

MATH 272, QUIZ 1
DUE FEBRUARY 5TH AT THE END OF CLASS

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 6 TOTAL PROBLEMS.

Problem 1. For the following, describe the domain D and codomain C for the given type of function.

- (a) **(1 pts.)** A curve is a function $\vec{\gamma}: D \rightarrow C$.
- (b) **(1 pts.)** A scalar field is a function $f: D \rightarrow C$.
- (c) **(1 pts.)** A vector field is a function $\vec{V}: D \rightarrow C$.

Problem 2. Let $\vec{\gamma}$ be a curve defined by

$$\vec{\gamma}(t) = \begin{pmatrix} e^t \\ e^{-t} \\ t \end{pmatrix} \quad \text{for } t \in [0, 1].$$

- (a) **(2 pts.)** Compute the tangent vector at time t , $\dot{\vec{\gamma}}(t)$.
- (b) **(2 pts.)** Compute the speed at time t , $|\dot{\vec{\gamma}}(t)|$ (do not feel the need to simplify).
- (c) **(2 pts.)** Let $f(x, y, z) = xyz$. Set up, but do not compute,

$$\int_{\vec{\gamma}} f(\vec{\gamma}) d\vec{\gamma}$$

(do not worry about simplifying this fully).

Problem 3. Let $f(x, y, z) = \sin(x) \sin(z) + e^{yz}$.

- (a) **(2 pts.)** Compute all first order partial derivatives of f .
- (b) **(2 pts.)** Compute the laplacian of f .

Problem 4. (3 pts.) Consider the vector field \vec{V} defined by

$$\vec{V}(x, y) = \begin{pmatrix} x^2 \\ xy \end{pmatrix}.$$

Plot and label the vector field at the following points.

- i. $\vec{x}_1 = (1, 2)$;
- ii. $\vec{x}_2 = (0, -3)$;
- iii. $\vec{x}_3 = (-1, -1)$.

Problem 5. Here is a plot of a 3-dimensional vector field \vec{V} when viewing from a few different angles.

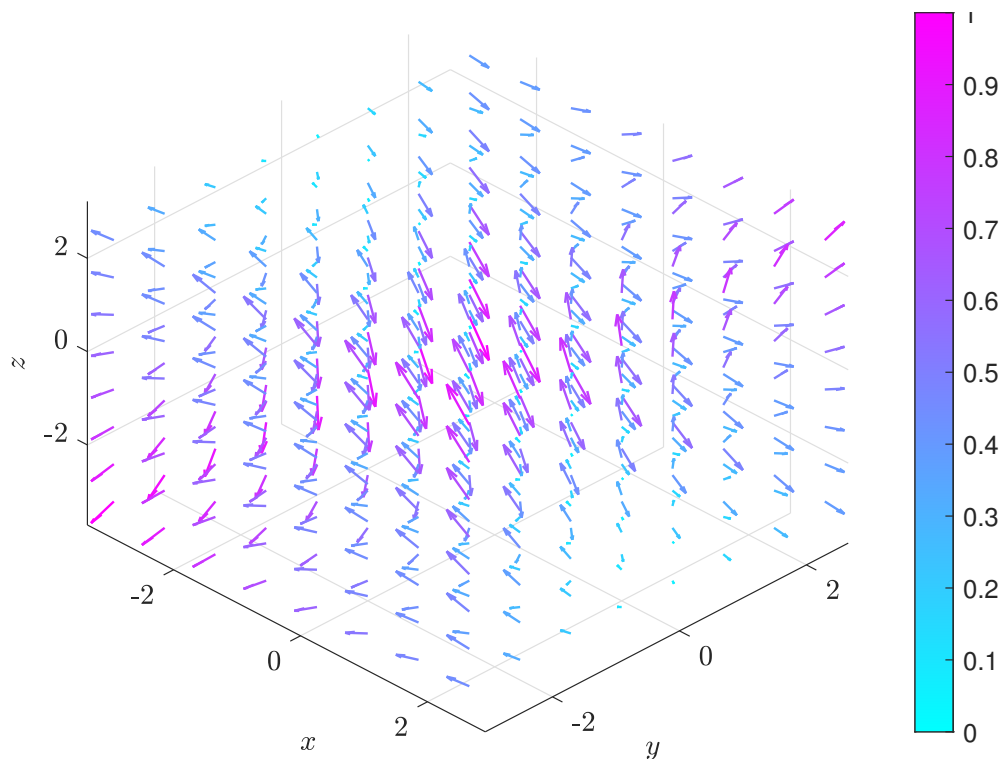
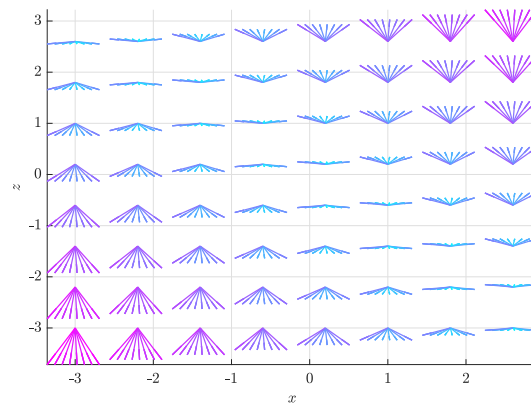
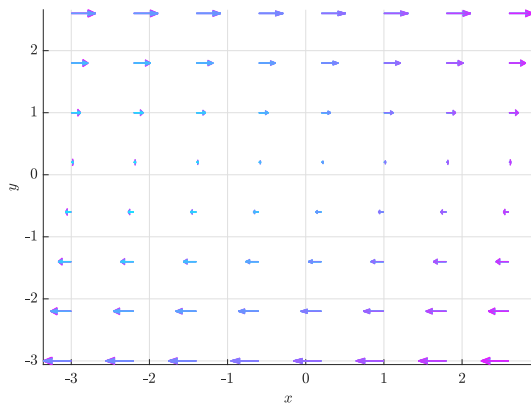


FIGURE 1. A view of the vector field \vec{V} . Note that the colorscaling on this plot represents the length of the vectors. The next plots use the same colorscale.



(I) \vec{V} when looking towards the xy -plane. (II) \vec{V} when looking towards the xz -plane.

- (a) **(2 pts.)** Does this vector field have divergence? Explain.
 (b) **(2 pts.)** Does this vector field have curl? Explain.

Problem 6. (2 pts.) Let f be a scalar field with gradient $\vec{\nabla}f$. Suppose as well that $\vec{\nabla}f$ has no divergence. Explain or show why $\Delta f = 0$.