

MATH 271, WORKSHEET 3  
FIRST ORDER AUTONOMOUS AND LINEAR EQUATIONS. CHEMICAL KINETICS.

**Problem 1.** Let's revisit Newton's law of cooling and describe the equilibria of the system.

- (a) Is the system autonomous? Explain.
- (b) Draw a phase line for the system.
- (c) Can you explain why the value you find for the equilibrium makes sense? Is this equilibrium be stable? Does this make sense? Explain.

**Problem 2.** Are the following ODEs separable, autonomous, linear, nonlinear, or none of the above? Keep in mind that some may satisfy more than one property!

- (a)  $x' = \sin(tx)$ .
- (b)  $x' = \sin(x)$ .
- (c)  $x' = t^2x$ .
- (d)  $e^tx' + tx = e^{-kt} \cos(\omega t)$ .

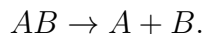
**Problem 3.** For the following problems, show that the equation is linear by writing the equation in a recognizable form.

- (a)  $\frac{x'}{x} = t$ .
- (b)  $2xx' + x^2 = xt$ .
- (c)  $\tan(t)x' + \sin(t)x = \ln(t)$ .

**Problem 4.** For the above linear equations, determine the integrating factor (even if you cannot compute the integral) and determine the solution  $x(t)$  (again, even if you cannot compute the integral).

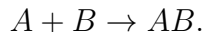
**Problem 5.** Find the solution to the equation  $x' + x = t^2$ .

**Problem 6.** Consider the dissociation chemical reaction



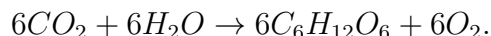
Write down an equation for each species  $A$ ,  $B$ , and  $AB$ . Find a solution.

**Problem 7.** Compare and contrast the equations for the above reaction and the synthesis reaction



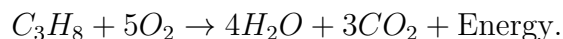
*Hint: maybe this is a bit of an odd way to think, but is one just the reversal in time of the other?*

**Problem 8.** Photosynthesis is an extremely important chemical reaction where plants convert carbon dioxide and water into glucose and oxygen. That is,



- (a) Write down the equations that describe the above equation.
- (b) Do we have any techniques to solve this type of equation (yet)?
- (c) The reaction also depends on the intensity of light. How can we implement this into the equation?

**Problem 9.** Everybody loves combustion. It keeps us warm and it looks cool on the 4<sup>th</sup> of July! Anyhow, combustion of propane in your grill at home is given by the equation



- (a) Write down the equations describing the above reaction (not including the energy term).
- (b) If a specific amount of energy is given off by each reaction (in this case, 2043kJ of energy per mole), include an equation for the total energy created if we begin with 1kg of propane in an abundance of oxygen.
- (c) Can you give some analogy to how much energy this is so that we may understand the usefulness a bit more? E.g., how many hot dogs could I cook on the 4<sup>th</sup> of July?