Math 474 HW #6

Problem 1(b)

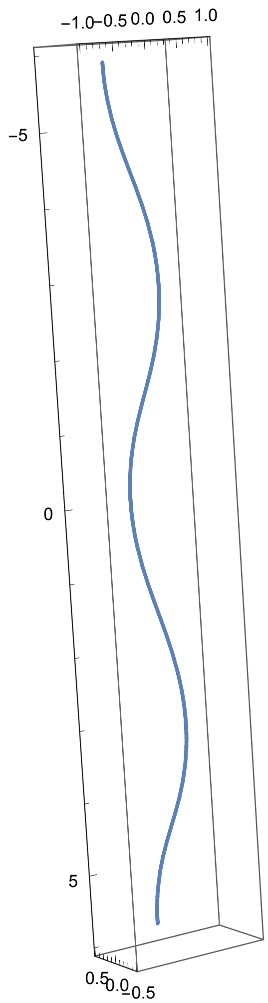
Letting C<1 (Here I picked .5) means that -2<=Sinv<=2, which means that v is unbounded. I allow v to go from -2Pi-2Pi to illustrate what the curve looks like.

```
phi0 = .5 * Cos[v];
```

```
In[10]:= psi0 = Integrate[Sqrt[1 - .25 * Sin[t]^2], {t, 0, v}];
```

In[52]:=
```
ParametricPlot3D[{phi0, 0, psi0}, {v, -2 * Pi, 2 * Pi}]
```

Out[52]=



Now it is obvious that, since v is unbounded,the surface of revolution of this curve will not close up ever. The parts that appear to touch the Z-Axis and could potentially make a closed surface are not intersecting at a perpendicular--so the surface wouldn't be regular anyways.

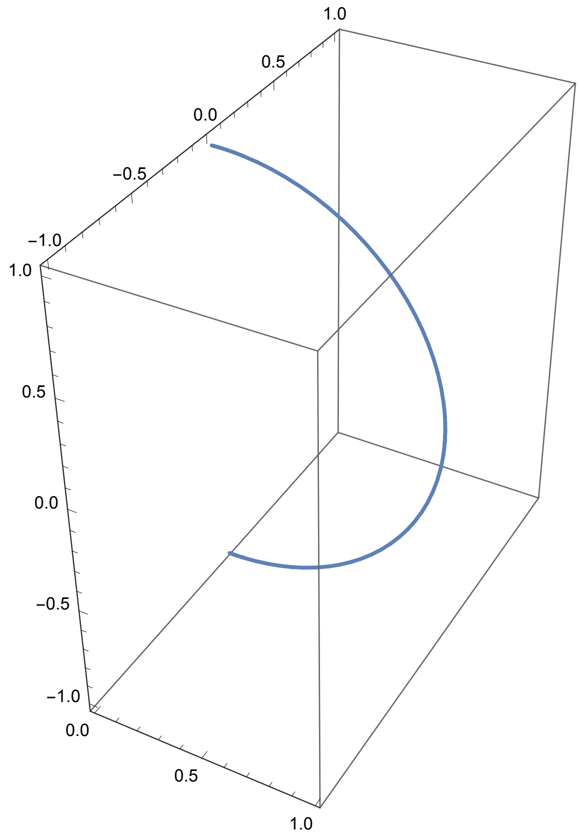
Now, let's let C = 1

In[12]:=
```
phi1 = Cos[v];
```

In[13]:=
```
psi1 = Integrate[Sqrt[1 - Sin[t]^2], {t, 0, v}];
```

In[15]:= `ParametricPlot3D[{phi1, 0, psi1}, {v, -Pi / 2, Pi / 2}]`

Out[15]=



In[56]:= `Limit[{phi1, 0, psi1}, v → -Pi / 2]`

Out[56]= `{0, 0, -1}`

this shows the bottom portion intercepts the z axis perpendicularly

In[57]:= `Limit[{phi1, 0, psi1}, v → Pi / 2]`

Out[57]= `{0, 0, -1}`

same with the top portion.

So we can see that the surface of revolution when C=1 is a sphere (as the points at the top and bottom forced me to take a limit as well). The sphere is obviously regular and compact.

Now, let C=2>1, this means the bounds are -Pi/6 to Pi/6

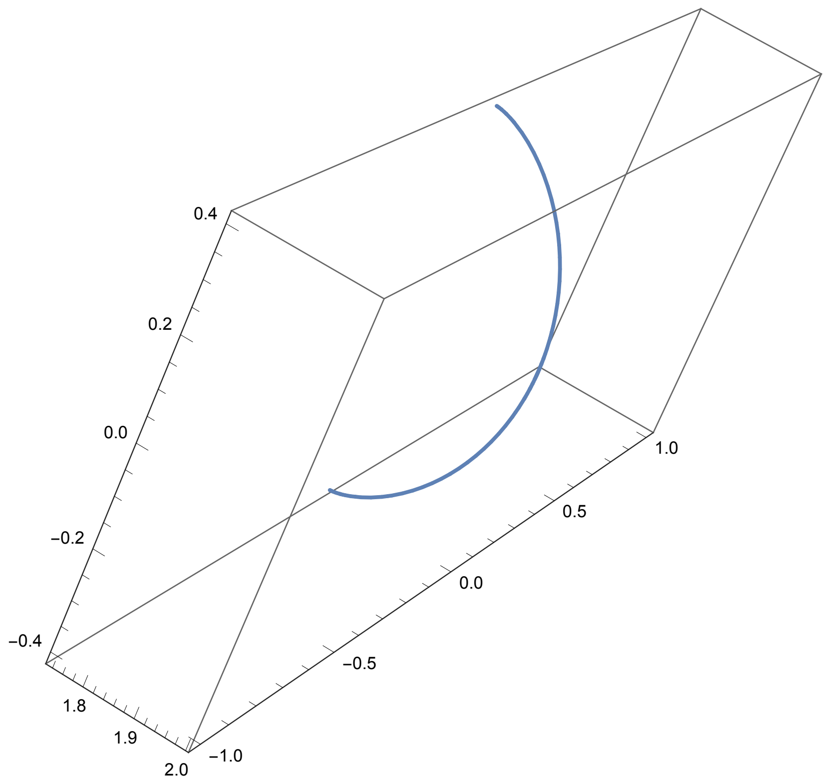
In[16]:= `phi2 = 2 * Cos[v];`

In[23]:= `psi2 = Integrate[Sqrt[1 - 4 * Sin[t] ^2], {t, 0, v}]`

Out[23]= $\text{ConditionalExpression}\left[\text{EllipticE}[v, 4], \text{Cos}[2\,v] \geq \frac{1}{2}\right]$

In[48]:=
```
ParametricPlot3D[{phi2, 0, psi2}, {v, -Pi / 6, Pi / 6}]
```

Out[48]=



In[62]:=
```
Re[{phi2, 0, psi2} /. v → Pi / 6] // N
```

Out[62]=
```
{1.73205, 0., 0.406299}
```

In[63]:=
```
Re[{phi2, 0, psi2} /. v → -Pi / 6] // N
```

Out[63]=
```
{1.73205, 0., -0.406299}
```

Above shows the vector that is at the end point of each side of this curve. I took the real portion to get rid of the tiny imaginary component to the Z component of the vector (no idea why it was there. It was on order of 10^-16). It's obvious that these are not intersecting the Z-axis in an orthogonal way and the surface of revolution looks more like a football shape with pointed ends rather than a smooth end. So this surface of revolution cannot be regular.

Part C Plots

first, Phi=Coshv, with domain (since C=1) ArcSinh[-1]<v<ArcSinh[1]

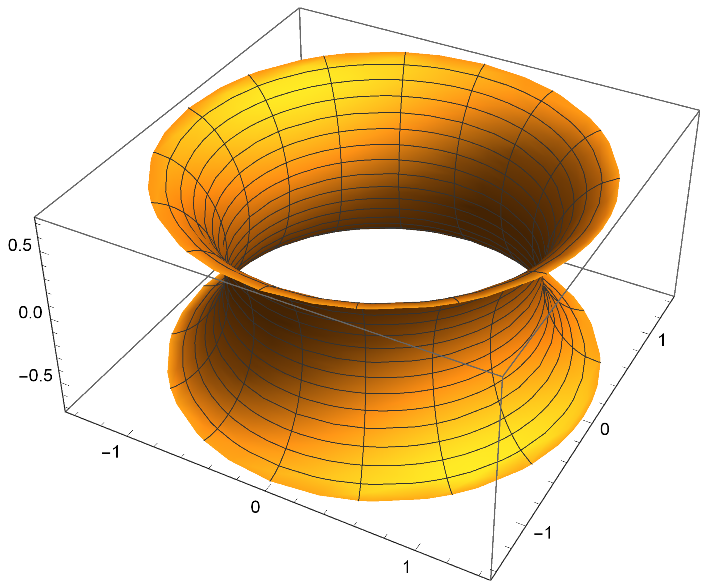
In[170]:=
```
Clear[u, v]
```

In[171]:=
```
store1 = Integrate[Sqrt[1 - Sinh[t]^2], {t, 0, v}]
```

Out[171]=
```
ConditionalExpression[-i EllipticE[i v, -1], -1 ≤ Sinh[v] ≤ 1]
```

In[175]:=
```
ParametricPlot3D[{Cosh[v] * Cos[u], Cosh[v] * Sin[u], store1},
  {u, 0, 2 * Pi}, {v, ArcSinh[-1], ArcSinh[1]}]
```

Out[175]=



So now we want to look at Phi=.5Sinhv, with domain 0<v<ArcCosh[2] (since Cosh is always positive)

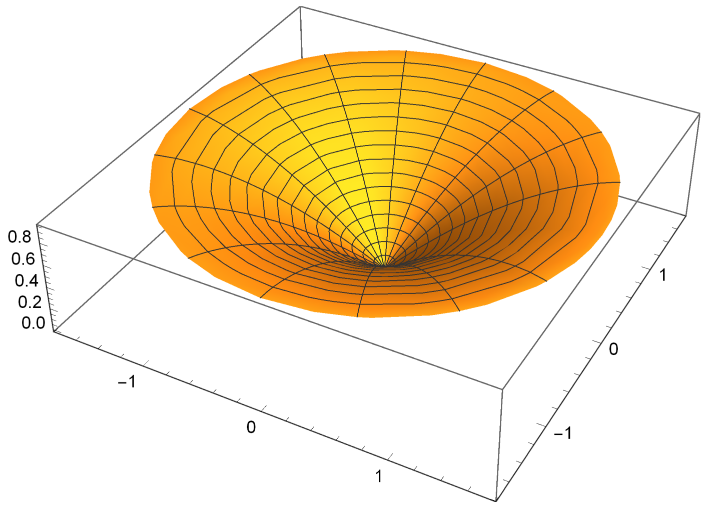
In[185]:=
```
Clear[u, v]
```

In[189]:=
```
store2 = Integrate[Sqrt[1 - .25 * Cosh[t]^2], {t, 0, v}]
```

Out[189]=
```
ConditionalExpression[
  (0. - 0.866025 i) EllipticE[(0. + 1. i) v, -0.333333], -1.73205080756887734160 ≤
    Sinh[1.00000000000000000000 v] ≤ 1.73205080756887734160 &&
   -2.00000000000000000000 ≤ Cosh[v] ≤ 2.00000000000000000000]
```

In[190]:=
```
ParametricPlot3D[{Sinh[v] * Cos[u], Sinh[v] * Sin[u], store2},
  {u, 0, 2 * Pi}, {v, 0, ArcCosh[2]}]
```

Out[190]=



Lastly, let Phi=e^v, and our domain is then v<0

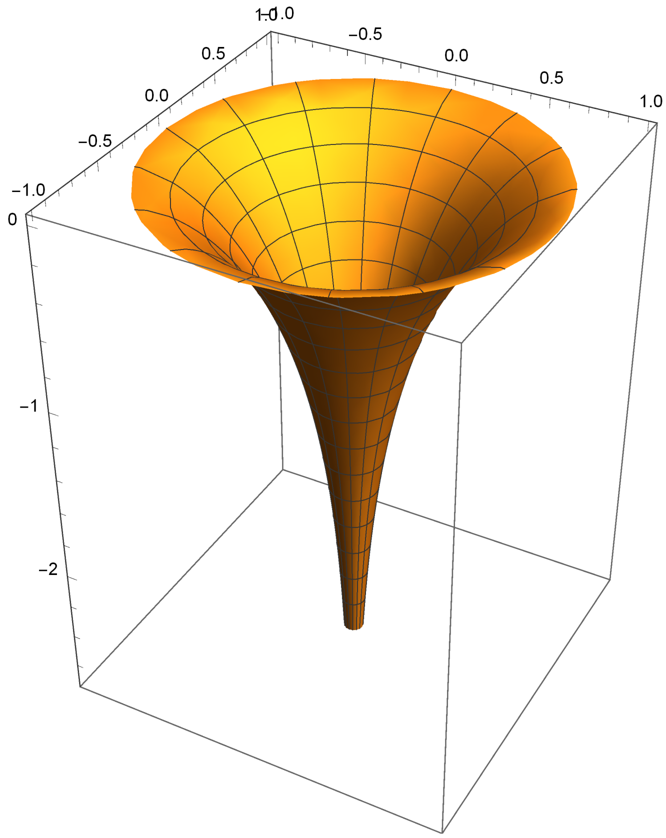
In[191]:=
```
Clear[u, v]
```

In[192]:=
```
store3 = Integrate[Sqrt[1 - Exp[2 * t]], {t, 0, v}]
```

Out[192]= ConditionalExpression$\left[\sqrt{1 - e^{2\,v}} - \text{ArcTanh}\left[\sqrt{1 - e^{2\,v}}\right],\ e^{2\,v} \le 1\right]$

In[194]:=
```
ParametricPlot3D[{Exp[v] * Cos[u], Exp[v] * Sin[u], store3}, {u, 0, 2 * Pi}, {v, 0, -3}]
```

Out[194]=



I think it is pretty interesting that when Phi=Sinh[v] there is only one point, ArcCosh[1], that exists. I didn't notice anything like this for the other surfaces, but that gave me some grief.

These are cool though!