Ohm and Power Laws

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MP 222

1 Introduction

In this lab we use a constant voltage power supply, voltmeter, and ammeter to test Ohm's Law on two different resistors. We verify that

$$V = IR \tag{1}$$

by measuring voltage across and current to our resistor and fitting a linear model. We then progress to verifying the power law for blackbodies,

$$I \propto V^{0.5882} \approx V^{3/5} \tag{2}$$

by measuring current and voltage across a lightbulb. We apply a a non-linear fit $I = cV^d$ to our data and a linear fit to the natural logarithm of our data. We compare these fits with the theoretical model of a blackbody.

2 Methodology

First we created a circuit diagram with which to base our experiment. Our goal was to measure current and voltage across our load component.

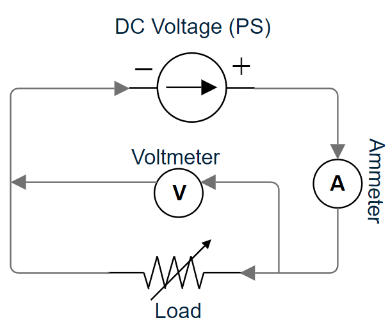

Figure 1: Circuit diagram for the experiment

We used a Keysight ES36103B DC Power Supply (PS), two Keysight U1272A Multimeters, five Pomona connecting wires, and the provided LCR Breakout Box.

We used one multimeter as an ammeter, in the $mA \cdot A$ mode, and the other as a voltmeter in the DC voltage V mode. In each experiment, we connected the PS across the resistor or lightbulb (load), with an ammeter in series with the load and a voltmeter in parallel with the load. In each trial, we set the PS to a constant voltage (CV). We measured the voltage across the load using the voltmeter, and the current to the load using the ammeter.

Across trials we varied the voltage output by the PS, from $3 - 10V$ in intervals of $0.5V$ for experiment 1 and $5 - 15V$ in intervals of $0.5V$ for experiment 2. For experiment 3, we noticed the bulb started to glow at some voltage between 8V and 8.5V so we took measurements across $8 - 20V$ in intervals of $0.5V$.

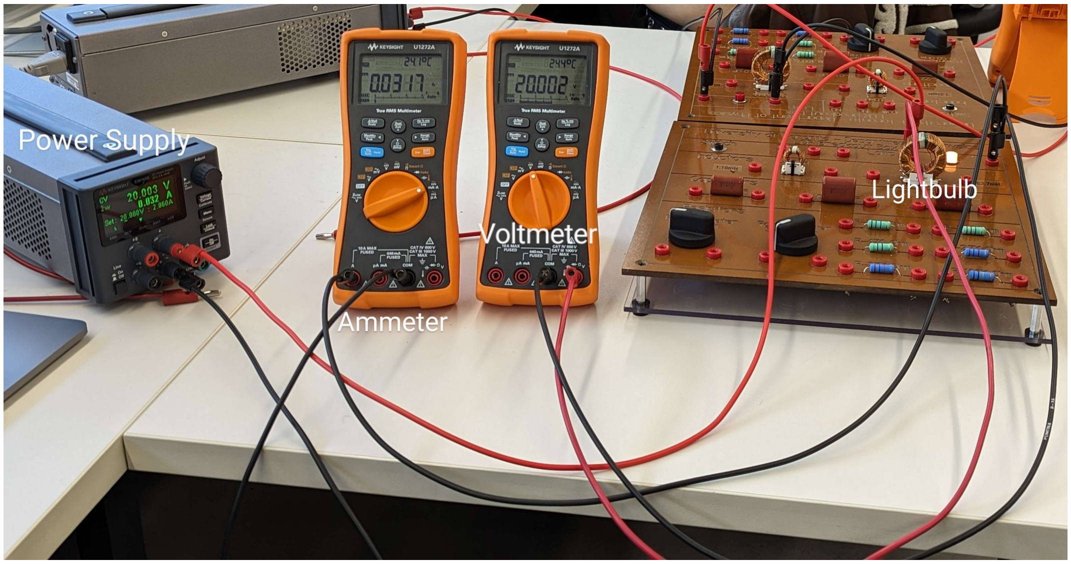

Figure 2: Photograph of our experimental set up

In experiments 1 and 2, after performing the measurements, we disconnected our circuit and switched one of the multimeters to ohmmeter mode. We then directly measured the resistance of the resistor.

3 Results

3.1 Figures

For the 100Ω resistor, we used curve_fit from the SciPy package to generate the line of best fit.

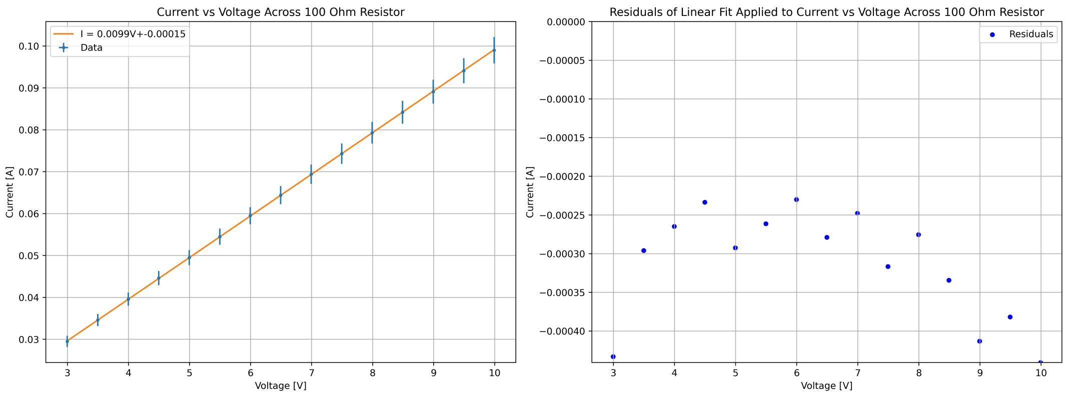

Figure 3: Left: Blue points are data points. Orange line is line of best fit. We see a strong linear correlation in our data. Right: Residuals of our linear fit. We see some structure within the residuals, but know that a linear model best represents our data.

We again fit a line of best fit to our 220Ω resistor.

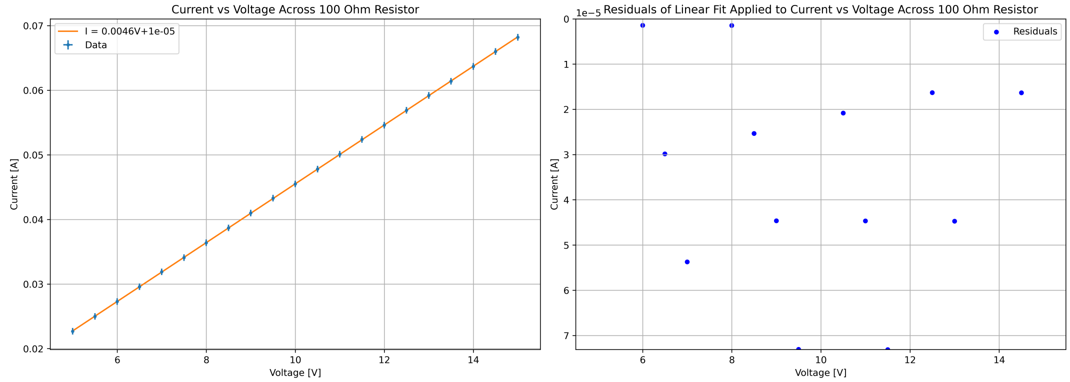

Figure 4: Left: Blue points are data points. Orange line is line of best fit. We see a strong linear correlation in our data. Right: Residuals of our linear fit. We see no structure in our residuals, indicating a linear model fits well to the data. Note that y axis is scaled $10^{-5}A$.

For our lightbulb, we compare our theoretical model with our two curve fitting mechanisms. We plot the data along with the various models on both linear and logarithmic scale.

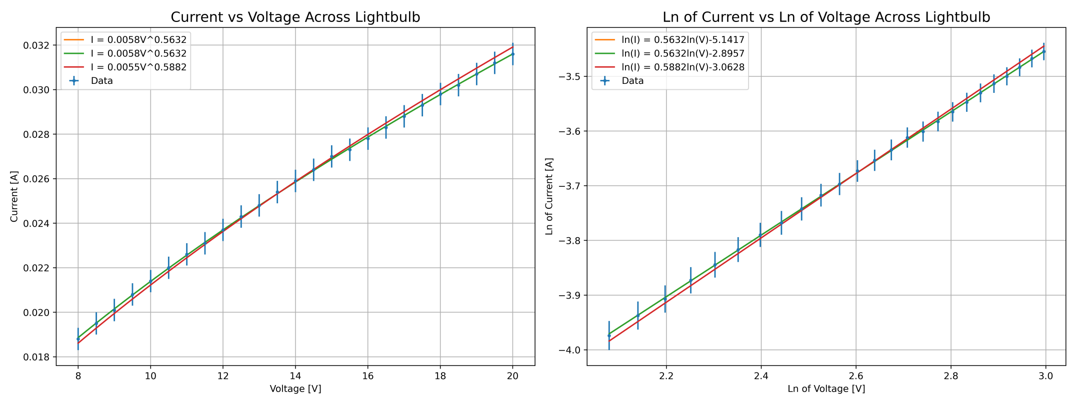

Figure 5: Blue points are data points. Orange curve is model calculated using linear fit to natural logarithm of our data. Green curve is nonlinear fit to our data. Red curve is the theoretical relationship in our data. The orange curve is hidden behind the green curve, as both models return the same parameter values.

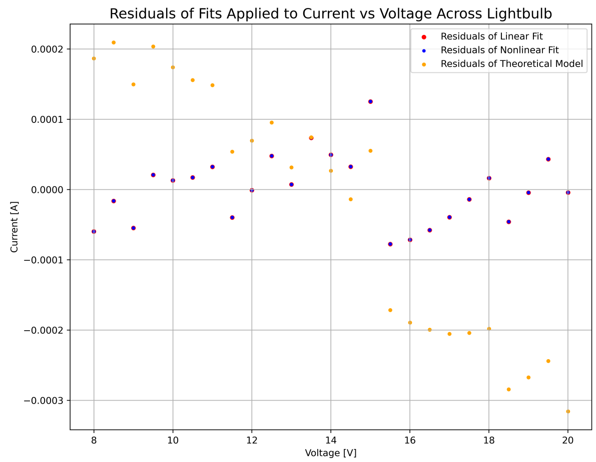

Figure 6: Red points are residuals of linear fit. Blue points are residuals of nonlinear fit. Orange points are residuals of theoretical model. We see our theoretical model does not fit the data as well as the other models. Linear and nonlinear fits have almost identical residuals.

3.2 Sources of Uncertainty

Our main sources of uncertainty are from the resistances of the connecting wires and the uncertainties within the multimeters. We assume all wires in our circuit have no resistance. This is only an approximation, but as we used only 5 wires in our circuit, the effect of the resistances of our wires should be negligible. Our largest source of uncertainty comes from the multimeters. For each measurement, the multimeter has uncertainty in precision and uncertainty in accuracy, as specified by the multimeter data sheet. We account for these uncertainties using the method outlined in section 6.2. One point of difference between the resistor and the lightbulb is that the resistors are much less susceptible to fluctuation in resistance due to heating over longer periods of usage, as they are better regulated for prolonged use when manufactured.

3.3 Values and Uncertainties

For resistors 1 and 2 our uncertainties were calculated as per the manufacturer's ratings as follows— (0.05% + 2 counts) for values measured in the $3V$ to $30V$ range for the DC Voltmeter setting, and (0.2% + 2 counts) for values measured in $0.03A$ to $0.3A$ range for the DC Ammeter setting. When measuring resistances, our uncertainties were (0.2% + 5 counts).

Using the SciPy package in Python, we used the curve_fit function to calculate the line of best fit for our data. For the 100Ω rated resistor, we get a slope of 0.00994 and intercept of -0.0002 in the I vs V graph, giving us a resistance of $100.6\Omega \pm 0.7\Omega$. Performing the linear fit again, this time with the y-intercept set to 0, we get a slope of 0.00991 instead, and a resistance of $100.9\Omega \pm 0.2\Omega$. We compare this with a measured value of $101.5\Omega \pm 0.2\Omega$ and the manufacturer specified $100\Omega \pm 5\Omega$.

We see all values with uncertainties overlap, except for the linear fit with no y-intercept and the measured value of resistance. However, our $y = mx + b$ model gives a resistance that agrees with the measured resistance. Moreover, all values are well within the manufacturer specifications.

For the 220Ω rated resistor, our linear model returns a slope of 0.00455 and intercept 5.85×10^{-6} in the I vs V graph, giving us a resistance of $219.7\Omega \pm 0.4\Omega$. Setting the y-intercept to 0, we again get a slope of 0.00455 (approx), and a resistance of $219.6\Omega \pm 0.1\Omega$. We compare this with a measured $218.8\Omega \pm 0.4\Omega$ and a manufacturer specification of $220\Omega \pm 11\Omega$. For this resistor, neither of our calculated resistances agree with the measured value. Accounting for uncertainties, our minimum calculated value is 219.3Ω which is greater than our largest measured value 219.2Ω. However, both measured and calculated values still fall within the manufacturer specified range.

For the lightbulb, our non-linear curve fit gave us parameter values of $c = (585 \pm 5) \times 10^{-5} A/V^d$ and $d = 0.563 \pm 0.003$ with χ_r^2 value of 0.001. Here, the units of our constant, A/V^d, are dependent on the exponent our model fits. The unit is mostly unimportant, as it just serves to convert a quantity in volts to the power d to a quantity in amperes. Our linear model gave the same constant, exponent, χ_r^2, and uncertainty values as the non-linear fit. The parameters calculated by our two models only differed for low order-of-magnitude decimal places. When we applied our theoretical model 10 to our data, we found a constant of $c = 548 \times 10^{-5} A/V^d$ and an exponent of $d = 0.588$ with $\chi_r^2 = 0.135$. As both linear and non-linear fits returned the same parameter values, neither gave an exponent closer to the expected value.

4 Analysis

4.1 Uncertainty Propagation

For both resistors, the calculated resistance of our two linear models (with y-intercept and without) were very close. This is because our initial linear models return very small, but nonzero y-intercepts anyway. The reason our y-intercepts are nonzero is likely measurement error, perhaps a small constant offset within one of our multimeters. Interestingly the linear model with no y-intercept had lower uncertainty. Perhaps we can attribute this to $y = mx$ being a better representation of the resistance than $y = mx + b$.

For the lightbulb, both of the models gave us an exponent value of 0.563 with uncertainty 0.003. These are very close to the estimated exponent of 0.6 in equation 2 and even closer to the theoretical value of 0.5882, but are not strictly within the standard error range. This is probably due to higher sensitivity to errors when dealing with exponential values as even small deviations in V lead to large deviation in I. Additionally the lightbulb heats up during its use, leading to less accurate measurements. Also it is not known if the bulb is made of pure tungsten in an isolated environment. It may have degraded over time with use.

4.2 Goodness of Fit

Our residual plots for both resistors (see 3.1), are fairly randomly spread and of very small magnitude compared to our measurements. This indicates that our models are a good fit for our data. For the lightbulb, we see that both fits returned almost identical residuals which were randomly located and very small in magnitude. However, the theoretical model returned larger residuals that clearly varied with voltage, indicating that our theoretical model did not fit our data as well as the regression models.

Both our models for the lightbulb have χ_r^2 values of 0.001. This is much lower than the theoretical model's χ_r^2 value of 0.135, meaning that our models fit the data better than the theoretical equation. Furthermore, both of our fits returned the same parameters, while the theoretical model's parameters were different, and did not fit the data as well. This does not indicate that our theoretical equation is wrong, but rather that it makes some assumptions or approximations of our physical system. Most notably, the equation assumes a pure tungsten filament inside the lightbulb. It is possible that the filament is mostly tungsten, but mixed with some other metals, or that the filament inside has degraded with use.

In all three experiments we had low χ_r^2 values. Typically this would imply that our models were over-fit to the data. However, in this lab we are verifying well-established principles. We know the form of the models that will best fit our data— linear for the resistors and exponential in for the lightbulb. In this lab we are instead comparing different methods of fitting curves to our data. We use the χ_r^2 values to see that linear regression fit our resistor data well and that both regression models fit our lightbulb data equally well. Our low χ_r^2 values also mean that our data is consistent and free from large experimental error.

5 Discussion

The experiment verifies both Ohm's Law and the Power law as presented in the introduction. For Ohm's law in particular, we were quite close to the expected values and well within the rated tolerance of the resistors. For the power law, however we were not as accurate with the findings. This difference is explained by section 4.1. In short, this experiment wasn't perfectly regulated due to the lightbulb itself being unrated and potentially impure or degraded over time with use. However, our calculated power law parameters are still reasonably close with the theoretical values, hence we conclude that our experiment verifies the power law.

6 Appendix

6.1 Data

The data collected for each of our three experiments.

Table 1: Experiment 1 Data. Measured current and voltage across resistor due to CV from PS

CV From PS [V]	Voltage Measured [V]	Current Measured[A]
3.000	2.997 (± 0.002)	0.030 (± 0.001)
3.500	3.496 (± 0.003)	0.035 (± 0.001)
4.000	3.996 (± 0.003)	0.040 (± 0.002)
4.500	4.496 (± 0.003)	0.045 (± 0.002)
5.000	4.995 (± 0.003)	0.050 (± 0.002)
5.500	5.495 (± 0.003)	0.055 (± 0.002)
6.000	5.995 (± 0.004)	0.060 (± 0.002)
6.500	6.493 (± 0.004)	0.064 (± 0.002)
7.000	6.993 (± 0.004)	0.070 (± 0.002)
7.500	7.493 (± 0.004)	0.074 (± 0.002)
8.000	7.992 (± 0.004)	0.079 (± 0.003)
8.500	8.491 (± 0.005)	0.084 (± 0.003)
9.000	8.992 (± 0.005)	0.089 (± 0.003)
9.500	9.492 (± 0.005)	0.094 (± 0.003)
10.000	9.991 (± 0.005)	0.099 (± 0.003)

Table 2: Experiment 2 Data. Measured current and voltage across resistor due to CV from PS

CV from PS [V]	Voltage Measured [V]	Current Measured [A]
5.000	4.998 (± 0.003)	0.0227 (± 0.0005)
5.500	5.498 (± 0.003)	0.0250 (± 0.0005)
6.000	5.998 (± 0.004)	0.0273 (± 0.0005)
6.500	6.497 (± 0.004)	0.0296 (± 0.0005)
7.000	6.997 (± 0.004)	0.0319 (± 0.0005)
7.500	7.498 (± 0.004)	0.0341 (± 0.0005)
8.000	7.997 (± 0.004)	0.0364 (± 0.0005)
8.500	8.497 (± 0.005)	0.0387 (± 0.0005)

9.000	8.998 (± 0.005)	0.0410 (± 0.0005)
9.500	9.497 (± 0.005)	0.0433 (± 0.0005)
10.000	9.997 (± 0.005)	0.0455 (± 0.0005)
10.500	10.497 (± 0.006)	0.0478 (± 0.0005)
11.000	10.997 (± 0.006)	0.0501 (± 0.0005)
11.500	11.496 (± 0.006)	0.0524 (± 0.0005)
12.000	11.996 (± 0.006)	0.0546 (± 0.0005)
12.500	12.497 (± 0.007)	0.0569 (± 0.0005)
13.000	12.996 (± 0.007)	0.0592 (± 0.0005)
13.500	13.496 (± 0.007)	0.0614 (± 0.0005)
14.000	13.996 (± 0.007)	0.0637 (± 0.0005)
14.500	14.496 (± 0.008)	0.0660 (± 0.0005)
15.000	14.996 (± 0.008)	0.0682 (± 0.0005)

Table 3: Experiment 3 Data. Measured current and voltage across lightbulb due to CV from PS

CV from PS [V]	Voltage Measured [V]	Current Measured [A]
8.000	7.998 (± 0.004)	0.0188 (± 0.0005)
8.500	8.499 (± 0.005)	0.0195 (± 0.0005)
9.000	8.999 (± 0.005)	0.0201 (± 0.0005)
9.500	9.500 (± 0.005)	0.0208 (± 0.0005)
10.000	9.999 (± 0.005)	0.0214 (± 0.0005)
10.500	10.499 (± 0.006)	0.0220 (± 0.0005)
11.000	11.000 (± 0.006)	0.0226 (± 0.0005)
11.500	11.500 (± 0.006)	0.0231 (± 0.0005)
12.000	12.000 (± 0.006)	0.0237 (± 0.0005)
12.500	12.500 (± 0.007)	0.0243 (± 0.0005)
13.000	12.999 (± 0.007)	0.0248 (± 0.0005)
13.500	13.500 (± 0.007)	0.0254 (± 0.0005)
14.000	14.000 (± 0.007)	0.0259 (± 0.0005)
14.500	14.501 (± 0.008)	0.0264 (± 0.0005)
15.000	15.000 (± 0.008)	0.0270 (± 0.0005)
15.500	15.502 (± 0.008)	0.0273 (± 0.0005)
16.000	16.002 (± 0.008)	0.0278 (± 0.0005)
16.500	16.501 (± 0.008)	0.0283 (± 0.0005)

17.000	17.002 (± 0.009)	0.0288 (± 0.0005)
17.500	17.502 (± 0.009)	0.0293 (± 0.0005)
18.000	18.003 (± 0.009)	0.0298 (± 0.0005)
18.500	18.502 (± 0.009)	0.0302 (± 0.0005)
19.000	19.00 (± 0.01)	0.0307 (± 0.0005)
19.500	19.50 (± 0.01)	0.0312 (± 0.0005)
20.000	20.00 (± 0.01)	0.0316 (± 0.0005)

6.2 Uncertainty Calculations

There are inherent errors of accuracy and precision in every measurement performed by the multimeter. The manufacturer specifies these errors in the data sheet of the instrument. The error of accuracy is calculated as some percent of the measurement, and the error of precision as a count. The numerical value of the count, placed in the last decimal place of the reading, represents the error of precision. We combine these two sources of uncertainty into one. Take, for example, the first voltage measured in experiment 1. The data sheet lists an error of accuracy of 0.05% and a count of 2. Thus the total uncertainty in this measurement is calculated

$$u(x_i) = \sqrt{u_{\text{precision}}(x_i)^2 + u_{\text{accuracy}}(x_i)^2} = \sqrt{(0.0005 \cdot 2.9966)^2 + (0.002)^2} = 0.0024989804 \qquad (3)$$

We round each uncertainty value to one significant figure. We then report each measurement to the decimal point of its uncertainty. In the example above, we record an uncertainty of $0.002V$ and report the measured voltage as $2.997V \pm 0.002V$

6.3 Derivation of Theoretical Model

The power law for tungsten filament is given as

$$P = A\sigma\epsilon(T)T^4 = A\sigma\epsilon_0 T^{4.663} \qquad (4)$$

where $\epsilon(T) = 1.731 \times 10^{-3} \times T^{0.663}$. We let $\epsilon_0 = 1.731 \times 10^{-3}$. The resistance of tungsten filament as a function of temperature is given

$$R = c_1 T^{1.209} \qquad (5)$$

where c_1 is a constant of proportionality. Letting $k = \frac{4.663}{1.209}$ and substituting $T^{4.663} = (R/c_1)^k$ we obtain the expression

$$P = A\sigma\epsilon_0 (R/c_1)^k \qquad (6)$$

We use $P = IV$ and $V = IR$ to write this is

$$IV \propto R^k = \frac{V^k}{I^k} \tag{7}$$

Thus we can solve

$$I^{k+1} \propto V^{k-1} \implies I \propto V^{(k-1)/(k+1)} = V^{0.5882} \tag{8}$$

Here we round $(k+1)/(k-1)$ to four significant digits in order to match the significant digits in the exponents of equations 4 and 5. We calculate the proportionality constant in equation 10 by calculating

$$c_i = \frac{I_i}{V_i^{0.5882}} \tag{9}$$

for all measurements i. We then take the average of all c_i's and use this value as our final constant of proportionality, giving us our final theoretical model:

$$I = \overline{c_i} V^{0.5882} \tag{10}$$

6.4 Linear Fit to Lightbulb Data

To fit a linear model to the lightbulb data, we first took the natural logarithms of both current and voltage values. We then used SciPy curve_fit function to fit a linear model

$$ln(I) = m \cdot ln(V) + b \tag{11}$$

to this data. Once fit, we transform this back into a power law of

$$I = e^b V^m \tag{12}$$

6.5 Division of Responsibilities

Aditya and Alek worked together to set up the circuits and take the data. Alek and Aditya both worked on the methodology. Alek performed the data analysis, writing all of the python code and doing the figures section. Alek wrote all parts of the appendix. Aditya wrote sections 3.2-5 himself. Alek made final revisions to Aditya's writing and submitted the report.