The story of Fourier and his theory of heat and trigonometric series

Colin Roberts

Colorado State University

Overview

1. **Fourier's life**
2. **The propagation of heat in solid bodies**
3. **Fourier series**
4. **Reception**

Section 1

Fourier's life

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- He was one of twelve children and was orphaned at age 10.
- He first received education at a local convent and moved to the École Royale Militaire of Auxerre after recommendation.

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- He was pardoned after the death of Maximilien Robespierre.

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- A few years later, Fourier joined Napoleon's army as a scientific advisor for the invasion of Egypt.
- Fourier continued to teach until 1801 when Napoleon appointed him as prefect in Grenoble.

Section 2

The propagation of heat in solid bodies

problem is now to integrate
imply

$$\frac{dv}{dt} = k\frac{d^2v}{dx^2} - hv,$$

ANALYTICAL THEORY OF HEAT

BY

Jean Baptiste JOSEPH FOURIER.

TRANSLATED, WITH NOTES,

BY

ALEXANDER FREEMAN, M.A.,

FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

Idea

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- Fourier assumed that heat moved linearly through a medium.

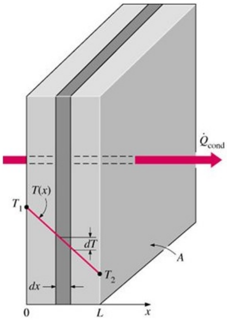
Conduction

- Let T_1 and T_2 be temperatures of two slabs.
- Heat moves through the slabs via

$$\dot{Q} = kA\frac{T_1 - T_2}{L}.$$

- In the infinitesimal case,

$$\dot{Q} = -kA\frac{dT}{dx}$$



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$$(\dot{Q}(x+\Delta x) - \dot{Q}(x))A - \dot{q}A\Delta x = \rho c \frac{\partial T}{\partial t} A\Delta x.$$

- In the infinitesimal case we get the heat equation

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \dot{q}.$$

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- He described the dimensions and meaning of all the constants above

$$k = \text{conductivity}$$

$$c = \text{heat capacity}$$

$$\rho = \text{density}$$

Section 3

Fourier series

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- He found a set of solutions as $e^{-n^2\pi^2 t}\cos(2\pi nx)$ and $e^{-n^2\pi^2 t}\sin(2\pi nx)$.
- He questioned whether any initial condition could be written using these solutions.

Trigonometric series

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- He attempts to solve

$$1 = \sum_{n \text{ odd}} a_n \cos(nx)$$

for which he found

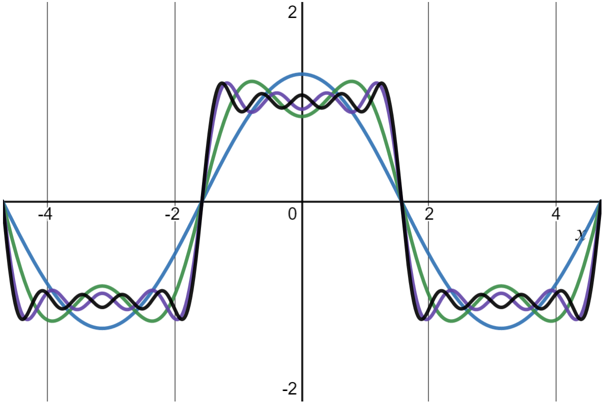
$$a_1 = \frac{3^2}{3^2 - 1^2} \cdot \frac{5^2}{5^2 - 1^2} \cdots \qquad a_3 = \frac{1^2}{1^2 - 3^2} \cdot \frac{5^2}{5^2 - 3^2} \cdots \ldots$$

and so on, then used Wallis' formula to show

$$a_n = (-1)^n \frac{4}{n\pi}$$

Eventually, he realizes

$$\sum_{n\text{ odd}} (-1)^n \frac{4}{n\pi} \cos(nx) = \begin{cases} \frac{\pi}{4} & 0 < x < \frac{\pi}{2} \\ -\frac{\pi}{4} & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$



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- Ultimately, he determines that one can determine coefficients in the series

$$f(x) = c_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

via integrals

$$a_n = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} f(x) \cos(nx)\, dx \qquad b_n = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} f(x) \sin(nx)\, dx.$$

Section 4

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- Biot and Poisson both attacked the theory as well. Fourier proved Biot's alternate route false. Poisson claimed to have another theory.
- Fourier eventually published in 1822.

And the rest is history...