MATH 271, Quiz 2

Due September 18th at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 4 TOTAL PROBLEMS.

Problem 1. Describe the following ODEs using the adjectives separable, autonomous, linear, or nonlinear, and provide the order of the equation as well. Then explain what you must specify as initial data in order to have a well-defined initial value problem.

(a) **(2 pts.)** $x'''' = k^4 x$.

(b) **(2 pts.)** $\frac{1}{t}x' + \sin(t)x = \frac{\sin(t)}{t}$.

(c) **(2 pts.)** $mx'' + \mu x' + kx = \sin(x)$.

Problem 2. For the following chemical reactions, determine the rate of change of the concentration for species A. Mention whether the equation is linear or nonlinear. *Do not couple the reactions together; treat them as separate entities.*

(a) **(2 pts.)** $3A \to B$.

(b) **(2 pts.)** $B + C \to 2A$.

Problem 3. Consider the following initial value problem

$$\begin{cases} x' = t & \text{as the ODE} \\ x(0) = 0 & \text{as the initial data.} \end{cases}$$

(a) **(2 pts.)** Find the general solution to the ODE.

(b) **(2 pts.)** Find a particular solution satisfying the initial value problem.

Problem 4. Consider the following initial value problem

$$\begin{cases} x'' + 3x' + 2x = 0 & \text{as the ODE} \\ x(0) = 1,\ x'(0) = 0 & \text{as the initial data.} \end{cases}$$

Note that the particular solution is $x(t) = 2e^{-t} - e^{-2t}$.

(a) **(2 pts.)** Show that x solves the ODE without using the characteristic polynomial.
(b) **(1 pt.)** Show that x satisfies the initial conditions.
(c) **(2 pts.)** Find the characteristic polynomial for the ODE.
(d) **(2 pts.)** Find the roots to the characteristic polynomial and write down a general solution.
(e) **(1 pt.)** What happens with this system if we wait a *very* long time?