Diagrams and algebraic expressions at order 3 in BMBPT

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Valid diagrams: 23
2N valid diagrams: 8
2N canonical diagrams for the energy: 1
2N canonical diagrams for a generic operator only: 1
2N non-canonical diagrams: 6
3N valid diagrams: 7
3N canonical diagrams for the energy: 7
3N canonical diagrams for a generic operator only: 3
3N non-canonical diagrams: 5

Contents

1 Time-structure diagrams

1.1 Tree diagrams

Time-structure diagram T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{1}$$

Number of related Feynman diagrams: 21.

Related Feynman diagrams: 23, 22, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 6, 5, 4, 3, 2, 1.

Time-structure diagram T2:

$$\mathrm{T2} = \frac{1}{a_1 a_2} \tag{2}$$

Number of related Feynman diagrams: 2.

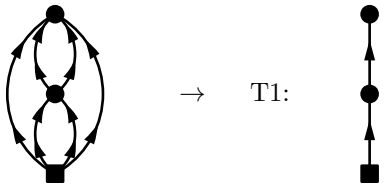
Related Feynman diagrams: 21, 7.

2 Two-body diagrams

2.1 Two-body energy canonical diagrams

Diagram 1:

$$\begin{aligned}\mathrm{PO3.1} &= \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^3} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{22}_{k_5k_6k_1k_2} \Omega^{04}_{k_5k_6k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_5k_6}_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}} \\ &= \frac{(-1)^2}{(2!)^3} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{22}_{k_5k_6k_1k_2} \Omega^{04}_{k_5k_6k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_3k_4k_5k_6}}\end{aligned} \tag{3}$$



$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{4}$$

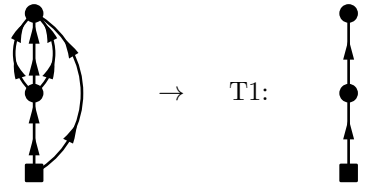
$$a_1 = \epsilon^{k_5k_6}_{k_1k_2}$$
$$a_2 = \epsilon_{k_3k_4k_5k_6}$$

2.2 Two-body canonical diagrams for a generic operator only

Diagram 2:

$$\mathrm{PO3.2} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{20}_{k_1k_2}\Omega^{31}_{k_3k_4k_5k_1}\Omega^{04}_{k_3k_4k_5k_2} \int_0^\tau \mathrm{d}\tau_1\mathrm{d}\tau_2\theta(\tau_2-\tau_1)e^{-\tau_1\epsilon^{k_3k_4k_5}_{k_1}}e^{-\tau_2\epsilon_{k_2k_3k_4k_5}}$$

$$= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{20}_{k_1k_2}\Omega^{31}_{k_3k_4k_5k_1}\Omega^{04}_{k_3k_4k_5k_2}}{\epsilon_{k_1k_2}\ \epsilon_{k_2k_3k_4k_5}} \tag{5}$$



$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{6}$$

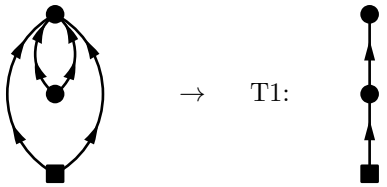
$$a_1 = \epsilon^{k_3k_4k_5}_{k_1}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5}$$

2.3 Two-body non-canonical diagrams

Diagram 3:

$$\mathrm{PO3.3} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} O^{20}_{k_1k_2}\Omega^{20}_{k_3k_4}\Omega^{04}_{k_3k_4k_1k_2} \int_0^\tau \mathrm{d}\tau_1\mathrm{d}\tau_2\theta(\tau_2-\tau_1)e^{-\tau_1\epsilon^{k_3k_4}}e^{-\tau_2\epsilon_{k_1k_2k_3k_4}}$$

$$= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{O^{20}_{k_1k_2}\Omega^{20}_{k_3k_4}\Omega^{04}_{k_3k_4k_1k_2}}{\epsilon_{k_1k_2}\ \epsilon_{k_1k_2k_3k_4}} \tag{7}$$



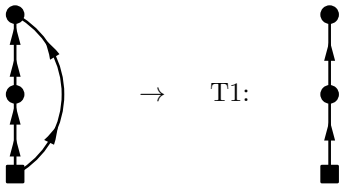
$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{8}$$

$$a_1 = \epsilon^{k_3k_4}$$
$$a_2 = \epsilon_{k_1k_2k_3k_4}$$

Diagram 4:

$$\mathrm{PO3.4} = \lim_{\tau\to\infty} (-1)^2 \sum_{k_i} O^{20}_{k_1k_2}\Omega^{11}_{k_3k_1}\Omega^{02}_{k_3k_2} \int_0^\tau \mathrm{d}\tau_1\mathrm{d}\tau_2\theta(\tau_2-\tau_1)e^{-\tau_1\epsilon^{k_3}_{k_1}}e^{-\tau_2\epsilon_{k_2k_3}}$$

$$= (-1)^2 \sum_{k_i} \frac{O^{20}_{k_1k_2}\Omega^{11}_{k_3k_1}\Omega^{02}_{k_3k_2}}{\epsilon_{k_1k_2}\ \epsilon_{k_2k_3}} \tag{9}$$

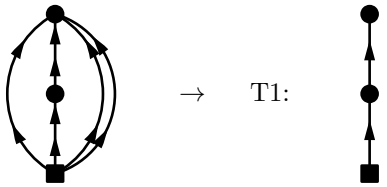


$$\text{T1} = \frac{1}{(a_1+a_2)a_2} \tag{10}$$

$$a_1 = \epsilon_{k_1}^{k_3}$$
$$a_2 = \epsilon_{k_2k_3}$$

Diagram 5:

$$\text{PO3.5} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{11}_{k_5k_1} \Omega^{04}_{k_5k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon_{k_1}^{k_5}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5}}$$
$$= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{11}_{k_5k_1} \Omega^{04}_{k_5k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_2k_3k_4k_5}} \tag{11}$$

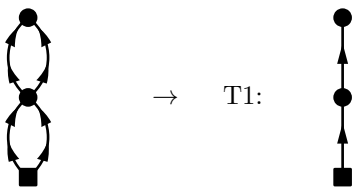


$$\text{T1} = \frac{1}{(a_1+a_2)a_2} \tag{12}$$

$$a_1 = \epsilon_{k_1}^{k_5}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5}$$

Diagram 6:

$$\text{PO3.6} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{22}_{k_3k_4k_1k_2} \Omega^{02}_{k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon_{k_1k_2}^{k_3k_4}} e^{-\tau_2 \epsilon_{k_3k_4}}$$
$$= \frac{(-1)^2}{(2!)^2} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{22}_{k_3k_4k_1k_2} \Omega^{02}_{k_3k_4}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4}} \tag{13}$$



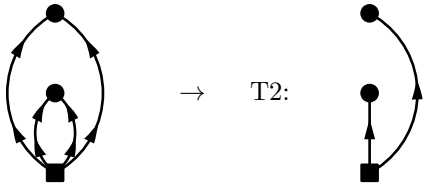
$$\text{T1} = \frac{1}{(a_1+a_2)a_2} \tag{14}$$

$$a_1 = \epsilon_{k_1k_2}^{k_3k_4}$$
$$a_2 = \epsilon_{k_3k_4}$$

Diagram 7:

$$\mathrm{PO3.7} = \lim_{\tau\to\infty} \frac{(-1)^2}{2(2!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{02}_{k_1k_2} \Omega^{02}_{k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 e^{-\tau_1 \epsilon_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4}}$$

$$= \frac{(-1)^2}{2(2!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{02}_{k_1k_2} \Omega^{02}_{k_3k_4}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4}} \tag{15}$$



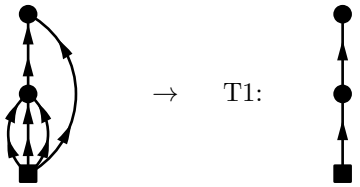
$$\mathrm{T2} = \frac{1}{a_1 a_2} \tag{16}$$

$$a_1 = \epsilon_{k_1k_2}$$
$$a_2 = \epsilon_{k_3k_4}$$

Diagram 8:

$$\mathrm{PO3.8} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{13}_{k_5k_1k_2k_3} \Omega^{02}_{k_5k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5}}$$

$$= \frac{(-1)^2}{(3!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{13}_{k_5k_1k_2k_3} \Omega^{02}_{k_5k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_4k_5}} \tag{17}$$



$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{18}$$

$$a_1 = \epsilon^{k_5}_{k_1k_2k_3}$$
$$a_2 = \epsilon_{k_4k_5}$$

3 Three-body diagrams

3.1 Three-body energy canonical diagrams

Diagram 9:

$$\mathrm{PO3.9} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{31}_{k_5k_6k_7k_1} \Omega^{06}_{k_5k_6k_7k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5k_6k_7}}$$

$$= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{31}_{k_5k_6k_7k_1} \Omega^{06}_{k_5k_6k_7k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_2k_3k_4k_5k_6k_7}} \tag{19}$$

$\rightarrow$ T1:

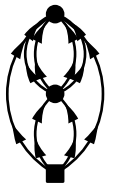
$$\text{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{20}$$

$$a_1 = \epsilon_{k_1}^{k_5 k_6 k_7}$$
$$a_2 = \epsilon_{k_2 k_3 k_4 k_5 k_6 k_7}$$

Diagram 10:

$$\text{PO3.10} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{40}_{k_1 k_2 k_3 k_4} \Omega^{42}_{k_5 k_6 k_7 k_8 k_1 k_2} \Omega^{06}_{k_5 k_6 k_7 k_8 k_3 k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_5 k_6 k_7 k_8}_{k_1 k_2}} e^{-\tau_2 \epsilon_{k_3 k_4 k_5 k_6}}$$

$$= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{40}_{k_1 k_2 k_3 k_4} \Omega^{42}_{k_5 k_6 k_7 k_8 k_1 k_2} \Omega^{06}_{k_5 k_6 k_7 k_8 k_3 k_4}}{\epsilon_{k_1 k_2 k_3 k_4}\ \epsilon_{k_3 k_4 k_5 k_6 k_7 k_8}} \tag{21}$$



$\rightarrow$ T1:

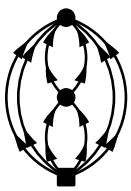
$$\text{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{22}$$

$$a_1 = \epsilon_{k_1 k_2}^{k_5 k_6 k_7 k_8}$$
$$a_2 = \epsilon_{k_3 k_4 k_5 k_6 k_7 k_8}$$

Diagram 11:

$$\text{PO3.11} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{60}_{k_1 k_2 k_3 k_4 k_5 k_6} \Omega^{22}_{k_7 k_8 k_1 k_2} \Omega^{06}_{k_7 k_8 k_3 k_4 k_5 k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_7 k_8}_{k_1 k_2}} e^{-\tau_2 \epsilon_{k_3 k_4 k_5 k_6 k_7 k_8}}$$

$$= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{60}_{k_1 k_2 k_3 k_4 k_5 k_6} \Omega^{22}_{k_7 k_8 k_1 k_2} \Omega^{06}_{k_7 k_8 k_3 k_4 k_5 k_6}}{\epsilon_{k_1 k_2 k_3 k_4 k_5 k_6}\ \epsilon_{k_3 k_4 k_5 k_6 k_7 k_8}} \tag{23}$$



$\rightarrow$ T1:

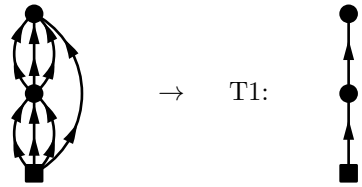
$$\text{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{24}$$

$$a_1 = \epsilon_{k_1 k_2}^{k_7 k_8}$$
$$a_2 = \epsilon_{k_3 k_4 k_5 k_6 k_7 k_8}$$

Diagram 12:

$$\mathrm{PO3.12} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{33}_{k_5k_6k_7k_1k_2k_3} \Omega^{04}_{k_5k_6k_7k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6k_7}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}}$$

$$= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{33}_{k_5k_6k_7k_1k_2k_3} \Omega^{04}_{k_5k_6k_7k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_4k_5k_6k_7}} \tag{25}$$



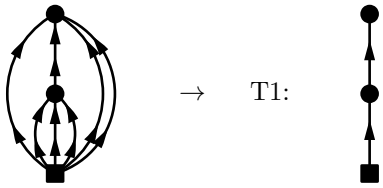
$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{26}$$

$$a_1 = \epsilon^{k_5k_6k_7}_{k_1k_2k_3}$$
$$a_2 = \epsilon_{k_4k_5k_6k_7}$$

Diagram 13:

$$\mathrm{PO3.13} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^2} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{13}_{k_7k_1k_2k_3} \Omega^{04}_{k_7k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}}$$

$$= \frac{(-1)^2}{(3!)^2} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{13}_{k_7k_1k_2k_3} \Omega^{04}_{k_7k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_4k_5k_6k_7}} \tag{27}$$



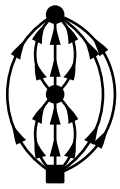
$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{28}$$

$$a_1 = \epsilon^{k_7}_{k_1k_2k_3}$$
$$a_2 = \epsilon_{k_4k_5k_6k_7}$$

Diagram 14:

$$\mathrm{PO3.14} = \lim_{\tau\to\infty} \frac{(-1)^2}{(3!)^3} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{33}_{k_7k_8k_9k_1k_2k_3} \Omega^{06}_{k_7k_8k_9k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7k_8k_9}_{k_1k_2k_3}} e^{-\tau_2 \epsilon_{k_4k_5k_6k_7}}$$

$$= \frac{(-1)^2}{(3!)^3} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{33}_{k_7k_8k_9k_1k_2k_3} \Omega^{06}_{k_7k_8k_9k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_4k_5k_6k_7k_8k_9}} \tag{29}$$



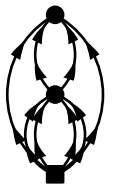
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{30}$$

$$a_1 = \epsilon^{k_7 k_8 k_9}_{k_1 k_2 k_3}$$
$$a_2 = \epsilon_{k_4 k_5 k_6 k_7 k_8 k_9}$$

Diagram 15:

$$\mathrm{PO3.15} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{24}_{k_7k_8k_1k_2k_3k_4} \Omega^{04}_{k_7k_8k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_7k_8}_{k_1k_2k_3k_4}} e^{-\tau_2 \epsilon_{k_5k_6k_7k_8}}$$
$$= \frac{(-1)^2}{(2!)^2(4!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{24}_{k_7k_8k_1k_2k_3k_4} \Omega^{04}_{k_7k_8k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_5k_6k_7k_8}} \tag{31}$$



$\rightarrow$ T1:

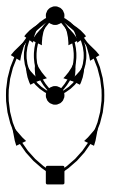
$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{32}$$

$$a_1 = \epsilon^{k_7 k_8}_{k_1 k_2 k_3 k_4}$$
$$a_2 = \epsilon_{k_5 k_6 k_7 k_8}$$

3.2 Three-body canonical diagrams for a generic operator only

Diagram 16:

$$\mathrm{PO3.16} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{20}_{k_1k_2} \Omega^{40}_{k_3k_4k_5k_6} \Omega^{06}_{k_3k_4k_5k_6k_1k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3k_4k_5k_6}} e^{-\tau_2 \epsilon_{k_1k_2k_3k_4k_5k_6}}$$
$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{20}_{k_1k_2} \Omega^{40}_{k_3k_4k_5k_6} \Omega^{06}_{k_3k_4k_5k_6k_1k_2}}{\epsilon_{k_1k_2}\ \epsilon_{k_1k_2k_3k_4k_5k_6}} \tag{33}$$



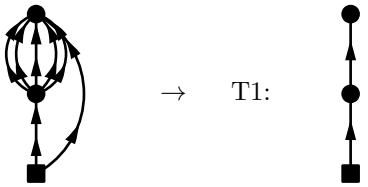
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{34}$$

$$a_1 = \epsilon^{k_3 k_4 k_5 k_6}$$
$$a_2 = \epsilon_{k_1 k_2 k_3 k_4 k_5 k_6}$$

Diagram 17:

$$\mathrm{PO3.17} = \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{20}_{k_1 k_2} \Omega^{51}_{k_3 k_4 k_5 k_6 k_7 k_1} \Omega^{06}_{k_3 k_4 k_5 k_6 k_7 k_2} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3 k_4 k_5 k_6 k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2 k_3 k_4 k_5 k_6 k_7}}$$
$$= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{20}_{k_1 k_2} \Omega^{51}_{k_3 k_4 k_5 k_6 k_7 k_1} \Omega^{06}_{k_3 k_4 k_5 k_6 k_7 k_2}}{\epsilon_{k_1 k_2}\ \epsilon_{k_2 k_3 k_4 k_5 k_6 k_7}} \tag{35}$$



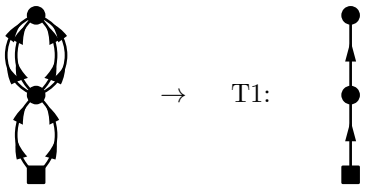
$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{36}$$

$$a_1 = \epsilon^{k_3 k_4 k_5 k_6 k_7}_{k_1}$$
$$a_2 = \epsilon_{k_2 k_3 k_4 k_5 k_6 k_7}$$

Diagram 18:

$$\mathrm{PO3.18} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{20}_{k_1 k_2} \Omega^{42}_{k_3 k_4 k_5 k_6 k_1 k_2} \Omega^{04}_{k_3 k_4 k_5 k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_3 k_4 k_5 k_6}_{k_1 k_2}} e^{-\tau_2 \epsilon_{k_3 k_4 k_5 k_6}}$$
$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{20}_{k_1 k_2} \Omega^{42}_{k_3 k_4 k_5 k_6 k_1 k_2} \Omega^{04}_{k_3 k_4 k_5 k_6}}{\epsilon_{k_1 k_2}\ \epsilon_{k_3 k_4 k_5 k_6}} \tag{37}$$



$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2)a_2} \tag{38}$$

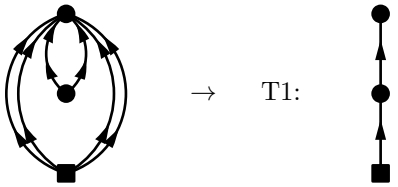
$$a_1 = \epsilon^{k_3 k_4 k_5 k_6}_{k_1 k_2}$$
$$a_2 = \epsilon_{k_3 k_4 k_5 k_6}$$

3.3 Three-body non-canonical diagrams

Diagram 19:

$$\mathrm{PO3.19} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{40}_{k_1k_2k_3k_4} \Omega^{20}_{k_5k_6} \Omega^{06}_{k_5k_6k_1k_2k_3k_4} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_5k_6}} e^{-\tau_2 \epsilon_{k_1k_2k_3k_4k_5k_6}}$$

$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{40}_{k_1k_2k_3k_4} \Omega^{20}_{k_5k_6} \Omega^{06}_{k_5k_6k_1k_2k_3k_4}}{\epsilon_{k_1k_2k_3k_4}\ \epsilon_{k_1k_2k_3k_4k_5k_6}} \tag{39}$$



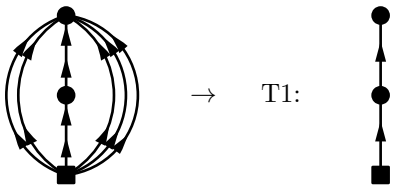
$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{40}$$

$$a_1 = \epsilon^{k_5k_6}$$
$$a_2 = \epsilon_{k_1k_2k_3k_4k_5k_6}$$

Diagram 20:

$$\mathrm{PO3.20} = \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{11}_{k_7k_1} \Omega^{06}_{k_7k_2k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2-\tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1}} e^{-\tau_2 \epsilon_{k_2k_3k_4k_5k_6k_7}}$$

$$= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{11}_{k_7k_1} \Omega^{06}_{k_7k_2k_3k_4k_5k_6}}{\epsilon_{k_1k_2k_3k_4k_5k_6}\ \epsilon_{k_2k_3k_4k_5k_6k_7}} \tag{41}$$



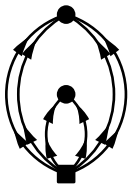
$$\mathrm{T1} = \frac{1}{(a_1+a_2)a_2} \tag{42}$$

$$a_1 = \epsilon^{k_7}_{k_1}$$
$$a_2 = \epsilon_{k_2k_3k_4k_5k_6k_7}$$

Diagram 21:

$$\mathrm{PO3.21} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{02}_{k_1k_2} \Omega^{04}_{k_3k_4k_5k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 e^{-\tau_1 \epsilon_{k_1k_2}} e^{-\tau_2 \epsilon_{k_3k_4k_5k_6}}$$

$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{60}_{k_1k_2k_3k_4k_5k_6} \Omega^{02}_{k_1k_2} \Omega^{04}_{k_3k_4k_5k_6}}{\epsilon_{k_1k_2}\ \epsilon_{k_3k_4k_5k_6}} \tag{43}$$



$\rightarrow$ T2:

$$\mathrm{T2} = \frac{1}{a_1 a_2} \tag{44}$$

$$a_1 = \epsilon_{k_1 k_2}$$
$$a_2 = \epsilon_{k_3 k_4 k_5 k_6}$$

Diagram 22:

$$\mathrm{PO3.22} = \lim_{\tau\to\infty} \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} O^{40}_{k_1 k_2 k_3 k_4} \Omega^{24}_{k_5 k_6 k_1 k_2 k_3 k_4} \Omega^{02}_{k_5 k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_5 k_6}_{k_1 k_2 k_3 k_4}} e^{-\tau_2 \epsilon_{k_5 k_6}}$$
$$= \frac{(-1)^2}{(2!)(4!)} \sum_{k_i} \frac{O^{40}_{k_1 k_2 k_3 k_4} \Omega^{24}_{k_5 k_6 k_1 k_2 k_3 k_4} \Omega^{02}_{k_5 k_6}}{\epsilon_{k_1 k_2 k_3 k_4}\ \epsilon_{k_5 k_6}} \tag{45}$$

$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2) a_2} \tag{46}$$

$$a_1 = \epsilon^{k_5 k_6}_{k_1 k_2 k_3 k_4}$$
$$a_2 = \epsilon_{k_5 k_6}$$

Diagram 23:

$$\mathrm{PO3.23} = \lim_{\tau\to\infty} \frac{(-1)^2}{(5!)} \sum_{k_i} O^{60}_{k_1 k_2 k_3 k_4 k_5 k_6} \Omega^{15}_{k_7 k_1 k_2 k_3 k_4 k_5} \Omega^{02}_{k_7 k_6} \int_0^\tau \mathrm{d}\tau_1 \mathrm{d}\tau_2 \theta(\tau_2 - \tau_1) e^{-\tau_1 \epsilon^{k_7}_{k_1 k_2 k_3 k_4 k_5}} e^{-\tau_2 \epsilon_{k_6 k_7}}$$
$$= \frac{(-1)^2}{(5!)} \sum_{k_i} \frac{O^{60}_{k_1 k_2 k_3 k_4 k_5 k_6} \Omega^{15}_{k_7 k_1 k_2 k_3 k_4 k_5} \Omega^{02}_{k_7 k_6}}{\epsilon_{k_1 k_2 k_3 k_4 k_5 k_6}\ \epsilon_{k_6 k_7}} \tag{47}$$

$\rightarrow$ T1:

$$\mathrm{T1} = \frac{1}{(a_1 + a_2) a_2} \tag{48}$$

$$a_1 = \epsilon^{k_7}_{k_1 k_2 k_3 k_4 k_5}$$
$$a_2 = \epsilon_{k_6 k_7}$$