MATH 272, Homework 6
Due March 16$^{\text{TH}}$

Problem 1. Let

$$\vec{\gamma}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix}, \quad f(x,y,z) = x^2 + y^2 - 2z^2, \quad \vec{V}(x,y,z) = \begin{pmatrix} x - y \\ y + x \\ z \end{pmatrix}.$$

Compute derivatives of the following composite functions.

(a) $f(\vec{\gamma}(t))$.

(b) $\vec{V}(\vec{\gamma}(t))$.

(c) $\vec{\gamma}(\vec{V}(x,y,z))$.

Problem 2. Show that for any fields $f(x,y,z)$ and $\vec{V}(x,y,z)$ that

(a) $\vec{\nabla} \times \left(\vec{\nabla} f \right) = 0$;

(b) $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{V} \right)$.

Problem 3. Let

$$\vec{U}(x,y,z) = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \qquad \text{and} \qquad \vec{V}(x,y,z) = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix},$$

be vector fields.

(a) Explain why there exists no potential function $f(x,y,z)$ for the vector field $\vec{U}$.

(b) Explain why there does exist a potential function $f(x,y,z)$ for the field $\vec{V}$.

(c) Compute the potential function for $\vec{V}$.

Problem 4. Parameterize the following either implicitly or explicitly. In Cartesian coordinates, find the parameterization of the normal vector as well.

(a) The plane perpendicular to the vector $\vec{v} = \hat{x} + \hat{y} + \hat{z}$.

(b) The upper half of the unit circle in $\mathbb{R}^2$.

(c) The surface of the unit sphere in $\mathbb{R}^3$.

Problem 5. In cylindrical coordinates (either implicitly or explicitly), parameterize the following objects.

(a) A cylinder with radius 3 and height 5 along with end-caps.

(b) An infinite cone with a vertex angle of $\pi/4$.

(c) A helical curve with constant radius 1 and pitch 1.

(d) A hyperboloid of one sheet.