Radioactive Decay

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1 Abstract

In this lab we indirectly measure in internal resistance of a power supply and a cell battery. We connect varying loads to a circuit to draw out differing currents. We calculate the discrepancy between open circuit voltage and measured voltage, and use the current flowing through the circuit to calculate the internal resistance. We use two different circuit designs, finding that one is better than the other. We also are able to estimate the resistance of our ammeter and current flowing through our voltmeter.

2 Introduction

In an ideal circuit of no resistance, the source of electromotive force generates a voltage difference of V_∞ across the circuit. With a load connected, the voltage across the circuit will be lower than V_∞. The current will encounter some resistance within the source itself, called the internal resistance. The voltage drop across our external circuit is $V_\infty - V_{\text{int}}$ in the presence of internal resistance. In this lab we try to measure the internal resistance of a power supply and a battery (source). We use the equation:

$$V = V_\infty - R_{\text{int}} I \tag{1}$$

which relates the voltage across our circuit (V), the voltage difference across our source (V_∞) the internal resistance of our source (R_{int}), and the current through our circuit (I). We connect our source to some external resistor R_{ext} to draw the current I out of the source. We then measure V and I through our circuit. We apply a linear fit to our data, and use the parameters of the line of best fit to infer the values of V_∞ and R_{int}.

3 Methodology

We used a Keysight ES36103B DC Power Supply (PS) and a Datalex NP12-6(6V12AH/20HR) cell battery as our sources of voltage. We used two Keysight U1272A Multimeters, one as a voltmeter and one as an ammeter. We used Pomona connecting wires, and the resistors on the provided LCR Breakout Box.

To draw the current I out of the source, we connect an external resistor to our source. To measure both the current and the voltage across our circuit, we must place the ammeter in series with the external resistor, and the voltmeter in parallel. There are two different ways to do this, as pictured in figure 1. We used both options of circuit on both sources. While the circuits are theoretically identical, the fact that the ammeter and voltmeter are not ideal makes the circuits different in practice.

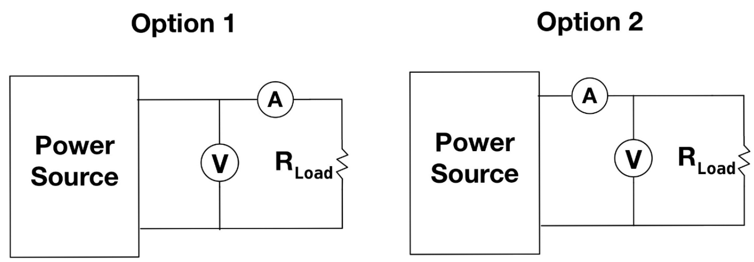

Figure 1: Circuit diagram for the experiment (Vahabi et al.). See 9.4 for picture of experiment setup.

We began by directly measuring the voltage across the cell battery and the resistances of each resistor. We then connected our circuit using circuit option 1, the battery, and an external resistor of 570Ω. We measured the voltage and current across the circuit, and then progressed to the next resistor. We used the resistances $570, 690, 790, 2700$, and 2800Ω. We then repeated this same process using the PS. Once finished we set up our circuit using option 2 and repeated these same measurements for the battery and PS again.

Near the end of the lab, we performed two final experiments in an attempt to get a more accurate final results of R_{int}. We used circuit option 2 and, in addition to the resistors we had previously used, added resistors $68.75, 82.456, 100$, and 220Ω. We carried out this experiment on the battery and the PS set to $20V$.

4 Results

Our results for our two most accurate experiments are shown here. The remaining results can be found in 9.2.

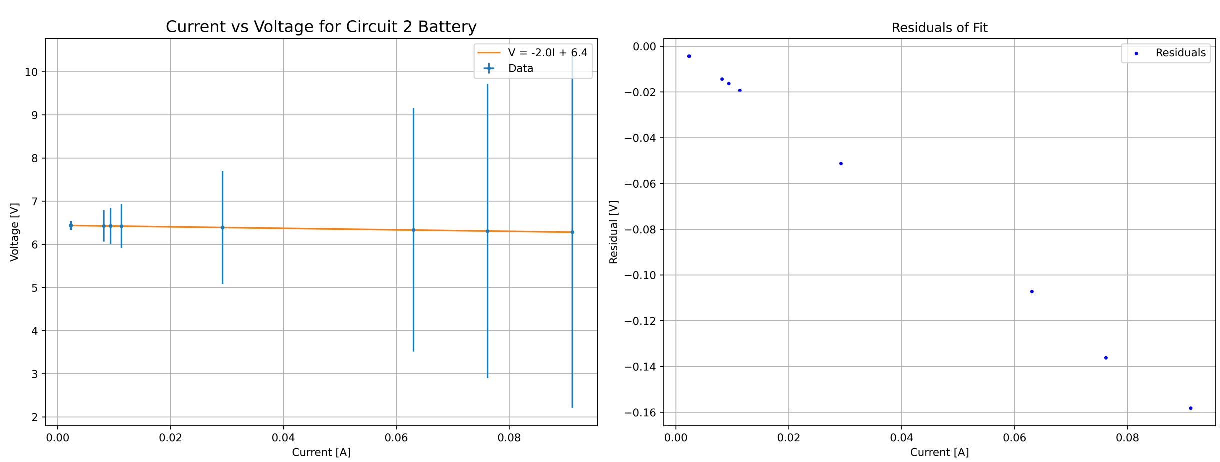

Figure 2: Line of best fit applied to our data. We see is slight negative slope as expected. We see large error bars, particularly for the larger currents. We see small residuals, but there is definite linear structure in the residuals.

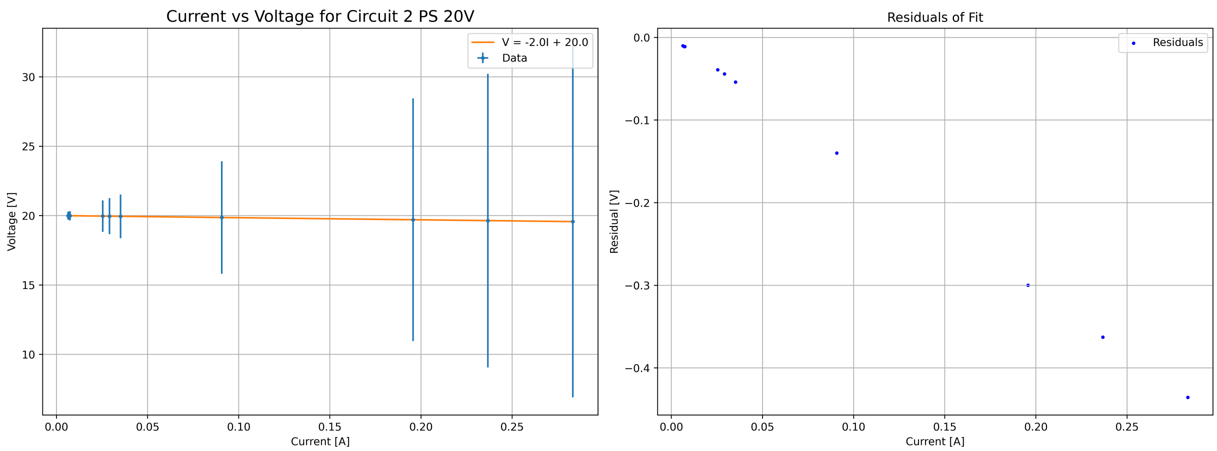

Figure 3: Small negative slope and large error bars for larger currents. Small residuals with a clear linear structure. Slope of the line of best fit is noticeably shallower than in figure 3.

The internal resistances we calculate for each circuit and source combination are: Circuit 1 Battery: $0.8000 \pm 0.0001\Omega$. Circuit 2 Battery: $1.74622 \pm 0.00001\Omega$. Circuit 1 PS: $-0.9476 \pm 0.0001\Omega$. Circuit 2 PS: $1.5431 \pm 0.0002\Omega$. Circuit 2 PS with $20V$: $1.53674 \pm 10^{-5}\Omega$.

5 Analysis

5.1 Uncertainty Analysis

The largest sources of uncertainty in this lab are the internal resistance of the ammeter and the current drawn by the voltmeter. We can say that the current through the voltmeter is on the order $100\mu A$. We can compare tables 2 with 3 and 4 with 5. We see that for each resistor and each source, the recorded current did not vary over the two different circuit options. For example, we recorded $11.3 \pm 0.5mA$ for the 570Ω resistor connected to the battery for both circuits 1 and 2. This applies for every pair of measurements we took in this lab. Thus any current that is drawn by the voltmeter will be within the uncertainty in current, so of order $0.1mA$. We also see that in circuit 1 we always measured a larger voltage than for the same measurement taken with circuit 2. The only difference between these circuits is the voltage drop over the ammeter. We subtract corresponding measurements to calculate the voltage drop over the ammeter and divide by the current through the circuit (as the current is the same between circuits 1 and 2). This gives a measurement of the resistance of the ammeter for each trial. Taking the average of these values over all trials, we find $R_{\text{ammeter}} = 7 \pm 2m\Omega$. However, in performing this calculation just for the battery data, we find $R_{\text{ammeter}} = 13 \pm 2m\Omega$ and for the PS data we find $R_{\text{ammeter}} = 1 \pm 0.05m\Omega$. These values differ by a factor of 10, so these estimations of R_{ammeter} are not precise.

Other uncertainties in this lab include uncertainty in each measurement on the multimeters, internal resistance of the wires, and heating of the resistors. The uncertainties in the multimeter measurements are accounted for by the process explained in 9.3. In this lab the internal resistance of the wires may be a non-negligible (although still small) source of error. This is because we were required to use 6-7 wires to connect our all components of our circuits. It is impossible to estimate this type of error, but we expect it to be small compared to the current through the voltmeter and resistance of the ammeter. Heating of our resistors should be a negligible source of uncertainty. In each measurement we connected the circuit for a matter of seconds to take a measurement. Then each resistor had ample time to cool down while we cycled through the other resistors and took measurements.

5.2 Goodness of Fit

We calculate the χ_r^2 values of each line of best fit, using eq 3. We find

Table 1: Calculated χ_r^2 values of each line of best fit

Circuit	χ_r^2
Option 1 Battery	$1.6 \cdot 10^{-4}$
Option 2 Battery	$2.2 \cdot 10^{-6}$
Option 1 PS	$5.6 \cdot 10^{-2}$
Option 2 PS	$2.9 \cdot 10^{-7}$
Option 2 PS 20V	$1.0 \cdot 10^{-7}$

Typically we would aim to have χ_r^2 values near 1. Having such low χ_r^2 usually indicates that our model is overfit to the data. In this case, we do not draw such a conclusion. We designed our experiment so that we would have low currents flowing through our circuit, and thus would be in the most linear regime of the V vs I graph. Thus it is no surprise that our data does indeed turn out to be very linear. Furthermore, we only took 5-10 data points per experiment. If we had taken many data points and they all lay perfectly on the same line, this may be cause for suspicion. However, seeing 5 data points lying almost on the same line is more of an indication of good experimental design rather than overfitting of our model.

In our residual plots, we see clear structure. In almost every residual plot we see a line of negative slope. Seeing this structure in the residuals indicates that we are lacking some terms in our model, in other words that our model is underfit to the data.

We have two indications of the goodness of our fit. One indication (χ_r^2) tells us that we are overfit, while the other (residual plots) indicates that we are underfit. With these contradictory results, we are forced to look for other indicators of the goodness of our fit. The theory behind internal resistance is sound. It is not as though we are testing some unproven theory. Thus we accept that our data is meant to follow a line of negative slope and conclude that our linear models fit our data well. Having decided that a line is indeed the best model for our data, we can say that the experiments with lowest χ_r^2 values, option 2 battery and option 2 PS with $20V$ are the most accurate experiments.

6 Discussion

6.1 Choice of Resistors

We chose our initial 5 values of resistance as we expected them to draw out the least possible currents that our ammeter could precisely measure. We knew that the linearity in equation 1 applied best for low currents. We also knew that by using low currents we could reduce heating in the resistors, which we expected to be a large source of error. However, we still required an accurate reading of current. Given the limitations of our ammeter, we wanted a current on the order of mA through our circuit which our ammeter could measure with low uncertainty. Given that we were going to apply a voltage of $\approx 6.5V$ across our circuits, we calculated that these resistors were optimal.

After taking this data and performing some preliminary analysis during the lab, we noticed that our voltage measurements were very constant. When we drew different currents out of the source, we saw minimal change in the voltage across our external resistor. Given the large uncertainties in some of our measurements, it would have been difficult to say what the line of best fit through our data was for such a small slope. Thus it would be difficult to extract a measurement of R_{int}.

We set out to use the remaining time in the lab to re-do the appropriate experiments to get a more accurate value of R_{int}. We expected that circuit 2 would give a more accurate results, so we used only this circuit for both the battery and PS. We also used a wider array of resistances, from 68.75 to 3170Ω. Such a wide range of resistances would draw a wide range of currents out of the source. The voltage difference we see in the external circuit is $R_{\text{int}}I$. Such a large range of current, then, would give larger voltage fluctuations. As these voltage fluctuations are what we fit a line of best fit to, larger fluctuations meant a more accurate value of the slope of this line which gives R_{int}.

For the PS, we further decided to increase the output voltage to $20V$. As the voltage fluctuations $\propto I$, a large voltage out of the PS would increase the voltage fluctuations. Furthermore, our ammeter is better equipped to measure larger currents. The one downside to this experiment is the heating of the resistors due to the large current flowing through them. However, as each resistor had time to cool down while we did our preliminary data analysis and each resistor was only connected to the circuit for a matter of seconds, we expect this source of error to be minimal.

6.2 Comparison of Circuits

The difference between the two circuits is the positioning of the ammeter. We predict that the one where the voltmeter is in parallel with both the ammeter and load will be less precise than the one where it is only across the resistor. This is because the ammeter also has some internal resistance, which will cause it to actually measure the voltage across the power source instead, which is not what we are looking for. Hence option 2 should give better results. From the discussion in 5.2 we can say that the experiment with the lower χ_r^2 is more accurate. Then, across both the battery and the PS, circuit option 2 gave more accurate results. Furthermore, circuit option 1 for the PS gives an internal resistance of $-0.9476 \pm 0.0001\Omega$. The internal resistance of the power supply cannot be negative. Then we conclude that this experiment is flawed. Thus we can conclude that circuit 2 is more appropriate.

6.3 Comparison with Theory

We see a good degree of agreement with theory within this experiment. The theory predicts that our data will follow equation 1. We see that most of our experiments follow this trend, with the exception of circuit 1 for the PS, in which we get a positive slope. We can attribute this experiment to our use of the circuit 1 design, as we established it will not give the most accurate results. We also know that the internal resistance of the power supply will be smaller than the battery as the power supply will work to output as close as possible to the voltage it is set to. Thus it is not too surprising that in a poor set-up (i.e. circuit 1) we measure a slightly negative quantity as a slightly positive quantity. An indication that we have good agreement with theory is an analysis of the y-intercepts of the lines of best fit. We would expect the y-intercept to be the free circuit voltage of the source. This is the voltage we set the PS to or the voltage difference across the terminals of the battery which we measured to be $6.444 \pm 0.004V$. In figure 3 we see the line of best fit has y-intercept at about $20V$. In figure 6 we see y-intercept near $6.5V$. We do not consider figure 5 as we already know that this experiment was flawed. In figures 4 and 2 we see y-intercepts very close to $6.444V$. In this lab we do not have any direct measurement of R_{int} to compare our results to. We also do not want to compare results across the two circuits, as we know that circuit 1 is prone to error. We can however compare the two experiments ran with circuit 2 PS, one with $6.5V$ and one with $20V$. The calculated internal resistances are $1.5431 \pm 0.0002\Omega$ and $1.53674 \pm 0.00001\Omega$ respectively. While not within uncertainty of eachother, they are still rather close.

7 Conclusion

Our experiment to indirectly measure the internal resistance of a cell battery and a PS using two different circuit designs tells us that option 2 is more accurate than option 1. Our line of best fit on the graph of V vs I for circuit 2 fits the data well and we observe good agreement between experiment and theory, but have no direct measurements of R_{int} to compare to. In the process of this analysis, we are able to give rough estimates of the resistance of the ammeter $\approx 7 \pm 2m\Omega$ and can say that the current flowing through the voltmeter is of order $100\mu A$.

8 Referneces

Vahabi, T., Horsley, E., Harlick, A., &; Serbanescu, R. M. (2022). The Output Resistance of a Power Supply. Toronto, Ontario; University of Toronto.

9 Appendix

9.1 Data

Table 2: Circuit 1 Battery Data

Resistor (Ω)	Current(mA)	Voltage(V)
570	11.3 ($\pm$0.5)	6.437 ($\pm$0.004)
690	9.4 ($\pm$0.4)	6.436 ($\pm$0.004)
790	8.2 ($\pm$0.4)	6.433 ($\pm$0.004)
2700	2.4 ($\pm$0.1)	6.439 ($\pm$0.004)
2800	2.3 ($\pm$0.1)	6.440 ($\pm$0.004)

Table 3: Circuit 2 Battery Data

Resistor (Ω)	Current(mA)	Voltage(V)
68.75	91 ($\pm$4)	6.284 ($\pm$0.004)
82.456	76 ($\pm$3)	6.306 ($\pm$0.004)
100	63 ($\pm$2)	6.335 ($\pm$0.004)
220	29 ($\pm$1)	6.391 ($\pm$0.004)
570	11.3 ($\pm$0.5)	6.423 ($\pm$0.004)
690	9.4 ($\pm$0.4)	6.426 ($\pm$0.004)
790	8.2 ($\pm$0.4)	6.428 ($\pm$0.004)
2700	2.4 ($\pm$0.1)	6.438 ($\pm$0.004)
2800	2.3 ($\pm$0.1)	6.438 ($\pm$0.004)

Table 4: Circuit 1 Power Supply Data

Resistor (Ω)	Current(mA)	Voltage(V)
570	11.4 ($\pm$0.5)	6.500 ($\pm$0.004)
690	9.5 ($\pm$0.4)	6.500 ($\pm$0.004)
790	8.3 ($\pm$0.4)	6.500 ($\pm$0.004)
2700	2.4 ($\pm$0.1)	6.500 ($\pm$0.004)
2800	2.3 ($\pm$0.1)	6.500 ($\pm$0.004)

Table 5: Circuit 2 Power Supply Data

Resistor (Ω)	Current(mA)	Voltage(V)
570	11.4 (±0.5)	6.483 (±0.004)
690	9.5 (±0.4)	6.486 (±0.004)
790	8.3 (±0.4)	6.488 (±0.004)
2700	2.4 (±0.1)	6.497 (±0.004)
2800	2.3 (±0.1)	6.497 (±0.004)

Table 6: Circuit 2 Power Supply Data 20V

Resistor (Ω)	Current(mA)	Voltage(V)
68.75	280 (±10)	19.57 (±0.01)
82.456	240 (±10)	19.64 (±0.01)
100	196 (±9)	19.70 (±0.01)
220	91 (±4)	19.86 (±0.01)
570	35 (±2)	19.95 (±0.01)
690	29 (±1)	19.96 (±0.01)
790	25 (±1)	19.97 (±0.01)
2700	7.4 (±0.3)	19.99 (±0.01)
2800	7.1 (±0.3)	19.99 (±0.01)
3170	6.3 (±0.3)	19.99 (±0.01)

Table 7: Measured Resistances of Resistors

Expected Resistance (Ω)	Measured Resistance (Ω)
68.75	68.9 (±0.1)
82.456	82.8 (±0.2)
100	100.6 (±0.2)
220	218.8 (±0.4)
470	467 (±1)
570	567 (±1)
690	685 (±1)
790	786 (±2)
2700	2700 (±5)
2800	2799 (±8)
3170	3166 (±8)

We measured the battery to have open circuit voltage $6.444 \pm 0.004V$.

9.2 Results

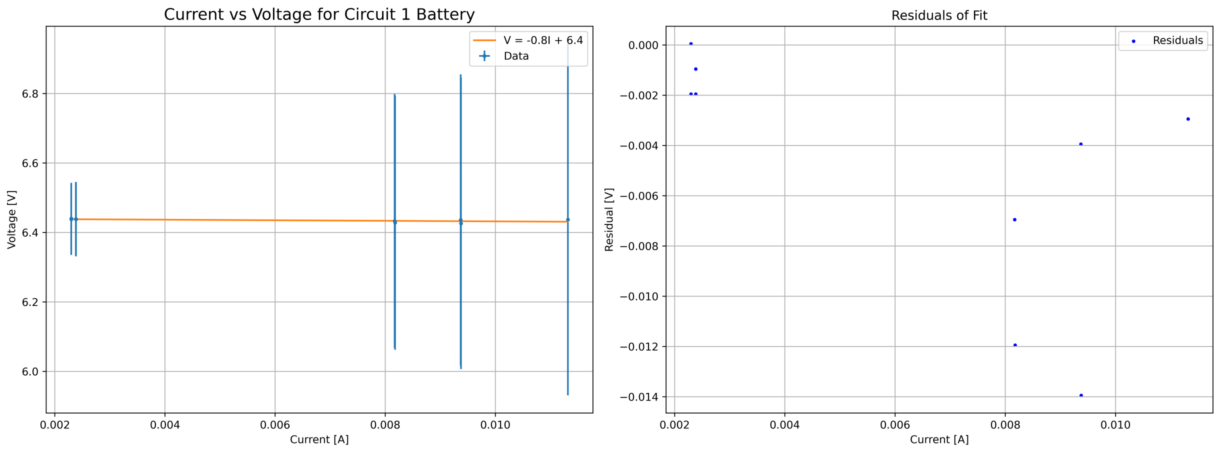

Figure 4: Almost zero slope in our line of best fit. Large error bars for data points with large uncertainties. Small residuals with little structure.

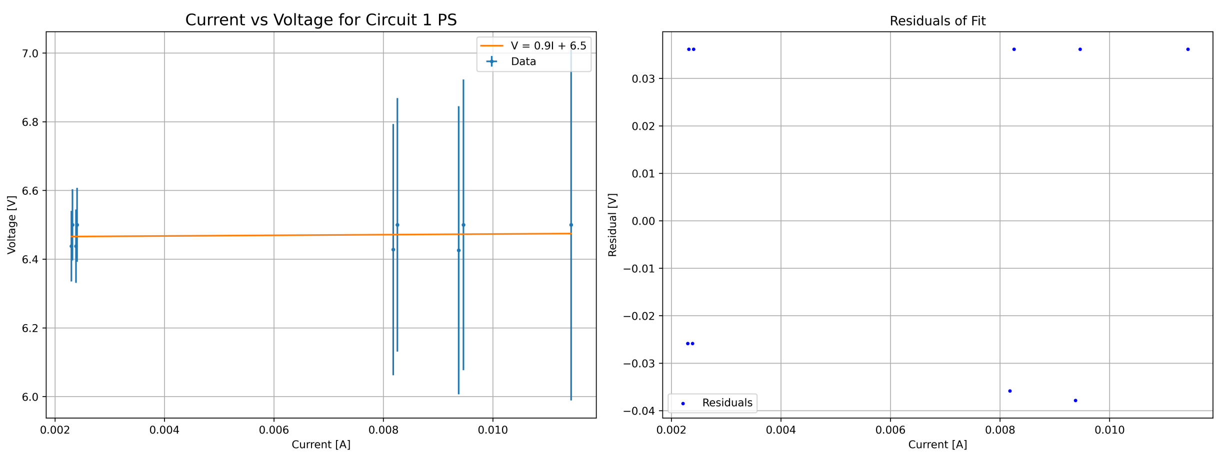

Figure 5: Slightly positive slope in line of best fit which indicates a negative internal resistance, which is impossible. Similar pattern in error bars as other experiments. Residuals are small, but seem to have some structure as they are always about $\pm 0.03V$

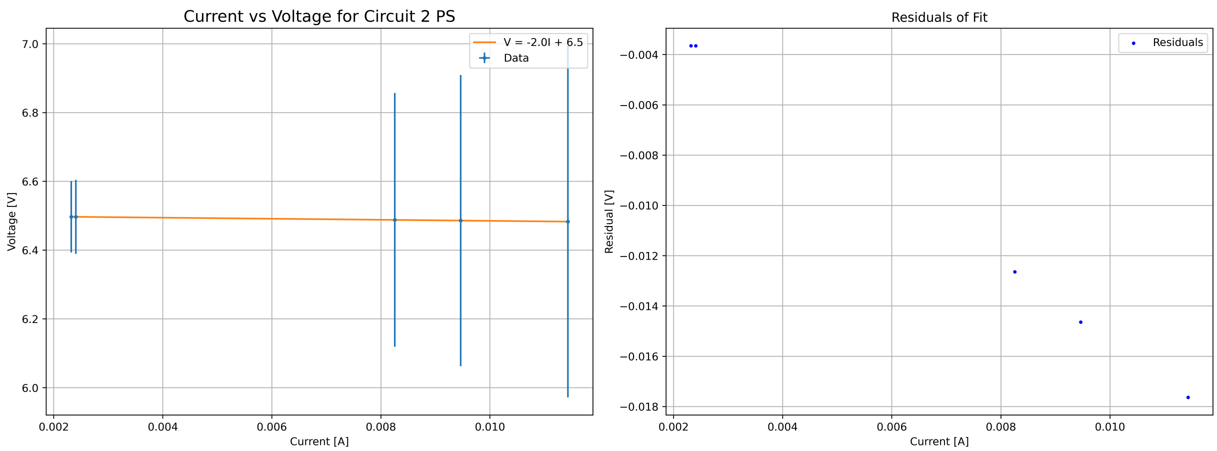

Figure 6: Similar to figure 3. Slight negative slope in line of best fit. Large error bars for higher currents. Definite linear structure in residuals.

9.3 Calculations

There are inherent errors of accuracy and precision in every measurement performed by the multimeter. The manufacturer specifies these errors in the data sheet of the instrument. The error of accuracy is calculated as some percent of the measurement, and the error of precision as a count. The numerical value of the count, placed in the last decimal place of the reading, represents the error of precision. We combine these two sources of uncertainty into one. Take, for example, the first voltage measured in experiment 1. The data sheet lists an error of accuracy of 0.05% and a count of 2. Thus the total uncertainty in this measurement is calculated

$$u(x_i) = \sqrt{u_{\text{precision}}(x_i)^2 + u_{\text{accuracy}}(x_i)^2} = \sqrt{(0.0005 \cdot 6.437)^2 + (0.002)^2} = 0.003789293107 \quad (2)$$

We round each uncertainty value to one significant figure. We then report each measurement to the decimal point of its uncertainty. In the example above, we record an uncertainty of $0.004V$ and report the measured voltage as $6.438V \pm 0.004V$ [1]

For measuring the goodness of fit, we use the χ_r^2 equation:

$$\chi_r^2 = \frac{1}{N-m} \sum \frac{(y_i - y(x_i))^2}{u(y_i)} \quad (3)$$

Where N is the the number of data points, m is the number of parameters, y_i is the data point, $y(x_i)$ is the value at that point from the fitted curve, and $u(y_i)$ is the uncertainty in that measurement.

[1]Note this section is taken from Aditya and Alek's Ohm's and Power Law's report. The uncertainty calculations are exactly the same in this experiment and the Ohm's and Power Law's experiment. Professor Lee approved us copying our uncertainty calculations section into this report.

9.4 Experiment Setup

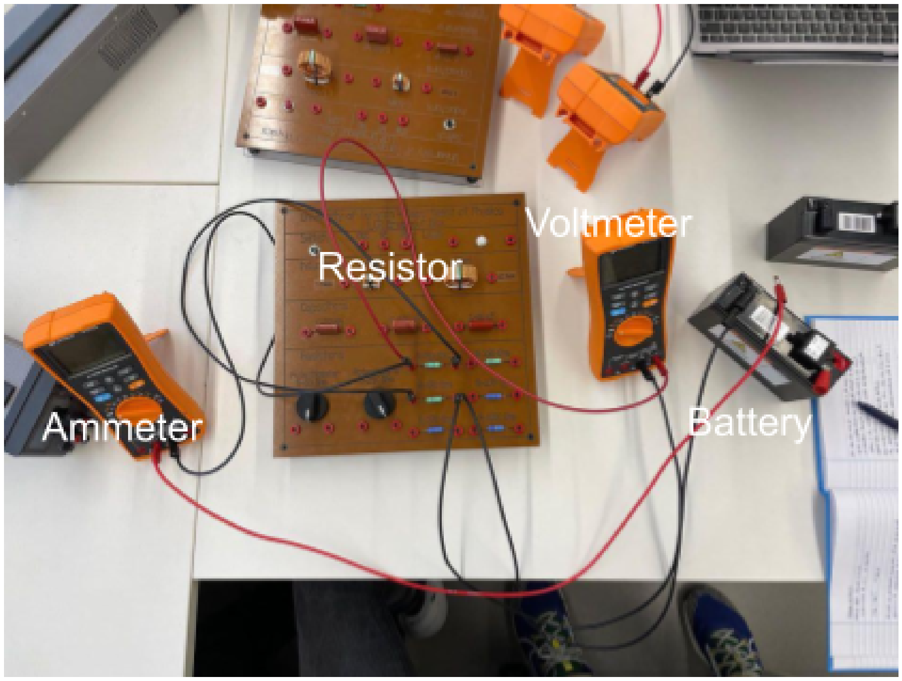

Figure 7: Experiment setup using the cell battery.

9.5 Lab Question

We see the maximum current I_{max} for any circuit (this is observed at a small resistance being placed across the supply) to vary proportionally with V_∞ as the internal resistance of the power supply is fixed, and should give a linear relation between the two values. (Intuitively, with a small resistance placed across, the internal resistance dominates as it is connected in series with the load)