MATH 271, Quiz 4

Due November 13^{th} at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 5 TOTAL PROBLEMS.

Problem 1. Consider the vectors $\vec{\boldsymbol{u}} = 3\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}$, $\vec{\boldsymbol{v}} = -\hat{\boldsymbol{y}}$, and $\vec{\boldsymbol{w}} = -\hat{\boldsymbol{x}} + 2\hat{\boldsymbol{y}}$.

(a) **(2 pts.)** Draw the vectors $\vec{\boldsymbol{u}}$, $\vec{\boldsymbol{v}}$, $\vec{\boldsymbol{w}}$, and $\vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$ in the plane.

(b) **(3 pts.)** Compute $\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}}$, $\|\vec{\boldsymbol{u}}\|$, and $\|\vec{\boldsymbol{v}}\|$.

(c) **(2 pts.)** What is the angle betwen $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$? (Do not worry about getting a numerical answer, just show the work to explain what the angle is.)

Problem 2. Consider the following functions.

i. $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x + 1$.

ii. $g\colon \mathbb{R}^2 \to \mathbb{R}$ given by $g\begin{pmatrix} x \\ y \end{pmatrix} = x + y$.

(a) **(2 pts.)** Is f linear or nonlinear? Explain.

(b) **(2 pts.)** Show that g is linear.

Problem 3. Consider the matrices

$$[A] = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad [B] = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 2 & -1 \end{pmatrix}, \quad [C] = \begin{pmatrix} -2 & -1 \\ 1 & 0 \\ 0 & 3 \end{pmatrix}.$$

(a) **(2 pts.)** Say which of the following products can you compute

$$[A][B], \quad [B][A], \quad [A][C], \quad [C][A], \quad [B][C], \quad [C][B].$$

(b) **(2 pts.)** What are n and m if we think of $[C]\colon \mathbb{R}^n \to \mathbb{R}^m$?

Problem 4. Consider the two matrices

$$[A] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad [B] = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}.$$

(a) **(2 pts.)** Find the solution(s) to the homogeneous equation $[A]\vec{\boldsymbol{x}} = \vec{\boldsymbol{0}}$.

(b) **(2 pts.)** What is the nullspace of $[A]$?
(c) **(2 pts.)** Can you find a solution to $[B]\vec{\boldsymbol{x}} = \vec{\boldsymbol{y}}$ with $\vec{\boldsymbol{y}} = 2\hat{\boldsymbol{x}} + 5\hat{\boldsymbol{y}}$? Explain.
(d) **(2 pts.)** Explain why $\det([B]) = 0$ without computing it.

Problem 5. Take the same two two matrices from Problem 4

$$[A] = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad [B] = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

(a) **(2 pts.)** Compute $\det([A])$ and $\det([B])$.
(b) **(1 pts.)** Compute $\det([A][B])$ without computing $[A][B]$.
(c) **(2 pts.)** Are the columns of $[A]$ linearly independent? Explain.