MATH 271, Quiz 2

Due October 9^{th} at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 6 TOTAL PROBLEMS.

Problem 1. For the following, say whether the statement is true or false. For full credit, justify your answer with an explanation.

(a) **(2 pts.)** If the sequence $a_n \to 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(b) **(2 pts.)** The power series $\sum_{n=1}^{\infty} x^n$ has an infinite radius of convergence.

Problem 2. (3 pts.) What is the derivative of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^n}$$

Problem 3. (3 pts.) What is the antiderivative of the power series

$$g(x) = \sum_{n=0}^{\infty} (n+1)x^n$$

Problem 4. (3 pts.) For what values of x does the above series $g(x)$ converge? In other words, what is the radius of convergence?

Problem 5.

(a) **(3 pts.)** Write down the first order approximation to $\tan(x)$ about the point $x = 0$.
(b) **(2 pts.)** Explain how you would determine a higher order approximation.

Problem 6. (2 pts.) Consider the differential equation

$$x' = t^2 + \tan(x).$$

Explain how you can use a first order approximation to $\tan(x)$ to approximate the above (nonlinear) equation as a first order linear equation. Should say something about initial conditions