The Calderón Problem

on Riemannian Manifolds

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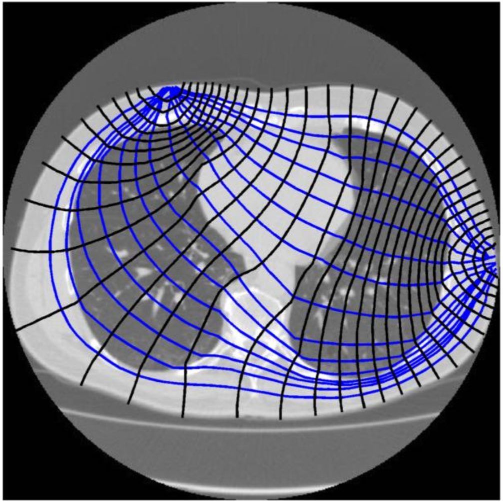
- This problem sparked interest due to its usefulness in geophysical and medical imaging.



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- Can we determine the conductivity matrix γ from Λ?

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- Recover g from knowing Λ.
- The EIT problem and the Calderón problem on manifolds are equivalent in dimensions $n \neq 2$.