A Multiscale approach to modeling the municipal spread of COVID-19

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Joint work with

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Note that the phrase,

“All models are wrong, but some are useful"

is in play.

Outline

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        ii Quarantining/presymptomatic.

Question

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How do schools and universities impact the spread of COVID-19 in the surrounding community?

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- Use a (***deterministic*** and ***homogeneous***) compartmental model approach.

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- Let agents interact with one another and keep track of the disease progression.

Examples

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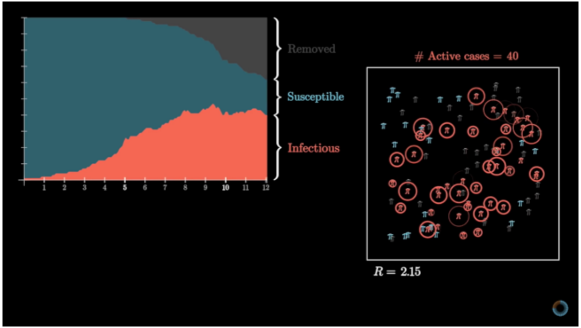
- Particle based simulations. (3Blue1Brown)



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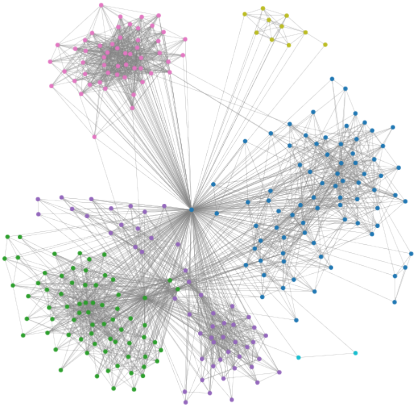
- Network based simulations. (covasim)



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- (Typically) less ad-hoc parameter tuning.

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- Stochastic nature requires ensembles to generate statistics.

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- Assume efficient and homogeneous mixing.

Example

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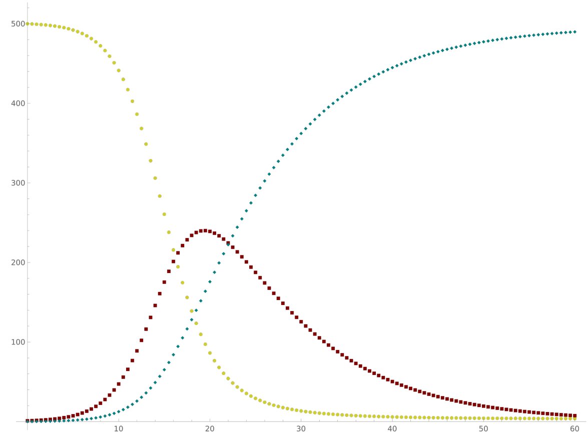
- SIR Model (Kermack and McKendrick, 1927)



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The SIR equations then read

$$\begin{pmatrix} \dot{S} \\ \dot{I} \\ \dot{R} \end{pmatrix} = \begin{pmatrix} -\beta\, C\, \frac{I}{N} \\ +\beta\, C\, \frac{I}{N} - \gamma I \\ +\gamma I \end{pmatrix},$$

where S, I, and R denotes the ***susceptible***, ***infected***, and ***removed*** populations respectively. Note, $N = S + I + R$ is the total (conserved) population size.

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- β is the likelihood of reaction.

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- $\gamma \in [0, \infty)$ is the recovery + death rate.

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- Ad-hoc parameter changes.

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- Couple together small and large scale models to study complicated systems.
- E.g., quantum mechanics $\to$ molecular dynamics $\to$ kinetic theory $\to$ statistical mechanics $\to$ thermodynamics.

Idea

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Can we couple an agent based model alongside a compartmental model to remove the drawbacks and gain benefits?

Hierarchical compartmental model

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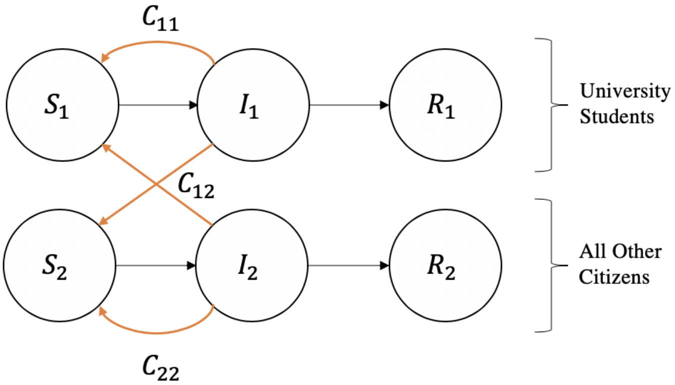

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In general, for n systems, we have the equations

$$\begin{pmatrix} \dot{S}_i \\ \dot{I}_i \\ \dot{R}_i \end{pmatrix} = \begin{pmatrix} -\beta S_i \sum_j C_{ij} \frac{I_j}{N_j} \\ +\beta S_i \sum_j C_{ij} \frac{I_j}{N_j} - \gamma I_j \\ +\gamma I_j \end{pmatrix}.$$

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In this case, the contact rate C becomes the ***contact matrix*** C_{ij} that describes the contact rate between the systems i and j. Note C_{ij} is symmetric.

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- This decomposition allows us to aggregate larger system dynamics by summing over relevant systems.

Adding complexity

SIR is lacking. We prefer the SEQIRD compartmental model governed by

$$\begin{pmatrix} \dot{S}_i \\ \dot{E}_i \\ \dot{Q}_i \\ \dot{I}_i \\ \dot{R}_i \\ \dot{D}_i \end{pmatrix} = \begin{pmatrix} -\beta S_i \sum_j C_{ij} \frac{I_j}{N_j} \\ \beta S_i \sum_j C_{ij} \frac{I_j}{N_j} - (\gamma_I - \gamma_Q) E_i \\ \gamma_I E_i - (\lambda + \kappa) I_i \\ \gamma_Q E_i - (\lambda + \kappa) Q_i \\ \lambda (I_i + Q_i) \\ \kappa (I_i + Q_i) \end{pmatrix}$$

- $S \equiv$ susceptible.
- $E \equiv$ exposed.
- $Q \equiv$ quarantined.
- $I \equiv$ infected.
- $R \equiv$ recovered.
- $D \equiv$ dead.

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- $\beta = 0.3 \equiv$ transmission rate.
- $C \equiv$ contact matrix.
- $\gamma = 0.07 \equiv$ latent time to infection.
- $\lambda = 0.1 \equiv$ recovery rate.
- $\kappa = 0.002 \equiv$ death rate.
- $q_{\text{percent}} \equiv$ percentage quarantining.
- $\gamma_Q = q_{\text{percent}} \gamma$.
- $\gamma_I = (1 - q_{\text{percent}}) \gamma$.

Determining parameters

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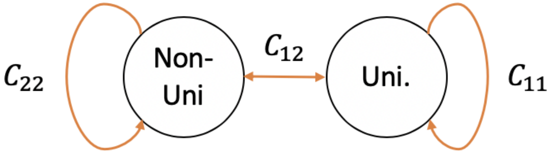

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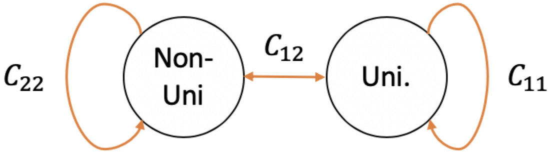

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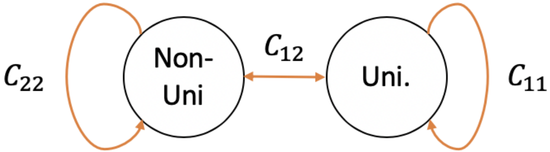

- All parameters but C were determined via measurements in other sources.
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- *This is where the model becomes multiscale.*

Assimilating data

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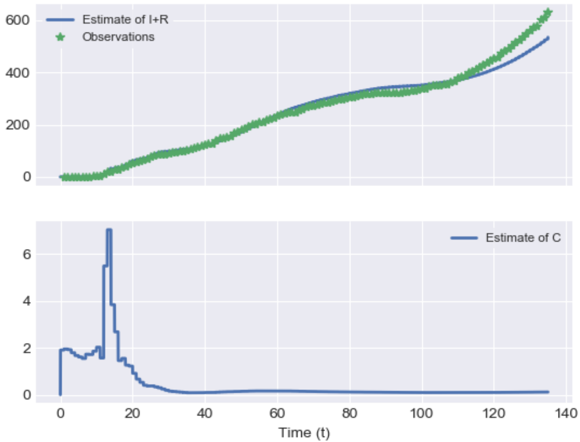
Using measured data from Fort Collins and Larimer County, we estimated a value for C assuming $\beta = 1$.



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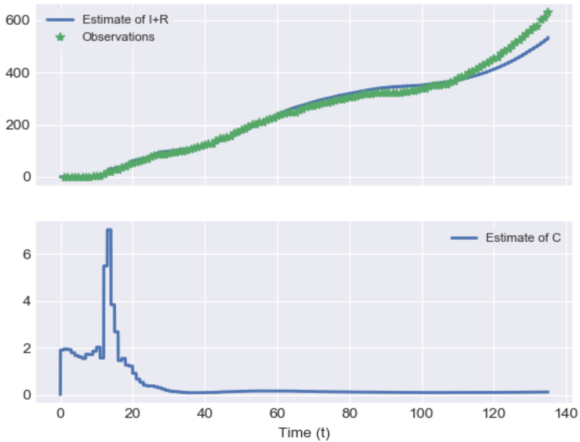

Counting contacts

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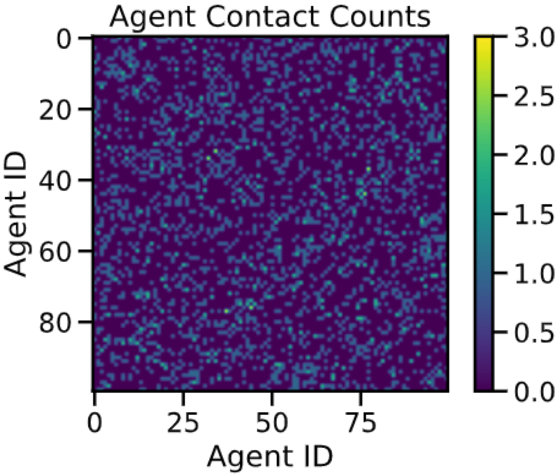
Idea: Agent based simulator tracks (various types of) contact between students each day.

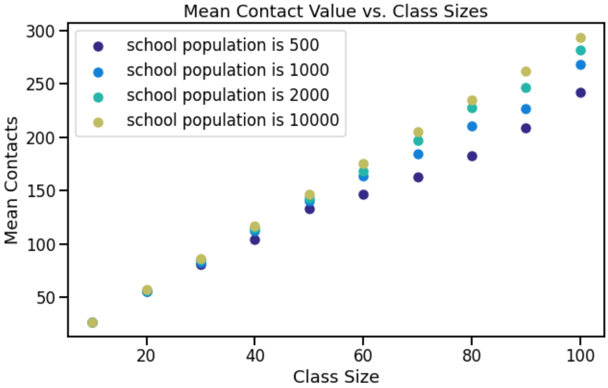
Adjustable parameters:

- Student body size.
- Number of class periods.
- Class sizes.
- Students per major.
- Schedule staggering.



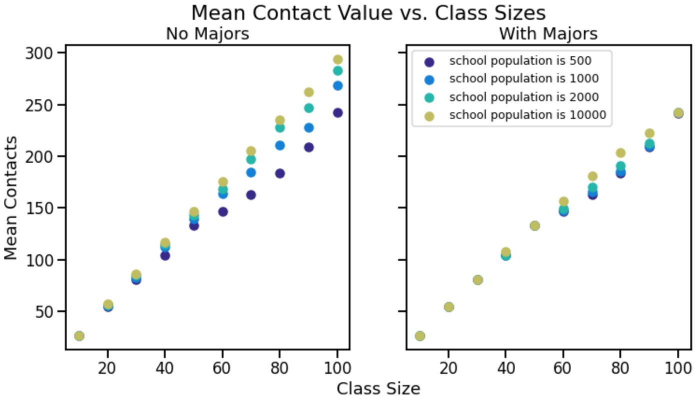
Contacts and school population

- Students are randomly assigned courses of a certain size.
- We count the number of unique contacts a student makes vs. class size for a three class period day.
- The growth is sublinear, but as the population increases we approach linear behavior.
- This is due to diminishing chances of having repeat students in other randomly drawn classes.



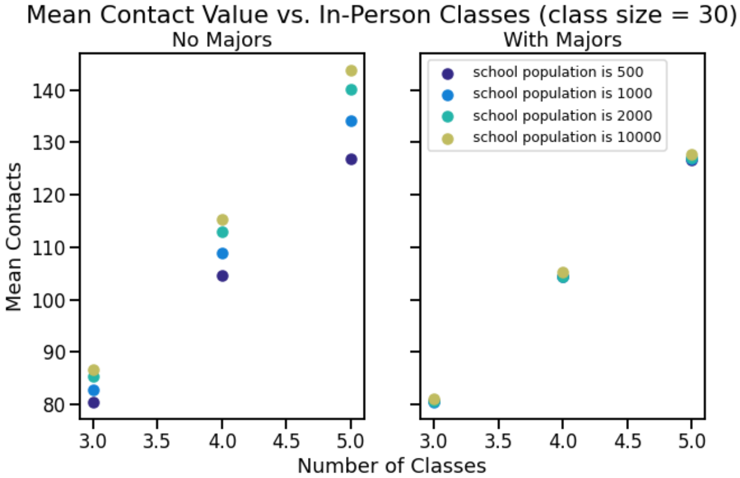
Contacts and major grouping

- To add realism or intervention, we can group students within majors.
- This leads to a decrease in new contacts and sublinear growth due to the effective population size decreasing to the size of the majors (500).



Contacts and class periods

- Increasing the number of in-person class periods increases contact rate almost linearly when class sizes are small (30).



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- This contact rate is used in the compartmental model and we integrate this ODE for a day.
- University members in the Q and D department are removed and the contact rate is recomputed

City-university coupling

- City-university coupling changes the strength of the off diagonal C_{21} element.
- With high coupling, $C_{21} = C_{22}$ and students mix just like typical city members.
- Coupling minimally affects the university, but greatly affects the city.

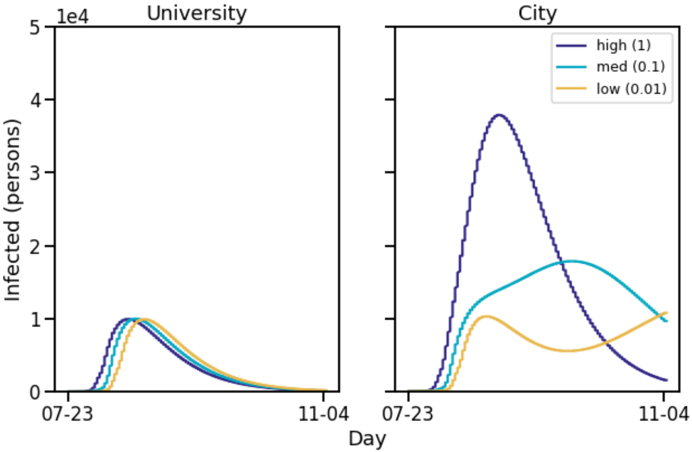

Figure: $I(t)$ for class size $= 15$, major size $= 500$, and no quarantining.

University quarantining

- Quarantining only delays infection within the university.
- Significantly alters aggregated city dynamics and prevents second wave.

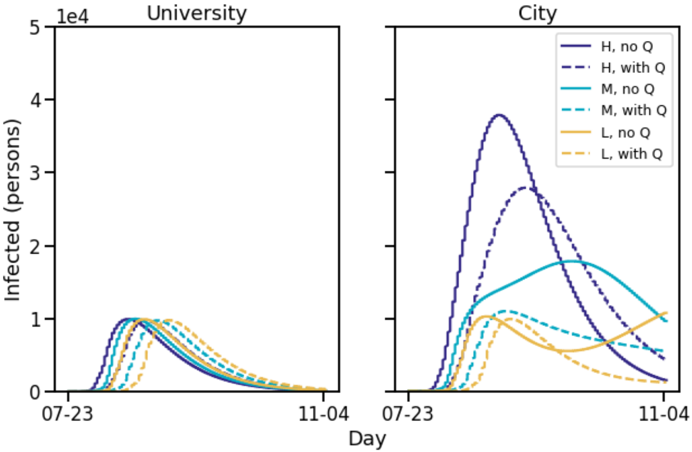

Figure: $I(t)$ with 50% quarantine rate, class size of 15, and major size

Staggering schedules

- Day and week staggering flatten the curve and reduce total infected.
- Week staggering performs slightly better due to latent time for infection.

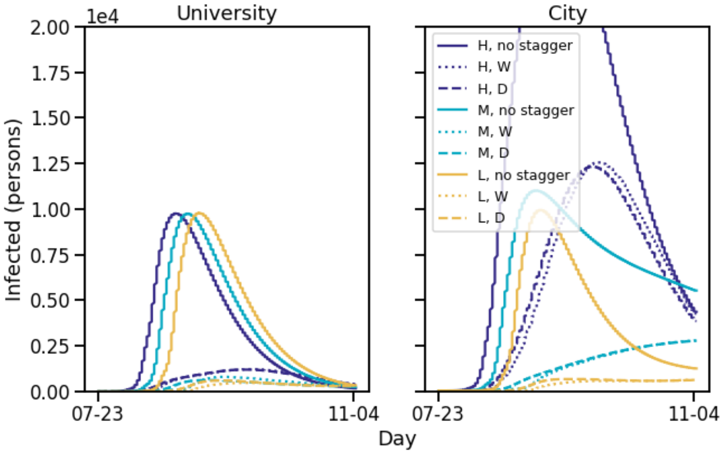

Figure: $I(t)$ with staggering, 50% of infected population quarantining,

Future directions

- Improve and extend the data assimilation for the compartmental model.
- Incorporate different types of contacts.
- Masks, hygiene, and other measures of contact severity.
- Quantify the stochasticity of the agent model.
- More heterogeneity from the agent model (scheduling, transportation, etc.).

Acknowledgements

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