

MATH 272, QUIZ 2
DUE FEBRUARY 26TH AT THE END OF CLASS

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 4 TOTAL PROBLEMS.

Problem 1. Decide if the following statements are **TRUE** or **FALSE**. For full credit, provide an explanation or proof for your answer.

- (a) **(2 pts.)** Let \vec{E} be a conservative vector field. Then

$$\int_{\vec{\gamma}} \vec{E} \cdot d\vec{\gamma} = 0$$

for all curves $\vec{\gamma}$.

- (b) **(2 pts.)** Let $f(x, y)$ be a scalar field, then

$$\int_0^1 \int_{-y}^y f(x, y) dx dy$$

computes the net volume under the graph of f and above the unit square in the xy -plane.

- (c) **(2 pts.)** Consider a vector field $\vec{V}(x, y, z) = \hat{x}$ and consider the surface Σ defined to be the surface of the unit cube. Then

$$\iint_{\Sigma} \vec{V} \cdot \hat{n} d\Sigma \neq 0.$$

Problem 2. Provide a parameterization of the following regions in any choice of coordinate system.

- (a) **(3 pts.)** A straight curve beginning at $x_0 = 1$, $y_0 = -1$, and $z_0 = 3$ and ending at $x_1 = 0$, $y_1 = 2$, and $z_1 = 3$.
- (b) **(3 pts.)** A solid cone with a vertex at the origin with a height of 1 above the xy -plane and a maximum radius of 1.
- (c) **(3 pts.)** A thick spherical shell with an inner radius of 1 and an outer radius of 2.

Problem 3. Consider a surface flux integral

$$\iint_{\Sigma} \vec{V} \cdot \hat{n} d\Sigma.$$

- (a) **(2 pts.)** Explain what \hat{n} represents geometrically. Does this depend in the position along the surface?
- (b) **(2 pts.)** What does the quantity $\vec{V} \cdot \hat{n}$ represent? Explain.
- (c) **(BONUS 3 pts.)** Explain what $d\Sigma$ represents geometrically. When would

$$d\Sigma \neq dxdy?$$

Explain.

Problem 4. Convert the following fields into cylindrical coordinates. Also, recall that

$$\hat{\rho} = \frac{x}{\sqrt{x^2 + y^2}} \hat{x} + \frac{y}{\sqrt{x^2 + y^2}} \hat{y}$$
$$\hat{\theta} = -\frac{y}{\sqrt{x^2 + y^2}} \hat{x} + \frac{x}{\sqrt{x^2 + y^2}} \hat{y}$$

- (a) **(3 pts.)** $f(x, y, z) = (x + y)^2$.
- (b) **(3 pts.)** $\vec{V}(x, y, z) = \frac{xz + yz}{(x^2 + y^2)^{3/2}} \hat{x} + \frac{yz - xz}{(x^2 + y^2)^{3/2}} \hat{y}$.