Electromagnetics Problem 1

Innova Lee(이상훈) gcccompil3r@gmail.com

균일한 선밀도를 가진 선전하의 전기장 문제

$$\begin{split} & \int \frac{1}{(a^2 + u^2)^{\frac{3}{2}}} du \\ & \sin^2 x + \cos^2 x = 1 \Leftrightarrow a^2 tan^2 x + a^2 = \frac{a^2}{\cos^2 x} = a^2 sec^2 x \\ & u^2 = a^2 tan^2 x \\ & u = atanx, \quad du = asec^2 x dx \\ & \frac{d}{dx} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\ & \frac{d}{dx} \left\{ tan(x) \right\} = \frac{d}{dx} \left\{ \frac{\sin(x)}{\cos(x)} \right\} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = sec^2(x) \\ & \int \frac{1}{(a^2 + u^2)^{\frac{3}{2}}} du = \int \frac{asec^2 x}{(a^2 + a^2 tan^2 x)^{\frac{3}{2}}} dx \\ & a^2 (1 + tan^2 x) = a^2 sec^2 x \qquad \left(\because 1 + tan^2 x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = sec^2 x \right) \\ & \int \frac{1}{(a^2 + u^2)^{\frac{3}{2}}} du = \int \frac{asec^2 x}{(a^2 + a^2 tan^2 x)^{\frac{3}{2}}} dx = \int \frac{asec^2 x}{(a^2 sec^2 x)^{\frac{3}{2}}} dx = \int \frac{asec^2 x}{a^3 sec^3 x} dx = \int \frac{1}{a^2} \frac{1}{sec(x)} dx = \frac{1}{a^2} \int \cos(x) \, dx = \frac{\sin(x)}{a^2} \\ & \frac{1}{a^2} \sin\left(tan^{-1} \left(\frac{u}{a}\right)\right) \qquad \left(\because \frac{u}{a} = tan(x), \qquad x = tan^{-1} \left(\frac{u}{a}\right) \right) \\ & \frac{1}{a^2} \sin(\theta) = \frac{1}{a^2} \frac{u}{\sqrt{u^2 + a^2}} = \frac{1}{a^2} \frac{u}{\sqrt{u^2 + a^2}} \end{split}$$