Math 425

Working with ∞, $-\infty$

Combinations of these are often BUT NOT ALWAYS indeterminate — meaning, Some alg is needed to help us find the limit (IF IT EXISTS)

No problem with

$$\infty + \infty$$

and $-\infty + -\infty$ (or $-\infty - \infty$).

"$\infty + \infty = \infty$" (Think: large positive + large positive = large positive)

"$-\infty + (-\infty) = -\infty$" (large negative + large negative = large negative)

"$-\infty - \infty = -\infty$" "same"

These ARE NOT indeterminate. →

Here are some examples with correct work shown:

(1) $\lim_{x \to -\infty} x^3 + x^5 =$ ______

$x \to -\infty$ given

<u>So</u> $x^3 \to -\infty$

and $x^5 \to -\infty$

Diagnosis: $(-\infty) + (-\infty)$

answer: $-\infty$

$$\boxed{\text{Answer: } \lim_{x \to -\infty} x^3 + x^5 = -\infty}$$

(2) $\lim_{x \to \infty} x^5 + x^4 =$ ______

$x \to \infty$ given

<u>So</u> $x^5 \to \infty$

and $x^4 \to \infty$

Diagnosis: $(+\infty) + (+\infty)$

Answer: $+\infty$

$$\boxed{\text{Answer: } \lim_{x \to \infty} x^3 + x^5 = \infty}$$

Here is another problem that we can answer quickly:

(3) $\lim_{x \to -\infty} x^5 - x^2 = \_\_\_\_$

$x \to -\infty$ given

so, $x^5 \to -\infty$

also $x^2 \to +\infty$

Diagnosis:

"$-\infty - \infty$"

$= (-\infty) + (-\infty)$

$= -\infty$ answer

Answer

$$\boxed{\lim_{x \to -\infty} x^5 - x^2 = -\infty.}$$

Now compare to the forms

$\left.\begin{array}{l} \infty - \infty \\ -\infty + \infty \end{array}\right]$ "risk" of cancellation, extra work needed!

(ex 1) $\lim_{x \to \infty} x^5 - x^2$

diagnosis: $\infty - \infty$!

Is it 0?

What is it?

Alg needed!

$$\lim_{x\to\infty} x^5 - x^2 = \lim_{x\to\infty} x^2(x^3-1).$$

Now: $x \to \infty$ given

$\therefore x^2 \to \infty.$

Also $x^3 \to \infty$

So $x^3-1 \to \infty.$

Now we have $(+\infty)\cdot(+\infty)$

$= +\infty$

Ans $\lim_{x\to\infty} x^5 - x^2 = \infty.$

Message here: $(+\infty)\cdot(+\infty)$ is OKAY, it is $+\infty$

Also: $(\infty)\cdot(-\infty)$ is OKAY, it is $-\infty$

calc1 team gets Answer $-\infty$

Pls, try this one:

ex2 $\lim_{x\to+\infty} x^2 - x^5$ (of Indeterminate form, $\infty - \infty$)

And this one:

ex3 $\lim_{x\to-\infty} x + x^4$

(of indet form $-\infty + \infty$)

Team gets Answer $+\infty$

Another Kind of indeterminate:

$\frac{\infty}{\infty}$, $\frac{-\infty}{\infty}$, $\frac{\infty}{-\infty}$, $\frac{-\infty}{-\infty}$] Call all of these "$\frac{\pm\infty}{\pm\infty}$"

These ALL require extra work to compute the limit.

(ex 4) $\lim_{x \to \infty} \frac{x^5 - x^7 + e}{\pi x^2 + 8}$ "$\frac{\pm\infty}{\pm\infty}$"

Method: divide by highest power of x in denom.

(ex 5) $\lim_{x \to -\infty} \frac{x^5 - x^7 + e}{\pi x^8 + 9}$

(ex 6) $\lim_{x \to \infty} \frac{2x^7 - x^6 + 5}{x - \sqrt{3}\, x^7}$

Calc Team Answers: ex 4, $-\infty$
ex 5, 0
ex 6, $-2/\sqrt{3}$

(See why we call these in determinate? Answer can be anything!)

We just handled ANY quotient of polynomials as $x \to \infty$ or $x \to -\infty$.

How about $\lim_{x \to -\infty} \dfrac{7x}{\sqrt{8x^2+3}}$ ← Not a polynomial!

NOTICE: $7x$ is negative bec. $x \to -\infty$

A sq. root is positive (or zero).

Our answer MUST be negative!

ALG! $\lim_{x \to -\infty} \dfrac{7x}{\sqrt{8x^2(1+\frac{3}{8x^2})}}$

Why do this step? To isolate x^2 term, the term that is ~~important~~.

$$= \lim_{x \to -\infty} \frac{7x}{\sqrt{8x^2}\sqrt{1+\frac{3}{8x^2}}}$$

Use here: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$$= \lim_{x \to -\infty} \frac{7x}{(-\sqrt{8}\,x)\sqrt{1+\frac{3}{8x^2}}}$$

Why that minus sign? Remember $x < 0$!

If you write $\sqrt{x^2} = x$ you're saying a sq. root is negative: No!

Now cancel:

$$= \lim_{x \to -\infty} \frac{7}{-\sqrt{8}} \frac{1}{\sqrt{1+\frac{3}{8}x^2}}$$

$$\boxed{= \frac{-7}{\sqrt{8}}}$$

$\sqrt{1+\frac{3}{8}x^2} \to \infty$

p6.