Exoplanet Detection Methods: What Works?

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1 Motivation

By discovering and studying exoplanets, astronomers are able to gain insight into the properties of the Universe. This is far from an easy process though, as several methods have been developed for detecting exoplanets with varying degrees of success. By calculating the state-of-the-art limitations of three exoplanet detection methods (Radial Velocity, Transit, and Direct Imaging), we can determine how effective they are for detecting a Jupiter-like planet orbiting around a Sun-like star.

2 Methods

2.1 Radial Velocity

When a star is moving away from an observer, the light is red-shifted, whereas light is blue-shifted for stars moving towards an observer. By measuring shifts in the spectra of the star, periodic, radial motion can be detected that implies the existence of an exoplanet. The "radial velocity signal" K is given by

$$K = \left(\frac{M_{\text{exo}}}{M_\star}\right)\sqrt{\frac{GM_\star}{a}}\sin i$$

where G is the gravitational constant, a is the semi-major axis of the exoplanet, and M_{exo} and $M_\star$ are the masses of the exoplanet and host star respectively. However, if we assume that the inclination is 90°($\sin i = 1$), the exoplanet is orbiting around a Sun-like star ($M_\star \approx 1.989 \times 10^{30}$ kg), and we use the modern-day standard of K= 0.5 m/s (Wang, 2022), we can create the following scaling relationship using an Earth-like planet as our reference:

$$M_{\text{exo}} = 5.59 M_\oplus \left(\frac{a}{1\ \text{AU}}\right)^{1/2}$$

This scaling relationship can then be plotted over modern exoplanet data to see if discovered planets correspond with this detection limit, as shown in Figure 1.

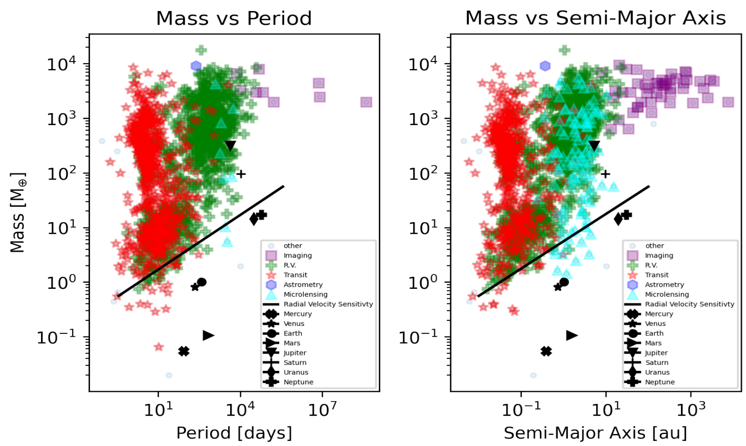

Figure 1: Radial Velocity Limit Against Modern Data

2.2 Transit

If an exoplanet's orbit falls directly between its host star and an observer, then the luminosity of that star measured will briefly dip down at a consistent, periodic rate. This is because the exoplanet blocks an appreciable amount of the stars light when it passes in front of, or transits, the star. However, the probability of an exoplanet transiting in front of its host star is low; the exoplanet's orbit must fall within small space for an observer to perceive it. With the assumption that the exoplanet is much smaller than its host star ($\mathrm{R_{exo}} << \mathrm{R_\star}$), this probability is given by

$$\mathbb{P}_{\text{transit}} = \frac{\mathrm{R}_\star}{a} \approx 0.005 \left(\frac{\mathrm{R}_\star}{\mathrm{R}_\odot}\right) \left(\frac{a}{1\ \mathrm{AU}}\right)^{-1}$$

for approximately circular orbits. When a transit does occur, the probability that the detected luminosity fluctuations are an exoplanet versus background noise is calculated. This "signal to noise ratio" (SNR) is given by

$$\frac{\mathrm{S}}{\mathrm{N}} = \frac{\delta}{\sigma_{\mathrm{CDPP}}} \sqrt{\frac{n_{\mathrm{tr}} \cdot t_{\mathrm{dur}}}{3\ \mathrm{hr}}}.$$

Borrowing the definitions of these variables and their corresponding assumption (Howard et al., 2012), we have that the photometric depth δ is given by

$$\delta = \frac{\mathrm{R}_{\mathrm{exo}}^2}{\mathrm{R}_\star^2}$$

where $\mathrm{R_{exo}}$ is the radius of an exoplanet orbiting about a star of radius $\mathrm{R}_\star$, σ_{CDPP} is the Combined Differential Photometric Precision, n_{tr} is the number of transits measured over a 90 day observational period ($n_{\rm tr} = \frac{90 \text{ days}}{P}$), and $t_{\rm dur}$ is the duration of the transit given by

$$t_{\rm dur} = (3.91 \text{ hours}) \left(\frac{\rho_\star}{\rho_\odot}\right)^{-1/3} \left(\frac{P}{10 \text{ days}}\right)^{1/3} \sqrt{1-b^2}$$

where $\rho_\star$ is the density of the host star, P is the period of the exoplanet, and b is the impact parameter ($b = \frac{a\cos(i)}{R_\star}$) (Ricker et al., 2014). Note that b goes to zero under the most ideal circumstances ($\cos(90^\circ) = 0$) (Wilson, 2016). If we assume the host star has Sun-like density, then $t_{\rm dur}$ becomes

$$t_{\rm dur} = (3.91 \text{ hours}) \left(\frac{P}{10 \text{ days}}\right)^{1/3}.$$

It is frequently cited that the boundary SNR that distinguishes an exoplanet candidate from background noise is SNR=7.1 (Howard et al., 2012). In order to maximize the effectiveness of this detection method, we take the idealized value of $\sigma_{\rm CDPP} = 30$ ppm for a 3 hour detection period which yields

$$R_{\rm exo} = (0.366\ R_\oplus) \left(\frac{P}{\text{hr}}\right)^{1/6}$$

This scaling relationship is then over-plotted on modern-day exoplanet data, as shown in Figure 2.

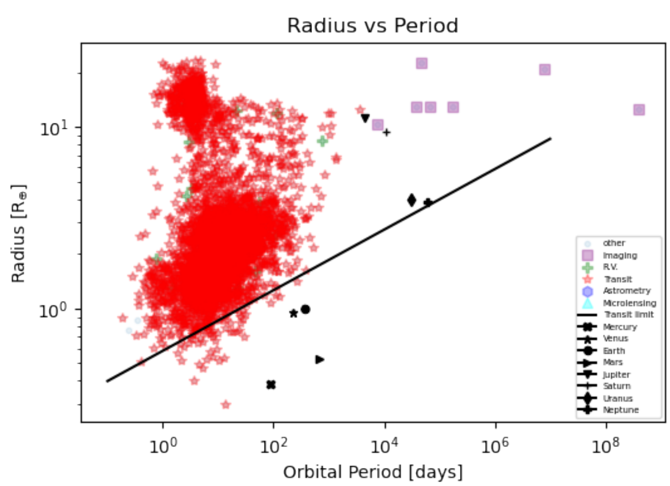

Figure 2: Transit Method Limit Against Modern Data

2.3 Direct Imaging

One of the most conceptually simplistic methods for detecting exoplanets involves directly taking a photo of the exoplanet. However, exoplanets are typically washed out by the light from their host star. This severely limits astronomers' ability to detect exoplanets this way without favorable conditions.

In order to resolve the planet and star into distinct objects they must obey the Rayleigh limit, which gives the smallest angular separation θ (in units of radians) for which two light emitting objects can still be resolved individually (this equation gives us the lower bound of detection for semi-major axis). This separation is given by

$$\theta \approx 1.22\frac{\lambda}{D}$$

where λ is the observation wavelength and D is the diameter of the aperture of the telescope being used. We can optimize this separation by adopting the peak emission wavelength of a blackbody, with a temperature of 130K, of $\lambda \approx 22.3\ \mu m$ and the largest telescope diameter available on Earth today (D = 10.4 m for the Gran Telescopio Canarias). This yields a minimal angular separation of $\theta = 0.54$ arcseconds to be detected. The ability for an exoplanet to be distinguishable from its host star also depends on the star-planet contrast f given by

$$f = \left(\frac{R_p}{R_\star}\right)^2 \cdot \frac{\exp\left[\frac{hc}{\lambda k_B T_\star}\right] - 1}{\exp\left[\frac{hc}{\lambda k_B T_{\text{exo}}}\right] - 1}$$

The contrast required identify an exoplanet is commonly cited as $f = 10^{-7}$ (although values as low as $f = 10^{-10}$ have been occasionally cited) (Li, 2021). So assuming that $T_\star = T_\odot$ and $T_{\text{exo}} = 130$ K for a Jupiter-like exoplanet, we obtain a lower bound for the radius of the exoplanet (1.23 $R_\oplus$). Using the Rayleigh limit, we also obtain a lower bound for the semi-major axis (5.79 AU). These two bounds are plotted on a graph of the exoplanet radius (in terms of $R_\oplus$) versus the semi-major axis (in AU), as shown in figure 3.

3 Results

By visual inspection, each detection limit in Figures 1-3 (shown by a black line) is approximately correct. The largest discrepancy comes from the direct imaging limit, which has a much lower limiting radius (solid black line) than the visually implied limit (dotted black line). This is believed to be the result of improper assumptions about the most ideal detection conditions (i.e., temperature of exoplanets, peak emission wavelength, etc.). However, these calculated detection

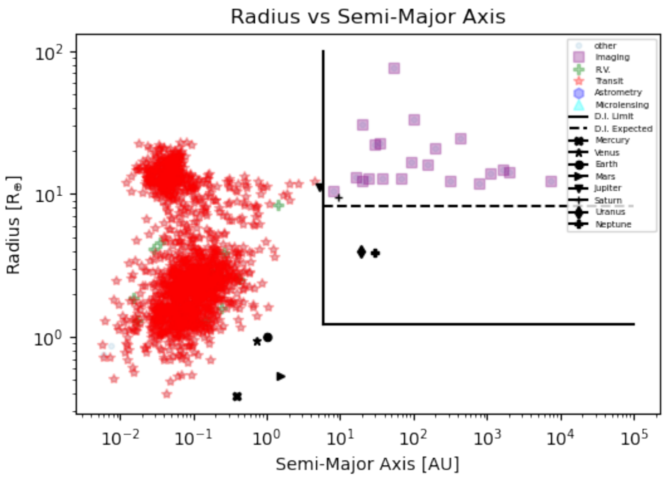

Figure 3: Direct Imaging Limit Against Modern Data

limits provide a good enough approximation for us to test them against a simulated system. The signal associated with a Jupiter-like planet orbiting around a Sun-like star is then detected using each of the above detection methods.

Using the radial velocity method, the described system produces a "radial velocity signal" of $K \approx 12\frac{\mathrm{m}}{\mathrm{s}}$. Notice that this is greater than the state-of-the-art detection limit of $K = 7.1\frac{\mathrm{m}}{\mathrm{s}}$ from above, so the described exoplanet could be detected using the radial velocity method.

The transit method has two major limiting factors: the probability of the transit being observable, and the strength of signal needed to be detected. There is a probability $\mathbb{P} \approx 0.094\%$ that the Jupiter-like orbit is possible to view during this method. However, if we assume that measuring the transit is possible, then the described system would produce an SNR of 7851. This signal is well above the state-of-the-art detection limit, so this system would be possible to detect using this method.

The direct imaging technique requires the exoplanet in question to surpass two detection limits. Firstly, the exoplanet must be at least 0.54 arcseconds away from its host star to satisfy the Rayleigh limit. Secondly, the star-planet contrast f must be greater than $f = 10^{-7}$ is order for the exoplanet to be visually distinct from its host star. If we assume that this Jupiter-like planet has the same semi-major axis as our Jupiter (a = 5.2 AU), and the host star is as close to the observer as the closest exoplanet discovered using direct imaging (d = INSERT), then this Jupiter-like planet would not satisfy the Rayleigh limit. However, if the system were closer to the observer or the Jupiter-like planet further from its host star, then its simulated star-planet contrast comes to $f = 8.79 * 10^{-6}$. This is greater than the state-of-the-art detection limit, so we would be able to detect this Jupiter-like planet.

4 Conclusion

Radial velocity exoplanet detection appears to be the most consistently reliable detection method. The probability of an exoplanet's transit being detectable is remarkably low, meaning that it cannot be reliably used for exoplanet detection. Additionally, direct imaging requires remarkably generous conditions for it to be viable, meaning that the majority of exoplanets cannot be detected in this way. This assumption is also confirmed by the fact that the vast plurality of exoplanets have been detected using the radial velocity method.
Beyond these results, more research could go into specifically determining the limiting factors of the direct imaging method. There is a clear discrepancy between our calculated limit and the expected limit for the direct imaging technique. We believe that this could be the result of assuming the wrong peak detection wavelength, poor assumptions for the temperature of the exoplanet and host star, or something else entirely.

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