MATH 272, Exam 1

Take Home Portion

Due February 19$^{\text{th}}$ at the start of class

Name ________________________________

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha). for this portion of the exam. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct. **Print out this sheet and staple your solutions to it. Use a new page for each problem.**

Problem 1 ____/10 **Problem 2** ____/15

Note, these problems span two pages.

In the heart of quantum mechanics lies probability theory. Since we can only compute the likelihood of observing something, we have to start thinking in this framework.

Problem 1. Consider a free particle constrained to the infinite line $\mathbb{R}$. Heisenberg's uncertainty principle states that one can never simultaneously know the position and momentum of a particle.

Suppose that we measure the *exact* position of a particle at time $t = 0$ to be at x_0. Then the wavefunction for this particle is given by

$$\Psi(x) = \delta(x - x_0).$$

Thus, since we know the position exactly, we expect to know *nothing* about the particle's momentum.

(a)

Problem 2. Probability is described via probability distribution functions (PDFs) $p(x)$. Intuitively, the value of $p(x)$ at the point x describes the weighting of that point x. We require that

- $p(x) \geq 0$ for all $x \in \mathbb{R}$. (*There is no negative probability.*)
- $\displaystyle\int_{-\infty}^{\infty} p(x)dx = 1$. (*The total probability is 1.*)

We can then define the probability of measuring an outcome in the region $[a, b]$ by

$$\mathcal{P}_{[a,b]}[p(x)] = \int_a^b p(x)dx.$$