Math 425

Problems of the diagnosis $\frac{"c"}{0}$

where numerator $c \neq 0$ (So we can't factor-and-cancel OR rationalize!)

ex 1 $\quad \lim\limits_{x \to 3} \dfrac{-7}{x-3}$

diagnosis: $\dfrac{"-7"}{0} \longrightarrow$ Answer could be $-\infty, +\infty,$ or DNE!

Notice $\quad x$ near 3 means
$(x-3)$ could be small POSITIVE or small NEGATIVE, left and right limits needed!

Solution $\quad \lim\limits_{x \to 3^+} \dfrac{-7}{x-3} = \dfrac{?}{0}$

This is required.

$x \to 3 \quad$ so $\quad x-3 \to 0$
$x > 3 \quad$ so $\quad x-3 > 0$

so $\quad \dfrac{1}{x-3} \to +\infty$

and $\quad \dfrac{-7}{x-3} \to -\infty$

Conclude $\dfrac{-7}{x-3} < 0$

Answer
$\lim\limits_{x \to 3^+} \dfrac{-7}{x-3} = -\infty$

p1.

$$\lim_{x \to 3^-} \frac{-7}{x-3} = \underline{}$$

This is required,

$$x \to 3 \quad so \quad x-3 \to 0$$
$$x < 3 \quad so \quad x-3 < 0$$
$$so \quad \frac{1}{x-3} \to -\infty$$
$$and \quad \frac{-7}{x-3} \to (-7)(-\infty) = \infty$$

conclude $\frac{-7}{x-3} > 0$

so our answer IS POSITIVE

Answer

$$\lim_{x \to 3^-} \frac{-7}{x-3} = +\infty$$

Since $\lim_{x \to 3^+} \frac{-7}{x-3} \neq \lim_{x \to 3^-} \frac{-7}{x-3}$

Conclude $\lim_{x \to 3} \frac{-7}{x-3}$ DNE.

ex2 $\lim_{x \to 4} \frac{-5}{x^2 - 8x + 16}$

Can we direct-sub? Put $x = 4$
into $x^2 - 8x + 16$, get 0, NOPE
CanNOT
dir sub!

p2

ex 2, Continued.

Look at the denom, a polynomial
$$p(x) = x^2 - 8x + 16$$

We just saw: If you sub $x = 4$,
YOU GET 0

This means $x - 4$ is a factor
[and this will help. A LOT!]

Factor: $x^2 - 8x + 16 = (x-4)^2$.

So our problem is
$$\lim_{x \to 4} \frac{-5}{(x-4)^2} = \underline{\quad}$$

Required
$$\begin{array}{l} x \to 4 \\ x - 4 \to 0 \ (\overset{pos \ or}{neg}) \\ (x-4)^2 \to 0, \ (x-4)^2 > 0 \end{array}$$

$$\frac{1}{(x-4)^2} \to +\infty$$

$$\frac{-5}{(x-4)^2} \to -\infty$$

So $\dfrac{-5}{(x-4)^2} < 0$

Our answer
will be NEGATIVE

Ans:
$$\lim_{x \to 4} \frac{-5}{(x-4)^2} = -\infty.$$

Some for you to try:

ex1

$$\lim_{X \to 7} \frac{X+2}{X^2 - 6X - 7} \underset{\substack{\text{helpful} \\ \text{step}}}{=} \lim_{X \to 7} \frac{X+2}{(X-7)(X+1)}$$

$X+2$ on top; $X+2 \to 9$ N.P.

$X+1$ in denom: $X+1 \to 8$. N.P.

$X-7$ in denom $\to 0$, might be NEGATIVE
might be POSITIVE

left and right hand limits NECESSARY

Calc 1 team says: this limit dne!

ex2

$$\lim_{X \to 7} \frac{X-9}{X^2 - 14X + 49}$$

Calc 1 team
gets $-\infty$
as answer.
Please check!

ex3

$$\lim_{X \to -4} \frac{X-4}{16 - X^2} \underset{ALG}{=} \lim_{X \to -4} \frac{X-4}{(4-X)(4+X)}$$

$$\underset{cancel}{=} \lim_{X \to -4} \frac{-1}{4+X}$$

Calc team gets
DNE for answer

Please check!

p4