Examining Na Abundance in Stars Using Spectral Data

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1 Motivation

The elemental composition of a star provides key insight into its metallicity, aspects of the star's formation, and the composition of exoplanets orbiting it. To measure this composition, astronomers must analyze the spectra of the star and look for distinct absorption lines as indicators of various elements. The strength of these absorption lines can then be used to determine the stellar abundance of those same elements. In this paper, we aim to showcase this process by analyzing stellar spectra to uncover details about the sodium content within our own sun.

2 Methods

2.1 Equivalent Width and Ground State Atoms

To analyze the sodium content in the sun we begin by examining solar spectral data depicted in Figure 1 below. This data contains two distinct peaks at wavelengths $\lambda = 5890\mathring{A}$ and $\lambda = 5896\mathring{A}$. We are examining the peak at $\lambda = 5890\mathring{A}$.

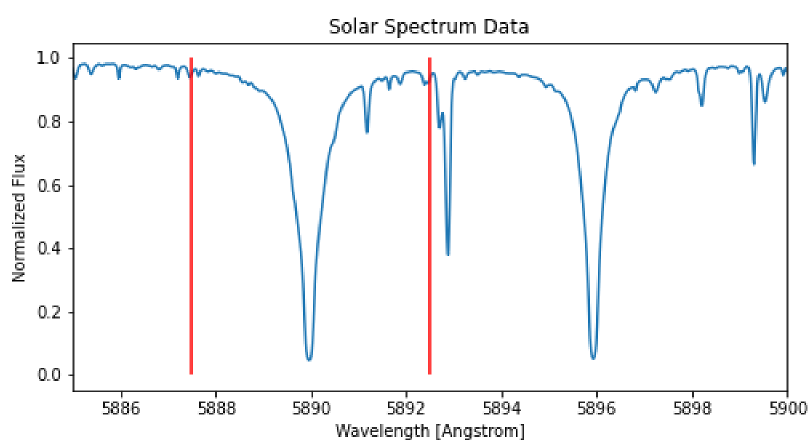

Figure 1: Normalized Solar Spectrum Plot

Prior to our examination of the spectrum data, through visual inspection, limits were set (shown in red in Figure 1) to aide in determining the equivalent width. The equivalent width can be defined as the area between the data and the line for where Normalized Flux = 1, which is visually represented in Figure 2. We aim to calculate the equivalent width surrounding the peak at $\lambda = 5890\mathring{A}$. We calculate this area using a Riemann Sum:

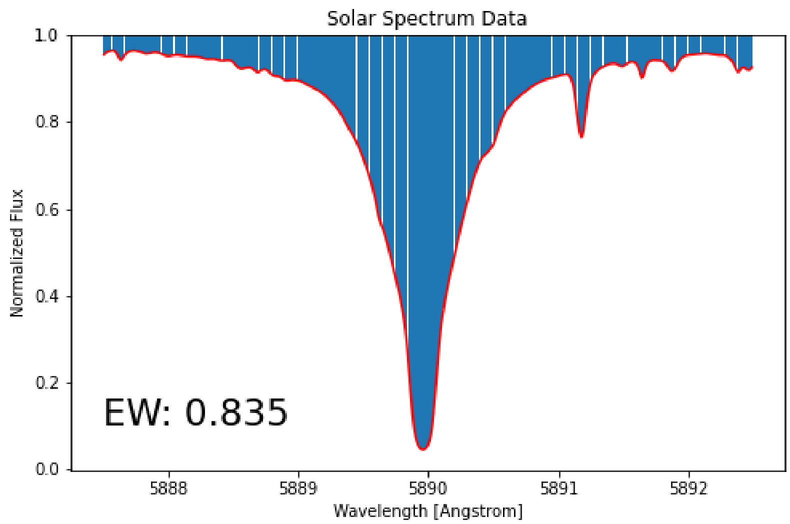

Figure 2: Visual representation of equivalent width, calculated using a Riemann Sum

Then we obtain an equivalent width of $0.835\mathring{A}$ for the specific frequency $\lambda = 5890\mathring{A}$. We can trace this value on the general curve of growth for the sun shown in Figure 3, and estimate the corresponding x-axis value.

We can see by visual inspection that the corresponding x-axis value is approximately 14.8. In order to solve for the column density, N, we set our estimated x-value equal to the expression for the x-axis values:

$$14.8 \approx \log\left(Nf\left(\frac{\lambda}{5000\mathring{A}}\right)\right)$$

We assume the wavelength of interest ($\lambda = 5890$ Angstroms) and an oscillator strength $f = 0.65$, where f expresses how likely a transition between energy levels of an atom is to occur [4]. We can then rearrange the above equation to find that the column density N of ground state sodium atoms. After performing this calculation we find $N = 8.24 \cdot 10^{24}$ atoms/cm^2.

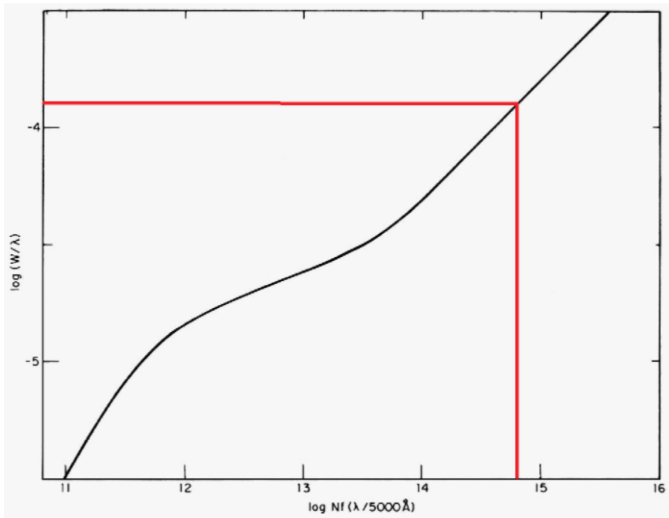

Figure 3: General curve of growth for the sun relating equivalent width and number of absorbing atoms. The red line uses the calculated equivalent width (EW = $0.835\mathring{A}$) to find the corresponding x-axis value (≈ 14.8).

2.2 Boltzmann Equation

We now wish to find the ratio of excited sodium atoms to those in the ground state. The Boltzmann Equation can be used to estimate this ratio. The Boltzmann equation can be written as

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

where N_n is the number density, g_n is the number of degenerate states for a quantum number n, E is energy at that state, k is Boltzmann constant, and T is surface temperature. For our purposes we have adopted the surface temperature of the sun, 5778K, for T.
The subscripts on g and N indicate which energy state the sodium electrons are in. Convention holds that $n = 1$ indicates the ground state while $n = 2$ corresponds to the first excited state.
For systems such as this one, g_n is given by $g_n = 2n^2$ for a given quantum number n. This means that g_1 and g_2 are 2 and 8 respectively [5]. Their corresponding energies are $E_1 = E_{3s} \approx -5.14eV$ and $E_2 = E_{3p} \approx -3.04eV$ [3]. Using these values we find that the ratio of excited sodium atoms to those in the ground state is

$$\frac{N_2}{N_1} = 0.05892$$

2.3 Saha Equation

In a star such as our sun, atoms will not always be in their neutral state. Therefore, we must apply The Saha Equation to estimate the ratio of neutral sodium atoms to ionized sodium atoms. The Saha Equation is given as:

$$\frac{Na_{II}}{Na_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\exp\left(-\frac{\chi}{kT}\right)$$

where k is the Boltzmann constant, T is the temperature, h is Planck's constant, and m_e is the electron mass. For sodium, we have adopted partition functions $Z_I = 2.4$, $Z_{II} = 1.0$, as well as electron pressure $P_e = n_e kT = 1.0N \cdot m^{-2}$, and ionization energy $\chi = 5.1eV$. Using these values we can calculate the ratio of neutral to ionized sodium atoms:

$$\frac{N_{II}}{N_I} = 2509.164$$

2.4 Column Density

Using the results from the previous three calculations, we can compute the total column density of sodium atoms in the photosphere of the sun. The total number of sodium atoms, N_{tot}, is given by:

$$N_{tot} = N_1 \times \left(1 + \frac{N_2}{N_1}\right) \times \left(1 + \frac{Na_{II}}{Na_I}\right)$$

Note that we have already found $N_1 = 8.24 * 10^{14}$ atoms/cm^2 by analyzing solar spectrum data, $\frac{N_2}{N_1} = 0.059$ using the Boltzmann equation, and $\frac{Na_{II}}{Na_I} = 2509.164$ using the Saha Equation. Therefore, we have all of the components required to calculate the column density. These values yield a column density of $2.197 \cdot 10^{18}$ particles/cm^2.

2.5 Relative Abundance

Finally, given the Column Density of Hydrogen atoms to be about $6.6 \cdot 10^{23}$, we calculate the abundance of sodium relative to hydrogen in both a physicist's and an astronomer's terminology. According to Dr. Wang, the values of N1/N2 or NI/NII can be either column density or number density, due to the fact that the ratio is unitless. Number density measures the concentration of a substance per unit volume, in this case atoms per cm^2. Thus, if we take the ratio of column density of Na to the column density of H we will find the ratio of abundances. Astronomers and physicists typically express the relative abundance in different ways, which we outline below. Physicists write the abundance using the mole ratio between sodium and hydrogen is can be written as

$$\frac{N_{Na}}{N_H}$$

Which we have found to be $3.329 \cdot 10^{-6}$. Astronomers working with galaxies communicate the relative abundance using the log of the mole ratio with an offset of 12.

$$12 + \log\left(\frac{N_{Na}}{N_H}\right)$$

In an astronomer's terms, specifically for astronomers working on galaxies, we have found a relative abundance of -0.613.
Lastly, astronomers working with stars typically examine the log of the mole ratio between sodium and hydrogen, with respect to the mole ratio of the sun:

$$\log\left(\frac{\frac{N_{Na}}{N_H}}{(\frac{N_{Na}}{N_H})_\odot}\right)$$

In an astronomer's terms, specifically for astronomers working with stars, we have computed a relative abundance of 0.222.

3 Results

Using the methods outlined above we have found the total column density of sodium atoms in the sun's photosphere ($2.197 \cdot 10^{18}$ atoms/cm^2), and the number density of sodium in the ground state ($N_1 = 8.240 \cdot 10^{14}$). Using our previous calculations, we have also found the number densities of neutral and ionized sodium. The number density of neutral sodium was found to be $Na_I = 8.726 \cdot 10^{14}$. The number density of ionized sodium atoms is $Na_{II} = 2.196 \cdot 10^{18}$.
Additionally, using our total number density we were able to compare the abundance of sodium to that of hydrogen in the sun. As discussed above, the mole ratio of sodium to hydrogen utilized by physicists is $3.329 \cdot 10^{-6}$. This abundance relationship led us to conclude that the abundance of sodium is vastly surpassed by the abundance of hydrogen. Expressing this abundance ratio as a log-mole ratio utilized by astronomers we find [Na/H] = 0.222.

4 Conclusion

Through this process we have developed reasonable values for the number density of sodium within our sun. Interestingly, we can conclude that majority of the sodium atoms within the sun are ionized, as the total number density of sodium is $2.197 \cdot 10^{18}$atoms/cm^2 while the ionized number density is $Na_{II} = 2.196 \cdot 10^{18}$. A few conclusions could be drawn from our abundance ratios as well, the first being that when compared to hydrogen the fraction of sodium in the sun is minimal, $\frac{N_{Na}}{N_H} = 3.329 \cdot 10^{-6}$. We can also see that there could be some error in our calculations from the [Na/H] ratio. We would expect [Na/H] to be zero, as this relationship involves a scaling relationship to with the sun, so when utilizing solar data we should see a $log_{10}(1) = 0$. Our value is nearly zero, so our answer is approximately correct with only slight discrepancies. In the future we could refine our methods by utilizing more exact measurements of wavelength and determining the width of our sodium peaks with greater precision. Regardless of discrepancies, we have shown that these methods do in fact work to find the abundance of materials in stars, and can be employed in future projects when analyzing other stars or searching for different elements within the sun. The next logical step would be to apply the methods outlined above to a different spectral line, in order to obtain the abundance of another element.

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