
APPLIED MATHEMATICS FOR CHEMISTS II

VECTOR FIELDS, PARTIAL DIFFERENTIATION, CYLINDRICAL AND SPHERICAL COORDINATES,
MULTIPLE INTEGRALS, LINE INTEGRALS, THE WAVE AND THE SCHRÖDINGER EQUATIONS,
SEPARATION OF VARIABLES METHOD. INNER PRODUCT SPACES. FOURIER SERIES.

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Part V

Further Topics in Linear Algebra

1

Hilbert Spaces

2

Linear Operators

spectral theory with fourier and with 1st and second order derivatives and matrices. different domains for different spectra, schrodinger equation with quadratic potential QHO. Start with complex functions since they introduce multivariate functions and inner products nicely. Can talk about $U(1)$?

Part VI

Multivariate Calculus

3

Functions

1 Overview of multivariate functions

Now that we have covered enough of the complex numbers, we will move back into the vector space \mathbb{R}^n and analyze the types of functions we can have with this space. Specifically, we will concentrate on \mathbb{R}^3 (or \mathbb{R}^2) and functions of the form:

$$\gamma: \mathbb{R} \rightarrow \mathbb{R}^3 \quad (\text{Eq. 3.1.1})$$

$$f: \mathbb{R}^3 \rightarrow \mathbb{R} \quad (\text{Eq. 3.1.2})$$

$$\mathbf{v}: \mathbb{R}^3 \rightarrow \mathbb{R}^3. \quad (\text{Eq. 3.1.3})$$

Abstractly, I could call each one of these functions a *field* (in the physics sense). However, I'll refrain from this (and let the mathematicians breathe a sigh of relief).