Math 425, 10/27

Good morning ALL!

Could everyone please MUTE?

Thanks.

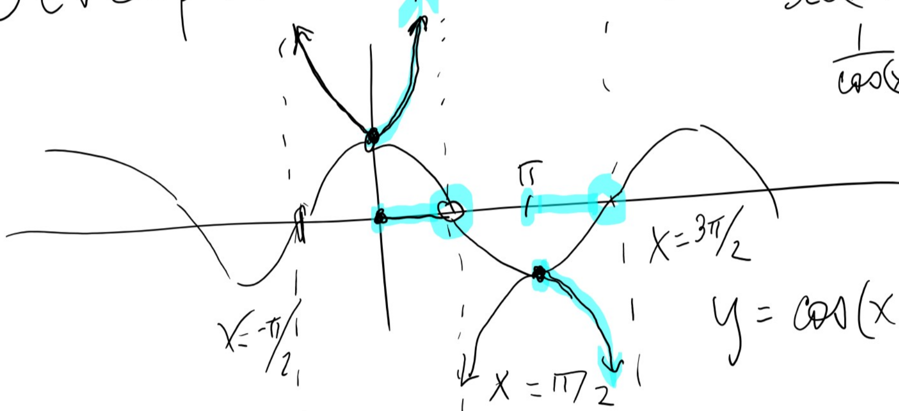
I will watch the CHAT for your questions.

Quiz tomorrow on material from last week: implicit dif and $\sin^{-1}(x)$

Mon: $\tan^{-1}(x)$ Today: $\sec^{-1}(x)$

Development of $\sec^{-1}(x)$

$\sec(x) = \dfrac{1}{\cos(x)}$



Notice: Output of $y = \sec(x)$ is $y \geq 1$, $y \leq -1$

We restrict x, our input, to $0 \leq x < \frac{\pi}{2}$, $\pi \leq x < \frac{3\pi}{2}$

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defn $y = \sec^{-1}(x)$ means

input of $\sec^{-1}$ is $x \ge 1$, $x \le -1$

"y is the angle whose secant value is x"

$\sec(y) = x$

If $x \ge 1$ choose $0 \le y < \pi/2$

If $x \le -1$ choose $\pi \le y < \frac{3\pi}{2}$

evaluations

1) $\sec^{-1}(-1) = \underline{\pi}$

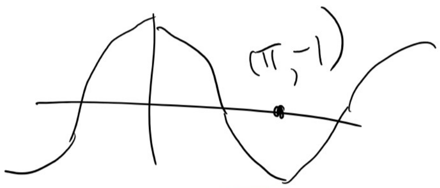
Solution $y = \sec^{-1}(-1)$

$\sec(y) = 1$

$\frac{1}{\cos(y)} = -1$

$-1 = \cos(y)$

$\boxed{y = \pi}$



$\cos(\pi) = -1$

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ex2 $\sec^{-1}(2/\sqrt{3}) = \underline{\pi/6}$

$y = \sec^{-1}(2/\sqrt{3})$

$$\sec(y) = 2/\sqrt{3}$$

$$\frac{1}{\cos(y)} = 2/\sqrt{3}$$

$$\sqrt{3}/2 = \cos(y)$$

$$y = \pi/6$$

$$\cos \pi/6 = \sqrt{3}/2$$

The inverse relationship:

$$\sec(\underbrace{\sec^{-1}(x)}) = x.$$

<u>Goal</u> $\frac{d}{dx}[\sec^{-1}(x)]$.

Goal: Let, $y = \sec^{-1}(x)$,

$\longrightarrow$ Find $y' = \frac{d}{dx}[y]$. $\longleftarrow$

$$y = \sec^{-1}(x)$$

$$\sec(y) = \sec(\sec^{-1}(x))$$

$$\longrightarrow \boxed{\sec(y) = x} \longleftarrow$$

$$\longrightarrow \frac{d}{dx}[\sec(y)] = \frac{d}{dx}[x] \longleftarrow$$

<u>reminder</u> $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

$$\frac{d}{dx}[\sec(g(x))] =$$

$$\frac{d}{dx}\left[\sec(g(x))\right] = \sec(g(x))\tan(g(x))\,g'(x)$$

"$y = g(x)$"

$$\frac{d}{dx}\left[\sec(y)\right] = \frac{d}{dx}[x]$$

$$\sec(y)\tan(y)\,y' = 1$$

$$\boxed{y' = \frac{1}{\sec(y)\tan(y)}}$$

Convert back to x

$$y' = \frac{1}{x\sqrt{x^2-1}}$$

Fact $\frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{x\sqrt{x^2-1}}$,

Trig Inverse Functions

Function	Input, x	Output, y	Deriv
$y=\sin^{-1}(x)$	$-1 \le x \le 1$	$-\pi/2 \le y \le \pi/2$	$\frac{1}{\sqrt{1-x^2}}$, $-1<x<1$
$y=\tan^{-1}(x)$	$-\infty < x < \infty$	$\frac{-\pi}{2} < y < \pi/2$	$\frac{1}{1+x^2}$
$y=\sec^{-1}(x)$	$x \ge 1 \dashrightarrow 0 \le y < \pi/2$ $x \le -1 \dashrightarrow \pi \le y < \frac{3\pi}{2}$		$\frac{1}{x\sqrt{x^2-1}}$

Back to $\ln(x)$

Back to $\ln(x)$

New on Monday

What is $\tan(y)$?

$$\tan^2(y) + 1 = \sec^2(y)$$

$$\tan^2(y) = \sec^2(y) - 1$$

$$\tan^2(y) = x^2 - 1$$

$$\tan(y) = \sqrt{x^2 - 1}$$

Fact $\dfrac{d}{dx}\left[\ln(x)\right] = \dfrac{1}{x} \quad (x > 0)$

— applications —

$$\frac{d}{dx}\left[\ln(g(x))\right] = \frac{g'(x)}{g(x)}$$

New (Last) method for computing derivatives.

Logarithmic differentiation

Idea: ① Apply ln to simplify the problem.

(2) Then compute derivative.

We will use

(3) $\frac{d}{dx}[\ln(y)] = \frac{1}{y} y'$

Think of y as $g(x)$ $\leftarrow \frac{1}{g(x)} g'(x)$

ex. Know $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ $\ln(x) = \log_e(x)$

What is $\frac{d}{dx}[\log_a(x)]$? base is a

$a \neq 1$?
$a > 0$?

$y = \log_a(x)$

$\rightarrow$ y is the power needed on base a to get x $\leftarrow$

$a^y = x$

$\boxed{\ln(a^y)} = \ln(x)$ $\ln(a^b) = b\ln(a)$

$y \ln(a) = \ln(x)$ So $y = \frac{\ln(x)}{\ln(a)}$

$\log_a(x) = y = \frac{\ln(x)}{\ln(a)}$

Conversion $\log_a(x) = \frac{\ln(x)}{\ln(a)}$

$\ln(a)$

$$\frac{d}{dx}\left[\log_a(x)\right] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(a)}\right]$$

$$= \frac{1}{\ln(a)}\frac{d}{dx}\left[\ln x\right]$$

$$= \frac{1}{\ln(a)}\frac{1}{x}$$

$$\frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{x}\frac{1}{\ln(a)}$$

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ex.2) Know $\frac{d}{dx}\left[e^x\right] = e^x$

Find $\frac{d}{dx}\left[a^x\right] = ?$

a is our base
$a \neq 1$
$a > 0$

$$\boxed{y = a^x}$$

$$\ln(y) = \ln\left[a^x\right]$$

$$\ln(y) = x\ln(a)$$

$\ln(a)$ is "just a number"

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[x\ln(a)\right]$$

$$\frac{d}{dx}\left[3x\right] = 3$$

$$\frac{1}{y}y' = \ln(a)$$

$$\frac{d}{dx}\left[\ln(a)\cdot x\right]$$

$\frac{1}{y} y' = \ln(a)$

$y' = y \cdot \ln(a)$

$= a^x \ln(a)$

$$\frac{d}{dx}[a^x] = a^x \ln(a)$$

$\frac{d}{dx}[\ln(a) \cdot x]$

$= \ln(a)$

$\frac{d}{dx}[5x] = 5$

$\frac{d}{dx}[\ln(a) \cdot x] =$

$\ln(a)$

ex 3 $\quad \frac{d}{dx}\left[x^{3x}\right] =$ ______

Notice: This is NOT exponential...

base x is a variable!

This is NOT a power function...

power x is a variable.

Goal $\frac{d}{dx}\left[x^{3x}\right]$.

$y = x^{3x}$

$\ln(y) = \ln\left(x^{3x}\right)$ apply ln to simplify

$\ln(y) = 3x \ln(x)$ variable power $3x$ moved down

$\frac{d}{dx}[\ln y] = \frac{d}{dx}[3x \cdot \ln(x)]$ ← product rule needed on

$\frac{1}{y} y' = 3x \frac{d}{dx}[\ln x] + \ln(x) \frac{d}{dx}[3x]$

$$\frac{1}{y} y' = 3x \frac{d}{dx}[\ln x] + \ln(x) \frac{d}{dx}[3x]$$ needed on right

$$\frac{1}{y} y' = 3x \cdot \frac{1}{x} + 3 \ln(x)$$

Our goal: $y' = ?$

multiply by y:

$$y' = y\left(3x \cdot \frac{1}{x} + 3 \ln x\right)$$

$$= y\left(3 + 3\ln(x)\right)$$

$$= 3^x\left(3 + 3\ln(x)\right)$$