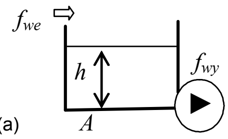
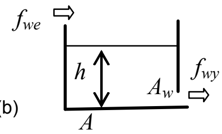
Otwarte układy hydrauliczne

(a)



(b)



1) Zawartość magazynu $V(t) = Ah(t)$

2) Zmiana zawartości magazynu $\frac{dV(t)}{dt} = A\frac{dh(t)}{dt} = A\dot{h}(t)$

3) Bilans strumieni wpływających i wypływających [m3/s] $A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$

(a) $f_{wy}(t)$

(a) $A\dot{h}(t) = f_{we}(t) - f_{wy}(t)$

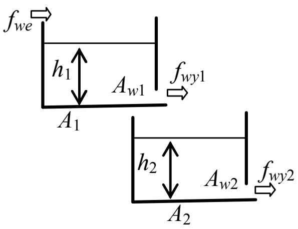
(b) $f_{wy}(t) = A_w\sqrt{2gh(t)} \approx ah(t)$

(b_1) $A\dot{h}(t) = f_{we}(t) - A_w\sqrt{2gh(t)}$

(b_2) $A\dot{h}(t) = f_{we}(t) - ah(t)$

4) Zmienne wejściowe i wyjściowe, kompletność modelu

Otwarte układy hydrauliczne



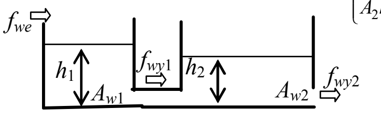
$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - f_{wy1}(t) \\ A_2\dot{h}_2(t) = f_{wy1}(t) - f_{wy2}(t) \end{cases}$$

$$f_{wy1}(t) = A_{w1}\sqrt{2gh_1(t)} \approx a_1h_1(t)$$

$$f_{wy2}(t) = A_{w2}\sqrt{2gh_2(t)} \approx a_2h_2(t)$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - A_{w1}\sqrt{2gh_1(t)} \\ A_2\dot{h}_2(t) = A_{w1}\sqrt{2gh_1(t)} - A_{w2}\sqrt{2gh_2(t)} \end{cases}$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - a_1h_1(t) \\ A_2\dot{h}_2(t) = a_1h_1(t) - a_2h_2(t) \end{cases}$$



$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - f_{wy1}(t) \\ A_2\dot{h}_2(t) = f_{wy1}(t) - f_{wy2}(t) \end{cases}$$

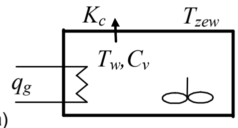
$$f_{wy1}(t) = A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} \approx a_1\left(h_1(t) - h_2(t)\right)$$

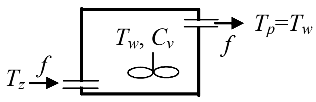
$$f_{wy2}(t) = A_{w2}\sqrt{2gh_2(t)} \approx a_2h_2(t)$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} \\ A_2\dot{h}_2(t) = A_{w1}\sqrt{2g\left(h_1(t) - h_2(t)\right)} - A_{w2}\sqrt{2gh_2(t)} \end{cases}$$

$$\begin{cases} A_1\dot{h}_1(t) = f_{we}(t) - a_1\left(h_1(t) - h_2(t)\right) \\ A_2\dot{h}_2(t) = a_1\left(h_1(t) - h_2(t)\right) - a_2h_2(t) \end{cases}$$

Obiekty cieplne

(a) 

(b) 

Założenie o doskonałym mieszaniu

1) Zawartość magazynu $Q(t)=c_p\rho VT(t)=C_V T(t)$

2) Zmiana zawartości magazynu $\frac{dQ(t)}{dt}=C_V\frac{dT_w(t)}{dt}=C_V\dot{T}_w(t)$

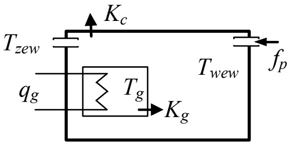
3) Bilans strumieni wpływających i wypływających [W] $C_v\dot{T}_w(t)=q_{we}(t)-q_{wy}(t)$

$$C_v\dot{T}_w(t)=q_g(t)-K_c\left(T_w(t)-T_{zew}(t)\right)$$

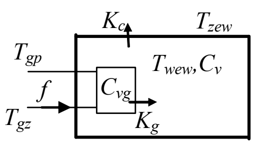
$$C_v\dot{T}_w(t)=c_p\rho\, f(t)T_z(t)-c_p\rho\, f(t)T_w(t)$$

4) Zmienne wejściowe i wyjściowe, kompletność modelu

Obiekty cieplne



$$\begin{cases}C_{vw}\dot{T}_{wew}(t)=K_g\left(T_g(t)-T_{wew}(t)\right)-K_c\left(T_{wew}(t)-T_{zew}(t)\right)-c_{pp}\rho_p f_p(t)\left(T_{wew}(t)-T_{zew}(t)\right)\\ C_{vg}\dot{T}_g(t)=q_g(t)-K_g\left(T_g(t)-T_{wew}(t)\right)\end{cases}$$



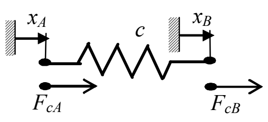
$$\begin{cases}C_{vg}\dot{T}_{gp}(t)=c_{pw}\rho_{pw}f(t)T_{gz}(t)-c_{pw}\rho_{pw}f(t)T_{gp}(t)-K_g\left(T_{gp}(t)-T_{wew}(t)\right)\\ C_{vw}\dot{T}_{wew}(t)=K_g\left(T_{gp}(t)-T_{wew}(t)\right)-K_c\left(T_{wew}(t)-T_{zew}(t)\right)\end{cases}$$

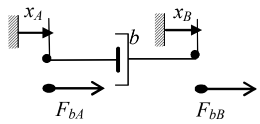
$$\begin{cases}C_{vg}\dot{T}_{gp}(t)=c_{pw}\rho_{pw}f(t)\left(T_{gz}(t)-T_{gp}(t)\right)-K_g\left(T_{gp}(t)-T_{wew}(t)\right)\\ C_{vw}\dot{T}_{wew}(t)=K_g\left(T_{gp}(t)-T_{wew}(t)\right)-K_c\left(T_{wew}(t)-T_{zew}(t)\right)\end{cases}$$

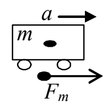
Proste układy mechaniczne

Założenie – jeden kierunek działania sił

1) Opis działania układu za pomocą idealnych elementów





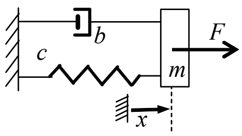


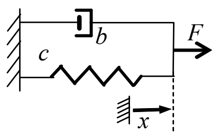
$$F_{cA}(t) = c\big(x_A(t) - x_B(t)\big)$$
$$F_{cB}(t) = c\big(x_B(t) - x_A(t)\big)$$

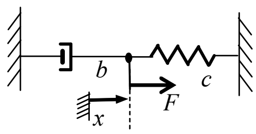
$$F_{bA}(t) = b\big(\dot{x}_A(t) - \dot{x}_B(t)\big)$$
$$F_{bB}(t) = b\big(\dot{x}_B(t) - \dot{x}_A(t)\big)$$

$$F_m(t) = m\ddot{x}(t)$$

2) Punkt bilansowania sił







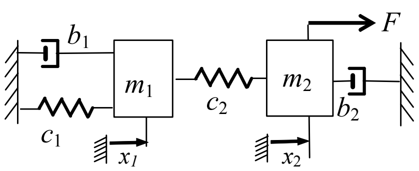
3) Bilans sił [N]

$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$

$$b\dot{x}(t) + cx(t) = F(t)$$

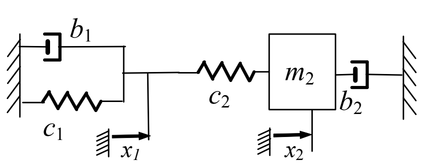
$$b\dot{x}(t) + cx(t) = F(t)$$

Proste układy mechaniczne



$$\begin{cases} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

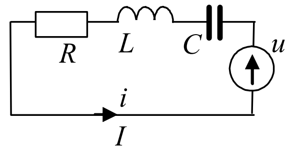
(2 punkty, 2 masy)



$$\begin{cases} 0 = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

(2 punkty, 2 masy, bez zewnętrzbej siły)

Proste obwody elektryczne



(1) $j\omega L\,I + R\,I + \frac{1}{j\omega C} I = U$

(2) $L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$ (3) $L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = u(t)$

(4) $sL\,i(s) + R\,i(s) + \frac{1}{sC}i(s) = u(s)$ (5) $i(s) = \frac{sC}{s^2LC + sRC + 1}u(s)$

$u(t) = U\sin(\omega t)$ $s = j\omega$ $i(t) = \frac{dq(t)}{dt}$

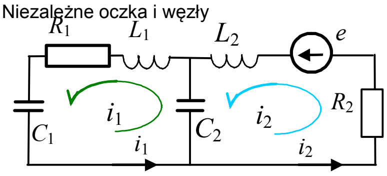
$i(t) = I\sin(\omega t + \varphi)$ $i(s) = sq(s)$

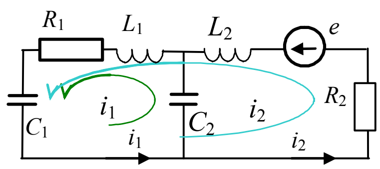
Proste obwody elektryczne

1) Opis działania układu za pomocą idealnych elementów

	Opis napięciowo-prądowy $u(i)$		O.prąd.-napięciowy $i(u)$	$u(q)$	Impedancje $Z(s)$	$Z(j\omega)$
rezystor (R)	$u(t) = Ri(t)$	$u(s) = Ri(s)$	$i(t) = Gu(t)$	$u(t) = R\dot{q}(t)$	R	R
kondensator (C)	$u(t) \quad \frac{1}{C}\int i(t)dt$	$u(s) \quad \frac{1}{sC}i(s)$	$i(t) \quad C\frac{du(t)}{dt}$	$u(t) \quad \frac{1}{C}q(t)$	$\frac{1}{sC}$	$\frac{1}{j\omega C}$
cewka (L)	$e_L(t) = -L\frac{di(t)}{dt}$	$u(s) \quad sLi(s)$	$i(t) = \frac{1}{L}\int u(t)dt$	$u(t) \quad L\ddot{q}(t)$	sL	$j\omega L$

2) Niezależne oczka i węzły





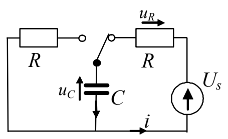
3) Bilans napięć w oczkach obwodu i/lub prądów (ładunków) w węzach [V, A]

$$\begin{cases} e = sL_2 i_2 + R_2 i_2(s) + \frac{i_2 - i_1}{sC_2} \\ 0 = sL_1 i_1 + R_1 i_1 + \frac{i_1}{sC_1} + \frac{i_1 - i_2}{sC_2} \end{cases}$$

$$\begin{cases} e = L_2\frac{di_2}{dt} + R_2 i_2 + \int\frac{i_2 - i_1}{C_2}dt \\ 0 = L_1\frac{di_1}{dt} + R_1 i_1 + \int\frac{i_1}{C_1}dt + \int\frac{i_1 - i_2}{C_2}dt \end{cases}$$

$$\begin{cases} e = L_2\ddot{q}_2 + R_2\dot{q}_2 + \frac{q_2 - q_1}{C_2} \\ 0 = L_1\ddot{q}_1 + R_1\dot{q}_1 + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_2} \end{cases}$$

Ładowanie/rozładowanie kondensatora



$u_R(t)+u_C(t)=U_s$

$Ri(t)+\frac{q(t)}{C}=U_s$

$R\dot{q}(t)+\frac{1}{C}q(t)=U_s \quad , q(0)=0$

r.s.) $R\lambda+\frac{1}{C}=0 \;\rightarrow\; \lambda=-\frac{1}{RC}$

$q_s(t)=Ae^{-\frac{1}{RC}t}$

r.w.) $\frac{1}{C}q(t)=U_s$

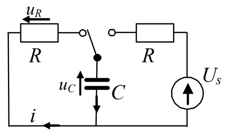
$q_w(t)=CU_s=q_{\max}$

r.o.) $q(t)=Ae^{-\frac{1}{RC}t}+CU_s$

w.p.) $0=Ae^{-\frac{1}{RC}0}+CU_s \;\rightarrow\; A=-CU_s$

r.s.) $\boxed{q(t)=CU_s\left(1-e^{-\frac{1}{RC}t}\right)}$

$i(t)=\frac{dq(t)}{dt}=\frac{U_s}{R}e^{-\frac{1}{RC}t} \quad , u_C(t)=\frac{q(t)}{C}=\frac{U_s}{RC}e^{-\frac{1}{RC}t}$



$u_R(t)+u_C(t)=0$

$Ri(t)+\frac{q(t)}{C}=0$

$R\dot{q}(t)+\frac{1}{C}q(t)=0 \;, q(0)=q_{\max}=CU_s$

$\frac{1}{C}q(t)=0$

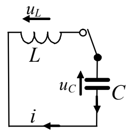
$q_w(t)=0$

$q(t)=Ae^{-\frac{1}{RC}t}$

$CU_s=Ae^{-\frac{1}{RC}0}{}_s \;\rightarrow\; A=CU_s$

$\boxed{q(t)=CU_se^{-\frac{1}{RC}t}}$

$i(t)=\frac{dq(t)}{dt}=-\frac{U_s}{R}e^{-\frac{1}{RC}t} \quad , u_C(t)=\frac{q(t)}{C}=\frac{U_s}{RC}e^{-\frac{1}{RC}t}$



$u_L(t)+u_C(t)=0$

$L\frac{di(t)}{dt}+\frac{1}{C}\int i(t)dt=0$

$L\ddot{q}(t)+\frac{1}{C}q(t)=0$

$\ddot{q}(t)+\frac{1}{LC}q(t)=0$

$\omega=\sqrt{\frac{1}{LC}}$