MATH 272, Homework 2

Due February 11$^{\text{th}}$

Problem 1.

Problem 2.

Problem 3.

Problem 4.

Problem 5. Consider maybe the momentum operator $i\frac{d}{dx}$ instead and show how you can build the Hamiltonian with this Consider the differential operator $\frac{d}{dx}$ acting on (sufficiently) differentiable complex valued functions with input $[0, L]$.

(a) Using the inner product for complex valued functions

$$\langle f, g \rangle = \int_0^L f(x) g^*(x) dx,$$

show that $\frac{d}{dx}$ is <u>not</u> Hermitian. *Hint: you will want to use integration by parts.*

(b) Indeed, this means the spectrum of $\frac{d}{dx}$ is not necessarily real. To see the spectrum for this operator, we can look at the eigenvalues λ that solve the equation

$$\frac{d}{dx} f(x) = \lambda f(x).$$

- What (complex) values can λ take on?
- What are the corresponding eigenfunctions for the derivative operator?

(c) Compare your result in (b) with the spectrum of the operator $\frac{d^2}{dx^2}$ that we see in the free Schrödinger equation (for either the particle in the 1-dimensional box or the particle on the ring)

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x) = E\Psi(x).$$