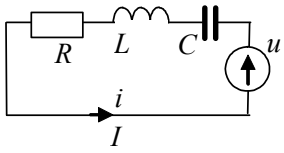
Modele obwodów elektrycznych



(1) $j\omega L\, I + R\, I + \frac{1}{j\omega C} I = U$

(2) $L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$ (3) $L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C}q(t) = u(t)$

(4) $sL\, i(s) + R\, i(s) + \frac{1}{sC} i(s) = u(s)$ (5) $i(s) = \frac{sC}{s^2 LC + sRC + 1} u(s)$

$u(t) = U\sin(\omega t)$

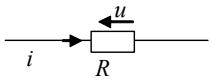
$i(t) = I\sin(\omega t + \varphi)$

$s = j\omega$

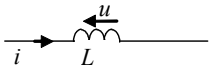
$i(t) = \frac{dq(t)}{dt}$

$i(s) = sq(s)$

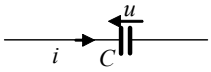
Obwody elektryczne



$u(t) = Ri(t)$



$u(t) = -L\frac{di(t)}{dt}$



$u(t) = \frac{1}{C}q(t)$

$u(t) = \frac{1}{C}\int i(t)dt$

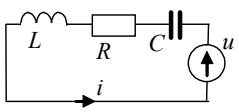
Tab. 1-6. Stosowane opisy podstawowych elementów elektrycznych

	$u(i)$		$i(u)$	$u(q)$	$Z(s)$
rezystor (R)	$u(t) = Ri(t)$	$u(s) = Ri(s)$	$i(t) = Gu(t)$	$u(t) = R\dot{q}(t)$	R
kondensator (C)	$u(t) = \frac{1}{C}\int i(t)dt$	$u(s) = \frac{1}{sC} i(s)$	$i(t) = C\frac{du(t)}{dt}$	$u(t) = \frac{1}{C}q(t)$	$\frac{1}{sC}$
cewka (L)	$e_r(t)$ $L\frac{di(t)}{dt}$	$u(s) = sLi(s)$	$i(t)$ $\frac{1}{L}\int u(t)dt$	$u(t) = L\ddot{q}(t)$	sL

$i(t) = \frac{dq(t)}{dt}$

$i(s) = sq(s)$

Obwody elektryczne



$$j\omega LI + RI + \frac{1}{j\omega C} I = U$$

$$sLi(s) + Ri(s) + \frac{1}{sC} i(s) = u(s)$$

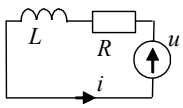
$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$$

$$L\ddot{q}(t) + R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$

$$q(s) = \left(\frac{C}{s^2 LC + sRC + 1}\right) u(s)$$

$$i(s) = \left(\frac{sC}{s^2 LC + sRC + 1}\right) u(s)$$

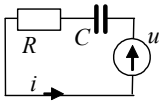
$$\boxed{i(s) = sq(s)}$$



$$L\frac{di(t)}{dt} + Ri(t) = u(t)$$

$$sLi(s) + Ri(s) = u(s)$$

$$i(s) = \left(\frac{1}{sL + R}\right) u(s)$$



$$Ri(t) + \frac{1}{C}\int i(t)dt = u(t)$$

$$Ri(s) + \frac{1}{sC} i(s) = u(s)$$

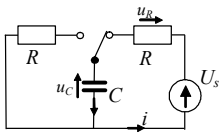
$$i(s) = \left(\frac{sC}{sRC + 1}\right) u(s)$$

$$R\dot{q}(t) + \frac{1}{C} q(t) = u(t)$$

$$sRq(t) + \frac{1}{C} q(t) = u(t)$$

$$q(s) = \left(\frac{C}{sCR + 1}\right) u(s)$$

Ładowanie/rozładowanie kondensatora



$$u_R(t) + u_C(t) = U_s$$

$$Ri(t) + \frac{q(t)}{C} = U_s$$

$$R\dot{q}(t) + \frac{1}{C} q(t) = U_s \quad , q(0) = 0$$

r.s.) $R\lambda + \frac{1}{C} = 0 \rightarrow \lambda = -\frac{1}{RC}$

$$q_s(t) = Ae^{-\frac{1}{RC}t}$$

r.w.) $\frac{1}{C} q(t) = U_s$

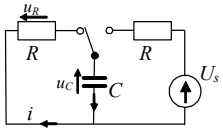
$$q_w(t) = CU_s = q_{\max}$$

r.o.) $q(t) = Ae^{-\frac{1}{RC}t} + CU_s$

w.p.) $0 = Ae^{-\frac{1}{RC}0} + CU_s \rightarrow A = -CU_s$

r.s.) $\boxed{q(t) = CU_s\left(1 - e^{-\frac{1}{RC}t}\right)}$

$$i(t) = \frac{dq(t)}{dt} = \frac{U_s}{R} e^{-\frac{1}{RC}t} \quad , \quad u_C(t) = \frac{q(t)}{C} = \frac{U_s}{RC} e^{-\frac{1}{RC}t}$$



$$u_R(t) + u_C(t) = 0$$

$$Ri(t) + \frac{q(t)}{C} = 0$$

$$R\dot{q}(t) + \frac{1}{C} q(t) = 0 \quad , q(0) = q_{\max} = CU_s$$

$$\frac{1}{C} q(t) = 0$$

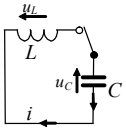
$$q_w(t) = 0$$

$$q(t) = Ae^{-\frac{1}{RC}t}$$

$$CU_s = Ae^{-\frac{1}{RC}0} \rightarrow A = CU_s$$

$$\boxed{q(t) = CU_s e^{-\frac{1}{RC}t}}$$

$$i(t) = \frac{dq(t)}{dt} = -\frac{U_s}{R} e^{-\frac{1}{RC}t} \quad , \quad u_C(t) = \frac{q(t)}{C} = \frac{U_s}{RC} e^{-\frac{1}{RC}t}$$



$$u_L(t) + u_C(t) = 0$$

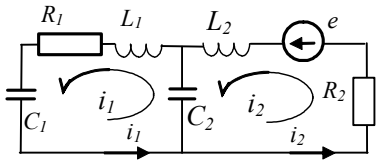
$$L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = 0$$

$$L\ddot{q}(t) + \frac{1}{C} q(t) = 0$$

$$\ddot{q}(t) + \frac{1}{LC} q(t) = 0$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Obwody elektryczne



$$\begin{cases} e = L_2\dfrac{di_2}{dt} + R_2 i_2 + \int \dfrac{i_2 - i_1}{C_2}\,dt \\ 0 = L_1\dfrac{di_1}{dt} + R_1 i_1 + \int \dfrac{i_1}{C_1}\,dt + \int \dfrac{i_1 - i_2}{C_2}\,dt \end{cases}$$

$$\begin{cases} e = sL_2 i_2 + R_2 i_2(s) + \dfrac{i_2 - i_1}{sC_2} \\ 0 = sL_1 i_1 + R_1 i_1 + \dfrac{i_1}{sC_1} + \dfrac{i_1 - i_2}{sC_2} \end{cases}$$

rząd?

$$\begin{cases} e = L_2\ddot{q}_2 + R_2\dot{q}_2 + \dfrac{q_2 - q_1}{C_2} \\ 0 = L_1\ddot{q}_1 + R_1\dot{q}_1 + \dfrac{q_1}{C_1} + \dfrac{q_1 - q_2}{C_2} \end{cases}$$

$$\begin{cases} sC_2 e = M_2 i_2 - i_1 \\ 0 = M_1 i_1 - C_1 i_2 \end{cases}$$

$$\begin{cases} C_2 e = M_2 q_2 - q_1 \\ 0 = M_1 q_1 - C_1 q_2 \end{cases}$$

$$M_2 = s^2 C_2 L_2 + sC_2 R_2 s + 1$$

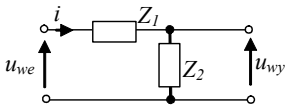
$$M_1 = s^2 C_1 C_2 L_1 + sC_1 C_2 R_1 + C_2 + C_1$$

$$i_1 = \frac{sC_1 C_2}{M_1 M_2 - C_1} e \qquad q_1 = \frac{C_1 C_2}{M_1 M_2 - C_1} e$$

$$i_2 = \frac{sC_2 M_1}{M_1 M_2 - C_1} e \qquad q_2 = \frac{C_2 M_1}{M_1 M_2 - C_1} e$$

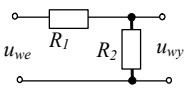
$$\boxed{i(s) = sq(s)}$$

Obwody elektryczne - czwórniki

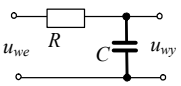


$$\begin{cases} u_{we} = (Z_1 + Z_2) i \\ u_{wy} = Z_2 i \end{cases}$$

$$u_{wy} = \left(\frac{Z_2}{Z_1 + Z_2}\right) u_{we}$$

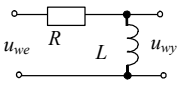


$$u_{wy}(s) = \left(\frac{R_2}{R_1 + R_2}\right) u_{we}(s)$$



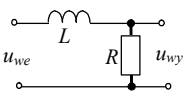
$$Z_1 = R$$
$$Z_2 = 1/sC$$

$$u_{wy}(s) = \left(\frac{1}{sCR + 1}\right) u_{we}(s)$$



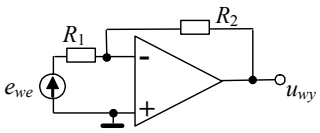
$$Z_1 = R$$
$$Z_2 = sL$$

$$u_{wy}(s) = \left(\frac{sL}{sL + R}\right) u_{we}(s)$$

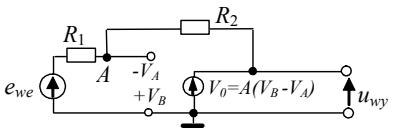


$$u_{wy}(s) = \left(\frac{R}{sL + R}\right) u_{we}(s)$$

Obwody elektryczne – wzmacniacze operacyjne

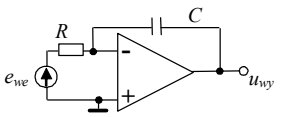


$$u_{wy} = \frac{-Z_2}{Z_1} e_{we}$$



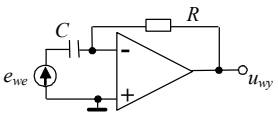
$$\begin{cases} \dfrac{V_A - e_{we}}{R_1} = \dfrac{V_0 - V_A}{R_2} \\ u_{wy} = V_0 = A(V_B - V_A) = -AV_A \end{cases}$$

$$u_{wy} = \frac{-R_2}{R_1 + \dfrac{R_1 + R_2}{A}} e_{we} \xrightarrow{A \to \infty} u_{wy} = \frac{-R_2}{R_1} e_{we}$$



$Z_1 = R$
$Z_2 = 1/sC$

$$u_{wy} = \frac{-1}{sRC} e_{we}$$

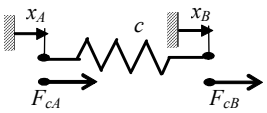


$Z_1 = 1/sC$
$Z_2 = R$

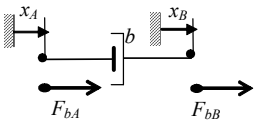
$$u_{wy} = -sRC\, e_{we}$$

[DBC: "Praktyczne wprowadzenie ... "]

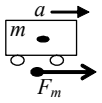
Układy mechaniczne



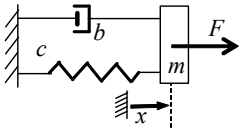
$$F_{cA}(t) = c\left(x_A(t) - x_B(t)\right)$$
$$F_{cB}(t) = c\left(x_B(t) - x_A(t)\right)$$



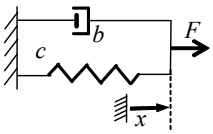
$$F_{bA}(t) = b\left(\dot{x}_A(t) - \dot{x}_B(t)\right)$$
$$F_{bB}(t) = b\left(\dot{x}_B(t) - \dot{x}_A(t)\right)$$



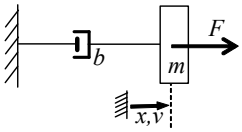
$$F_m(t) = m\ddot{x}(t)$$



$$m\ddot{x}(t) + b\dot{x}(t) + cx(t) = F(t)$$
$$ms^2x(s) + bsx(s) + cx(s) = F(s)$$



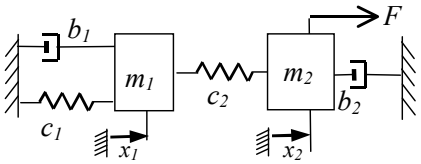
$$b\dot{x}(t) + cx(t) = F(t)$$
$$bsx(s) + cx(s) = F(s)$$
$$x(s) = \left(\frac{1}{sb + c}\right) F(s)$$



$$m\ddot{x}(t) + b\dot{x}(t) = F(t)$$
$$m\dot{v}(t) + bv(t) = F(t)$$
$$v(s) = \left(\frac{1}{sm + b}\right) F(s)$$

[DBC: "Praktyczne wprowadzenie ... "]

Układy mechaniczne



$$\begin{cases} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + c_2(x_2 - x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$

$$\begin{cases} F = s^2m_2x_2 + sb_2x_2 + c_2(x_2 - x_1) \\ 0 = s^2m_1x_1 + sb_1x_1 + c_1x_1 + c_2(x_1 - x_2) \end{cases}$$