Równanie równania n-tego rzędu

$$a_n x^{(n)} + \ldots + a_1 \dot{x} + a_0 x = 0$$

$$a_n \lambda^n + \ldots + a_1 \lambda + a_0 = 0 \longrightarrow a_n(\lambda - \lambda_k)\ldots.(\lambda^2 + b\lambda + c) = 0$$

Równanie 2. rzędu

$$\ddot{x}(t) + b_1 \dot{x}(t) + b_0 x(t) = u(t) \longleftarrow a_2 \ddot{x}(t) + a_1 \dot{x}(t) + a_0 x(t) = u_1(t)$$

$$b_0 \neq 0$$

$$\ddot{x}(t) + 2\xi\,\omega_n\,\dot{x}(t) + \omega_n^2 x(t) = u(t) \qquad \ddot{x}(t) + 2\xi\,\omega_n\,\dot{x}(t) - \omega_n^2 x(t) = u(t)$$

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0 \quad ,\omega_n > 0 \qquad \lambda^2 + 2\xi\omega_n\lambda - \omega_n^2 = 0 \quad ,\omega_n > 0$$

Równanie oscylacyjne

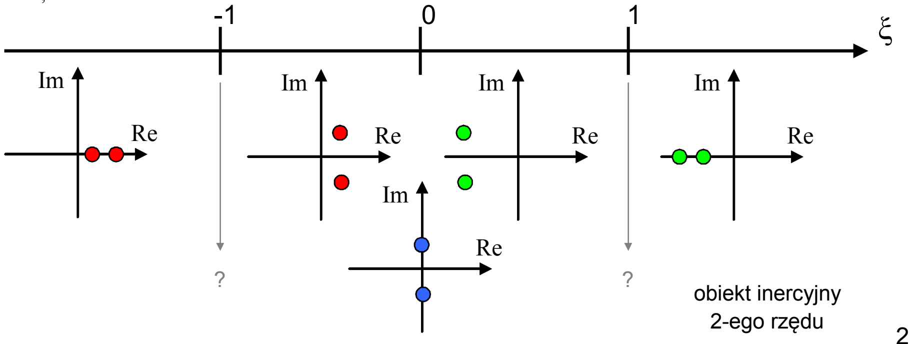
$\omega_n < 0$ - nie ma miejsca, bo ... $\qquad$ $\omega_n = 0$ - nie ma miejsca, bo ...

Równanie oscylacyjne

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0 \qquad \omega_n > 0$$

$$\lambda_{1,2} = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2} \longrightarrow \lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$\lambda_{1,2} > 0$ $\qquad$ $\mathrm{Re}(\lambda_{1,2}) > 0$ $\qquad$ $\mathrm{Re}(\lambda_{1,2}) < 0$ $\qquad$ $\lambda_{1,2} < 0$



obiekt inercyjny
2-ego rzędu

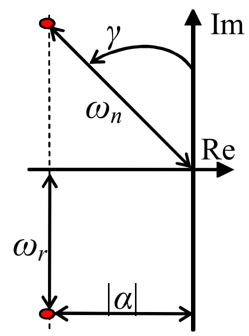
Położenie biegunów

$$\ddot{x}(t)+2\xi\,\omega_n\,\dot{x}(t)+\omega_n^2 x(t)=\omega_n^2 u(t)$$

$$s_{1,2}=\alpha \pm j\omega_r \qquad \alpha=-\xi\omega_n \qquad \omega_r=\omega_n\sqrt{1-\xi^2}$$

$$\sqrt{\alpha^2+\omega_r{}^2}=\sqrt{(-\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}=\omega_n$$

$$\sin\gamma=\frac{|\alpha|}{\sqrt{\alpha^2+\omega_r^2}}=\frac{\xi\omega_n}{\sqrt{\xi^2\omega_n^2+\omega_n^2(1-\xi^2)}}=\xi$$



Położenie biegunów a odpowiedź skokowa

$$\ddot{x}(t)+2\xi\,\omega_n\,\dot{x}(t)+\omega_n^2 x(t)=\omega_n^2 u(t)$$

$$\lambda_{1,2}=-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$\lambda_{1,2}=\alpha \pm j\omega_r \qquad \alpha=-\xi\omega_n \qquad \omega_r=\omega_n\sqrt{1-\xi^2}$$

Dla $0<\xi<1$ $(\rightarrow \alpha<0)$

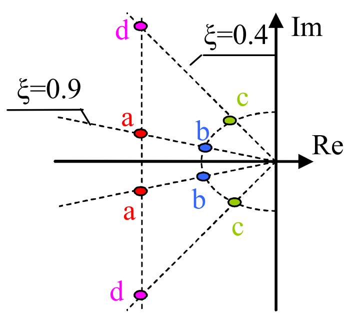
$$x_s(t)=A_1e^{(\alpha+j\omega_r)t}+A_2e^{(\alpha-j\omega_r)t}=$$

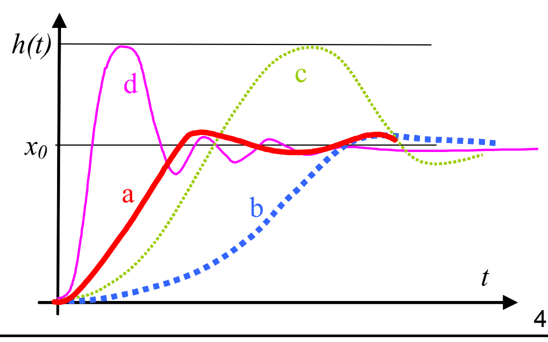
$$=e^{\alpha t}\left(B_1\cos\omega_r t+B_2\sin\omega_r t\right)=$$

$$=Ae^{\alpha t}\sin(\omega_r t+\varphi)$$

ω_n – pulsacja drgań własnych nietłumionych
ω_r – pulsacja drgań własnych tłumionych
ξ – współczynnik tłumienia względnego

$$u(t)=1(t) \qquad x(0)=0 \qquad \dot{x}(0)=0 \qquad x(t)=1+Ae^{\alpha t}\sin(\omega_r t+\varphi)$$





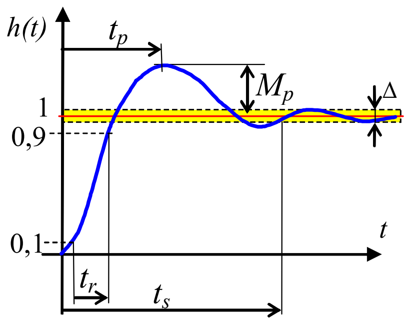
Równanie oscylacyjne a bezpośrednie wskaźniki jakości

$$\ddot{x}(t) + 2\xi\,\omega_n\,\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 u(t)$$

$$\lambda_{1,2} = \alpha \pm j\omega_r \qquad \alpha = -\xi\omega_n < 0$$

$$\omega_r = \omega_n\sqrt{1-\xi^2}$$

$$x(t) = 1 + Ae^{\alpha t}\sin(\omega_r t + \varphi)$$



Projektowanie własności układu

$a\ddot{x}(t) + 3\dot{x}(t) + 4x(t) = u(t)$

$a \neq 0$

$$\ddot{x}(t) + \frac{3}{a}\dot{x}(t) + \frac{4}{a}x(t) = \frac{u(t)}{a}$$

$\boxed{a > 0}$ $\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0,\ \omega_n > 0$

różne ξ

$\boxed{a < 0}$ $\lambda^2 + 2\xi\omega_n\lambda - \omega_n^2 = 0,\ \omega_n > 0$

$\boxed{a = 0}$ $3\dot{x}(t) + 4x(t) = u(t)$

$a\ddot{x}(t) + 3\dot{x}(t) + 4x(t) = u(t)$

$\Delta = 9 - 16a$

$$\lambda_{1,2} = \frac{-3 \pm \sqrt{\Delta}}{2a}$$

$a \neq 0$

$a = 0$

$3\dot{x}(t) + 4x(t) = u(t)$

$2\ddot{x}(t) + 3\dot{x}(t) + ax(t) = u(t)$

$$\ddot{x}(t) + \frac{3}{2}\dot{x}(t) + \frac{a}{2}x(t) = \frac{u(t)}{2}$$

$\boxed{a > 0}$ $\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0,\ \omega_n > 0$

różne ξ

$\boxed{a < 0}$ $\lambda^2 + 2\xi\omega_n\lambda - \omega_n^2 = 0,\ \omega_n > 0$

$\boxed{a = 0}$ $2\ddot{x}(t) + 3\dot{x}(t) = u(t)$

$2\ddot{x}(t) + 3\dot{x}(t) + ax(t) = u(t)$

$\Delta = 9 - 8a$

$$\lambda_{1,2} = \frac{-3 \pm \sqrt{\Delta}}{4}$$

$2\ddot{x}(t) + a\dot{x}(t) + 4x(t) = u(t)$

$$\ddot{x}(t) + \frac{a}{2}\dot{x}(t) + 2x(t) = \frac{u(t)}{2}$$

$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0,\ \omega_n > 0$

różne ξ

$2\ddot{x}(t) + a\dot{x}(t) + 4x(t) = u(t)$

$\Delta = a^2 - 32$

$$\lambda_{1,2} = \frac{-a \pm \sqrt{\Delta}}{4}$$

Portrety fazowe

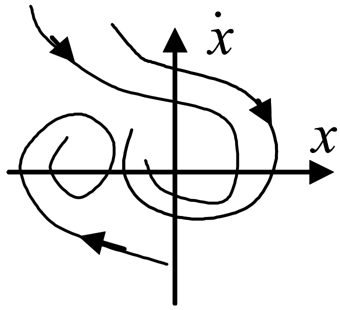
płaszczyzna fazowa $(\dot{x},\, x)$

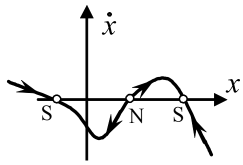
trajektoria fazowa (obraz ewolucji stanu)

- stałe wymuszenie

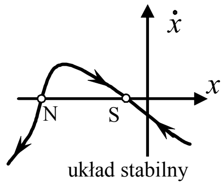
portret fazowy

- punkt(y) równowagi
- kierunek czasu
- przecięcie z osią *x*
- determinizm
- układy 1-2 rzędu
- układy liniowe/nieliniowe
- stabilność



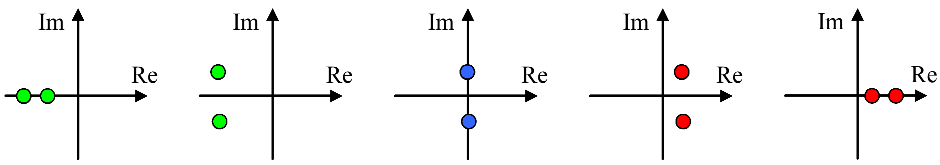


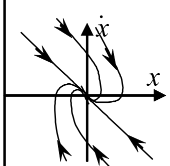
układ stabilny globalnie



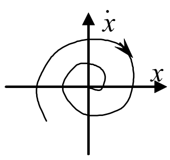
układ stabilny tylko lokalnie

Portrety fazowe

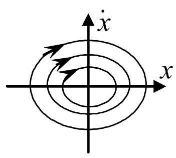




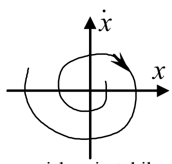
węzeł stabilny



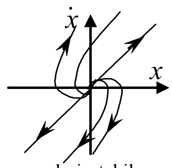
ognisko stabilne



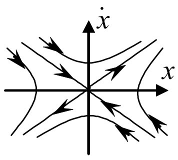
środek (centrum)



ognisko niestabilne

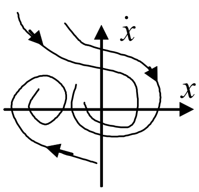


węzeł niestabilny

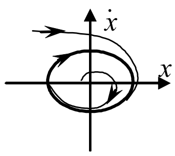


siodło

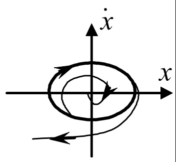
$(\lambda_1>0,\ \lambda_2<0)$



ukł.nieliniowy



cykl graniczny st.



cykl graniczny nst.