Autumn Ava Barnes
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What is PROC, and how to roll one and a half Dice

Abstract - What is this about?

In this guide/paper/resource I will be explaining what **PROC** is in game-design, how to roll one and a half dice, and how this is used in other games. **PROC** is found in anything in which there is a random chance for something to happen.

Who is this for?

I am mainly making this for hobby game-developers, and as such I'll make sure to give plenty of examples, and break down the math behind it in a *hopefully* easy to understand way. I want this to be for anyone no matter your expereince, because sometimes looking at big math can be overwhelming. This kind of stuff can be really fun, and I wanted to share my enjoyment of the subject with you, and maybe I'll even get you to laugh.

Why is this in a ***professional*** paper format?

I typically hate looking up something game design related, and then stumbling upon the god-awful confusing mess that is technical lingo where I spend more time deciphering it than I do learning from it. I wanted to give it a shot on my own and see if I can make something that is educational, easy to understand, as well as fun. This is also an excuse to get better in typesetting for my school work. Enjoy!

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yeah it is pretty short

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Rolling Dice

Lets say you have a random chance for an event to happen. Maybe a coin flip to see who starts first, or a dice roll to see if your knight lands their attack on the goblin perhaps.

The important part is that something happens as a result of a random number. In the wonderful world of computers, we roll virtual dice. We can generate random numbers from 1 to 6 (*like a six-sided dice*), or since it is usually easier to work with, we can generate a random number from 0 to 1.

Now why is random numbers from 0 to 1 easier to work with? It seems kinda counter-intuitive right? Well lets say that we want to have a %50 percent chance for something to happend, well with dice we would check if our dice roll is 4 or more. If it helps, you can think about it like this:

$$\text{if } (\text{d6} \geq 4) \text{ then}$$

Now this is kinda lame, the probability isn't easily recognizable just from looking at it. Also, if we are only generating integers (*meaning whole numbers, so no 7.5 or 3.8561*), then we lose some configurability because checking if the dice roll is, lets say, above 5.5, isn't really useful.

On the other hand, random numbers ranging from **any** number inbetween 0 and 1 make so much more sense. Here is what a %50 chance would look like:

$$\text{if } r < 0.5 \text{ then}$$

You can see that all we do is check to see if our random number r is below 0.5, which is the same thing as %50. Here is what %75 chance would look like:

$$\text{if } r < 0.75 \text{ then}$$

From here on out, when I say rolling dice, I am usually refering to a computer generated random number from 0 to 1.

Whats PROC?

The act of having a random chance of an event happening has a fun name too!

Procedural **R**andom **OC**currence
also known as **PROC**.

and just for the hell of it, we can also call succeding the dice roll and triggering the event **PROCCING**.

Now lets define a function for us to use, lets call it DoesPROC. It will take in a *probability* (*This is our 0.5 or %75*), do our dice roll, and will return **true** if the value is less than out *probability*.

This *probability* is also called the **coefficient**, though both terms are interchangeable in this case.

Lets implement it; my personal programming language of choice is **Lua**, Here is what that would look like :

```lua
function DoesPROC(coefficient)
  return math.random() < coefficient
end

-- Used like this:
if DoesProc(0.75) then
  --Do Event
end
```

Yeah, not too complex, and I bet you are bored at this point. But lets see something cool you can do with this.

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Lets Roll Two Dice!

Yeah I know it still sounds pretty boring.

What all of this is leading up to rolling multiple dice. More specifically, lets say you get to roll 2 dice, and if any of them meet the coefficient, then it returns true.

If that doesn't make sense, you can also view this as rolling multiple dice, and taking the best one. This method is used in a few games, but I discovered it from Risk of Rain

With this idea, we can up the players probability of succeeding, without having to increase the coefficient. You can see this as giving the player multiple chances, or as Risk of Rain uses it, controlling how lucky the player is.

Lets put this into code so you can see how it works :

```
function DoesPROC(coefficient, dice)
    local Rolls = {}
    for i=1, dice do
        Rolls[i] = math.random()
    end

    for i=1, #Rolls do
        if Rolls[i] < coefficient then
            return true
        end
    end

    return false
end
```

This code is unoptimized, there are better ways to do this that don't include keeping a table of the dice frolls, I just wanted to demonstrate it in a way that would be similar to how to would do it with real dice

Because of the fun world of probability, with a coefficient of 0.5 and rolling a single dice, we get a probability of %50, as expected. But if we roll 2 dice, then our chance of triggering our event jumps up to 0.75 percent! And again, just to clarify, all we are doing is rolling extra dice and seeing if any of them meet the requirement.

Cool right, maybe, I don't know about you but I'm stoked. Anyway, this method is kinda messy, and it still has a few limitations.

The Math AHHHH

That's right, unfortunately, there is math involved. Rather than doing this convoluted nonsense, we can turn it into a single equation.

Now why would we want an equation when the previous method works just fine? Well first of all, what if I wanted to roll **one and a half dice**? Like we can't just do that, that's just not possible with the previous method. Also, what the hell would you even do if you gave a negative number of dice to roll?! *You toss a negative dice onto the table and it just falls through?*

This is another thing that I our old method doesn't support

The second reason for this is that for a developer, it might be a bit harder to visual with the old method. If we had an equation, we could graph a nice curve that shows how an increase in dice rolls can effect the probability of Proccing.

Time for math, the equation is pretty small :

$$f(p, d) = 1 - (1 - p)^d$$

Where:

- p is our coefficient
- d is how many dice

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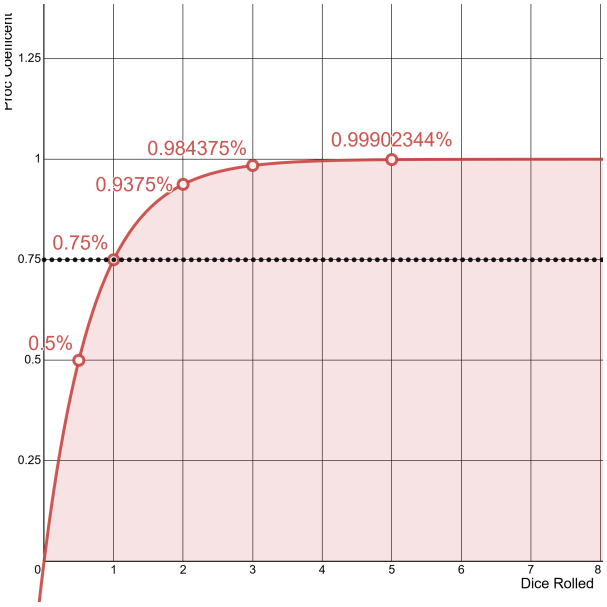
Visualization

Before we dive into the math, Lets just see what that would look like as a graph, and mess with it's values first:



Desmos Graph Link

You can move the `p` slider to see what different starting coefficient is. The x axis is how many dice are being rolled. Notice how the probability (*The curve*) never goes above 1.

You can also now know what happens when rolling negative dice! Turns out it will always Proc, a little lame - but that does give an idea, what if you had a negative coefficient? I'll let you have fun with that one though.

Understanding - How did we get here?

In this section, I'll explain how we got this function from our problem. If you find yourself getting lost because of the math, in the last section I break the the math step by step - you are welcome to go back and foruth from either section.

Lets redefine the problem as simply as possible :

We need a function that represents the **probability** of rolling **atleast 1 dice** that is under a given **coefficient**.

Cool, this is our goal that we are aiming for, but for now lets keep that in the back of our heads.

Lets say you flip a coin, there are only two posibilities :

1 Fail	1 Success

To get the probability of Success, it is as simple as

$$\frac{\#\text{ of Successful Combinations}}{\text{Total }\#\text{ of Combinations}}$$

In this sernario it is just $\frac{1}{2}$, meaning a 50% probability of success.

New problem, what is multiplying probabilities? Lets say you have two coin flips. Here is a table that represents every combination that can occur:

1 2	1 2
1 2	1 2

There are 4 possible outcomes. Lets say that both coins have to be black for success. This means that there is a $\frac{1}{4}$ (25%) probability of rolling that combination.

If you notice, each coin flip is a has a 50% chance, aka $\frac{1}{2}$. Lets see what happens when we multiply the probabilities of each coin flip together

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Crazy right? How that just perfectly lines up with the probability that we saw before.

To better explain the next part, I am going to switch from 2 coins to 2 six-sided dice. This si what our combination table looks like :

$1,1$	$1,2$	$1,3$	$1,4$	$1,5$	$1,6$
$2,1$	$2,2$	$2,3$	$2,4$	$2,5$	$2,6$
$3,1$	$3,2$	$3,3$	$3,4$	$3,5$	$3,6$
$4,1$	$4,2$	$4,3$	$4,4$	$4,5$	$4,6$
$5,1$	$5,2$	$5,3$	$5,4$	$5,5$	$5,6$
$6,1$	$6,2$	$6,3$	$6,4$	$6,5$	$6,6$

In this sernario, what if we wanted to get the probability of **EITHER** of the dice being 1? Well there a pretty simple two step method.

The chance for one dice succeeding is: $\frac{1}{6}$. First step is to get the chances of **NOT** rolling a 1, which believe it or not is acutally simpler, and it is as simple as flipping the probability :

$$1 - \left(\frac{1}{6}\right) = \left(\frac{5}{6}\right)$$

Now that we have the chances of not rolling a 1 on a single dice, we can apply the same method as before!

$(1-p)^d$	Formula for % of both not being 1
$\left(1 - \frac{1}{6}\right)^2$	Subsitute our p and d
$\left(\frac{5}{6}\right)^2$	Subtract by 1 to flip probability
$\frac{25}{36}$	Raise to the power of 2

Congrats, but we still have one last step. This number represents the chances of both dice not rolling a 1. *Guess what our last step is.*

FLIP IT!

$$1 - \left(\frac{25}{36}\right) = \left(\frac{11}{36}\right) \approx 0.3055556\%$$

Yeah that's it. This number now represents the probability of **ANY** of our dice rolling a 1, and to check to make sure that we got this correct:

We had 36 combinations, if we count up the combinations that are 1, then we get 11.

$$\frac{\#\text{ of Successful Combinations}}{\text{Total }\#\text{ of Combinations}} = \frac{11}{36}$$

When we put all of these steps together, here is our function:

$$f(p,d) = 1 - (1-p)^d$$

Where:
- p is our coefficient
- d is how many dice

Implementing - The code

```lua
function DoesPROC(coefficient, dice)
   local new_coefficient = 1 - (1 - coefficient) ^ dice
   return math.random() < new_coefficient
end
```

Not much too it code-wise, quite-boring. Now it is fine to just copy the function, plug it in and use it - but I feel like understanding why this function represents our dice problem is just as important.

If you are interested on how you get/derive this function from our problem, keep reading, I'll break it down bit by bit and build up the function from the start.

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Math Breakdown for those that need it

This section is optional, it explains how to solve the formula

Let's break it down bit by bit. $f(p, d)$ is a function, it takes the input of p and d. p is our coefficient and d is how many dice we are rolling.

This function is going to return the new probability that represents the chances **any** of **d** dice with a probability of **p** of succeeding

Lets start with the first step in the equation. This would be $(1 - p)$.

$(1 - p)$ will basically *flip the coefficient*. So if we had a value of 0.25, it would get flipped to 0.75. Here are some examples:

$$\begin{aligned} 1 - 0.25 &= 0.75 \\ 1 - 0.75 &= 0.25 \\ 1 - 1.0 &= 0 \\ 1 - 0.4 &= 0.6 \\ 1 - 0.5 &= 0.5 \end{aligned}$$

That's the first step done. Next would be the scary part, the exponent. What we are doing is taking our first step and raising it to the power of d (*how many dice we are rolling*).

Remember that raising a number to a power just means that we are multipling it by itself, so for three dice we are multiplying our first step by itself three times. So for example if we had 3^4 it would look like:

$$3 \cdot 3 \cdot 3 \cdot 3 = 81$$

Lets walk through it step by step if we had a coefficent of 0.25, and rolled two dice.

$(1 - p)^d$	Starting Formula
$(1 - 0.25)^2$	Subsitute our p and d values
$(0.75)^2$	Flip the coefficent
0.5625	Raise to the power of 2

That's our second step done! Just one last one, and we already have done it once. All we do is do the subtract it from 1 again.

So in the above equation all we would do is

$$1 - 0.5625 = 0.4375$$

Congrats! The math is over and our weary brain muscles can rest. What we are left with is a probability representing the probability of any of the dice succeeding.

As one last example :

$$\begin{aligned} f(p, d) &= 1 - (1 - p)^d \\ \\ f(0.1, 5) &= 1 - (1 - 0.1)^5 \\ f(0.1, 5) &= 1 - (0.9)^5 \\ f(0.1, 5) &= 1 - (0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9) \\ f(0.1, 5) &= 1 - 0.59049 \\ f(0.1, 5) &= 0.40951 \end{aligned}$$

There is a probability of 0.40951 or 40.951% chance that any of the 5 random numbers are below or equal to 0.1