Charge-to-Mass Ratio for the Electron
Lab Report

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Abstract

The authors of this paper set out to measure the ratio of charge to mass for an electron, without actually measuring either quantity, by measuring the radius of a loop formed by an electron beam passing through a magnetic field. Our experiment entails a setup of a electron gun beam through two Helmholtz coils. Our calculations result in a value very close to the known value for e/m_e and indicate the success of our experimental design.

1 Introduction

Quantum mechanics tells us that charge is quantized, which was observed in the Millikan oil drop experiment. If the electron has a fixed charge, then does it also have a fixed mass? It is difficult to measure each quantity, but in an isolated ideal system with a good electron beam, we know the electrons will follow a known trajectory when under the influence of a uniform magnetic field.

By inducing a uniform magnetic field, we expect the electrons to experience a tangential force of a certain magnitude, as each electron has a fixed charge. As laws of physics tell us about the nature of the motion, we can be certain that formulas for centrifugal force would be appropriate for the situation. By obtaining this data, we can obtain the charge-to-mass ratio for an electron.

Such ratios are important, as for small particles, it is extremely hard to get accurate measurements for fundamental properties. So by calculating ratios using out-of-the-box experiments, we can derive the other fundamental properties from just one of them.

Background Theory

The main idea is that in a coil, the moving electrons generate a field perpendicular to the current. So, the effective field will be in the direction of a line joining the centers of the two coils. So close to the center of the coils, we get a near uniform magnetic field in that direction.

When we have a linear electron beam in the field, the field will cause the electrons to feel a force perpendicular to their motion and result in them turning. In the glass bulb, this will form a circle under favorable conditions.

The main equation we use comes from combining the equation for force exerted on a beam of electrons from an external magnetic field and for centrifugal motion for non-relativistic approximations and get a relation for the charge-mass ratio for the electron:

$$e\vec{v} \times \vec{B} = evB = \frac{mv^2}{r} \quad \& \quad e\Delta V = \frac{1}{2}mv^2$$
$$\implies 1/r = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{\Delta V}}$$

Here e is the charge of an electron, v is the velocity of the electron, B is the magnetic field strength for the coils, m is the mass of an electron, r is the radius of curvature, and ΔV is potential needed to achieve the velocity.

For calculating the magnetic field strength, we use the following equation:

$$B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 n I}{R}$$

With $\mu_0 = 4\pi\ 10^{-7}$ T m A^{-1} as permeability of free space, n as number of turns in each coil, I as current in the coils, and R as radius of the coils.

We use two modifications to this to increase our accuracy:

$$\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4(0.6583 + 0.29\frac{\rho^2}{R^2})^2} \tag{1}$$

$$B_c = \frac{1}{2}\sqrt{\frac{2m}{e}\Delta V} - B_e \tag{2}$$

In equation 1, ρ is the off-axis distance, and R is the radius of the coils. This formula is used when the formed beam is not perfectly at the center. Equation 2 compensates for the external field due to the earth and other factors as B_e and calculates the exact field due to the coils to be B_c.

Lastly, we use $y = mx + b$ for fitting the plots to the data.

Experimental Setup

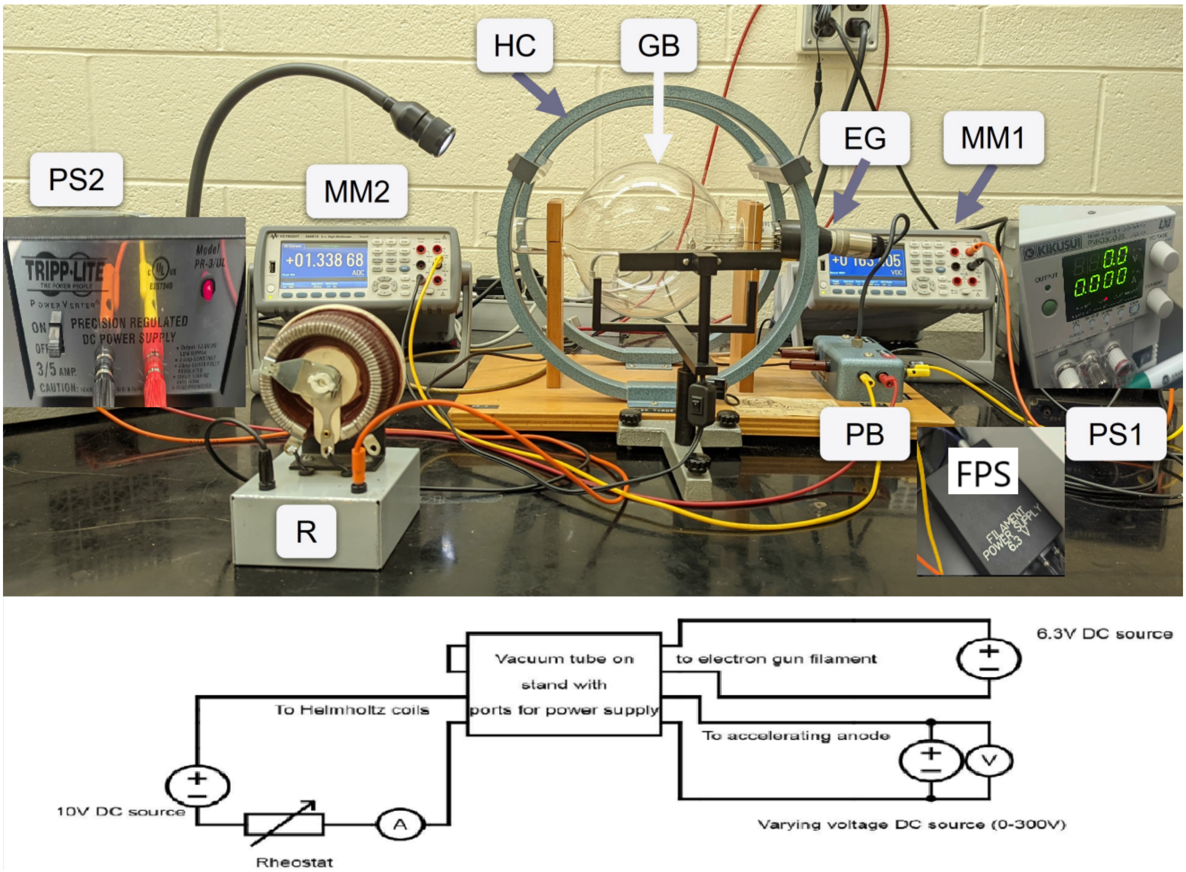

Figure 1: Experimental Setup Image alongside the circuit diagram from the lab manual with newer equipment (Zhan et. al., 2022). *Labels* explained below in setup configuration to avoid redundancy.

Our setup comprised connecting different elements to the coil-bulb apparatus. Our lab had a custom setup with 2 - 130 turns for each copper Helmholtz coil [HC] set parallel to each other with radius and separation $16.0 \pm 0.2cm$. These were positioned around a glass bulb [GB] filled with low-pressure hydrogen, with an unlabelled custom electron gun [EG] inside. There were three main power supplies: one for the coils [PS2] (Tripp-Lite PR-3/UL 13.8V DC with 3A constant supply), one for the electron gun filament [FPS] (Custom 6.3V AC Power Supply), and one for the anode [PS1] (Kikusui PMX320-0.2A Regulated DC Power Supply) inside the gun used to create the beam. These were all plugged into the board as per the circuit diagram above. We also used an OHMITE RLS10RE 3.873A 10Ω Rheostat [R] in series with the coil source to regulate the magnetic field, and used two Keysight 34461A Digital Multimeters, one in series with the coils as ammeter [MM2] and one in parallel with the anode as voltmeter [MM1]. Lastly, we used a 30cm wooden ruler and an unlabelled self-illuminated scale with a plastic reflector (see image in appendix).

2 Methods

Data Collection

We began by collecting measurements for the setup: distance between the coils, their radius, the plastic reflector, and the self-illuminated scale. We also took baseline measurements for the ammeter/voltmeters and made sure to zero them.

Before data collection, we turned the filament on first to let it heat up for 30 seconds. Then, we turned on the anode potential and turned it up until we saw the electron beam. Afterward, we turned the supply for the coils on and observed the beam curve. We positioned the multimeters as far away from the setup as possible and only turned the ammeter on.

We observed that after setting a fixed voltage from the anode power supply, the voltmeter reading fluctuated within 0.1 V of a measurement, regardless of the setting. So, we only turned the voltmeter on during every change in the supply setting to take the base reading to avoid accumulated charges and then turned it off.

For taking the distance measurement, we set the screen as far away from the base of the electron gun as the screen was from the self-illuminated scale. Then, while looking at the electron curve, we moved our heads to see if the scale's reflection appeared to move with the curve, and if not, we adjusted the scale's position until it did before taking the measurement. Then, we measured the diameter of the circle formed on the scale.

As we only had a few sessions, We took multiple measurements for 6 different voltages, with enough gap to get good data and variation enough where it was significant enough while still giving us a reasonable curve to measure. We also made sure to have a perfect loop of electrons that did not curve. Lastly, we took data for measuring diameter at 4 different angles ($0°, 45°, 90°, 135°$) in 3 cases of different voltages for large (11.5cm), medium (9.0cm), and small (6.5cm) diameters. We also took a sample for interference from external fields (mobile phone)

Data Analysis

Data analysis was done by plotting the different voltage curves on current vs diameter and current curves at voltage vs. diameter and fitting a linear fit using scipy.optimize.curve_fit, as we expect linear relation between the values we measure,

3 Results

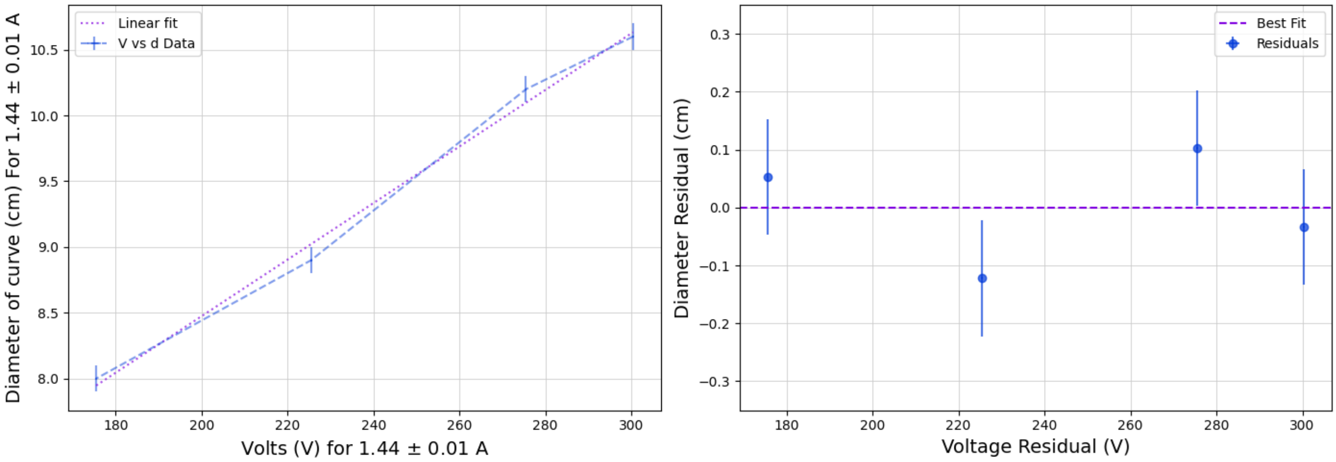

Figure 2: Diameter (cm) vs Accelerating potential (V) of electron ring for a setup with Helmholtz coil current 1.44 ± 0.01 A. Plots for data (blue) vs. fit (purple) and residuals from line of best fit. Line of best fit has slope 0.022 ± 0.002 cm V^{-1} with y-intercept 4.2 ± 0.4 cm. Horizontal errorbars plotted but very small due to the scale. Fit has χ^2 0.24 with probability 0.9, so values are reasonably good. Residuals are sort of evenly distributed (too few data-points). We claim the fit is a good one.

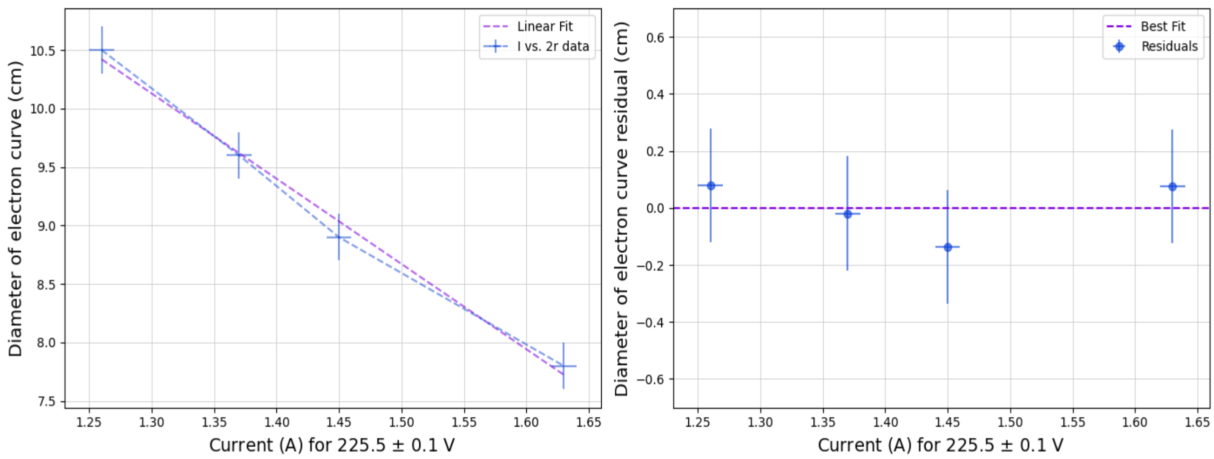

Figure 3: Diameter (cm) vs. Coil current (A) for electron ring with anode potential 225.5 ± 0.1 V. Plots for data (blue) vs. fit (purple) and residuals from line of best fit. Line of best fit has slope -7.3 ± 0.7 cm A^{-1} with y-intercept 19.6 ± 1.1 A. Residuals are balanced and unpatterned. Plots have a value χ^2 0.26 with probability 0.9 (probably due to the few number of values). So overall, the fit is pretty decent.

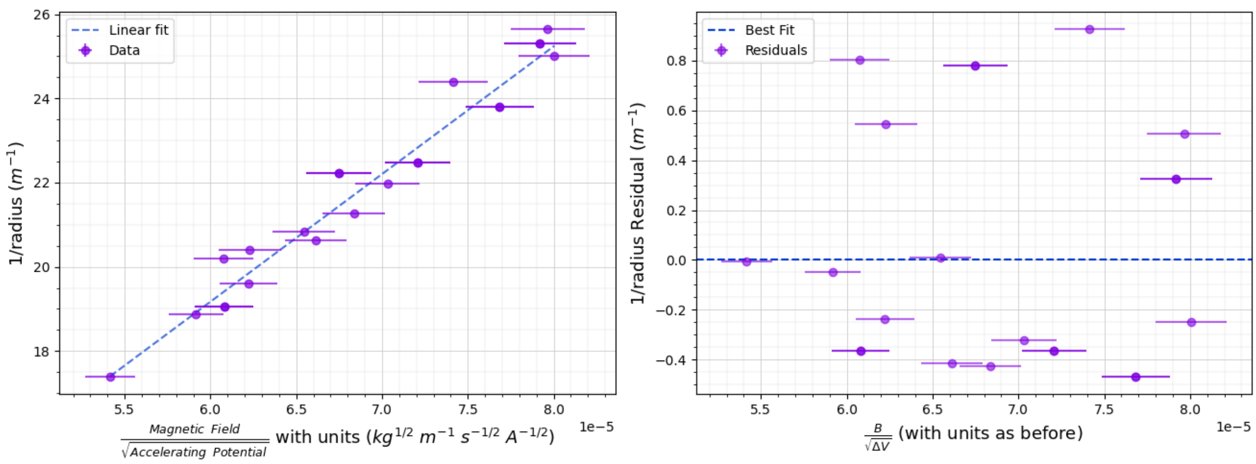

Figure 4: Best fit and residuals plot for 1/r vs $B/\sqrt{\Delta V}$ for the electron beam curve. Line of best fit (Blue) is a linear fit with slope $3.03 \times 10^5 \pm 0.13 \times 10^5$ $\text{A}^{\frac{1}{2}}$ $\text{s}^{\frac{1}{2}}$ $\text{kg}^{\frac{-1}{2}}$ with y-intercept 0.97 ± 0.87 m^{-1}. The residuals are slightly biased negatively, but are otherwise good and unpatterned. Plots have χ^2 1.02 and probability0.44, which are good. We thus deem the fit to be immaculate.

Sources of Error and Uncertainty Calculations

The main sources of error for this data were the angle of the curving beam and the measurement made using the self-illuminating scale. We tried our best to mitigate them, by ensuring the loop back hit the rough center of the back of the electron gun, and the self illuminated scale appeared as "stationary" as possible with respect to the electron ring. Inputs for current and voltage were pretty standard and had good resolution with low error in the measurement.

We also kept any ferromagnetic substances away from the bulb, and only turned on the voltmeter for taking the recording the voltage and turned it off before taking the measurement for the current and then the diameter.

The main uncertainty calculation was done using the accuracy specifications in the Keysight Manual (Link(1). For current range (3A), we had an uncertainty of $\pm[(0.0018 * reading) + (0.0006)A]$, and for voltage range (100-1000V), we had an uncertainty of $\pm[(0.00002 * reading) + (0.006)V]$.

However these uncertainties are very small and as we saw visible fluctuations of the voltage within 0.1V of the reading for the voltmeter, we chose that to be our upper bound for uncertainties on these measurements, and for current we saw the same but for 0.01A, and followed suit.

Another consideration was the unevenness of the field throughout the glass bulb, for positions more than 0.2 of the radius away from the center. In our observations, the curves tended to be very circular. So for the calculation using equation (1), we simply applied it evenly for the designated value of the radius. For the offset of the electron circle from the center, if some point's distance from the center is $p = r + k$, due to symmetry there is some point with distance $p' = r - k$, with radius r for some k. As R was calculated to be 16.0 ±0.2 cm, for all radii more than 4 cm, we did the calculation as per 1 by transposing the $B(0)$ term to the right.

For the radius of the coils and diameter of the observed circle, we used least count of the tool taken twice, once each for the start and end measurements, giving us ± 0.2 cm. For further calculations, we simply used propagation of uncertainties. No uncertainties were take for constants, 1 was takin for number of turns, formula for uncertainty calculation is provided in the appendix.

For small accelerating potential more than the threshold, and high coil current, we observed the electron beam to still form a circle, but highly skewed towards the electron gun. The diameter was around 6cm. It was still pretty circular, but we omitted some of the data for the least voltage and highest current (2 data points), as they were too large of an outlier in the final plot for $\frac{1}{r}$ vs $\frac{B}{\sqrt{\Delta V}}$.

4 Discussion

Overall, our experiment had very good data. For our observations of circularity, we observed it for large, medium and small radii, by tilting the scale to different angles and seeing the diameter value, which stayed the same.

We observed the influence of external ferromagnetic substances by bringing a phone near the glass bulb. To see maximum effect, we reduced the current as much as we could while maintaining a circle, and increased the anode voltage. In the appendix, figures 5, and 6 show that the phone's presence causes a distortion in the shape of the ring and consequentially the field.

We calculated the value of the external field by using the value of the x-intercept of the plot in figure 8. At $r = \infty$, we should have zero field, so whatever the value we get from x intercept $= -b/m = (-0.94 \pm 0.63) \times 10^{-3}$ Tesla. The minus sign indicates it's in teh direction opposite to the field of the coils. So the calculate value would be reduced by that much, but as it's so small, we consider it to be negligible.

Our final calculation for the slope of the plot in figure 4 is $(3.03 \pm 0.13) \times 10^5$ C$^{1/2}$ kg$^{-1/2}$. By squaring and then multiplying this by 2, we get experimental $e/m_e = (18.4 \pm 1.6) \times 10^{10}$ C/kg

For $e = 1.60218 \times 10^{-19}$ C and $m_e = 9.10938 \times 10^{-31}$ kg obtained from wikipedia, we get that $e/m_e = 17.59 \times 10^{10}$ C/kg which gives us $\sqrt{\frac{e}{2m_e}} = 2.97 \times 10^5C^{1/2}$ kg$^{-1/2}$, both of which are very close to our calculated value and validates our experimental calculations.

5 Conclusion

We conclude by saying our experiment to calculate the charge to mass ratio of the electron was a success. By introducing a magnetic field to an electron beam and measuring its radius, we were able to calculate the values for different forces at play (electromotive, magnetic, centrifugal). Our calculations resulted in $e/m = (18.4 \pm 1.6) \times 10^10$ C/kg, which is quite close to and within the error margin of the known value of the constant as 17.6×10^10 C/kg.

6 References

H. Zhan, E. Horsley, A. Harlick (2022). *Charge-to-mass ratio for the electron Lab Manual* (Revision: 63bbf38). https://www.physics.utoronto.ca/∼phy224_324/LabManuals/ChargeToMassRatio.pdf

Link(1) - https://www.keysight.com/ca/en/assets/7018-03846/data-sheets/5991-1983.pdf

Electron, Wikipedia (2024). Accessed October 23, 2024 via link: https://en.wikipedia.org/wiki/Electron.

7 Acknowledgement of AI Use

The authors of this paper did not use any AI tools, except maybe the built-in tools for spell-check in google docs.

Appendix

Additional Figures

Figure 5: Image from a clean ring when no external interference in the setup.

Figure 6: Image depicting a distorted ring when external interference (phone above the glass bulb) is present.

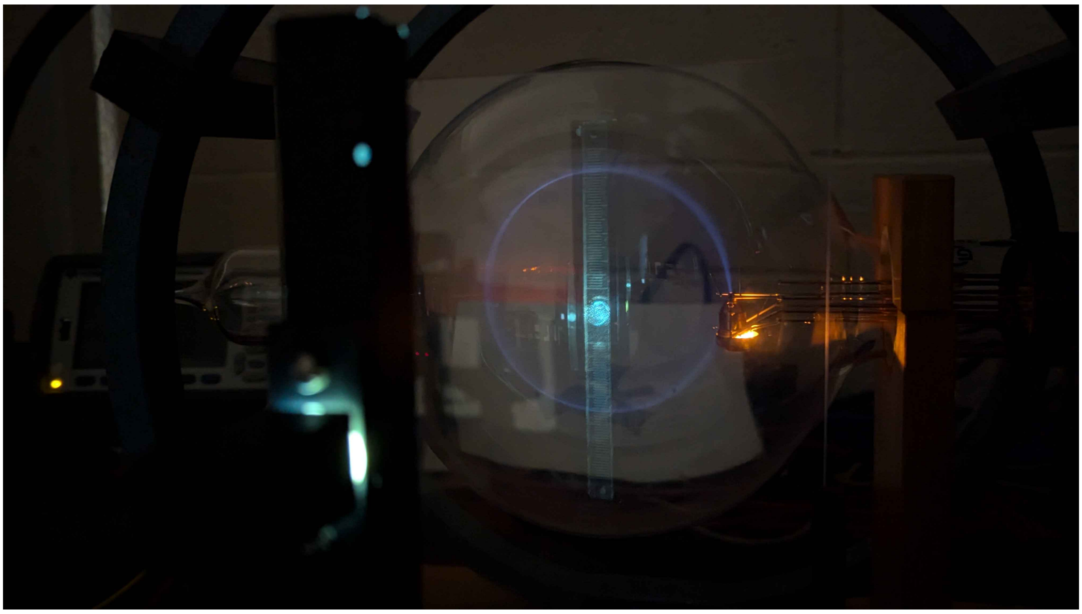

Figure 7: Experimental Setup Image for self-illuminated scale on the electron beam ring

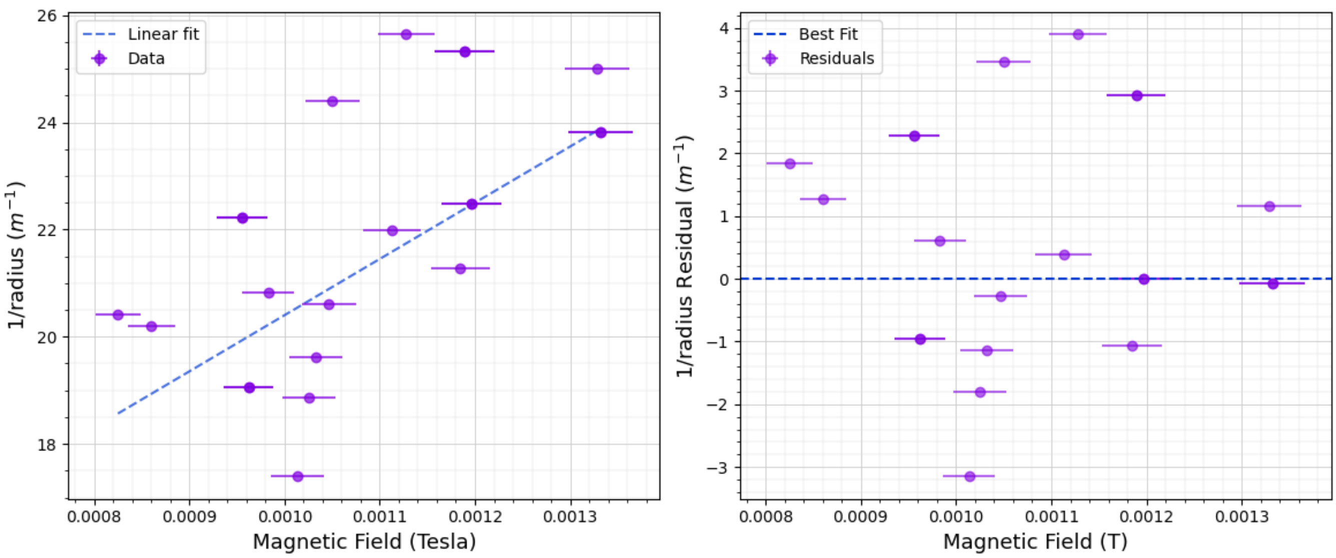

Figure 8: 1/r as a function of magnetic field with Slope 10500 ± 700 $\mathrm{m}^{-1}\mathrm{T}^{-1}$ and intercept 9.9 ± 0.8 m^{-1} with $\chi^2 = 16.75$ probability $= 0.00$.

Uncertainty Calculation Formula:

For $1/r$
$\Delta 1/r = 1/r \times \frac{\Delta r}{r}$

For $B = (4/5)^{3/2} \frac{\mu_0 n I}{R}$ and $\sqrt{\Delta V}$

$$\Delta \frac{B}{\sqrt{\Delta V}} = \frac{B}{\sqrt{\Delta V}} \times \frac{\Delta n}{n} + \frac{\Delta I}{I} + \left(\frac{1}{2} \times \frac{\Delta(\Delta V)}{(\Delta V)}\right) + \frac{\Delta R}{R}.$$

All the delta values were experimentally obtained.