MATH 271, Worksheet 2
Ordinary Differential Equations

Problems 1-6 are related.

Problem 1. (Newton's law of cooling) Write down a differential equation that models the following scenario:

The temperature of a substance in an ambient environment changes over time proportionally to the difference of the substance from the ambient environment.

Let $T(t)$ be the temperature of the substance, T_a be the ambient temperature, and k be the constant of proportionality.

Problem 2. With the equation found above, find a general solution.

Problem 3. With the parameter values $T_a = 100$, $k = 1$, and initial data $T(0) = 50$, find the particular solution.

Problem 4. Plot the particular solution to the problem and interpret what this means for the temperature over time.

Problem 5. What happens instead if the initial temperature is equal to the ambient temperature? Does your solution reflect this? Does this make physical sense?

Problem 6. Let $\delta = (T_a - T)$. Show that the equation you found in Problem 1 reduces to

$$\delta' = -k\delta.$$

What is this equation describing physically?

Problems 7-8 are related.

Problem 7. Show that $x = c_1 \sin(t) + c_2 \cos(t)$ is a general solution to the equation

$$x'' + x = 0.$$

Problem 8. Find the particular solution if $x(0) = 1$ and $x'(0) = 2$. Plot your solution in the t, x-plane and in the x, x'-plane.

Problem 9. Consider the differential equation

$$x' = \frac{x^2 + tx + t^2}{tx}.$$

(a) Let $f(x,t) = x^2 + tx + t^2$. Show that $f(\lambda x, \lambda t) = f(x,t)$.

(b) Use the substitution $u = \frac{x}{t}$ in order to make the original equation separable.

(c) Find the general solution to this separable equation in terms of u and t. You may use Wolfram Alpha to compute the necessary integral.

(d) Find the solution to the original equation using the substitution $u = \frac{x}{t}$ and your solution from (c).