

# Exoplanet Detection Methods: What Works?

B. Stephens, T. Armitage, L. Robinson, C. O'Brien, E. Fahimi

January 2022

## 1 Motivation

By discovering and studying exoplanets, astronomers are able to gain insight into the properties of the Universe. This is far from an easy process, however, and the methods developed for detecting exoplanets have varying degrees of success. By calculating the state-of-the-art limitations of three exoplanet detection methods (Radial Velocity, Transit, and Direct Imaging), we can determine how effective they are for detecting a Jupiter-like planet orbiting around a Sun-like star. By studying this simulation, we can get a glimpse into the overall efficacy of these methods.

## 2 Methods

### 2.1 Radial Velocity

When a star is moving away from an observer, the light is red-shifted, whereas light is blue-shifted for stars moving towards an observer. By measuring shifts in the spectra of the star, periodic, radial motion can be detected that implies the existence of an exoplanet. The “radial velocity signal”  $K$  is given by

$$K = \left( \frac{M_{\text{exo}}}{M_{\star}} \right) \sqrt{\frac{GM_{\star}}{a}} \sin i$$

where  $G$  is the gravitational constant,  $a$  is the semi-major axis of the exoplanet, and  $M_{\text{exo}}$  and  $M_{\star}$  are the masses of the exoplanet and host star respectively. However, if we assume that the inclination is  $90^\circ$  ( $\sin i = 1$ ), the exoplanet is orbiting around a Sun-like star ( $M_{\star} \approx 1.989 \times 10^{30}$  kg), and we use the modern-day standard of  $K = 0.5$  m/s (Wang, 2022), we can create the following scaling relationship using an Earth-like planet as our reference:

$$M_{\text{exo}} = 5.59 M_{\oplus} \left( \frac{a}{1 \text{ AU}} \right)^{1/2}$$

This scaling relationship can then be plotted over modern exoplanet data to see if discovered planets correspond with this detection limit, as shown in Figure 1.

### 2.2 Transit

If an exoplanet’s orbit falls directly between its host star and an observer, then the measured luminosity of that star will briefly dip at a consistent, periodic rate. This is because the

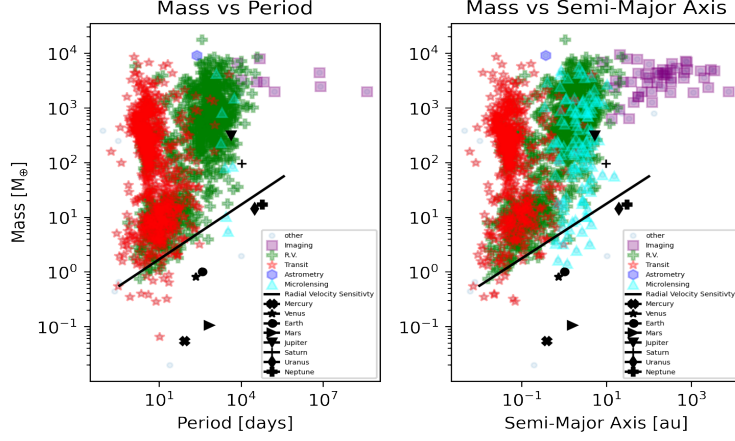


Figure 1: Radial Velocity Limit Against Modern Data

exoplanet blocks an appreciable amount of the stars light when it passes in front of, or transits, the star. However, the probability of an exoplanet transiting in front of its host star is low; the exoplanet's orbit must fall within small space for an observer to perceive it. With the assumption that the exoplanet is much smaller than its host star ( $R_{\text{exo}} \ll R_{\star}$ ), this probability is given by

$$\mathbb{P}_{\text{transit}} = \frac{R_{\star}}{a} \approx 0.005 \left( \frac{R_{\star}}{R_{\odot}} \right) \left( \frac{a}{1 \text{ AU}} \right)^{-1}$$

for approximately circular orbits. When a transit does occur, it is useful to write the ratio between the detected signal and background noise. This "signal to noise ratio" (SNR) is given by

$$\frac{S}{N} = \frac{\delta}{\sigma_{\text{CDPP}}} \sqrt{\frac{n_{\text{tr}} \cdot t_{\text{dur}}}{3 \text{ hr}}}.$$

Borrowing the definitions of these variables and their corresponding assumption (Howard et al., 2012), we have that the photometric depth  $\delta$  is given by

$$\delta = \frac{R_{\text{exo}}^2}{R_{\star}^2}$$

where  $R_{\text{exo}}$  is the radius of an exoplanet orbiting about a star of radius  $R_{\star}$ ,  $\sigma_{\text{CDPP}}$  is the Combined Differential Photometric Precision,  $n_{\text{tr}}$  is the number of transits measured over a 90 day observational period ( $n_{\text{tr}} = \frac{90 \text{ days}}{P}$ ), and  $t_{\text{dur}}$  is the duration of the transit given by

$$t_{\text{dur}} = (3.91 \text{ hours}) \left( \frac{\rho_{\star}}{\rho_{\odot}} \right)^{-1/3} \left( \frac{P}{10 \text{ days}} \right)^{1/3} \sqrt{1 - b^2}$$

where  $\rho_{\star}$  is the density of the host star,  $P$  is the period of the exoplanet, and  $b$  is the impact parameter ( $b = \frac{a \cos(i)}{R_{\star}}$ ) (Ricker et al., 2014). Note that  $b$  goes to zero under the most

ideal circumstances ( $\cos(90^\circ) = 0$ ) (Wilson, 2016). If we assume the host star has Sun-like density, then  $t_{\text{dur}}$  becomes

$$t_{\text{dur}} = (3.91 \text{ hours}) \left( \frac{P}{10 \text{ days}} \right)^{1/3}.$$

It is frequently cited that the boundary SNR that distinguishes an exoplanet candidate from background noise is  $\text{SNR}=7.1$  (Howard et al., 2012). In order to maximize the effectiveness of this detection method, we take the idealized value of  $\sigma_{\text{CDPP}} = 30 \text{ ppm}$  for a 3 hour detection period which yields

$$R_{\text{exo}} = (0.366 R_{\oplus}) \left( \frac{P}{\text{hr}} \right)^{1/6}$$

This scaling relationship is then over-plotted on modern-day exoplanet data, as shown in Figure 2.

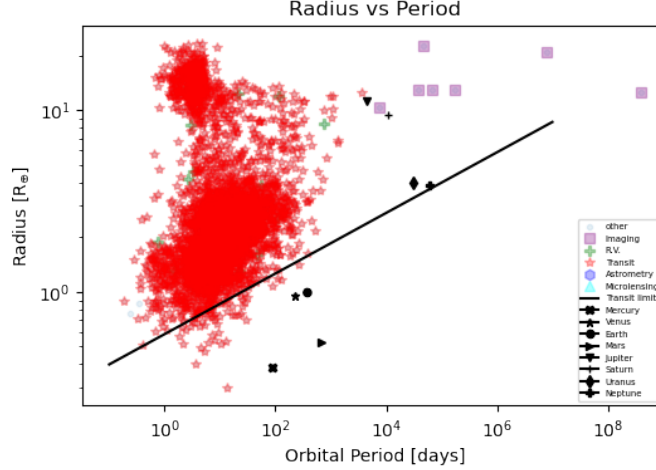


Figure 2: Transit Method Limit Against Modern Data

### 2.3 Direct Imaging

One of the most conceptually simplistic methods for detecting exoplanets involves directly taking a photo of the exoplanet. However, exoplanets are typically washed out by the light from their host star. This severely limits astronomers' ability to detect exoplanets this way without favorable conditions.

In order to resolve the planet and star into distinct objects they must obey the Rayleigh limit, which gives the smallest angular separation  $\theta$  (in units of radians) for which two light emitting objects can still be resolved individually (this equation gives us the lower bound of detection for semi-major axis). This separation is given by

$$\theta \approx 1.22 \frac{\lambda}{D}$$

where  $\lambda$  is the observation wavelength and  $D$  is the diameter of the aperture of the telescope being used. We can optimize this separation by adopting the peak emission wavelength of a blackbody, with a temperature of 130K, of  $\lambda \approx 22.3 \mu m$  and the largest telescope diameter available on Earth today ( $D = 10.4$  m for the Gran Telescopio Canarias). This yields a minimal angular separation of  $\theta = 0.54$  arcseconds to be detected. The ability for an exoplanet to be distinguishable from its host star also depends on the star-planet contrast  $f$  given by

$$f = \left( \frac{R_{exo}}{R_{\star}} \right)^2 \cdot \frac{\exp \left[ \frac{hc}{\lambda k_B T_{\star}} \right] - 1}{\exp \left[ \frac{hc}{\lambda k_B T_{exo}} \right] - 1}$$

The contrast required identify an exoplanet is commonly cited as  $f = 10^{-7}$  (although values as low as  $f = 10^{-10}$  have been occasionally cited) (Li, 2021). So assuming that  $T_{\star} = T_{\odot}$  and  $T_{exo} = 130$  K for a Jupiter-like exoplanet, we obtain a lower bound for the radius of the exoplanet ( $1.23 R_{\oplus}$ ). Using the Rayleigh limit, we also obtain a lower bound for the semi-major axis (5.79 AU). These two bounds are plotted on a graph of the exoplanet radius (in terms of  $R_{\oplus}$ ) versus the semi-major axis (in AU), as shown in figure 3.

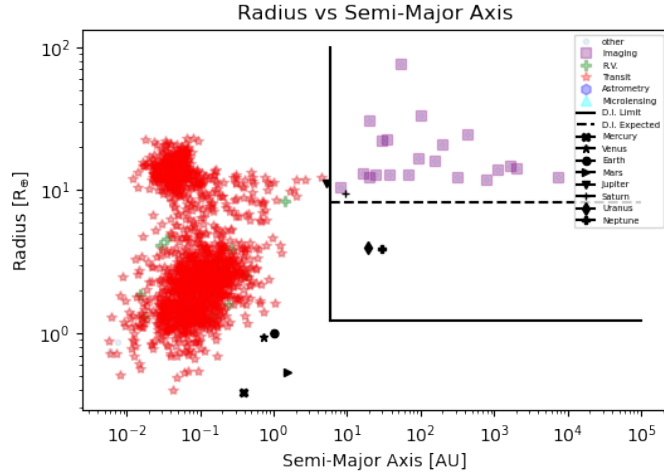


Figure 3: Direct Imaging Limit Against Modern Data

### 3 Results

By visual inspection, each detection limit in Figures 1-3 (shown by a black line) is approximately correct. The largest discrepancy comes from the direct imaging limit, which has a much lower limiting radius (solid black line) than the visually implied limit (dotted black line). We believe this to be the result of improper assumptions about the most ideal detection conditions (i.e., temperature of exoplanets, peak emission wavelength, etc.). However, these calculated detection limits provide an adequate approximation for us to test against a simulated system. The signal associated with a Jupiter-like planet orbiting

around a Sun-like star is examined in this section using each of the above detection methods.

Using the radial velocity method, the described system produces a "radial velocity signal" of  $K \approx 12 \frac{\text{m}}{\text{s}}$ . Notice that this is greater than the state-of-the-art detection limit of  $K = 0.5 \frac{\text{m}}{\text{s}}$  from above, so the described exoplanet could be detected using the radial velocity method.

The transit method has two major limiting factors: the probability of the transit being observable, and the strength of signal needed to be detected. There is a  $\text{IP} \approx 0.094\%$  probability that the Jupiter-like orbit has a measurable transit. However, if we assume that measuring the transit is possible, then the described system would produce an SNR of 163. This signal is well above the state-of-the-art detection limit, so this exoplanet would be detectable assuming a favorable transit. However, due to the low probability of a favorable transit, it is unlikely that the transit method is a viable form of detection.

The direct imaging technique requires the exoplanet in question to surpass two detection limits. Firstly, the exoplanet must be at least 0.54 arcseconds away from its host star to satisfy the Rayleigh limit. Secondly, the star-planet contrast  $f$  must be greater than  $f = 10^{-7}$  is order for the exoplanet to be visually distinct from its host star. For our simulation, we make two primary assumptions. First, we assume that this Jupiter-like planet has the same semi-major axis as our Jupiter ( $a = 5.2 \text{ AU}$ ). Second, we assume that the host star is as close to the observer as the closest exoplanet discovered using direct imaging ( $d \approx 11 \text{ pc}$ ). Utilizing these assumptions, this Jupiter-like planet would not satisfy the Rayleigh limit and thus would not be able to be differentiated from its host star. For sake of completeness, we have tested the second detection limit regardless. We found that the simulated star-planet contrast comes to  $f = 8.79 * 10^{-6}$ . This is greater than the state-of-the-art detection limit, so if the system had first satisfied the Rayleigh limit, it would be detectable. However, since the Rayleigh limit is not satisfied, the direct imaging technique would be impossible in this system.

## 4 Conclusion

Ultimately, we have found radial velocity exoplanet detection appears to be the most consistently reliable detection method. The probability of an exoplanet's transit being detectable is remarkably low, meaning that the transit method cannot be reliably used for exoplanet detection. Similarly, the direct imaging method requires remarkably generous conditions for it to be viable, meaning that the majority of exoplanets cannot be detected in this way. The radial velocity method was the only method we found to be successful in our detection test regarding a Jupiter-like exoplanet.

Beyond these results, more research could go into specifically determining the limiting factors of the direct imaging method. There is a clear discrepancy between our calculated limit and the expected limit for the direct imaging technique. We believe that this could be the result of assuming an incorrect peak detection wavelength or poor assumptions for the temperature of the exoplanet and host star. Outside of this discrepancy, we have found our results to be largely consistent with what should be expected. Our conclusion regarding the superiority of the radial velocity method is corroborated by the fact that the vast plurality of exoplanets have been detected using radial velocity exoplanet detection.

## 5 Acknowledgements

**Bailey Stephens** - *Paper Lead, Background Research, Slide Development, Code Editing*  
**Tayt Armitage** *Presentation Co-Lead, Code Lead (Part I), Background Research*  
**Lauren Robinson** *Presentation Co-Lead, Background Research, Paper Editor, Slide Editor*  
**Caitlin O'Brien** *Code Lead (Part II), Background Research, Paper Editor*  
**Ethan Fahimi** *Code Aid (Part I), Paper Editor, Slide Editor*

## References

- [1] Alan Gould. *About transits*. Oct. 2016. URL: <https://www.nasa.gov/kepler/overview/abouttransits>.
- [2] Andrew W. Howard et al. *IOPscience*. June 2012. URL: <https://iopscience.iop.org/article/10.1088/0067-0049/201/2/15>.
- [3] Michelle Kunimoto et al. *IOPscience*. Jan. 2018. URL: <https://iopscience.iop.org/article/10.3847/1538-3881/aaa005>.
- [4] Zhexing Li et al. *IOPscience*. June 2021. URL: <https://iopscience.iop.org/article/10.3847/1538-3881/abf831>.
- [5] A. Naumov and I. Khizhnyakov. *Thermal Emissions of Jupiter*. Mar. 1964. URL: [https://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle\\_query?bibcode=1965SvA.....9..480N&db\\_key=AST&page\\_ind=0&data\\_type=GIF&type=SCREEN\\_VIEW&classic=YES](https://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle_query?bibcode=1965SvA.....9..480N&db_key=AST&page_ind=0&data_type=GIF&type=SCREEN_VIEW&classic=YES).
- [6] George R. Ricker et al. *Transiting Exoplanet Survey Satellite*. URL: <https://www.spiedigitallibrary.org/journals/Journal-of-Astronomical-Telescopes-Instruments-and-Systems/volume-1/issue-01/014003/Transiting-Exoplanet-Survey-Satellite/10.1117/1.JATIS.1.1.014003.full?SSO=1>.
- [7] Sara Seager. *Exoplanets*. The Crown Publishing Group, 2010.
- [8] Paul A. Wilson. *The Exoplanet Transit Method*. Jan. 2017. URL: <https://www.paulanthonywilson.com/exoplanets/exoplanet-detection-techniques/the-exoplanet-transit-method/>.