MATH 271, Quiz 2

Due September 18th at the end of class

Instructions You are allowed a textbook, homework, notes, worksheets, material on our Canvas page, but no other online resources (including calculators or WolframAlpha) for this quiz. **Do not discuss any problem any other person.** All of your solutions should be easily identifiable and supporting work must be shown. Ambiguous or illegible answers will not be counted as correct.

THERE ARE 5 TOTAL PROBLEMS.

Problem 1. For the following, say whether the statement is true or false. For full credit, justify your answer with an explanation.

(a) **(2 pts.)** If x_1 and x_2 are solutions to a second order linear inhomogeneous equation, then $x = \alpha_1 x_1 + \alpha_2 x_2$ is also a solution.

(b) **(2 pts.)** The system given by the ODE $x'' + x' + x = 0$ exhibits decaying oscillatory behavior.

Problem 2. (6 pts.) Classify the following equations (e.g., n^{th} order, separable, linear, etc.) and explain your reasoning. Then put which method you would use to find a general solution.

(a) $x' + x = xt$.

(b) $x'' + 5x' + 2x = 0$.

(c) $x' + e^t x = e^t$.

Problem 3. (5 pts.) Consider the following autonomous equation

$$x' = (x-1)(x-2)(x-3)$$

(a) Draw the phase line for this system.

(b) Label the equilibria on your phase line.

(c) Which equilibria are stable? Also, explain what the different behaviors of the system will be over a long period of time. *Hint: consider initial conditions in the different regions of your phase line.*

Problem 4. (6 pts.) Let

$$x' = \frac{1}{xt}$$

Make a quick argument for why the above equation is separable. Then, find the particular solution given $x(1) = 1$.

Problem 5. (4 pts.) For the following chemical reactions, determine the rate of change of the concentration for species A. Mention whether the equation is linear or nonlinear.

(a) $3A \to B$.

(b) $B + C \to 2A$.