

MATH 271, HOMEWORK 2
DUE SEPTEMBER 11TH

Problem 1. Consider the autonomous (Hamilton) equation

$$x' = ix$$

with initial condition $x(0) = 1$. *Hint: Think about what this is saying – the velocity of the particle x' is a rotation by $\pi/2$ of the position x .*

- (a) Find the particular solution to this equation.
- (b) Plot the function $x(t)$ in the complex plane.
- (c) Show that your solution $x(t)$ solves the ODE

$$x'' = -x$$

which is essentially Worksheet 1 Problem 11. *It is worth thinking about this too – the acceleration of a particle x'' is in the opposite direction of the position x , i.e., centripetal motion.*

Problem 2. Consider the following autonomous equation.

$$x' = -x^2.$$

- (a) Draw the phase line for this system. What are the equilibrium point(s)? Which equilibria are stable? Which are unstable? Explain.
- (b) Find a general solution to the ODE.
- (c) Explain how your general solution fits the qualitative behavior expected from the phase line. That is, can you show that limits of your general solution match your qualitative analysis?
- (d) Can $x(0) = 0$ be an initial condition? Explain. *Hint: your analysis from the phase line may prove to be more useful than the general solution you found.*

Problem 3. Let y be a function of x and consider the following differential equation.

$$y' = y \cos(x).$$

- (a) What is the order of this equation? Is the equation separable? Explain.
- (b) Plot an approximation of the slope field for this equation using this Desmos link: <https://www.desmos.com/calculator/e93gkwtwfo>. *Note that you will have to modify the $g(x, y)$ equation in that page.*
- (c) Find the general solution to this equation.

- (d) Given the initial data $y(0) = 1$, find the particular solution.
- (e) Plot this function over your slope field. Explain how you could have approximated this solution using just the slope field.
- (f) Explain in words what the solution describes if we let $y(x)$ be the position of some object and x represents time.

Problem 4. Consider the differential equation

$$x' = \frac{x+t}{t}.$$

- (a) Let $f(x, t) = \frac{x+t}{t}$. Show that $f(x, t) = f(\lambda x, \lambda t)$.
- (b) Given (a) holds, use the change of variables $u = \frac{x}{t}$ to rewrite the differential equation as a separable equation in terms of u .
- (c) Find the general solution to the equation and write your solution in terms of the original variables t and x .

Problem 5. Find the general solution to the following equation.

$$tx' + 2x = \frac{\sin(t)}{t}.$$

Show that your solution is correct. (*Hint: can you use an integrating factor?*)