# The Calderón Problem

## on Riemannian Manifolds

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Section 1

# Introduction

# The Two Sides to the Problem

- Practitioners: Work with sparse and noisy data to recover information in the real world.
- Theorists: Work in ideal scenarios with chosen information to see the scope of possibilities.

Subsection 1

# Electrical Impedence Tomography

# EIT

**Idea:** Given a domain $\Omega$ which has an interior $\Omega^+$ we cannot probe, what can we learn from studying the boundary $\partial\Omega$? In particular...

- $\Omega^+$ is free of charges, hence $\Delta u = 0$ in $\Omega^+$ where $u$ is the electrostatic potential.
- Apply a known voltage $f$ at the boundary $\partial\Omega$. Hence $f = u|_{\partial\Omega}$.
- Measure the current flux $g$ through the boundary $\partial\Omega$. Hence, $g = \frac{\partial u}{\partial \nu}$.
- This defines the voltage-to-current map $\Lambda$ so that $\Lambda(f) = g$.
- What can we learn about $\Omega^+$ from $\Lambda$?
- Can we determine the conductivity matrix $\gamma$ from $\Lambda$?

# EIT

Use cases:

- Medical Imaging: $\Omega^+$ could be a portion of a human body. (AC Method)
- Geophysical Imaging: $\Omega^+$ could be a below the Earth's surface. (DC Method)
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Subsection 2

# Riemannian Manifolds

Subsection 3

# Challenges

Section 2

# Preliminaries

Subsection 1

# Smooth Manifolds