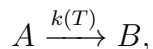


MATH 271, EXAM 1
ORAL EXAMINATION PROBLEMS
DUE ONE HOUR BEFORE YOUR EXAM TIME SLOT.

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page. You can use online tools such as Desmos and Wolfram Alpha to check your work, but you will need to explain how you arrived at your answers. You can work with other students and this is, in fact, encouraged! However, I will not be giving out direct help for these problems but can answer questions about previous problems and notes, for example. Ambiguous or illegible answers will not be counted as correct. Scan your solutions and submit them as a pdf on Canvas under Oral Exam 1.

Note, there are three total problems.

Problem 1. Consider the chemical reaction



where the rate of reaction $k(T)$ depends on the temperature of the solution T (in Kelvin, so $T > 0$).

- (a) Write down expressions for the rate of change of the concentration for species A and B .
- (b) Assuming, momentarily, $k(T)$ is constant, find the particular solution for the concentrations of A and B assuming $[A](0) = 1$ and $[B](0) = 0$.
- (c) Explain what happens to the concentrations as $t \rightarrow \infty$.
- (d) If the rate of reaction increases proportionally to T , then we can put $k(T) = \alpha T$ for some constant $\alpha > 0$. Explain how this changes the reaction. That is, will the reaction occur more quickly, more slowly, or something else? Why?
- (e) If instead $\alpha < 0$, how will this change the reaction?
- (f) Which case, $\alpha < 0$ or $\alpha > 0$ more closely represents an endothermic reaction? That is, an endothermic reaction is one that requires heat in order to form products.

Problem 2. Consider the autonomous equation

$$x' = \sin(x).$$

- (a) Draw the phase line for this system for x values in the interval $[-3\pi, 3\pi]$.
- (b) What are the equilibrium points for this system? (Consider all x values, not just the ones that you plotted in the previous part.)
- (c) Which equilibrium points are stable and which are unstable. Explain.

Problem 3. Consider the Schrödinger equation for a particle in a box of length 1,

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} = E\Psi,$$

with boundary conditions $\Psi(0) = 0$ and $\Psi(1) = 0$.

- (a) Find the general solution to the differential equation.
- (b) Apply the boundary conditions and write down a solution for each positive integer n . Recall that we call these solutions *states* and denote the states by ψ_n .
- (c) Determine the normalization constant for each ψ_n . Does this constant depend on n ?
- (d) Explain why a superposition of states is also a solution to the Schrödinger equation.