please unmute to say which # you want to go over.

10a, 1f, 4d, 16 c+d, 1b, 8a, 1d, 15c,
7b+c+d, 15 b+e, 8b, 8d, 13b, 14 a+c,
13d, 16a+b, 6b, 17c+d, 7e, 9 a+c, 1c

1. Find the domain of each function.

- (a) $f(x) = \frac{x^2-4}{x^2-5x+6}$
- (b) $g(x) = \frac{x+3}{\sqrt{x^2-4}}$
- (c) $k(x) = \tan(2x)$. $= \frac{\sin(2x)}{\cos(2x)}$
- (d) $L(x) = \ln(1-3x)$
- (e) $M(x) = \frac{\sqrt{3-x}}{\sqrt{x+1}}$.
- (f) $J(x) = e^{2x+\pi}$.

There is no restriction on what I can plug in for x

$\boxed{\text{Domain: } \mathbb{R} \text{ or } (-\infty, \infty)}$

1b.) Domain of $g(x) = \frac{x+3}{\sqrt{x^2-4}}$

- den $\neq 0$
- $\sqrt{y}$ $y \geq 0$

$x^2 - 4 \geq 0$

If $x^2 - 4 = 0$

then $\sqrt{x^2-4} = \sqrt{0} = 0$

$\Rightarrow$ den of 0.

$x^2 - 4 > 0$
$+4 \quad +4$

$x^2 > 4$

$|x| > 2$

$x > 2 \quad x < -2$

$\boxed{(-\infty, -2) \cup (2, \infty)}$

d.) domain: $\ln(1-3x)$

$\ln(y)$

$y > 0$

$\ln_e (1-3x) = k$

$e^k = 1-3x$

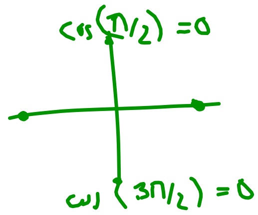
$e^k > 0$

$1 - 3x > 0$
$+3x \quad +3x$

$\frac{1}{3} > \frac{3x}{3}$

$\boxed{x < 1/3}$ or $(-\infty, 1/3)$

c.) domain $\frac{\sin(2x)}{\cos(2x)} \leftarrow$ den $\neq 0$



$\cos(2x) \neq 0$

$\frac{2x}{2} \neq \frac{\pi/2}{2}, \frac{3\pi/2}{2}, \frac{5\pi/2}{2}, \ldots$

$\boxed{x \neq \pi/4, 3\pi/4, 5\pi/4, \ldots}$

2. Graph the given function. Identify its domain and make sure your graph is consistent with your answer for the domain.

- (a)

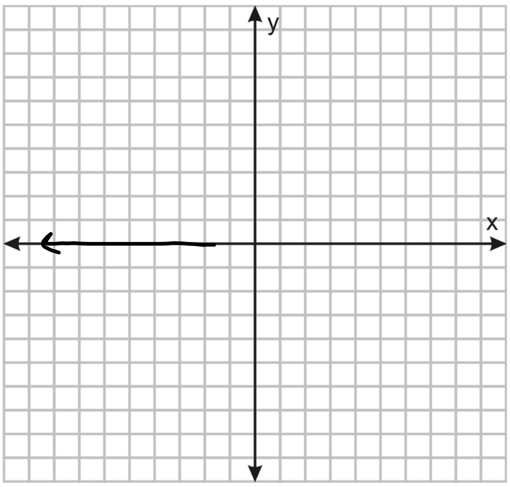
$\lim_{x \to 3} f(x)$ DNE

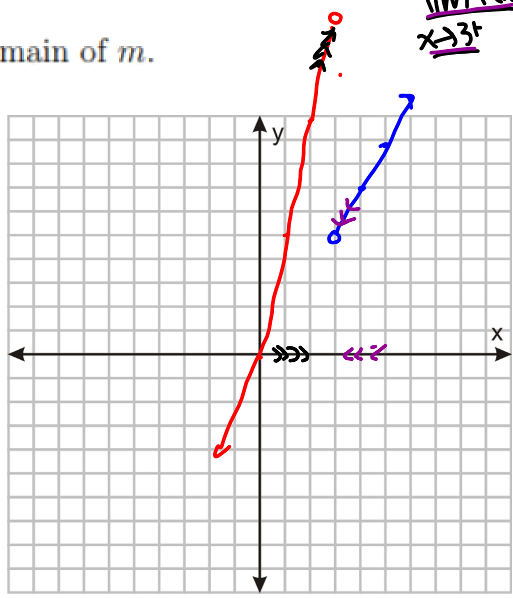
$$f(x) = \begin{cases} 2x-1, & \text{if } x > 3, \\ 5x, & \text{if } x < 3. \end{cases}$$

$\lim_{x \to 3^-} f(x) = 15$

$\lim_{x \to 3^+} f(x) = 5$

- (b) $m(x) = |x| + x$. Start by finding the domain of m.



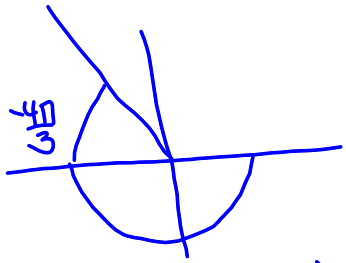


3. The monthly cost for a cell phone contract is a flat fee of \$25.00 plus \$1.25 per minute. Find a formula for the cost as a function of minutes used.

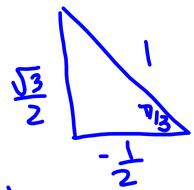
4. Evaluate all the trig values. Remember that you are allowed to use, without showing work, the trig values for the standard angles $\pi/6, \pi/4, \pi/3$ and $0, \pi/2, \pi, 3\pi/2, 2\pi$. For any other angle you must show work suitably.

- (a) $\sin(15\pi/3)$
- (b) $\cos(-3\pi)$
- (c) $\sec(4\pi)$
- (d) $\sin(-4\pi/3)$.

sin +
cos −



Quadrant 3



$$\boxed{\sin\left(\frac{-4\pi}{3}\right) = \frac{\sqrt{3}}{2}}$$

5. You are given that $\sin(\theta) = -7/8$ and $\pi < \theta < 3\pi/2$. Find the value of the following:

$$\boxed{\cos(\theta), \sec(\theta),} \tan(\theta), \sin(\theta - 4\pi).$$

Method 1

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{-7}{8}\right)^2 + \cos^2\theta = 1$

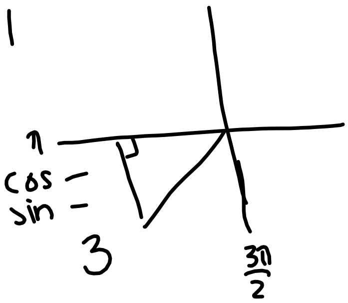
$\frac{49}{64} + \cos^2\theta = 1$

$-\frac{49}{64} \qquad -\frac{49}{64}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{15}{64}}$

$\cos\theta = \frac{\pm\sqrt{15}}{\sqrt{64}} = \frac{\pm\sqrt{15}}{8}$ must be neg in quad 3

$\boxed{\cos\theta = \frac{-\sqrt{15}}{8}}$

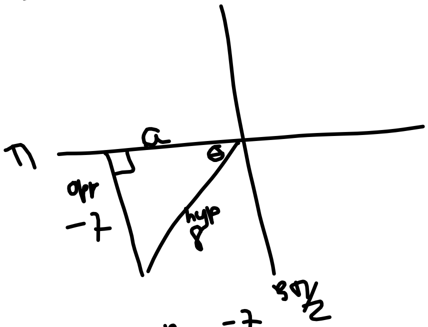


$\sec\theta = \frac{1}{\cos(\theta)} = \frac{1}{\frac{-\sqrt{15}}{8}}$

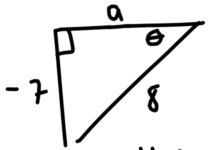
$= 1 \cdot \frac{8}{-\sqrt{15}} = \boxed{\frac{-8}{\sqrt{15}}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ . . . . .

Method 2



$\sin\theta = \frac{opp}{hyp} = \frac{-7}{8}$



By pyth thm $a^2 + (-7)^2 = 8^2$

$a^2 + 49 = 64$

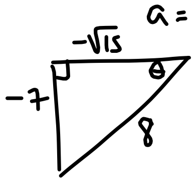
$\quad -49 \quad -49$

$\sqrt{a^2} = \sqrt{15}$

$a = \pm\sqrt{15}$

must be neg in quad 3

$a = -\sqrt{15}$



$\boxed{\cos\theta = \frac{adj}{hyp} = \frac{-\sqrt{15}}{8}}$

6. Solve the following equations.

- (a) $e^{3x-4} = 1$
- (b) $\ln(5x - 2) = 0$
- (c) $\ln(\ln(x)) = 0$
- (d) $e^{\ln(x^2-1)} = 3$

b.) $\ln_e(5x-2) = 0$

$e^0 = 5x-2$

$1 = 5x - 2$
$+2 \quad\quad +2$

$\frac{3}{5} = \frac{5x}{5}$

$\boxed{x = 3/5}$

7. Simplify fully, using laws of exponentials and logs. Note that some of the expressions may be in simplest possible terms.

- (a) $e^{(\ \ln(5)+\ln(2)\ )}$
- (b) $\ln(e^2+e^4)$ Hint: You can factor e^2 out of the expression e^2+e^4. You will then have the log of a product.
- (c) $\ln(1+e^2)$ > in simplest form
- (d) $\frac{\ln(5)}{\ln(2)}$
- (e) $\ln(\frac{5}{2})$ $= \boxed{\ln(5) - \ln(2)}$
- (f) $\ln(e^{\ln(2)})$

$\ln\left(\frac{5}{2}\right) = \ln(5) - \ln(2)$

b) $\ln(e^2+e^4) = \ln\left(e^2(1+e^2)\right)$

$= \ln(e^2) + \ln(1+e^2)$

$= 2\underbrace{\ln(e)}_{=1} + \ln(1+e^2)$

$\boxed{= 2 + \ln(1+e^2)}$

8. You are given $f(x) = \sqrt{4-x}$ and $g(x) = 2x + 3$.

- (a) Find the domain of the function $(f \circ g)(x)$. Find the formula for $(f \circ g)(x)$.
- (b) Find the domain of the function $(g \circ f)(x)$. Find the formula for $(g \circ f)(x)$.
- (c) What is the domain of the function $(g \circ g)(x)$? What is the formula?
- (d) What is the domain of the function $(f \circ f)(x)$? What is the formula for this function?

a.) $f \circ g(x)$

$f(g(x)) = \sqrt{4-(2x+3)} = \sqrt{4-2x-3} = \sqrt{1-2x}$

even root

$1-2x \geq 0$

$\frac{1}{2} \geq \frac{2x}{2}$

$\frac{1}{2} \geq x$ $\boxed{x \leq 1/2}$

b) $g \circ f(x) = g(f(x))$

$\boxed{g \circ f(x) : 2(\sqrt{4-x}) + 3}$

$4-x \geq 0$
$+x \quad +x$

$4 \geq x$

Domain $\boxed{x \leq 4 \qquad (-\infty, 4]}$

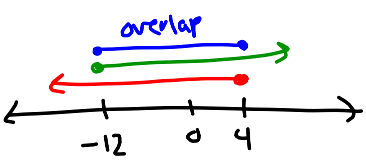
d.) $f \circ f(x) = f(f(x)) = \sqrt{4 - (\underbrace{\sqrt{4-x}}_{\geq 0})}$ ①

and ② $4-\sqrt{4-x} \geq 0$

① $4-x \geq 0$
$4 \geq x$
$x \leq 4$

② $4-\sqrt{4-x} \geq 0$
$+\sqrt{4-x} \quad +\sqrt{4-x}$
$4^2 \geq \sqrt{4-x}^2$
$16 \geq 4-x$
$+x \quad +x$

and $x+16 \geq 4$
$-16 \quad -16$
$x \geq -12$



$\boxed{-12 \leq x \leq 4}$ $[-12, 4]$

c) $g \circ g(x) = g(g(x))$

$2(2x+3) + 3$

$= 4x+6+3$

$g \circ g(x) = 4x+9$

Domain : $\mathbb{R}$

9. Compute the limits. Your answer can be ∞, $-\infty$, a finite number OR "DNE". Answer clearly. If you use a limit law, state it briefly. USE METHODS FROM OUR CLASS.

- (a) $\lim_{x\to 1} \frac{x^2-1}{x^2-4x+3}$.
- (b) $\lim_{x\to 2} \frac{x^2-4}{x+2}$
- (c) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

a.) By direct sub $\frac{0}{0}$ issue

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-3)} = \lim_{x\to 1} \frac{(x+1)}{(x-3)}$ by direct sub $\frac{2}{-2} = -1$

$\boxed{\lim_{x\to 1} \frac{x^2-1}{x^2-4x+3} = -1}$

c.) by direct sub $\frac{0}{0}$ issue!

$\lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} x+2$ by direct sub $= 4$

$\boxed{\lim_{x\to 2} \frac{x^2-4}{x-2} = 4}$

10. More limits, see directions above.

- (a) $\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3}$

- (b) $\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-3}$

11. More limits, see directions above.

- (a) $\lim_{x\to 1^+} \frac{x+2}{x-1}$
- (b) $\lim_{x\to 1^-} \frac{x+2}{x-1}$
- (c) $\lim_{x\to 1} \frac{x+2}{x-1}$.

by direct sub $\frac{0}{0}$ is an issue!

$$= \lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3} \frac{(\sqrt{x+6}+3)}{(\sqrt{x+6}+3)}$$

$$= \lim_{x\to 3} \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+6}+3)}$$

$$= \lim_{x\to 3} \frac{1}{\sqrt{x+6}+3}$$

by direct sub $= \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

$$\boxed{\lim_{x\to 3} \frac{\sqrt{x+6}-3}{x-3} = \frac{1}{6}}$$

12. What is

$$\lim_{x \to 3} f(x)$$

for the function in problem 2a, above?

13. Evaluate each limit. See directions at problem 9.

- (a) $\lim_{x \to 2} \dfrac{\pi x^2 + \sqrt{3}}{x^3}$
- (b) $\lim_{x \to -4} e^2 + \ln(5)x$ by direct sub $e^2 + (\ln(5))(-4)$ $e^2 - 4\ln(5)$
- (c) $\lim_{x \to 3} \sin(\frac{\pi}{7})\ x + \ln(4)$
- (d) $\lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ (In this problem, your answer will depend on x.)

careful w/ pareth.

$\cancel{\ln[5(-4)]}$

$[\ln(5)](-4)$

$$\boxed{b)\ \lim_{x \to -4} e^2 + \ln(5)x = e^2 - 4\ln(5)}$$

$\lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$ by direct sub I get $\frac{0}{0}$ issue!

$= \lim_{h \to 0} \dfrac{(x+h)(x+h) - x^2}{h} = \dfrac{\cancel{x^2} + 2xh + h^2 \cancel{-x^2}}{h}$

$\dfrac{2xh + h^2}{h} = \dfrac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+h$

$= \lim_{h \to 0} 2x + h = 2x$

$$\boxed{\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = 2x}$$

14. Evaluate each limit. See directions at problem 9.

- (a) $\lim_{x \to 0} \frac{|x|}{x}$
- (b) $\lim_{x \to 3} \frac{|x|}{x}$
- (c) $\lim_{x \to -2} \frac{|x|}{x}$.

by direct sub $\frac{|0|}{0} = \frac{0}{0}$ issue!

$\lim_{x \to 0^-} \frac{|x|}{x} = -1$ since $\frac{|x|}{x} = \frac{-x}{x}$ when x neg

$\lim_{x \to 0^+} \frac{|x|}{x} = \frac{x}{x} = 1$ when $x \in \mathbb{Z}^+$

left limit + right limit don't match!

$\boxed{\lim_{x \to 0} \frac{|x|}{x} \text{ DNE}}$

c.) $\lim_{x \to -2} \frac{|x|}{x}$ by direct sub $\frac{|-2|}{-2} = \frac{2}{-2} = -1$

$\boxed{\lim_{x \to -2} \frac{|x|}{x} = -1}$

15. Evaluate each limit. See directions at problem 9.

- (a) $\lim_{x \to 0} \frac{\sqrt{x^2+100}-10}{x^2}$
- (b) $\lim_{x \to 1} \frac{\sqrt{x^2+100}-10}{x^2}$
- (c) $\lim_{x \to \pi} \frac{x^2+\sqrt{3}}{4x+e}$
- (d) $\lim_{h \to 0} \frac{(4(x+h)+3)-(4x+3)}{h}$ Your answer will depend on x.
- (e) $\lim_{x \to 1} \sin(\pi/7)+\sqrt{3}x$

c.) $\lim_{x \to \pi} \frac{x^2+\sqrt{3}}{4x+e}$ by direct sub $\frac{\pi^2+\sqrt{3}}{4\pi+e}$

$$\boxed{\lim_{x \to \pi} \frac{x^2+\sqrt{3}}{4x+e} = \frac{\pi^2+\sqrt{3}}{4\pi+e}}$$

b) $\lim_{x \to 1} \frac{\sqrt{x^2+100}-10}{x^2}$ by direct sub $\frac{\sqrt{1+100}-10}{1^2} = \sqrt{101}-10$

$$\lim_{x \to 1} \frac{\sqrt{x^2+100}-10}{x^2} = \sqrt{101}-10$$

e.) $\lim_{x \to 1} \sin(\pi/7)+\sqrt{3}x$

By direct sub $\sin(\pi/7)+\sqrt{3}(1)$

$$\boxed{\lim_{x \to 1} \sin(\pi/7)+\sqrt{3}x = \sin(\pi/7)+\sqrt{3}}$$

a.) direct sub gets $\frac{0}{0}$

$$\frac{(\sqrt{x^2+100}-10)}{x^2} \cdot \frac{(\sqrt{x^2+100}+10)}{(\sqrt{x^2+100}+10)} = \frac{x^2+\cancel{100}-\cancel{100}}{x^2(\sqrt{x^2+100}+10)} = \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+100}+10)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x^2+100}+10} = \frac{1}{\sqrt{100}+10} = \frac{1}{20}$$

$$\boxed{\lim_{x \to 0} \frac{\sqrt{x^2+100}-10}{x^2} = \frac{1}{20}}$$

16. In this problem you are given:

$$\lim_{x\to1} f(x) = \sqrt{2} \text{ and } \lim_{x\to1} g(x) = 7.$$

Use limit laws from our class to compute the following limits. What laws are you using?

- (a) $\lim_{x\to1} 3f(x) - g(x)$
- (b) $\lim_{x\to1} \dfrac{f(x)}{g(x)}$
- (c) $\lim_{x\to1} \dfrac{f(x)}{3x^2} = \dfrac{\lim_{x\to1} f(x)}{\lim_{x\to1} 3x^2} = \dfrac{\sqrt{2}}{3}$
- (d) $\lim_{x\to1}(\pi + f(x))$

By direct sub

$$\boxed{\lim_{x\to1} \frac{f(x)}{3x^2} = \frac{\sqrt{2}}{3}}$$

$$\underbrace{\lim_{x\to1} \pi}_{\pi} + \underbrace{\lim_{x\to1} f(x)}_{\sqrt{2}}$$

$$\boxed{\lim_{x\to1} (\pi + f(x)) = \pi + \sqrt{2}}$$

a) $\lim_{x\to1} 3f(x) - g(x)$

$$\underbrace{\left(\lim_{x\to1} 3\right)}_{3}\underbrace{\left(\lim_{x\to1} f(x)\right)}_{\sqrt{2}} - \underbrace{\lim_{x\to1} g(x)}_{7}$$

$$\boxed{\lim_{x\to1} 3f(x) - g(x) = 3\sqrt{2} - 7}$$

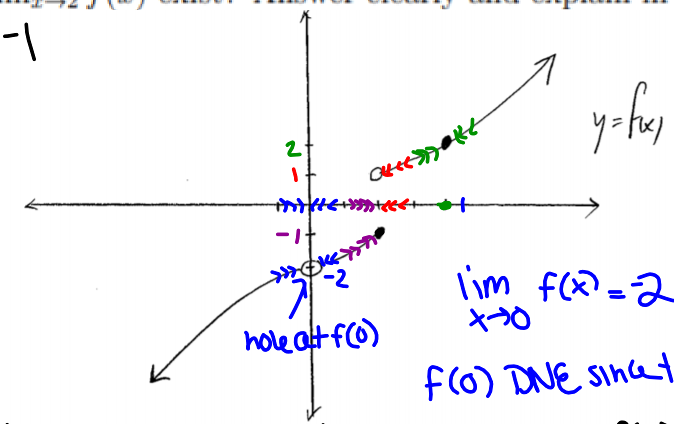
b) $\lim_{x\to1} \dfrac{f(x)}{g(x)}$

$$= \frac{\lim_{x\to1}(f(x))}{\lim_{x\to1} g(x)} = \frac{\sqrt{2}}{7}$$

$$\boxed{\lim_{x\to1} \frac{f(x)}{g(x)} = \frac{\sqrt{2}}{7}}$$

17. Use the graph below to answer the questions and give a 1 or 2-sentence explanation for your answer.

- (a) Does $\lim_{x\to 4} f(x)$ exist? Why or why not? If the limit exists, use the graph to find its value. yes $\lim_{x\to 4} f(x) = 2$ since $\lim_{x\to 4^+} f(x) = 2 = \lim_{x\to 4^-} f(x)$
- (b) Does $f(4)$ exist? yes $f(4) = 2$ there's no hole or V.A or discontinuity.
- (c) Find $\lim_{x\to 0} f(x)$ if possible. Answer as in (a).
- (d) Does $f(0)$ exist? Is it equal to the limit from part (c)?
- (e) Find $\lim_{x\to 2^+} f(x)$. 1 since the outputs of f(x) approach 1 as the inputs approach 2 from the right.
- (f) Find $\lim_{x\to 2^-} f(x)$. −1 " " " " − − − −1 " " " 2 from the left
- (g) Does $\lim_{x\to 2} f(x)$ exist? Answer clearly and explain in a sentence.
- (h) $f(2) = -1$



$\lim_{x\to 0} f(x) = 2$

f(0) DNE since there's a hole at f(0)

g) No $\lim_{x\to 2}$ DNE since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$