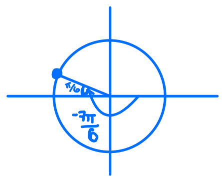
Homework 2 Solutions

① Evaluate the trig value below:

(a) $\sin\left(\frac{-7\pi}{6}\right)$

Solution



First, notice $\frac{-7\pi}{6}$ occurs in the 2^{nd} Quadrant,

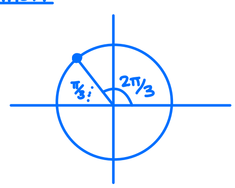
So, $\sin\left(\frac{-7\pi}{6}\right)$ will be a positive number.

We also know $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$

So, it follows that $\boxed{\sin\left(\frac{-7\pi}{6}\right)=\frac{1}{2}}$

(b) $\tan\left(\frac{2\pi}{3}\right)$

Solution



Recall, $\tan\left(\frac{2\pi}{3}\right)=\dfrac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}$

Since $\frac{2\pi}{3}$ is in the 2^{nd} Quadrant,
- $\sin\left(\frac{2\pi}{3}\right)$ will be positive
- $\cos\left(\frac{2\pi}{3}\right)$ will be negative

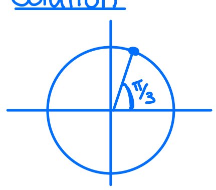
So, since $\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ and $\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$

We have $\sin\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$ and $\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$.

Thus, $\boxed{\tan\left(\frac{2\pi}{3}\right)=\dfrac{\sqrt{3}/2}{-1/2}=-\sqrt{3}}$

(c) $\cos\left(\frac{13\pi}{3}\right)$

Solution



We know $\cos(x)$ has a period of 2π, i.e. $\cos(x)=\cos(x+2\pi)$.

Notice, $2\pi=\frac{6\pi}{3}$.

So, $\cos\left(\frac{13\pi}{3}\right)=\cos\left(\frac{7\pi}{3}+\frac{6\pi}{3}\right)=\cos\left(\frac{7\pi}{3}\right)$.

Also, notice

$$\cos\left(\frac{7\pi}{3}\right)=\cos\left(\frac{\pi}{3}+\frac{6\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right).$$

So, $\cos\left(\frac{13\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$.

So, $\boxed{\cos\left(\frac{13\pi}{3}\right)=\frac{1}{2}}$.

② You are given $\pi < \theta < \frac{3\pi}{2}$ AND $\sin(\theta) = \frac{-12}{13}$. Find the value of $\cos(\theta)$.

Solution

Recall $1 = \sin^2(\theta) + \cos^2(\theta)$.

So, $1 = \left[\frac{-12}{13}\right]^2 + \cos^2(\theta)$

$$1 = \frac{144}{169} + \cos^2(\theta)$$

$$1 - \frac{144}{169} = \cos^2(\theta)$$

$$\frac{25}{169} = \cos^2(\theta)$$

$$\pm\frac{\sqrt{25}}{\sqrt{169}} = \cos(\theta)$$

$$\pm\frac{5}{13} = \cos(\theta)$$

Notice, $\pi < \theta < \frac{3\pi}{2}$, so θ is in the 3^{rd} Quadrant. Hence $\cos(\theta)$ is negative.

Thus, $\boxed{\cos(\theta) = \frac{-5}{13}}$.

③ For each function below, find the domain.

- $f(x) = \sqrt[4]{5-2x} \cdot \sqrt{3-x}$

Solution

We cannot have a negative number under an even root, so we need

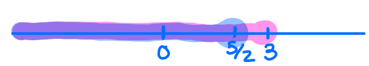
$5 - 2x \geq 0$ and $3 - x \geq 0$.

$5 \geq 2x$ $\qquad$ $3 \geq x$

$5/2 \geq x$

So, we need $x \leq 5/2$ and $x \leq 3$

To satisfy both conditions, we need $x \leq 5/2$.



$\boxed{\text{So, the domain of } f \text{ is } x \leq 5/2 \text{, OR } (-\infty, 5/2].}$

- $f(x) = \dfrac{x^2 + \pi}{\sin(x)}$

Solution

The numerator is a polynomial, so x can be any real number.

We need to make sure the denominator doesn't equal 0.

So, we need $\sin(x) \neq 0$.

We know $\sin(x) = 0$ when $x = 0$ OR $x = \pi$ OR $x = 2\pi$.

But sine is a periodic function, so $0 = \sin(\pi) = \sin(3\pi) = \sin(5\pi) = \ldots$

and $0 = \sin(0) = \sin(2\pi) = \sin(4\pi) = \ldots$

So, $\sin(x) = 0$ when x is any integer multiple of π, i.e. when $x = n\pi$ for n an integer.

$\boxed{\text{Therefore, the domain of } f \text{ is } x \neq n\pi \text{ where } n \text{ is an integer OR all real numbers except integer multiples of } \pi.}$

- $f(x) = \dfrac{\sqrt{3-x}}{x^2-1}$

Solution

In the numerator, we cannot have a negative number under the even root.

So, we need $3-x \geq 0$

$3 \geq x.$

So, we need $\underline{x \leq 3.}$

Also, we cannot have the denominator equal 0.

So, $x^2-1 \neq 0$

$x^2 \neq 1$

$x \neq \pm\sqrt{1}$

$x \neq \pm 1.$

So, we need $\underline{x \neq \pm 1.}$

Putting these together we get the domain of f is $x \leq 3$ and $x \neq \pm 1$

OR $(-\infty,-1)\cup(-1,1)\cup(1,3]$.

- $f(x) = \dfrac{\sqrt{3}}{2\cos(x)-1}$

Solution

We need to make sure we do not have a 0 in the denominator.

So, we need to find when $2\cos(x)-1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}.$

We know $\cos(x) = \frac{1}{2}$ when $x = \pi/3$ and $x = \frac{5\pi}{3}$.

Since cosine is a periodic function with period 2π, we get

$\frac{1}{2} = \cos(\pi/3) = \cos\left(\frac{\pi}{3}+2\pi\right) = \cos\left(\frac{7\pi}{3}\right) = \cos\left(\frac{7\pi}{3}+2\pi\right) = \cos\left(\frac{13\pi}{3}\right) = .$

$\frac{1}{2} = \cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{5\pi}{3}+2\pi\right) = \cos\left(\frac{11\pi}{3}\right) = \cos\left(\frac{11\pi}{3}+2\pi\right) = \cos\left(\frac{17\pi}{3}\right) = \cdots$

So, the domain is all real numbers except $x = \pi/3 + 2\pi n$, $x = 5\pi/3 + 2\pi n$ where n is an integer

OR Domain is all real numbers except $x = \ldots, \pi/3, 5\pi/3, \ldots$