO2.8.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{8} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_5 k_6 k_3}^{04} \Omega_{k_4 k_2 k_7 k_8}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_6 k_3 k_5}} e^{-\tau_2 \epsilon_{k_2 k_4 k_8 k_7}} \\
 &= \frac{(-1)^2}{8} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_5 k_6 k_3}^{04} \Omega_{k_4 k_2 k_7 k_8}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) R_{k_8 k_7}^{--}(\varphi)}{\epsilon_{k_1 k_6 k_5 k_3} \epsilon_{k_2 k_8 k_7 k_4}}
 \end{aligned} \tag{37}$$



→ T1:

$$T1 = \frac{1}{a_1 a_2}$$

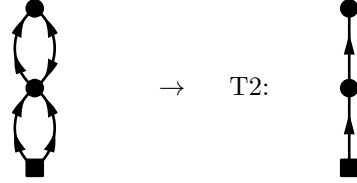
$$a_1 = \epsilon_{k_1 k_6 k_3 k_5}$$

$$a_2 = \epsilon_{k_2 k_8 k_7 k_4}$$

2.3 Two-body non-canonical diagrams

Diagram 1.2:

$$\begin{aligned}
 \text{PO2.1.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_2 k_4}^{13} \Omega_{k_3 k_5}^{02} R_{k_5 k_4}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2 k_4}^{k_3}} e^{-\tau_2 \epsilon_{k_3 k_5}} \\
 &= \frac{(-1)^2}{(2!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_2 k_4}^{13} \Omega_{k_3 k_5}^{02} R_{k_5 k_4}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4} \epsilon_{k_3 k_5}}
 \end{aligned} \tag{39}$$



→ T2:

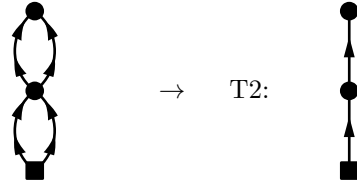
$$T2 = \frac{1}{(a_1 + a_2) a_2}$$

$$a_1 = \epsilon_{k_1 k_2 k_4}^{k_3}$$

$$a_2 = \epsilon_{k_3 k_5}$$

Diagram 1.1:

$$\begin{aligned}
 \text{PO2.1.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_2}^{22} \Omega_{k_3 k_4}^{02} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2}^{k_3 k_4}} e^{-\tau_2 \epsilon_{k_3 k_4}} \\
 &= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4 k_1 k_2}^{22} \Omega_{k_3 k_4}^{02}}{\epsilon_{k_1 k_2} \epsilon_{k_3 k_4}}
 \end{aligned} \tag{41}$$



→ T2:

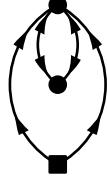
$$T2 = \frac{1}{(a_1 + a_2) a_2}$$

$$a_1 = \epsilon_{k_1 k_2}^{k_3 k_4}$$

$$a_2 = \epsilon_{k_3 k_4}$$

Diagram 2.3:

$$\begin{aligned}
 \text{PO2.2.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_5}^{02} \Omega_{k_6 k_4 k_1 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_3 k_5}} e^{-\tau_2 \epsilon_{k_1 k_2 k_4 k_6}} \\
 &= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_5}^{02} \Omega_{k_6 k_4 k_1 k_2}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_3 k_5} \epsilon_{k_1 k_2 k_4 k_6}}
 \end{aligned} \tag{43}$$



→ T1:



$$T1 = \frac{1}{a_1 a_2}$$

$$a_1 = \epsilon_{k_3 k_5}$$

$$a_2 = \epsilon_{k_1 k_2 k_4 k_6}$$

(44)

Diagram 2.2:

$$\begin{aligned}
 \text{PO2.2.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4}^{11} \Omega_{k_3 k_5 k_1 k_2}^{04} R_{k_5 k_4}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_4}^{k_3}} e^{-\tau_2 \epsilon_{k_1 k_2 k_3 k_5}} \\
 &= \frac{(-1)^2}{(2!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4}^{11} \Omega_{k_3 k_5 k_1 k_2}^{04} R_{k_5 k_4}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4} \epsilon_{k_1 k_2 k_3 k_5}}
 \end{aligned} \tag{45}$$



→ T2:



$$T2 = \frac{1}{(a_1 + a_2) a_2}$$

$$a_1 = \epsilon_{k_4}^{k_3}$$

$$a_2 = \epsilon_{k_1 k_2 k_3 k_5}$$

(46)

Diagram 2.1:

$$\begin{aligned}
 \text{PO2.2.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4}^{20} \Omega_{k_3 k_4 k_1 k_2}^{04} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_3 k_4}} e^{-\tau_2 \epsilon_{k_1 k_2 k_3 k_4}} \\
 &= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_4}^{20} \Omega_{k_3 k_4 k_1 k_2}^{04}}{\epsilon_{k_1 k_2} \epsilon_{k_1 k_2 k_3 k_4}}
 \end{aligned} \tag{47}$$



→ T2:



$$T2 = \frac{1}{(a_1 + a_2) a_2}$$

$$a_1 = \epsilon^{k_3 k_4}$$

$$a_2 = \epsilon_{k_1 k_2 k_3 k_4}$$

(48)

Diagram 3.1:

$$\begin{aligned}
\text{PO2.3.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_3}^{13} \Omega_{k_5 k_4}^{02} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_2 k_3}^{k_5}} e^{-\tau_2 \epsilon_{k_4 k_5}} \\
&= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1 k_2 k_3}^{13} \Omega_{k_5 k_4}^{02}}{\epsilon_{k_1 k_2 k_3 k_4} \epsilon_{k_4 k_5}}
\end{aligned} \tag{49}$$



\rightarrow T2:



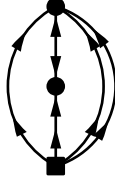
$$T2 = \frac{1}{(a_1 + a_2) a_2} \tag{50}$$

$$a_1 = \epsilon_{k_1 k_2 k_3}^{k_5}$$

$$a_2 = \epsilon_{k_4 k_5}$$

Diagram 4.2:

$$\begin{aligned}
\text{PO2.4.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_5}^{02} \Omega_{k_6 k_2 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_5}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_6}} \\
&= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_5}^{02} \Omega_{k_6 k_2 k_3 k_4}^{04} R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_5} \epsilon_{k_2 k_3 k_4 k_6}}
\end{aligned} \tag{51}$$



\rightarrow T1:



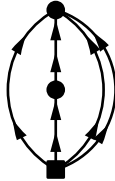
$$T1 = \frac{1}{a_1 a_2} \tag{52}$$

$$a_1 = \epsilon_{k_1 k_5}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_6}$$

Diagram 4.1:

$$\begin{aligned}
\text{PO2.4.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{(3!)} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1}^{11} \Omega_{k_5 k_2 k_3 k_4}^{04} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_5}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_5}} \\
&= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_5 k_1}^{11} \Omega_{k_5 k_2 k_3 k_4}^{04}}{\epsilon_{k_1 k_2 k_3 k_4} \epsilon_{k_2 k_3 k_4 k_5}}
\end{aligned} \tag{53}$$



\rightarrow T2:



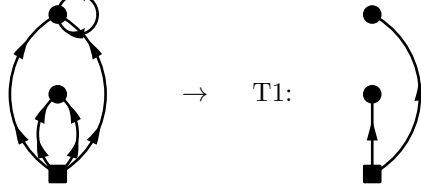
$$T2 = \frac{1}{(a_1 + a_2) a_2} \tag{54}$$

$$a_1 = \epsilon_{k_1}^{k_5}$$

$$a_2 = \epsilon_{k_2 k_3 k_4 k_5}$$

Diagram 6.3:

$$\begin{aligned}
 \text{PO2.6.3} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2}^{02} \Omega_{k_3 k_4 k_5 k_6}^{04} R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_2}} e^{-\tau_2 \epsilon_{k_3 k_4 k_6 k_5}} \\
 &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2}^{02} \Omega_{k_3 k_4 k_5 k_6}^{04} R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_2} \epsilon_{k_3 k_4 k_6 k_5}}
 \end{aligned} \tag{55}$$

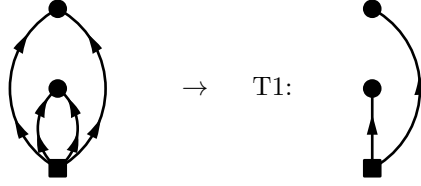


$$T1 = \frac{1}{a_1 a_2} \tag{56}$$

$$\begin{aligned}
 a_1 &= \epsilon_{k_1 k_2} \\
 a_2 &= \epsilon_{k_3 k_4 k_6 k_5}
 \end{aligned}$$

Diagram 6.1:

$$\begin{aligned}
 \text{PO2.6.1} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2}^{02} \Omega_{k_3 k_4}^{02} \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_2}} e^{-\tau_2 \epsilon_{k_3 k_4}} \\
 &= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2 k_3 k_4}^{40}(\varphi) \Omega_{k_1 k_2}^{02} \Omega_{k_3 k_4}^{02}}{\epsilon_{k_1 k_2} \epsilon_{k_3 k_4}}
 \end{aligned} \tag{57}$$

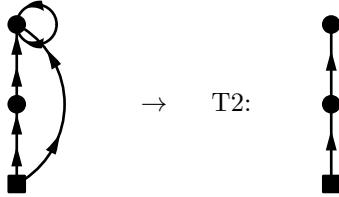


$$T1 = \frac{1}{a_1 a_2} \tag{58}$$

$$\begin{aligned}
 a_1 &= \epsilon_{k_1 k_2} \\
 a_2 &= \epsilon_{k_3 k_4}
 \end{aligned}$$

Diagram 8.7:

$$\begin{aligned}
 \text{PO2.8.7} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1}^{11} \Omega_{k_3 k_2 k_4 k_5}^{04} R_{k_5 k_4}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_3}} e^{-\tau_2 \epsilon_{k_2 k_3 k_5 k_4}} \\
 &= \frac{(-1)^2}{2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1}^{11} \Omega_{k_3 k_2 k_4 k_5}^{04} R_{k_5 k_4}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4} \epsilon_{k_2 k_3 k_5 k_4}}
 \end{aligned} \tag{59}$$

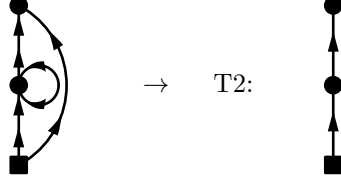


$$T2 = \frac{1}{(a_1 + a_2) a_2} \tag{60}$$

$$\begin{aligned}
 a_1 &= \epsilon_{k_1}^{k_3} \\
 a_2 &= \epsilon_{k_2 k_3 k_5 k_4}
 \end{aligned}$$

Diagram 8.5:

$$\begin{aligned}
 \text{PO2.8.5} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_5}^{13} \Omega_{k_3 k_2}^{02} R_{k_5 k_4}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1 k_5 k_4}^{k_3}} e^{-\tau_2 \epsilon_{k_2 k_3}} \\
 &= \frac{(-1)^2}{2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1 k_4 k_5}^{13} \Omega_{k_3 k_2}^{02} R_{k_5 k_4}^{--}(\varphi)}{\epsilon_{k_1 k_2 k_5 k_4} \epsilon_{k_2 k_3}}
 \end{aligned} \tag{61}$$



→ T2:



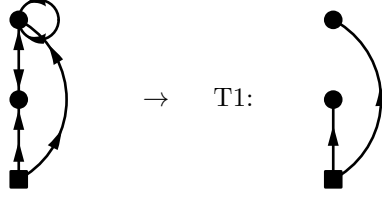
$$T2 = \frac{1}{(a_1 + a_2)a_2} \tag{62}$$

$$a_1 = \epsilon_{k_1 k_5 k_4}^{k_3}$$

$$a_2 = \epsilon_{k_2 k_3}$$

Diagram 8.4:

$$\begin{aligned}
 \text{PO2.8.4} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3}^{02} \Omega_{k_4 k_2 k_5 k_6}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_3}} e^{-\tau_2 \epsilon_{k_2 k_4 k_6 k_5}} \\
 &= \frac{(-1)^2}{2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3}^{02} \Omega_{k_4 k_2 k_5 k_6}^{04} R_{k_4 k_3}^{--}(\varphi) R_{k_6 k_5}^{--}(\varphi)}{\epsilon_{k_1 k_3} \epsilon_{k_2 k_6 k_5 k_4}}
 \end{aligned} \tag{63}$$



→ T1:



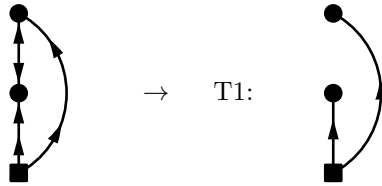
$$T1 = \frac{1}{a_1 a_2} \tag{64}$$

$$a_1 = \epsilon_{k_1 k_3}$$

$$a_2 = \epsilon_{k_2 k_4 k_6 k_5}$$

Diagram 8.2:

$$\begin{aligned}
 \text{PO2.8.2} &= \lim_{\tau \rightarrow \infty} \frac{(-1)^2}{2} \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3}^{02} \Omega_{k_4 k_2}^{02} R_{k_4 k_3}^{--}(\varphi) \int_0^\tau d\tau_1 d\tau_2 e^{-\tau_1 \epsilon_{k_1 k_3}} e^{-\tau_2 \epsilon_{k_2 k_4}} \\
 &= \frac{(-1)^2}{2} \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_1 k_3}^{02} \Omega_{k_4 k_2}^{02} R_{k_4 k_3}^{--}(\varphi)}{\epsilon_{k_1 k_3} \epsilon_{k_2 k_4}}
 \end{aligned} \tag{65}$$



→ T1:



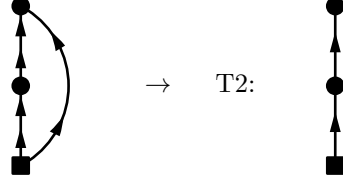
$$T1 = \frac{1}{a_1 a_2} \tag{66}$$

$$a_1 = \epsilon_{k_1 k_3}$$

$$a_2 = \epsilon_{k_2 k_4}$$

Diagram 8.1:

$$\begin{aligned}
 \text{PO2.8.1} &= \lim_{\tau \rightarrow \infty} (-1)^2 \sum_{k_i} \tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1}^{11} \Omega_{k_3 k_2}^{02} \int_0^\tau d\tau_1 d\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_3}} e^{-\tau_2 \epsilon_{k_2}^{k_3}} \\
 &= (-1)^2 \sum_{k_i} \frac{\tilde{O}_{k_1 k_2}^{20}(\varphi) \Omega_{k_3 k_1}^{11} \Omega_{k_3 k_2}^{02}}{\epsilon_{k_1 k_2} \epsilon_{k_2 k_3}}
 \end{aligned} \tag{67}$$



$$\text{T2} = \frac{1}{(a_1 + a_2)a_2} \tag{68}$$

$$a_1 = \epsilon_{k_1}^{k_3}$$

$$a_2 = \epsilon_{k_2 k_3}$$