

Mathematics of Plasma and the Coordinate Free Vlasov Equation

FColin Roberts* and Second B. Author Jr.[†]
Business or Academic Affiliation 1, City, State, Zip Code

Third C. Author[‡]
Business or Academic Affiliation 2, City, Province, Zip Code, Country

Fourth D. Author[§]
Business or Academic Affiliation 2, City, State, Zip Code

The kinematics and self interaction of charges and neutral particles is described by combining Maxwell's equations of electromagnetism with the Boltzmann equation of gas/fluid kinematics. In tandem, we refer to the set of these equations as the Vlasov equations. One can note that the theory of electromagnetism is purely topological and, in fact, through the lens of relativity, Maxwell's equations are purely topological. This perspective is immensely powerful, but it does not immediately ascend to providing us with a topological version of Vlasov's equation due to Boltzmann's equation. There have been versions of Vlasov's equations which are, at least, coordinate free.

Our goals are to understand the coordinate free Vlasov equations and seek to determine a topological version, if possible. We take two novel approaches. First, we consider relativistic phase space and construct the Boltzmann equations and investigate the symplectic and contact structures therein. Second, we take an approach to describing the relativistic motion of single charged particles via differential equations of spinors and attempt to extend this to a fluid of charges. An analogy forms – the equations of motion of lone charges immersed in an electromagnetic field follows paths in the Poincaré group much like the motion of rigid body can be seen as a path (in fact, geodesics) in the Euclidean group as described by Vladimir Arnol'd. In the same vein, Arnol'd determined that incompressible fluids follow geodesics on the infinite dimensional group of volume preserving diffeomorphisms. This begs the question of whether ideal charged fluids behave similarly but on, perhaps, some other group.

I. Nomenclature

A	=	amplitude of oscillation
a	=	cylinder diameter
C_p	=	pressure coefficient
C_x	=	force coefficient in the x direction
C_y	=	force coefficient in the y direction
c	=	chord
dt	=	time step
F_x	=	X component of the resultant pressure force acting on the vehicle
F_y	=	Y component of the resultant pressure force acting on the vehicle
f, g	=	generic functions
h	=	height
i	=	time index during navigation
j	=	waypoint index
K	=	trailing-edge (TE) nondimensional angular deflection rate

*Insert Job Title, Department Name, Address/Mail Stop, and AIAA Member Grade (if any) for first author.

[†]Insert Job Title, Department Name, Address/Mail Stop, and AIAA Member Grade (if any) for second author.

[‡]Insert Job Title, Department Name, Address/Mail Stop, and AIAA Member Grade (if any) for third author.

[§]Insert Job Title, Department Name, Address/Mail Stop, and AIAA Member Grade (if any) for fourth author (etc.).

II. Introduction

Plasma dynamics is a complicated problem with many facets of interest in many communities. Since plasmas consist of freely moving charged particles, the evolution of a plasma is tightly coupled to its self generated electromagnetic (EM) field. A consistent theory should provide a coupling of kinematic (or fluid) equations for the plasma to the self induced field via the Lorentz force under the constraints of Maxwell's equations. Examples of these equations are given by, but not limited to, magnetohydrodynamics and the Vlasov equation.

If we take for example, the Vlasov equation, we find that it is, in essence, a combination of Maxwell's equations (which describe the field produced by charges) with the collisionless Boltzmann equation (which describes the kinematics of these charges). Fundamentally, Maxwell's equations are topological. They solely require that the spacetime manifold M^4 admits a $3 + 1$ -foliation and do not hinge on a metric structure on spacetime [? ?]. It is a worthy question to ask if the Vlasov equations admit a purely topological understanding as well. Hence, we really seek to find a topological version of the collisionless Boltzmann equation.

Kinematics on spacetime is a touchy subject. Sadly, there is an inability to describe the worldlines of more than one particle. Sarbach and Zannias have produced a series of papers [? ? ?] that provide a relativistic version of Boltzmann's equation for a single particle which they claim is useful for describing the average properties of a gas. In fact, they even provide the equations for a charged and massive gas that includes self generated EM field and gravitation. In our case, we can ignore gravitational interactions since we wish to consider plasma on the small and low mass scale. Almost 50 years prior to Sarbach and Zannias, Bichteler produced the paper [?] which outlines a similar approach in the first few sections.

Colin: cite something here

III. Procedure for Paper Submission

All manuscripts are to be submitted electronically to the ScholarOne Abstracts site created for each conference. The manuscript upload will be enabled several weeks after acceptance notices have been sent. Presenting authors of accepted papers will receive an email with instructions when manuscript submission is open. It is important that presenting authors keep their email addresses up-to-date so they do not miss this notice.

Before completing manuscript submission, submitters must also select the copyright statement that will appear on the paper, and complete other acknowledgments. It is also necessary to click both the "Accept" and "Save" buttons to complete a submission. All completed manuscript submission will be confirmed by email. Completed submissions will also have a status of "Accepted" at the top of your manuscript submission page.

All files must be in pdf format. Please be sure that all security settings are removed from the pdf file before uploading to ensure proper processing of your manuscript file.

IV. General Guidelines

The following section outlines general (nonformatting) guidelines to follow, drawn from the original AIAA Manuscript Preparation Kit. These guidelines are applicable to all authors (except as noted), and include information on the policies and practices relevant to the publication of your manuscript.

A. Publication by AIAA

Your manuscript cannot be published by AIAA if:

- 1) It has been published previously or
- 2) The work contains copyright-infringing material or
- 3) An appropriate copyright statement has not yet been selected.

B. Paper Review and Visa Considerations

It is the responsibility of the author to obtain any required government or company reviews for their papers in advance of publication. Start early to determine if the reviews are required; this process can take several weeks.

If you plan to attend an AIAA Forum, technical conference or professional development course held in the United States and you require a visa for travel, it is incumbent upon you to apply for a visa with the U.S. embassy (consular division) or consulate with ample time for processing. To avoid bureaucratic problems, AIAA strongly suggests that you submit your formal application to U.S. authorities a minimum of 120 days in advance of the date of anticipated travel.

Prospective conference and course attendees requiring a visa to travel to the United States should first contact

AIAA to request an official letter of invitation. This letter and a copy of the conference call for papers should be presented along with the required documentation to the U.S. consular officials as part of the formal application process. AIAA cannot directly intervene with the U.S. Department of State, consular offices, or embassies on behalf of individuals applying for visas. A letter of invitation can be requested by completing the Visa Invitation Letter Request Form at <https://www.aiaa.org/Secondary.aspx?id=6258> or you may contact the Event Registrar at invitation@aiaa.org for more information.

C. Control ID Number vs Paper Number

Your paper was assigned a control ID number at the time you submitted your abstract. It is critical that you reference the tracking number and conference name when contacting AIAA regarding your submission. The control ID number is *not* the final AIAA paper number. The paper number, which appears in the format AIAA-20XX-XXXX, will be used to refer to your paper in the program and in any publication format. It will not be assigned until shortly before the conference. **Do not include a paper number anywhere on your paper, as this number will be stamped automatically in the top right corner of your paper at the time of processing.**

D. Copyright

Before AIAA can print or publish any paper, the copyright information must be completed in the submission system. Failure to complete the electronic form correctly could result in your paper not being published. The following fields must be completed:

- 1) Clearance Statement
- 2) Non-Infringement Statement
- 3) Publication Status Statement
- 4) One Copyright Assignment Statement (Select either A, B, C, or D)

Be sure to read the copyright statements carefully. AIAA requires a copyright transfer from the author(s) to AIAA or a license to publish and distribute your material; government authors can assert that the work is in the public domain. If you are not sure which copyright statement to use, contact your legal department. Refer to AIAA's Rights and Permissions page at www.aiaa.org for more information; AIAA cannot help you determine which statement to use. Do not include a copyright statement anywhere on your paper. A hard copy of the form is found in the Author Kit for your reference. As you will be completing this form online, you do not need to fill out the hard-copy form. Do not include a copyright statement anywhere on your paper, and do not upload a copyright form with your paper. The correct statement will be stamped automatically at the time of processing.

E. Submission Deadlines

Manuscripts will be accepted for upload to the system from the receipt of the email invitation until the deadline set for the conference. You will be notified of the specific manuscript submission deadline in your acceptance letter, and the deadline will also be listed on the conference web page at AIAA. Please do not upload a draft version of your manuscript with the intent to upload a final version later. **Please review your manuscript very carefully before completing your submission to ensure that your paper is complete and final in all respects. Once the manuscript deadline has passed you will be locked out of the manuscript site so it is critical that you upload a final, carefully proofed document.**

Online conference proceedings will be made accessible to attendees who have registered for the "full conference" when the conference opens. Once the proceedings are published online, the conference papers will be considered the version of record and may not be removed or replaced. Changes to published papers can be made available through the Crossmark feature, where corrections and updates are accessed by clicking the Crossmark icon available on every paper published in Aerospace Research Central.

The opportunity to submit Crossmark updates will be provided to presenting authors starting the first day of the conference through 2000 hrs/8 pm Eastern Time, seven business days after the last day of the conference. The proceedings will be updated with Crossmark updates shortly after that date.

AIAA will NOT accept changes and/or change requests that solely correct grammatical errors, spelling errors, or errors in formatting. All corrections should be for editorially significant changes where the change affects interpretation or crediting of the work.

To ensure conference quality, session chairs will enforce a "no paper, no podium" rule. This policy is intended

to eliminate no-shows, to improve the quality of the conference for all participants, and to ensure that the published proceedings accurately represent the presentations made at a conference.

V. Detailed Formatting Instructions

The styles and formats for the AIAA Papers Template have been incorporated into the structure of this document. If you are using \LaTeX , please use this template to prepare your manuscript. A Microsoft Word template is also available from AIAA's website if you prefer to use Microsoft Word 2001 or later. Regardless of which program you use to prepare your manuscript, please use the formatting instructions contained in this document as a guide.

If you are using the AIAA Meeting Papers \LaTeX Template file to prepare your manuscript, you can simply type your own text over sections of this document, or cut and paste from another document and use the available markup styles. If you choose to cut and paste, select the text from your original document and choose Edit>Copy. (Do not select your title and author information, since the document spacing may be affected. It is a simple task to reenter your title and author information in the template.) Open the \LaTeX template file. Place your cursor in the text area of the template and select Edit>Paste. Please note that special formatting (e.g., subscripts, superscripts, italics) may be lost when you copy your text into the template. Use italics for emphasis; do not underline. Use the compiled \LaTeX pdf to see the most accurate representation of how your final paper will appear.

A. Document Text

The default font for AIAA papers is Times New Roman, 10-point size. The first line of every paragraph should be indented, and all lines should be single-spaced. Default margins are 1" on all sides. In the electronic version of this template, all margins and other formatting is preset. There should be no additional lines between paragraphs.

Extended quotes, such as this example, are to be used when material being cited is longer than a few sentences, or the standard quotation format is not practical. In this \LaTeX template, the appropriate command environment is `\begin{quoting}... \end{quoting}`. Extended quotes are to be in Times New Roman, 9-point font, indented 0.4" and full justified.

NOTE: If you are using the electronic \LaTeX template to format your manuscript, the required spacing and formatting will be applied automatically.

B. Headings

The title of your paper should be typed in bold, 24-point type, with capital and lower-case letters, and centered at the top of the page. The names of the authors, business or academic affiliation, city, and state/province should follow on separate lines below the title. The names of authors with the same affiliation can be listed on the same line above their collective affiliation information. Author names are centered, and affiliations are centered and in italic type immediately below the author names. The affiliation line for each author is to include that author's city, state, and zip/postal code (or city, province, zip/postal code and country, as appropriate). The first-page footnotes (lower left-hand side) contain the job title and department name, street address/mail stop, and AIAA member grade for each author. Author email addresses may be included also.

Major headings ("sections" in the \LaTeX template commands) are bold 11-point font, centered, and numbered with Roman numerals.

Subheadings ("subsections" in the \LaTeX template commands) are bold, flush left, and numbered with capital letters.

Sub-Subheadings ("subsubsections" in the \LaTeX template commands) are italic, flush left, and numbered (1. 2. 3. etc.)

C. Abstract

The abstract should appear at the beginning of your paper. It should be one paragraph long (not an introduction) and complete in itself (no reference numbers). It should indicate subjects dealt with in the paper and state the objectives of the investigation. Newly observed facts and conclusions of the experiment or argument discussed in the paper must be stated in summary form; readers should not have to read the paper to understand the abstract. The abstract should be bold, indented 3 picas (1/2") on each side, and separated from the rest of the document by blank lines above and below the abstract text..

D. Nomenclature

Papers with many symbols may benefit from a nomenclature list that defines all symbols with units, inserted between the abstract and the introduction. If one is used, it must contain all the symbology used in the manuscript, and the definitions should not be repeated in the text. In all cases, identify the symbols used if they are not widely recognized in the profession. Define acronyms in the text, not in the nomenclature.

E. Footnotes and References

Footnotes, where they appear, should be placed above the 1" margin at the bottom of the page. To insert footnotes into the template, use the Insert>Footnote feature from the main menu as necessary. Numbered footnotes as formatted automatically in the template are acceptable, but superscript symbols are the preferred AIAA style, *, †, ‡, §, ¶, **, ††, ‡‡, §§, etc.

List and number all references at the end of the paper. Corresponding bracketed numbers are used to cite references in the text [?], including citations that are an integral part of the sentence (e.g., “It is shown in [?] that . . .”) or follow a mathematical expression: “ $A^2 + B = C$ (Ref. [?]).” For multiple citations, separate reference numbers with commas [? ?], or use a dash to show a range [? ? ? ?]. Reference citations in the text should be in numerical order.

In the reference list, give all authors’ names; do not use “et al.”. Papers that have not been published should be cited as “unpublished”; papers that have been submitted or accepted for publication should be cited as “submitted for publication.” Private communications and personal website should appear as footnotes rather than in the reference list.

References should be cited according to the standard publication reference style (for examples, see the “References” section of this template). Never edit titles in references to conform to AIAA style of spellings, abbreviations, etc. Names and locations of publishers should be listed; month and year should be included for reports and papers. For papers published in translation journals, please give the English citation first, followed by the original foreign language citation.

F. Images, Figures and Tables

All artwork, captions, figures, graphs, and tables will be reproduced exactly as submitted. Be sure to position any figures, tables, graphs, or pictures as you want them printed. AIAA will not be responsible for incorporating your figures, tables, etc. (Company logos and identification numbers are not permitted on your illustrations.)

Do not insert your tables and figures in text boxes. Figures should have no background, borders, or outlines. In the L^AT_EX template, use the “caption” command to type caption text. Captions are bold with a single tab (no hyphen or other character) between the figure number and figure description.

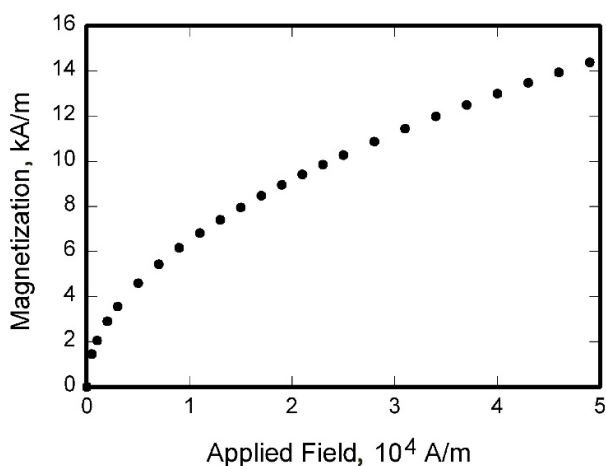


Fig. 1 Magnetization as a function of applied fields.

Place figure captions below all figures; place table titles above the tables. If your figure has multiple parts, include the labels “a),” “b),” etc. below and to the left of each part, above the figure caption. Please verify that the figures and tables you mention in the text actually exist. *Please do not include captions as part of the figures, and do not put captions in separate text boxes linked to the figures.* When citing a figure in the text, use the abbreviation “Fig.” except

at the beginning of a sentence. Do not abbreviate “Table.” Number each different type of illustration (i.e., figures, tables, images) sequentially with relation to other illustrations of the same type.

Figure axis labels are often a source of confusion. Use words rather than symbols. As in the example to the right, write the quantity “Magnetization” rather than just “M.” Do not enclose units in parenthesis, but rather separate them from the preceding text by commas. Do not label axes only with units. As in Fig. 1, for example, write “Magnetization, A/m,” not just “A/m.” Do not label axes with a ratio of quantities and units. For example, write “Temperature, K,” not “Temperature/K.”

Multipliers can be especially confusing. Write “Magnetization, kA/m” or “Magnetization, 10^3 A/m.” Do not write “Magnetization (A/m) \times 1000” because the reader would not then know whether the top axis label in Fig. 1 meant 16000 A/m or 0.016 A/m. Figure labels must be legible, and all text within figures should be uniform in style and size, no smaller than 8-point type.

G. Equations, Numbers, Symbols, and Abbreviations

Equations are numbered consecutively, with equation numbers in parentheses flush right, as in Eq. (1). Insert a blank line above and below the equation. To insert an equation into the \LaTeX document, use the `\begin{equation} . . . \end{equation}` command environment.

A sample equation is included here, formatted using the preceding instructions. To make your equation more compact, you can use the solidus (/), the exp function, or appropriate exponents. Use parentheses to avoid ambiguities in denominators.

$$\int_0^{r_2} F(r, \varphi) dr d\varphi = [\sigma r_2 / (2\mu_0)] \int_0^\infty \exp(-\lambda |z_j - z_i|) \lambda^{-1} J_1(\lambda r_2) J_0(\lambda r_i) \lambda d\lambda \quad (1)$$

Be sure that the symbols in your equation are defined before the equation appears, or immediately following. Italicize symbols (*T* might refer to temperature, but *T* is the unit tesla). Refer to “Eq. (1),” not “(1)” or “equation (1)” except at the beginning of a sentence: “Equation (1) is. . .” Equations can be labeled other than “Eq.” should they represent inequalities, matrices, or boundary conditions. If what is represented is really more than one equation, the abbreviation “Eqs.” can be used.

Define abbreviations and acronyms the first time they are used in the text, even after they have already been defined in the abstract. Very common abbreviations such as AIAA, SI, ac, and dc do not have to be defined. Abbreviations that incorporate periods should not have spaces: write “P.R.,” not “P. R.” Delete periods between initials if the abbreviation has three or more initials; e.g., U.N. but ESA. Do not use abbreviations in the title unless they are unavoidable (for instance, “AIAA” in the title of this article).

H. General Grammar and Preferred Usage

Use only one space after periods or colons. Hyphenate complex modifiers: “zero-field-cooled magnetization.” Avoid dangling participles, such as, “Using Eq. (1), the potential was calculated.” [It is not clear who or what used Eq. (1).] Write instead “The potential was calculated using Eq. (1),” or “Using Eq. (1), we calculated the potential.”

Insert a zero before decimal points: “0.25,” not “.25.” Use “cm²” not “cc.” Indicate sample dimensions as “0.1 cm \times 0.2 cm,” not “0.1 \times 0.2 cm².” The preferred abbreviation for “seconds” is “s,” not “sec.” Do not mix complete spellings and abbreviations of units: use “Wb/m²” or “webers per square meter,” not “webers/m².” When expressing a range of values, write “7 to 9” or “7–9,” not “7~9.”

A parenthetical statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within parenthesis.) In American English, periods and commas are placed within quotation marks, like “this period.” Other punctuation is “outside”! Avoid contractions; for example, write “do not” instead of “don’t.” The serial comma is preferred: “A, B, and C” instead of “A, B and C.”

If you wish, you may write in the first person singular or plural and use the active voice (“I observed that. . .” or “We observed that. . .” instead of “It was observed that. . .”). Remember to check spelling. If your native language is not English, please ask a native English-speaking colleague to proofread your paper.

Be aware of the different meanings of the homophones “affect” (usually a verb) and “effect” (usually a noun), “complement” and “compliment,” “discreet” and “discrete,” “principal” (e.g., “principal investigator”) and “principle” (e.g., “principle of measurement”). Do not confuse “imply” and “infer.”

The word “data” is plural, not singular (i.e., “data are,” not “data is”). The subscript for the permeability of vacuum μ_0 is zero, not a lowercase letter “o.” The term for residual magnetization is “remanence”; the adjective is “remanent”;

do not write “remnance” or “remnant.” The word “micrometer” is preferred over “micron” when spelling out this unit of measure. A graph within a graph is an “inset,” not an “insert.” The word “alternatively” is preferred to the word “alternately” (unless you really mean something that alternates). Use the word “whereas” instead of “while” (unless you are referring to simultaneous events). Do not use the word “essentially” to mean “approximately” or “effectively.” Do not use the word “issue” as a euphemism for “problem.” When compositions are not specified, separate chemical symbols by en-dashes; for example, “NiMn” indicates the intermetallic compound $\text{Ni}_{0.5}\text{Mn}_{0.5}$ whereas “Ni–Mn” indicates an alloy of some composition $\text{Ni}_x\text{Mn}_{1-x}$.

Be aware of the different meanings of the homophones “affect” (usually a verb) and “effect” (usually a noun), “complement” and “compliment,” “discreet” and “discrete,” “principal” (e.g., “principal investigator”) and “principle” (e.g., “principle of measurement”). Do not confuse “imply” and “infer.”

Prefixes such as “non,” “sub,” “micro,” “multi,” and “ultra” are not independent words; they should be joined to the words they modify, usually without a hyphen. There is no period after the “et” in the abbreviation “et al.” The abbreviation “i.e.,” means “that is,” and the abbreviation “e.g.,” means “for example” (these abbreviations are not italicized).

VI. Conclusion

A conclusion section is not required, though it is preferred. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions. *Note that the conclusion section is the last section of the paper that should be numbered. The appendix (if present), acknowledgment, and references should be listed without numbers.*

Appendix

An Appendix, if needed, should appear before the acknowledgments.

Acknowledgments

An Acknowledgments section, if used, **immediately precedes** the References. Sponsorship information and funding data are included here. The preferred spelling of the word “acknowledgment” in American English is without the “e” after the “g.” Avoid expressions such as “One of us (S.B.A.) would like to thank. . .” Instead, write “F. A. Author thanks. . .” Sponsor and financial support acknowledgments are also to be listed in the “acknowledgments” section.