

MATH 271, QUIZ 1
DUE SEPTEMBER 4TH AT THE END OF CLASS

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 7 TOTAL PROBLEMS.

Problem 1. (4 pts.) Given two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, draw a picture showing how to find the sum $z_1 + z_2$. Also, draw a picture of $-z_1$ and $2z_1$ and explain what these scaling operations do to a complex number. (You can pick specific values for z_1 and z_2 if this helps.)

Problem 2. (4 pts.) Given a distance r from the origin and an angle θ measured counter clockwise from the positive real axis, draw a picture and explain how you can find the real and imaginary part this number using trigonometry. Then, describe how you can write this number in polar form using Euler's formula.

Problem 3. (3 pts.) Write down some initial value problem that satisfies the following.

- The dependent variable (i.e., the solution function) is given by $x(t)$.
- The equation is first order.
- The equation is separable.
- The independent variable t appears *explicitly* in the ODE.

Problem 4. (2 pts.) Write down an example of a second order differential equation.

Problem 5. (4 pts.) Show that the function $x(t) = C_1e^{-t} + C_2te^{-t}$ is a general solution to the equation

$$x'' + 2x' + x = 0.$$

If you have trouble showing x is a solution, explain how you would go about doing so. *Hint: we don't yet know how to find solutions to equations like this, but you can still show that x is a solution!*

Problem 6. (4 pts.) Explain what it means to be a general solution to an ODE. Explain what it means to be a particular solution to an initial value problem. What are the key differences between general and particular solutions?

Problem 7. For the following, say whether the statement is true or false. For full credit, justify your answer with an explanation.

- (a) **(2 pts.)** The equation $0 = a_0 + a_1 z^1 + a_2 z^2 + \cdots + a_n^z$ *always* has n solutions when z is allowed to be a complex number.
- (b) **(2 pts.)** A second order ODE requires two initial conditions to have a unique particular solution.