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University of International Business and Economics

Actuarial Report on Improvement of Guarantee Products

Legal Representative: SmallFund

Responsible Actuary: Li Hua

Application Date: January 4, 2018

Contents

Summary

The scope of this report is to:

- Compare the option-based product against capital-based product and outline the benefits and drawbacks of the two products.
- Put forward some suggestions when designing products.

In order to conduct a convincing analysis, we have done a mass of calculations about return of different investment strategies and revenue of various products. Here are some critical conclusions:

- Compared to the capital-based products, the option-based products seem to have higher average payout and lower product fee.
- Benefits of the option-based product are that it can effectively ease the impact of economic depression and maintain the stability of investment returns. However, some new risks like default risk may also emerge from it. Correspondingly, the capital-based product has to take more risks from market fluctuations, but provides diversified investment choices.
- Therefore, a modified option-based product, which is actually a combination of the option-based and capital-based products, is the most profitable choice.

Based on above analysis, we provide the following suggestions:

- If any product uses capitals when necessary, the capital should be invested in the portfolio consists of 11.40% equity, 88.08% bond and 0.52% bill, with average yield rate being 5.648578% p.a, to ensure that the volatility of return is low enough and the average return rates is as high as possible.
- If you want to maximize the average payout to investors of these products, use the option-based product and set the strike price to be 0.96 is the best choice. The gap between actual stock yield rate and guarantee rate can be filled up by capital. The product fee is 14.40% and average payout to investors is 133.01%.
- If you want to further improve the product, you may need to consider other ways to reduce product fees, such as cutting down the cost of capital use or finding other financing methods with lower cost.

If there are some changes in the social condition, our assumptions and model may not be suitable, so there will be various risks and uncertainties. The main categories of these risks include:

- Inflation

Since our analysis do no take inflation into consideration, then if inflation is getting extremely worse during the three years, the accuracy of our recommendation may be influenced.

- Economic Depression

Although options play an important role on guarantee of profitability during economic downturns, few people are willing to be the seller of put option in a recession and the option-based product will not work.

- Emergencies

Emergencies, such as a natural disaster or war can cause devastating effects on a small country. Financial markets will be hit hard due to the scarcity of materials and the need for reconstruction, which lead to return of these product being great deviated from expectations. This could lead to similar consequences as those when economic depression occurs.

R codes of all the process is provided in the appendix.

Declaration

As a member of the organization, I volunteer to write the analysis report as an investment actuary, and I am committed to being responsible for the authenticity and accuracy of the report, as well as the corresponding consequences. All of my work is based on analysis of historical data without any fabrication or fraud.

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1 Introduction

1.1 Background

Country A has a well-developed financial sector. Recently, the fund managers in country A are allowed to sell products using options. SmallFund is a fund management organization in Country A. In the past, the most popular products provided by SmallFund have been investments with guarantee.

However, the cost of capital has increased from 13% to 17% due to poor market outcomes, which means that the fees of guarantee products will increase, which may lead to reduction of investors for such products. So SmallFund is exploring the possibility of developing a guarantee product using options, by which both the option-based product and capital-based product will be optional for investors.

1.2 Scope of the problem

The scope of this report is to:

- Compare the option-based product against capital-based product and outline the benefits and drawbacks of the two products.
- Put forward some suggestions when designing products.

1.3 Assumptions

All the assumptions used in our report are listed below:

- **Assumption I**: The probability of capital backing of the capital-based products is 99%.
- **Assumption II**: For simplification, we suppose that both the amount in the investors' funds and the stock price is 1 at our starting time point T, i.e. we only invest in 1 unit of stock at time T.
- **Assumption III**: Since the term of the product is only three years, we do not take inflations into consideration.
- **Assumption IV**: After fitness of original data, we assume that the yield rates of equity are normally distributed, and the yield rates of bond and cash have a Weibull distribution to generate the investment return rate in the future.

Detailed discussion of Assumption IV, will be showed later in Part 2 Analysis of products.

1.4 Overview of our work

Our work can be divided into three parts: analysis of the capital-based product and the option-based product, improvement of the two products, and comparison between them. Specific procedures of each part are briefly summarized below.

1.4.1 Analysis Part

As for the capital-based product:

- Simulate the annual return rate of equities, bonds and cash using Assumption III. The three-year cumulative yield rate of stock is calculated accordingly.
- Calculate capital amount required for filling the gap between actual stock yield rate and guaranteed rate under each simulation, as well as its 99% quantile due to Assumption I.
- Assume that all the capital is invested in bond to calculate product fee, and figure out the final payout from the difference between gross return (actual stock yield rate or guaranteed rate) and fee.

As for the option-based product:

- Given the strike price of 114.9369% (identical to the guaranteed rate), calculate the price of the European put option.
- Calculate the product fee using the option price, the transaction cost and the target profit, then get the final payout from the difference between gross return and fee.

1.4.2 Improvement Part

In order to promote the capital-based product, we try different proportions of investment approach and identify the best portfolio that maximizes the final payout with certain accuracy. As for the option part, we change the strike price to either higher or lower to maximize the final payout and pick the appropriate strike price out.

1.4.3 Comparison Part

According to the consequences from previous calculations, we list the corresponding fee, the amount of final payout and the volatility of the capital-based product under the best portfolio and the option-based product under the suitable strike price separately. Additionally, we give some analysis and further suggestions based on the comparing table.

2 Analysis of Products

2.1 Capital-Based Product

In this section, we first predict the yield rates of investment to analysis the capital-based product, and then try to maximize the final payout of the product.

2.1.1 Yield Rates Prediction

In this part, we figure out the reasonable return rates of bill, bond and equity based on the historical data.

Considering that time series analysis may be applicable to the process, we first use time series method to predict future investment return rates. R codes of this process is provided in the appendix.

However, we found the prediction are not ideal. For bill and bond, the yield curves tend to be flat and yield rates remain in a fairly low level. This may be explained by the fact that observations which are nearer to present time point account for a higher proportion in the model, which is a characteristic of the method to analysis time series, and that recently the yield rates of bond and bill kept in a bad shape. More importantly, the predicted return rates cannot fully reflect the high volatility of the return on equity. Figure 1 shows the forecasts of equity yield rates. Notice that the predicted yield rates tend to be constant after year 2020, and it can’t reflect the volatility of stock returns.

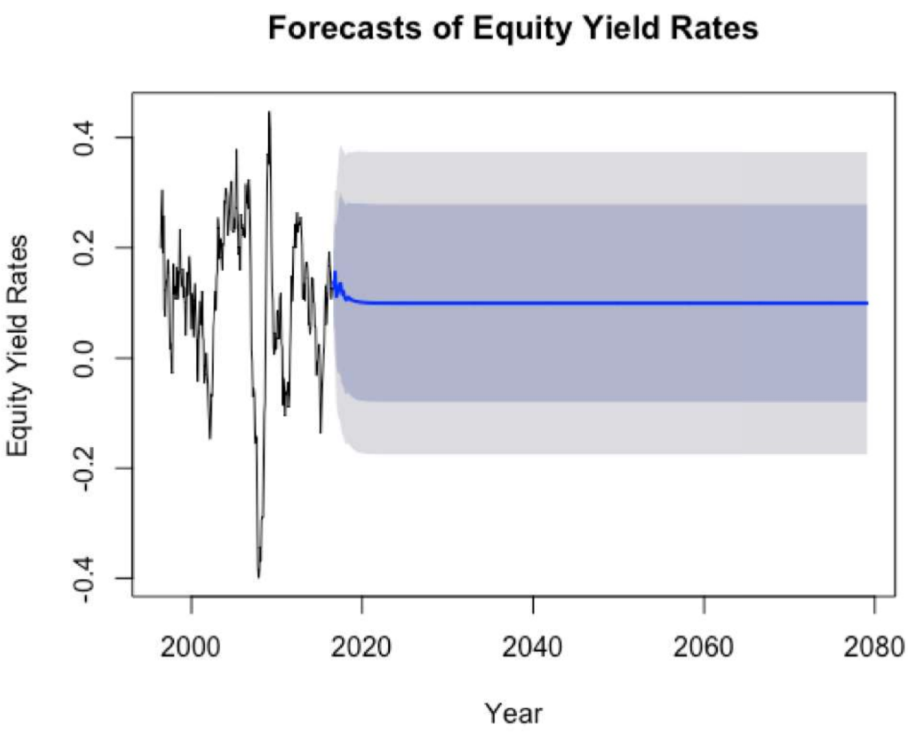

Figure 1 Forecasts of Equity Yield Rates

Therefore, we simply assume that the yield rates of the above three follow a certain distribution, and use the fitted distribution to proceed later work.

Take bond as the example of prediction process. We can obviously find that there's a sharp decrease around Year 1996 of bond yield rate by observing the raw data. And comparing with the bond return rate in the past, the yield has been fluctuating at a lower level. To be more conservative, we choose the data in recent 20 years with lower mean for analysis to predict lower yields.

Figure 2 shows five kinds of fitted density functions against the histogram and the empirical density function of the raw data. Notice that the Weibull distribution seems fits the data the best and other statistics also implies that. Therefore, we choose it for the most proper distribution of the yield rate of bond to predict future bond yields.

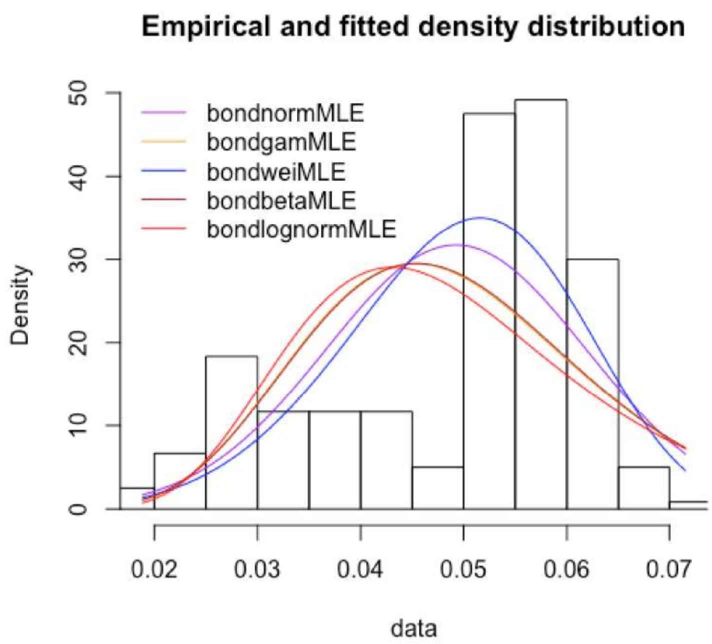

Figure 2 Fitted Distributions of Government Bond Yield

The prediction of bill and equity yields is similar to above process, and finally we choose Weibull distribution for predicting bill yield rates and normal distribution for equity. It should be noted that the skewness coefficient of historical equity yield rates is 0.3366177, which indicates that the distribution of stock returns is right-skewed, but the degree of skewness is not great. This is another reason why we choose the symmetric normal distribution for equity.

Additionally, in order to obtain information related to the comparison of the two products, we simulated yield rates of equity in 3 successive years for 10000 times by the fitted normal distribution and calculate its three-year cumulative rate of return. Meanwhile, we calculate the corresponding three-year cumulative guaranteed rate of return based on the given data, which equals to 1.149369.

2.1.2 The Basic Capital-Based Product

In the basic capital based product, we simply assume that all the capital is invested in bond and the yield rate is the average value of the historical data, which equals to 4.925458%. The cost of capital and profit target of your organization are 17%, 0.4% p.a. respectively. Calculating by R, the average final payout is 125.4494% and product fee is 22.2591%.

Emphasize again that the values expressed by percentages are based on the assumption that both the amount in the investors' funds and the stock price is 1 at our starting time point T. If the amount in the investors' funds is X and the stock price is S_T at the start, values of payout and fee will change proportionally.

The final result tells that only the product fee exceeds 22.2591% can the profit target of the Board be achieved and the probability of capital backing is sufficiently high. The average payout that your clients can received is 125.4494% of their origin input.

2.1.3 Product Improvement

In order to reduce the product fee and maximize the final payout of the product, we consider other investment strategies of capital. The aim is to find portfolios that provide better return rates, so we consider two criteria to get the best one:

- The average yield rates should be as high as possible.
- The volatility of yield rates should be low enough.

To match these requirements, we screen out portfolios that the probability of their yield rates being higher than the risk-free rate is the top 5% among all portfolios, and try to find the portfolio with the highest average yield rates among those selected ones. Holding all the assumptions and conditions remaining the same, and keep changing the weight of equity, bond and bill, we get an ideal portfolio consists of 11.40% equity, 88.08% bond and 0.52% bill, with average yield rate being 5.648578% p.a.

Next we use this portfolio as our new investment strategy and develop a better product. Its final payout and product fee are showed in Table 1 compared to the basic product, where:

- "mean eq yield rates" denotes the average yield rates of equity in three years.
- "guarantee rate" is the amount that the product is guaranteed pay to your clients.

- "product fee" is obviously the fee your organization should charge from your clients.
- "mean payout" is the average value of the final payout to your clients.
- "sd payout" is the standard deviation of the final payout to your clients.

Table 1 Comparison of The Basic and Modified Capital-Based Product

	mean eq yield rates	guarantee rate	product fee	mean payout	sd payout
Basic capital-based product	141.7144%	114.9369%	22.2591%	125.4494%	35.7690%
Modified capital-based product	141.7144%	114.9369%	20.9229%	126.7856%	35.7690%

By changing investment strategy, the product fee has a reduction of 1.3362% and the average final payout has an increase of 1.3362% accordingly. Thus, we choose this modified product as the best capital-based product.

2.2 Option-Based Product

In this section, we use the European put option as the base of product, since it ensures that we will get at least an amount of money equal to the strike price of the option on expiration date.

2.2.1 The Basic Option-Based Product

First of all, we suppose that strike price equals to the guarantee rate (114.9369%). Let S_T be the stock price on expiration date, K be the strike price and G be the guarantee rate. There are two cases that may happen on expiration date.

First, when $S_T < K$, you should exercise the option and get an amount of K, which equals to the guarantee rate G. Second, when $S_T \geqslant K$, no option should be executed at this time. Sell the stock share directly and you can get an amount of S_T, which is able to cover the guarantee return to your client.

In this case, price of the option is 13.37989% calculated by the Black-Scholes formula. With an additional 0.15% transaction cost per option trade and a profit target being 0.4% p.a., the product fee should be 16.8493% and the average final payout equals to 130.8592%.

2.2.2 Product Improvement

In order to reduce the product fee and maximize the final payout of the product, we consider two ways to improve the product:

- Increase strike price "K", which means K > guarantee rate G.
- Reduce strike price "K", which means K < guarantee rate G, and use capital to fill in the gap between guarantee rate and actual stock yield rate on expiration date if necessary. Investment strategy of capital is the portfolio we have discussed in Section 2.1.2, which consists of 11.40% equity, 88.08% bond and 0.52% bill, with average yield rate being 5.648578% p.a.

1.Reduce Strike Price

When you decrease strike price "K", there are three cases that may happen on expiration date.

Firstly, when $S_T < K < G$, you should exercise the option and get an amount of K. In this case, you may need capital to complement the difference between K and G, and final payout equals to the guarantee rate less product fee.

Secondly, when $K < S_T < G$, no option should be executed at this time. You should sell the stock share directly and just get an amount of S_T. Capital is needed to complement the difference between S_T and G, final payout also equals to the guarantee rate less product fee.

Thirdly, when $K < G < S_T$, also no option should be executed. Sell the stock share directly and you can get an amount of S_T. No need for capital in this case, final payout equals to S_T less product fee.

Through analysis, we can conclude that this product is actually a combination of the option-based and capital-based products (we call it the mixed product in the following section). Its product fee consists of the following three parts:

- Cost of capital to cover the gap of guarantee rate and actual stock yield rate
- Cost of option trading, including the price of option and 0.15% transaction cost per trade
- Profit target of SmallFund, which is 0.4% p.a.

We use above analysis to figure out the product fee and average payout under different strike price. Figure 3 shows the graph of different strike prices and their corresponding average final payout. From it we can see that as the strike price K decreases, the average final payout first increase to the maximum, then decrease to the minimum, and slightly increase to a stable level. The maximum and minimum values are obtained when K is about 0.96 and 0.6, respectively, calculated by R.

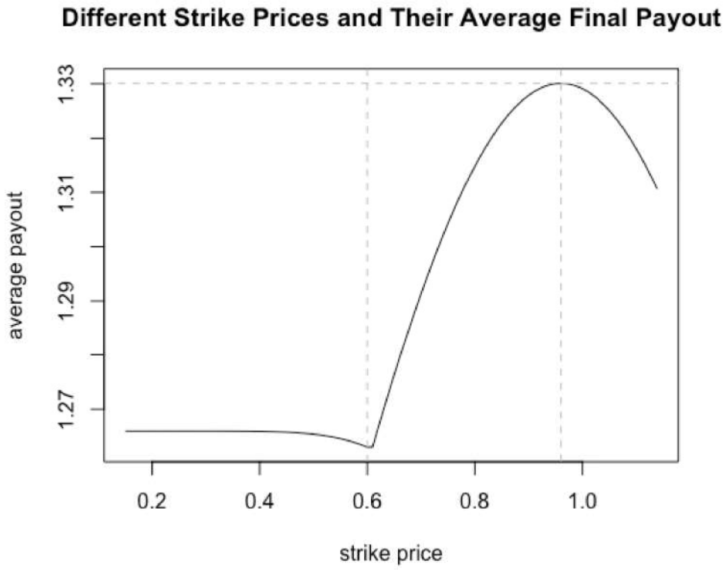

Figure 3 Different Strike Prices and Their Average Final Payout When Reducing Strike Price

Obviously, there is a strike price that can maximize the average final payout. Therefore, we set strike price of the option being 0.96 to develop a better product. Its final payout and product fee are showed in Table 2 compared to the basic product.

Table 2 Comparison of The Basic and Modified Option-Based Product

	mean eq yield rates	guarantee rate	product fee	mean payout	sd payout
Basic option-based product	141.7144%	114.9369%	16.8493%	130.8592%	35.7690%
Modified option-based product	141.7144%	114.9369%	14.7005%	133.0080%	35.7690%

By reducing strike price, the product fee declines 2.1488% and the average final payout increases 2.1488% accordingly. This modified product promotes the profitability of option-based products significantly.

Moreover, we can see in Figure 3 that the average final payout tends to be constant as the strike price is lower than 0.4. This is because that when the strike price is low enough, the option price is close to zero and the cost of option trading equals to the transaction cost (0.15%). Additionally, the probability of exercising options is near zero, since stock price S_T are unlikely to be lower than strike price K. Capital is used to fill in the gap between S_T and G. In this case, the mixed product is roughly equivalent to the capital-based product, except for the 0.15% transaction cost. Table 3 provides some examples. Notice that when the strike price is lower than 0.4, the average payout is 126.60%, which is close to 126.79% of the capital-based product.

Table 3 Different Strike Prices and Their Comparison

strike price	option trading cost	product fee	average payout
15.00%	0.1500%	21.1090%	126.5995%
25.00%	0.1500%	21.1090%	126.5995%
35.00%	0.1508%	21.1100%	126.5984%
45.00%	0.1655%	21.1282%	126.5802%
65.00%	0.6026%	20.0751%	127.6334%
75.00%	1.4388%	17.2820%	130.4264%
85.00%	3.0230%	15.4295%	132.2789%
95.00%	5.5374%	14.7068%	133.0017%
105.00%	9.0579%	15.1907%	132.5177%
114.00%	13.0689%	16.6446%	131.0639%

2.Increase Strike Price

When strike price "K" is greater than the guarantee rate, there is no need to fill the gap between guarantee rate and actual stock yield rate with capital. The product fee consists of only two parts: the cost of option trading and a profit target of 0.4% p.a.

Similarly, we draw a plot about different strike prices and their corresponding average final payout, as showed in Figure 4.

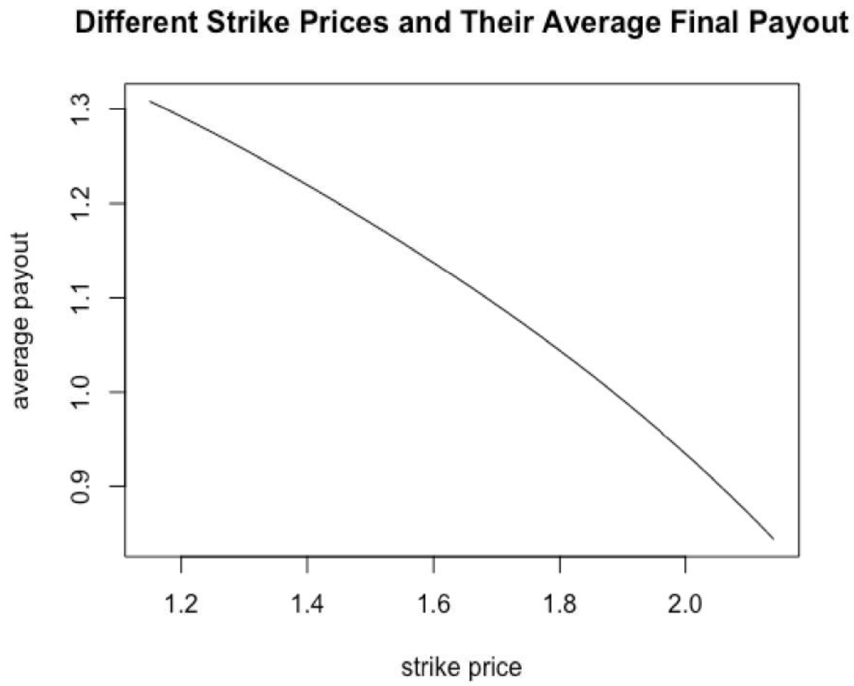

Figure 4 Different Strike Prices and Their Average Final Payout When Increasing Strike Price

As can be seen intuitively from the figure, with the increase of strike price K, the average payout decreases. Therefore, this method cannot be used to improve the product.

2.3 Comparison

The comparison of capital-based products and option-based products is showed in Table 4. From the table, we can know that the option-based products have lower product fee and higher average payout compared to the capital-based products. Standard deviation of payout of the two products is the same since the trend of stock price is identical in both cases.

Table 4 Comparison of Basic and Modified Products

	mean eq yield rates	guarantee rate	product fee	mean payout	sd payout
Basic capital-based product	141.71%	114.94%	22.26%	125.45%	35.77%
Modified capital-based product	141.71%	114.94%	20.92%	126.79%	35.77%
Basic option-based product	141.71%	114.94%	16.85%	130.86%	35.77%
Modified option-based product	141.71%	114.94%	14.70%	133.01%	35.77%

Obviously, benefits of the option-based product are that it can effectively ease the impact of economic depression and maintain the stability of investment returns. However, some new risks may emerge from it, like moral hazard of default, even though it reduces the loss caused by market volatility.

More specific types of risks, such as decreasing willingness to sell put options during depression, will be discussed in the Section 4 Risks and Uncertainties. Correspondingly, the capital-based product has to take more risks from market fluctuations, but it also provides diversified investment choices, which is actually what the option-based product lacks.

Therefore, the modified option-based product, which is actually a combination of the option-based and capital-based products, has advantages of the two products, and seems to be the most profitable choice for your organization.

3 Recommendations

Based on above analysis, we provide the following suggestions:

- If any product uses capitals when necessary, the capital should be invested in the portfolio consists of 11.40% equity, 88.08% bond and 0.52% bill, with average yield rate being 5.648578% p.a., to ensure that the volatility of return is low enough and the average return rates is as high as possible.
- If you want to maximize the average payout to investors of these products, use the option-based product and set the strike price to be 0.96 is the best choice, as discussed in Section 2.2&2.3. The gap between actual stock yield rate and guarantee rate can be filled up by capital, which makes the product be a combination of the option-based and capital-based products. The product fee is 14.40% and average final payout to investors is 133.01%.
- If you want to further improve the product, you may need to consider other ways to reduce product fees, such as cutting down the cost of capital use or finding other financing methods with lower cost. You may also take a more aggressive investment strategy, which may increase final payout to investors when the market is in good shape, but it will also increase risks. Thus, to be more conservative, we do not recommend this approach.

4 Risks and Uncertainties

Even though we choose prudent assumptions to do the work, there are still some circumstances our recommendations may not work.

4.1 Inflation

Since above analysis do no take inflation into consideration according to Assumption III, if inflation is getting extremely worse during the three years, the accuracy of our analysis may be influenced.

4.2 Economic Depression

If a major economic depression occurs, like what happened in the Great Depression in America in 1930s, it will first trigger a series of direct reactions: bank failures, factory closures, unemployment, and poverty. As a result, domestic stocks will be sold off in large numbers and investment rate will decline.

Although options play an important role on guarantee of profitability during economic downturns, few people are willing to be the seller of put option in a recession and the option-based product will not work.

4.3 Emergencies

Emergencies, such as a natural disaster or war can cause devastating effects on a small country. Financial markets will be hit hard due to the scarcity of materials and the need for reconstruction, which lead to return of these product being great deviated from expectations. This could lead to similar consequences as those when economic depression occurs.

5 Appendix

```
#============================================================================
#Part 1 ：  Capital based product
#============================================================================

times=10000##模拟次数

#============================================================================

#分别列出 equities，bonds，cash 的 investment expense
iexpense<-c(0.002,0.0015,0.001)

#============================================================================

#预测 bill 收益率
#使用近 20 年数据，因为在 1996 年前后，bill 的收益率有较为明显的变化
billdata=read.csv(file.choose()) #导入 bill 近 20 年.csv
length(billdata$BANK.BILL.YIELDS)
y0=billdata$BANK.BILL.YIELDS/100
summary(y0)
mean(y0)
sd(y0)
hist(y0,breaks = 50)
lines(density(y0),lty=1)
length(y0)
length(y0[y0<0]) #没有小于零的值，可以采用许多分布进行拟合

library("fitdistrplus")
#用 MLE 进行正态分布拟合
billnormMLE<-fitdist(y0,"norm",method = "mle")
denscomp(billnormMLE,fitcol = "purple")
lines(density(y0),lty=1)

#Gamma 分布拟合
billgamMLE<-fitdist(y0,"gamma",method = "mle")
denscomp(billgamMLE,fitcol = "orange")
lines(density(y0),lty=1)

###Weibull 分布拟合
billweiMLE<-fitdist(y0,"weibull",method = "mle")
denscomp(billweiMLE,fitcol = "blue")
lines(density(y0),lty=1)

#Beta 分布拟合
billbetaMLE<-fitdist(y0,"beta",method = "mle")
denscomp(billbetaMLE,fitcol = "brown")
lines(density(y0),lty=1)

#lognormal 拟合
billlognormMLE<-fitdist(y0,"lnorm",method = "mle")
denscomp(billlognormMLE,fitcol = "red")
lines(density(y0),lty=1)

txt <- c("billnormMLE","billgamMLE","billweiMLE","billbetaMLE","billlognormMLE")
```

```
denscomp(list(billnormMLE,billgamMLE,billweiMLE,billbetaMLE,billlognormMLE),legendtext=t
xt,
              fitlty=1,fitcol = c("purple","orange","blue","brown","red"),
              main="Empirical and fitted density distribution")
lines(density(y0),lty=1)

#goodness-of-fit tests
gofstat(list(billnormMLE,billbetaMLE,billweiMLE,billlognormMLE,billgamMLE),
            discrete =
TRUE,fitnames=c("billnormMLE","billbetaMLE","billweiMLE","billlognormMLE","billgamMLE"
))

#选用近 20 年的数据，Weibull 的 AIC,BIC 最小
summary(billweiMLE)

#根据拟合结果
#bill 服从 weibull 分布，parameter 为 shape=3.46767905,scale=0.05082414
#生成该情况下 bill 的收益率矩阵
billyield20<-matrix(NA,ncol = 3,nrow = times)#4 年
for (i in 1:times){
   set.seed(i)
   billyield20[i,]<-rweibull(3,shape=3.46767905,scale=0.05082414)
   i=i+1
}
billyield20<-billyield20 - iexpense[3]
summary(billyield20)
mean(billyield20)
sd(billyield20)

billexpense=0.0010    #investment expenses for bill
billyield20=billyield20-billexpense
billyr3=mean(billyield20)
a=(1+billyr3)^3-1
#==============================================================

#预测 bond 收益率，选用近 20 年的数据
bonddata=read.csv(file.choose())   #导入 bond 近 20 年.csv
length(bonddata$GOVERNMENT.BOND.YIELDS)
y0=bonddata$GOVERNMENT.BOND.YIELDS/100
meanbond <- mean(y0)
meanbond
hist(y0,breaks = 50)
lines(density(y0),lty=1)

length(y0)
length(y0[y0<0])#没有小于零的值，可以采用许多分布进行拟合

#用 MLE 进行正态分布拟合
bondnormMLE<-fitdist(y0,"norm",method = "mle")
denscomp(bondnormMLE,fitcol = "purple")
lines(density(y0),lty=1)

#Gamma 分布拟合
bondgamMLE<-fitdist(y0,"gamma",method = "mle")
denscomp(bondgamMLE,fitcol = "orange")
lines(density(y0),lty=1)

###Weibull 分布拟合
```

```
bondweiMLE<-fitdist(y0,"weibull",method = "mle")
denscomp(bondweiMLE,fitcol = "blue")
lines(density(y0),lty=1)

#Beta 分布拟合
bondbetaMLE<-fitdist(y0,"beta",method = "mle")
denscomp(bondbetaMLE,fitcol = "brown")
lines(density(y0),lty=1)

##lognormal 拟合
bondlognormMLE<-fitdist(y0,"lnorm",method = "mle")
denscomp(bondlognormMLE,fitcol = "red")
lines(density(y0),lty=1)

txt <- c("bondnormMLE","bondgamMLE","bondweiMLE","bondbetaMLE","bondlognormMLE")
denscomp(list(bondnormMLE,bondgamMLE,bondweiMLE,bondbetaMLE,bondlognormMLE),legen
dtext=txt,xlegend = "topleft",
              fitlty=1,fitcol = c("purple","orange","blue","brown","red"),
              main="Empirical and fitted density distribution")
lines(density(y0),lty=1)

#goodness-of-fit tests

gofstat(list(bondnormMLE,bondbetaMLE,bondweiMLE,bondlognormMLE,bondgamMLE),
            discrete =
TRUE,fitnames=c("bondnormMLE","bondbetaMLE","bondweiMLE","bondlognormMLE","bondga
mMLE"))

#选用近 20 年的数据，Weibull 的 AIC,BIC 最小
summary(bondweiMLE)

#根据拟合结果
#bond 服从 weibull 分布，parameter 为 shape=5.02201861,scale=0.05390222
#生成该情况下 bond 的收益率矩阵
bondyield20<-matrix(NA,ncol = 3,nrow = times)
for (i in 1:times){
   set.seed(i)
   bondyield20[i,]<-rweibull(3,shape=5.02201861,scale=0.05390222)
   i=i+1
}
bondyield20<-bondyield20 - iexpense[2]
summary(bondyield20)
mean(bondyield20)
sd(bondyield20)

#=========================================================================

eqdata=read.csv(file.choose()) #导入 eq.csv
length(eqdata$EQUITIES.YILEDS)
y0=eqdata$EQUITIES.YILEDS
hist(y0,breaks=50)
lines(density(y0),lty=1)

#由于除了正态分布之外的其他分布都定义在大于零的区间内，
#因此这里加上一个能使 y3 均为正的数用于拟合，拟合完毕后再减去这个数
summary(y0)     #y0 的最小值为-0.39952
y4=y0+0.4
y4[y4<0]         #y4 中已经没有小于零的值了
```

```
#正态分布拟合
eqnormMLE<-fitdist(y4,"norm",method = "mle")
denscomp(eqnormMLE,fitcol = "purple")
lines(density(y4),lty=1)

###Gamma 分布拟合
eqgamMLE<-fitdist(y4,"gamma",method = "mle")
denscomp(eqgamMLE,fitcol = "orange")
lines(density(y4),lty=1)

#Weibull 分布拟合
eqweiMLE<-fitdist(y4,"weibull",method = "mle")
denscomp(eqweiMLE,fitcol = "blue")
lines(density(y4),lty=1)

#lognorm 拟合
eqlognormMLE<-fitdist(y4,"lnorm",method = "mle")
denscomp(eqlognormMLE,fitcol = "red")
lines(density(y4),lty=1)

txt <- c("eqnormMLE","eqgamMLE","eqweiMLE","eqlognormMLE")
denscomp(list(eqnormMLE,eqgamMLE,eqweiMLE,eqlognormMLE),legendtext=txt,
         fitlty=1,fitcol = c("purple","orange","blue","brown"),
         main="Empirical and fitted density distribution")
lines(density(y0),lty=1)

#对比四种分布的 AIC 和 BIC
gofstat(list(eqnormMLE,eqlognormMLE,eqgamMLE,eqweiMLE),
        discrete =
TRUE,fitnames=c("eqnormMLE","eqlognormMLE","eqgamMLE","eqweiMLE"))

#正态分布的 AIC 和 BIC 最小，因此选用正态分布拟合 equity 的收益率
summary(eqnormMLE)

#根据拟合结果
#equity 服从正态分布，parameter 为 mean=0.5246394,sd=0.1923657
#生成 eq 的收益率矩阵
eqyield2<-matrix(NA,ncol = 3,nrow = times)
for (i in 1:times){
  set.seed(i)
  eqyield2[i,]<-rnorm(3,mean=0.5246394,sd=0.1923657)
  i=i+1
}
eqyield=eqyield2-0.4-iexpense[1]#为了拟合方便，将拟合数据全部加上 0.4，所以预测 equity 时
减去 0.4
eqyield
mean(eqyield)
sd(eqyield)

#equity3 年累计收益率
eqyield3<-matrix(NA,ncol = 1,nrow = times)
for (i in 1:times){
  set.seed(i)
  eqyield3[i]<-(1+eqyield[i,1])*(1+eqyield[i,2])*(1+eqyield[i,3])
  i=i+1
}
eqyield3
```

```
#收益率模拟结束
#===============================================================================

#根据每年 RPI 求出 3 年累计的应保证的收益率
RPI <- c(0.025,0.0275,0.03)
y1gur <- 1+0.02+RPI[1]
y2gur <- 1+0.02+RPI[2]
y3gur <- 1+0.02+RPI[3]
gurrate <- y1gur*y2gur*y3gur
gurrate
gurrate <- matrix(rep(gurrate ,length(eqyield3)), nrow = length(eqyield3))

#计算 capital
capital<-matrix(NA,ncol = 1,nrow = times)
for (i in 1:times){
  set.seed(i)
  if(eqyield3[i]<gurrate[i]){
    capital[i]<-gurrate[i]-eqyield3[i]
  }else{
    capital[i]<-0}
  i=i+1
}
capital
##99%分位点的 capital
qc<-quantile(as.numeric(capital),0.99)
qc

#计算仅以 bond 为投资方式的 fee
return <- 0.004
feeyear0 <- (0.17-meanbond)*(as.numeric(qc)) + return
feeyear0
feeyear03 <- (1+feeyear0)^3-1
feeyear03 #累积三年的费用

#求 payout
payout0 = matrix(rep(NA,length(eqyield3)), nrow=length(eqyield3))
for (i in 1:length(eqyield3)) {
  payout0[i,]=max(gurrate[i,],eqyield3[i,])-feeyear03
  i=i+1
}
payout0
mean(payout0)

#构建选用 capital-based 产品的股票收益率、保证收益率、费用和最终支出比较表
capitalplan0 = data.frame(eqyield3,gurrate,feeyear03,payout0)
capitalplan0
lapply(capitalplan0,mean)


##==================建议部分：计算最大投资收益组合==========================

#生成预期投资收益率的矩阵(此处以近 20 年的数据为准)
bondfore1<-bondyield20
billfore1<-billyield20
eqfore1<-eqyield
```

```
times2=10000#模拟次数
##定义函数计算投资收益率最大的 portfolio
f<-function(start1,end1,start2,end2,power){
  #start1,end1,start2,end2 分别为 portfolio 中 equity 和 bond 的比例区间限定，power 为方便调整结果精度设计的 parameter
  portfolio<-matrix(0,nrow=times2,ncol=3)
  eppt<-rep(NA,times2)
  bppt<-rep(NA,times2)
  cppt<-rep(NA,times2)
  for (i in 1:times2){
    set.seed(i)
    eppt[i]=sample(start1:end1,1)/(10^power)
    bppt[i]=sample(start2:end2,1)/(10^power)
    cppt[i]=0
    if(eppt[i]+bppt[i]<=1){
      cppt[i]<-1-(eppt[i]+bppt[i])
    }else{
      cppt[i]=NA
    }
    kkk<-c(eppt[i],bppt[i],cppt[i])
    portfolio[i,]<-kkk
  }
  #筛选掉第三列(Cash 权重)为 NA，以及权重重复的投资组合，得到所有权重为 10%的投资组合 portfolio2_dup
  portfolio2<-portfolio[is.na(portfolio[,3])==FALSE,]
  portfolio2_dup<-portfolio2[!duplicated(portfolio2),]
  #计算每个 portfolio 的平均收益率
  j=0
  result=0
  prob=0
  repeat{
    j=j+1
    ir<-portfolio2_dup[j,1]*eqfore1+portfolio2_dup[j,2]*bondfore1+portfolio2_dup[j,3]*billfore1
    yr<-matrix((1+ir[,1]))*matrix((1+ir[,2]))*matrix((1+ir[,3]))-1
    probability=length(yr[which(yr>a)])/length(yr)##求每组 portfolio 的收益率比 bill 高的概率，作为后续筛选条件

    meanyr<-mean(yr)
    result<-c(result,meanyr)
    prob<-c(prob,probability)

    if(j>=nrow(portfolio2_dup)) break
  }
  result<-result[2:length(result)]##去掉第一个赋值的 0
  prob=prob[2:length(prob)]##去掉第一个赋值的 0
  qc2=quantile(prob,0.95)##取 0.95 分位点（如果取 0.99 只剩一组数据）
  portfolio2_dup=portfolio2_dup[which(prob>qc2),]##筛选出高于概率的 95%分位点的 portfolio
  result=result[which(prob>qc2)]
  #求出使平均收益率最小化的 portfolio 的下标
  zuihou=matrix(portfolio2_dup[which(result==max(result)),],c(1,3))
  colnames(zuihou)<-c("eppt","bppt","cppt")

  maxyr=max(result)
  list=list(zuihou,maxyr)
  names(list)=c("portfolio","mean yeild rate")

  return(list)
}
```

```
#划定 eppt、bppt 在 portfolio 中比例的区间均为（0，1），计算收益率最大的 portfolio
v1<-f(0,10,0,10,1)
v1
#提高精度
v2<-f(0,100,0,100,2)
v2
#提高精度
v3<-f(0,1000,0,1000,3)
v3
#提高精度
v4<-f(0,10000,0,10000,4)
v4
#计算最佳 portfolio 的三年总收益
yr3<-v4$`mean yeild rate`
#计算每年平均收益率
yr1<-(1+yr3)^(1/3)-1
yr1

#计算 fee
return<-0.004
feeyear<-(0.17-yr1)*(as.numeric(qc))+return
feeyear
feeyear3<-(1+feeyear)^3-1
feeyear3   #最佳投资组合下的 fee（一个产品有且仅有一个 fee 的数额）

#求 payout
cappayout=matrix(rep(NA,length(eqyield3)),nrow=length(eqyield3))

for (i in 1:length(eqyield3)) {
   cappayout[i,]=max(gurrate[i,],eqyield3[i,])-feeyear3
   i=i+1
}

cappayout
capsdpayout <- sd(cappayout)

#构建选用 capital-based 产品的股票收益率、保证收益率、费用和最终支出比较表
capitalplan=data.frame(eqyield3,gurrate,feeyear3,cappayout)
capitalplan

#=====================================================================
#Part 2 : Option based product
#=====================================================================

##1.计算当 K= guarantee = 1.1494 时的 final payout

#在 capital 部分中已得 equity 的 1000 次模拟累积收益 eqyield3
eqyield3
equity=eqdata$EQUITIES.YILEDS
bill20=billdata$BANK.BILL.YIELDS

#导入计算期权价格所需数据
r=mean(bill20)/100
T=3
S0=1
K=gurrate[1]
sigma=sd(equity)
tranc=0.0015
```

```
#计算期权价格
d1=(log(S0/K)+(r+sigma^2/2)*T)/(sigma*sqrt(T))
d2=d1-sigma*sqrt(T)
price0=K*(exp(-r*T))*pnorm(-d2)-S0*pnorm(-d1)
price=tranc+price0

#计算年金使得三年累计为 option 价格
f1<-function(x) x*(1/(1+r)+1/(1+r)^2+1/(1+r)^3)-price
annufee=uniroot(f1,c(0,1e10))$root
annufee

#计算 product fee
profit=0.004
productfee=annufee+profit
productfee
fee=(1+productfee)^3-1

#计算 payout
option <- cbind(eqyield3,gurrate,fee)   #gurrate 是前文计算出的 guaranteed return
colnames(option)<-c(1,2,3)
payout <- rep(0,times)
for (i in 1:times){
   payout[i]<-ifelse(option[i,1]>option[i,2],option[i,1],option[i,2])-fee
   i=i+1
}
payout
var(payout)
finalpayout=mean(payout)
finalpayout

#列表 option 及每种情况下的 payout
optionplan <- data.frame(option, payout)
colnames(optionplan)<-c("eqyield3","gurrate","fee","payout")
optionplan
lapply(optionplan,mean)



##2.建议部分
###(1)减少 strike price-"K"，此时存在三种情况。
#1.ST<K<guarantee  此时实行期权，需要 capital 补足 K 和 guarantee 之间的差值。
#2.K<ST<guarantee  此时不实行期权，需要 capital 补足 ST 和 guarantee 之间的差值。
#3.K<guarantee<ST  此时不实行期权，不需要 capita 补足。

#将 K 值定为 0.15 至 1.14 之间的等间隔的 100 个数
times3=100
K0=15:114/100

#求 final payout
prices=rep(0,times3)
annufee=rep(0,times3)

cappre=matrix(rep(0,times),nrow=times,ncol=1)
cappre99=rep(0,times3)
kpayout=rep(0,times)
averkpayout=rep(0,times3)
sdkpayout=rep(0,times3)
```

```
capannualcost=rep(0,times3)
opbestannualfee=rep(0,times3)
opbestfee=rep(0,times3)

for(k in 1:times3){
  d11 = (log(S0/K0[k])+(r+sigma^2/2)*T)/(sigma*sqrt(T))
  d12 = d11-sigma*sqrt(T)
  prices0 = K0[k]*(exp(-r*T))*pnorm(-d12)-S0*pnorm(-d11)
  prices[k]=tranc+prices0
  fs<-function(x) x*(1/(1+r)+1/(1+r)^2+1/(1+r)^3)-prices[k]
  annufee[k]=uniroot(fs,c(0,1e10))$root
  #对每个 K 值计算需要补充的 caiptal(使用之前模拟出的 eqyield3)
  for(i in 1:times){
    cappre[i,] <- gurrate[i]-min(gurrate[i],max(K0[k],eqyield3[i]))
    i = i+1
  }
  #计算每个 K 值对应的 99%分位点的 capital
  cappre99[k]=quantile(cappre,0.99)
  #计算每个 K 值对应的 capital 的年成本
  capannualcost[k]=(0.17-yr1)*cappre99[k]
  #计算特定 K 值下的 fee
  opbestannualfee[k]=annufee[k] + capannualcost[k] + profit
  #总费用
  opbestfee[k]=(1+opbestannualfee[k])^3-1
  #计算特定 K 值下的 final payout
  for (i in 1:times) {
    kpayout[i]=max(gurrate[i],eqyield3[i])-opbestfee[k]
    i=i+1
  }
  averkpayout[k]<-mean(kpayout)
  sdkpayout[k]<-sd(kpayout)
  k=k+1
}

#K 值与期权价格，产品费用，payout 均值和 payout 方差的集合
Kprices<-data.frame(K0,prices,opbestfee,averkpayout,sdkpayout)
Kprices
plot(K0,averkpayout,type="l")

#找出使 payout 最大化的 K 值
maxk<-which(Kprices$averkpayout==max(Kprices$averkpayout))
Kprices[maxk,]

##K0 取 0.43 左右及更小值时，期权的价格趋近于 0，同时计算出的 payout 均值与 capital
based 产品相同
##重点：论文中应说明 K0 减小到一定程度时，该期权与资本结合的产品基本等同于资本产
品

###（2）增加 strike price,即 K>guarantee
#此时不需要 capital 补足期权执行的差值，且即使 S>K 不执行期权，收益也大于 guarantee
#1.K>guarantee>S,final payout=guarantee-fee
#2.K>S>guarantee  或  S>K>guarantee  时,final payout=S-fee

K2<-115:214/100

times4<-100

#求 final payout
```

```
prices2=rep(0,times4)
annufee2=rep(0,times4)
kpayout2=rep(0,times)
averkpayout2=rep(0,times4)
sdkpayout2=rep(0,times4)
opbestannualfee2=rep(0,times4)
opbestfee2=rep(0,times4)

for(k in 1:times4){
  d11 = (log(S0/K2[k])+(r+sigma^2/2)*T)/(sigma*sqrt(T))
  d12 = d11-sigma*sqrt(T)
  prices02 = K2[k]*(exp(-r*T))*pnorm(-d12)-S0*pnorm(-d11)
  prices2[k]=tranc+prices02
  fs<-function(x) x*(1/(1+r)+1/(1+r)^2+1/(1+r)^3)-prices2[k]
  annufee2[k]=uniroot(fs,c(0,1e10))$root
  #计算特定 K 值下的 fee
  opbestannualfee2[k] = annufee2[k]+profit
  opbestfee2[k] = (1+opbestannualfee2[k])^3-1
  #计算特定 K 值下的 final payout
  for (i in 1:times) {
    kpayout2[i]=max(K2[k],eqyield3[i])-opbestfee2[k]
    i=i+1
  }
  averkpayout2[k]<-mean(kpayout2)
  sdkpayout2[k]<-sd(kpayout2)
  k=k+1
}

#K 值与对应价格，折合年金，average payout 的集合
Kprices2<-data.frame(K2,prices2,opbestfee2,averkpayout2,sdkpayout2)
Kprices2
plot(K2,averkpayout2,type = "l")

##由于 K 增加时，price 增加的幅度较大，出售股票所得增加的幅度相对较小，因此当 K 增
加时，
#payout 会出现持续减小的趋势


####两产品最终对比表
capitalplanbest=c(as.numeric(lapply(capitalplan,mean)), capsdpayout)
optionplanbest=c(mean(eqyield3),gurrate[1],Kprices[maxk,]$opbestfee,Kprices[maxk,]$averkpayout
, Kprices[maxk,]$sdkpayout)
compare=as.data.frame(rbind(capitalplanbest,optionplanbest))
colnames(compare) <- c("eq yield rate","guaranteed rate", "fee", "mean payout","sd payout")
compare

# ==========================================
# Alternatively，根据时间序列拟合 bond 和 bill 的收益率（最终没有采用）
# ==========================================
# 由于时间序列本身性质，距离原时间点更近的变量占比更大，而今年附近的收益率均处低
迷状态，
# 因此预测出 bill，bond 的收益率远小于实际收益率，且由于算法性质，在预测的后 30 年中
收益率趋于定值，不合理
# 最终得出的使 levy rate 最小化的投资比例大约为：eq80%，bond20%，bill0%,显然不适用于
实际，考虑拟合分布分析
# 此处论文应包括：时间序列的合理性与局限性，时间序列拟合收益率结果及得到的投资
比，
# 收益率与投资比不合理的原因，以及选择分布拟合的合理性
```

#---------------------模拟出 investment rate (ir)--------------------#
#--------对 bond 和 bill 采用时间序列方法；对 eq 采用模拟分布函数的方法-----------#

```
#读取 bond 的收益率
bonddata=read.csv(file.choose())   #导入 bond.csv
y1=bonddata$GOVERNMENT.BOND.YIELDS
x1=bonddata$INVESTMENT.DATA
plot(x1,y1)   #可以看出 bond 的收益率是一个趋势较为明显的时间序列
summary(y1)
mean(y1)
sd(y1)

#读取 bill 的收益率
billdata=read.csv(file.choose()) #导入 bill.csv
y2=billdata$BANK.BILL.YIELDS
x2=billdata$INVESTMENT.DATA
plot(x2,y2)
summary(y2)
mean(y2)
sd(y2)

#读取 eq 的收益率
eqdata=read.csv(file.choose()) #导入 eq.csv
x3=eqdata$INVESTMENT.DATA
y3=eqdata$EQUITIES.YILEDS
plot(x3,y3)
summary(y3)
mean(y3)
sd(y3)
#eq 的收益率较为均匀 可以看成白噪声（？？）

#安装时间序列分析程序包
install.packages("forecast")
library(forecast)

###用时间序列研究 bond 收益率
bondts=ts(y1,frequency = 12,start = c(1979,12))#将 bond 的收益率及其时间转化成时间序列形式
plot.ts(bondts)
abline(lm(bondts~time(bondts)))#添加线性拟合曲线，可以看出随着时间的推进，收益率不断减少

#先设置 train 即用来拟合的数据，test 即用来检验拟合数据是否准确的数据
train1<-window(bondts,start=c(1979,12),end=c(2015,12))
test1<-window(bondts,start=c(2016,1),end=c(2016,12))

#先使用 Holt-Winters 算法分析该数据
stl(bondts,s.window="periodic")
#使用 Holt-Winters 算法作出 bond 的时间序列图像、季节性图像、趋势图像、remainder 图像
plot(stl(bondts,s.window="periodic"))
#使用 Holt-Winters 算法拟合数据
bondhw<-hw(train1,h=12,seasonal='multiplicative')
bondhw
#用 accuracy 检验拟合程度，选用 rmse 作为标准
accuracy(predict(bondhw,h=12),test1)          #rmse=0.6706962

#再使用 auto.arima 算法分析该数据
bondarima=auto.arima(train1)
```

```
accuracy(forecast(bondarima,h=12),test1)      #rmse=0.4956113

#由以上结果可以得出选用 auto.arima 算法分析该时间序列最为准确，因为其 rmse 值最小

#使用 auto.arima 算法预测未来 63 年的 bond 收益率
bondarimaT=auto.arima(bondts)
forecast(bondarima,h=750)
#因为预测值里自 2030 年开始出现负值（因为整体呈现递减趋势），因此选用最近 20 年的数据作为预测依据再次进行预测
bondts1=window(bondts,start=c(1996,6))
bondts1arima=auto.arima(bondts1)
bondfore=forecast(bondts1arima,h=750)
bondfore1=bondfore$mean
bondfore1   #这里预测出来的是每月的收益率，计算年收益率的过程在后面

#bond 年收益率
bondforetable<-as.data.frame(bondfore1)
bondforetable
B=bonddata[c(446:451),]
b=c(rep(seq(1,63,1),each=12))
c=as.character(b)
e=c(B$GOVERNMENT.BOND.YIELDS,bondforetable$x)
bondyield=data.frame(c,e)
attach(bondyield)
bondyield1=aggregate(bondyield$e,by=list(c),FUN=mean)
bondyield2=bondyield1[order(as.numeric(bondyield1$Group.1)),]
colnames(bondyield2)<-c("Year","Bond Yield Rate")
bondyield=bondyield2$`Bond Yield Rate`
bondyield
summary(bondyield)
mean(bondyield)
sd(bondyield)
#最后导出来的数据是用月份来计算的年均收益率，第二列是第几年，第三列是收益率


###bill 的估算
billts=ts(y2,frequency = 12,start = c(1979,12))#将 bill 的收益率及其时间转化成时间序列形式
plot.ts(billts)
abline(lm(billts~time(billts)))#添加线性拟合曲线，可以看出随着时间的推进，收益率不断减少

#使用近二十年数据，（因为在 1996 年前后，bill 的收益率有较为明显的变化）先设置 train 即用来拟合的数据，test 即用来检验拟合数据是否准确的数据
train2<-window(billts,start=c(1996,12),end=c(2015,12))
test2<-window(billts,start=c(2016,1),end=c(2016,12))

#先使用 Holt-Winters 算法分析
stl(billts,s.window="periodic")
#使用 Holt-Winters 算法作出 bill 的时间序列图像、季节性图像、趋势图像、remainder 图像
plot(stl(billts,s.window="periodic"))
#使用 Holt-Winters 算法拟合数据
billhw<-hw(train2,h=12,seasonal='multiplicative')
billhw
#用 accuracy 检验拟合程度，选用 rmse 作为标准
accuracy(predict(billhw,h=12),test2)          #rmse=0.3639

#auto.arima 分析该数据
billarima=auto.arima(train2)
accuracy(forecast(bondarima,h=12),test2)   #rmse=0.7717
```

```
#由以上结果可以得出选用 Holt-Winters 算法分析该时间序列最为准确，因为其 rmse 值最小，
#但在正式估算时因所选时间跨度过短，故使用 ARIMA

#使用 ARIMA 算法预测未来 63 年的 bond 收益率
#因为预测值里自 2030 年开始出现负值，因为整体呈现递减趋势，因此选用最近 20 年的数据
作为预测依据再次进行预测
billts1=window(billts,start=c(1996,6))
billARIMA=auto.arima(billts1)
billfore=forecast(billARIMA,h=750)
billfore1=billfore$mean
billfore1   #这里预测出来的是每月的收益率，下面计算 bill 的年收益率
billforetable<-as.data.frame(billfore1)

#bill 年收益率
b=c(rep(seq(1,63,1),each=12))
c=as.character(b)
A=billdata[c(446:451),]
d=c(A$BANK.BILL.YIELDS,billfore1)
billyield=data.frame(c,d)
attach(billyield)
billyield1=aggregate(billyield$d,by=list(c),FUN=mean)
billyield2=billyield1[order(as.numeric(billyield1$Group.1)),]
colnames(billyield2)<-c("Year","Bill Yield Rate")

billyield=billyield2$`Bill Yield Rate`
billyield
summary(billyield)
mean(billyield)
sd(billyield)

# ============================================
#对 eq 进行时间序列分析
eqts=ts(y3,frequency = 12,start = c(1980,1)) #将 eq 的收益率及其时间转化成时间序列形式
plot.ts(eqts)
abline(lm(eqts~time(eqts)))#添加线性拟合曲线，可以看出随着时间的推进，收益率不断减少

#使用近 20 年数据，因为在 1996 年前后，bill 的收益率有较为明显的变化
#先设置 train 即用来拟合的数据，test 即用来检验拟合数据是否准确的数据
train3<-window(eqts,start=c(1996,12),end=c(2015,12))
test3<-window(eqts,start=c(2016,1),end=c(2016,12))

#该数据偏向于稳态，因此选用 ARIMA 估计
#auto.arima 分析该数据
eqarima=auto.arima(train3)
accuracy(forecast(eqarima,h=12),test3)

#使用 ARIMA 算法预测未来 63 年 eq 收益率
#因为预测值里自 2030 年开始出现负值（因为整体呈现递减趋势），因此选用最近二十年的
数据作为预测依据再次进行预测
eqts1=window(eqts,start=c(1996,6))
eqARIMA=auto.arima(eqts1)
eqfore=forecast(eqARIMA,h=750)
eqfore1=eqfore$mean
eqfore1
plot(eqfore)
# ============================================
```

Declaration

As a member of the organization, I volunteer to write the analysis report as an investment actuary, and I am committed to being responsible for the authenticity and accuracy of the report, as well as the corresponding consequences. All of my work is based on analysis of historical data without any fabrication or fraud.