MATH 272, Worksheet 1

Curves, scalar fields, and partial derivatives.

Problem 1. Consider the curve

$$\vec{\gamma}(t) = \begin{pmatrix} (5 + 3\cos(8t))\cos(t) \\ (5 + 3\cos(8t))\sin(t) \\ 3\sin(8t) \end{pmatrix}.$$

(a) Plot this curve from $t = 0$ to $t = 2\pi$.

(b) Compute the tangent (velocity) vector $\dot{\vec{\gamma}}(t)$.

(c) Compute the normal (acceleration) vector $\ddot{\vec{\gamma}}(t)$.

(d) Compute the following

$$\int_{\vec{\gamma}} \left|\ddot{\vec{\gamma}}(t)\right| dt,$$

which is closely related to the total curvature of the curve. Indeed, this would be the total force applied to an object of mass $m = 1$.

Problem 2.

(a) Write an equation for a curve

$$\vec{\gamma}\colon \mathbb{R} \to \mathbb{R}^3,$$

satisfying:

- Starts with $\vec{\gamma}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.
- Ignoring the z-component, makes a spiral emanating from the origin.
- Moves upward at a constant rate in the z-direction.

Plot this curve that you made to verify that it is correct.

(b) Find the tangent vector $\dot{\vec{\gamma}}(t)$ to this curve.

(c) Find the normal vector $\ddot{\vec{\gamma}}(t)$ to this curve.

Problem 3. Consider the two dimensional scalar function

$$f(x, y) = e^{\frac{xy}{x^2+y^2-1}}.$$

(a) Plot the graph of this function $(x, y, f(x, y))$ for $x^2 + y^2 < 1$.

(b) Plot the graph for $x^2 + y^2 > 1$.

(c) Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(d) Compute all second partial derivatives of f.

Problem 4. Write the equation for a scalar function

$$f\colon \mathbb{R}^2 \to \mathbb{R},$$

satisfying,

- Has positive $\frac{\partial f}{\partial x}$ everywhere.
- Has negative $\frac{\partial f}{\partial y}$ everywhere.

(*Hint: it may help to try adding single variable functions together. That is, let* $f(x, y) = u(x) + v(y)$.)

Problem 5. For the following functions, plot the level sets for $c = -1$, $c = 0$, and $c = 1$.

(a) For just $c = 1$, plot the level set for $E(x, y, z) = \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9}$.

(b) $f(x, y, z) = xyz$.

(c) $g(x, y, z) = e^x - y^2 - z^2$.

(d) $h(x, y, z) = \sin(x) + \cos(y) - \tanh(z)$.

(e) $p(x, y, z) = \sin^2(x) + \sin^2(y) - \frac{1}{2}\sin(z)$.

(f) $q(x, y, z) = x^2 + xy + y^2 + sin(yz)$.

(g) One of your own choosing.