Homework 5 Solutions

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We continue working with the World Top Incomes Database [http://topincomes.g-mond.parisschoolofeconomics.eu], and the Pareto distribution, as in the lab. We also continue to practice working with data frames, manipulating data from one format to another, and writing functions to automate repetitive tasks. We saw in the lab that if the upper tail of the income distribution followed a perfect Pareto distribution, then

$$\left(\frac{P99}{P99.9}\right)^{-a+1} = 10$$

$$\left(\frac{P99.5}{P99.9}\right)^{-a+1} = 5$$

$$\left(\frac{P99}{P99.5}\right)^{-a+1} = 2$$

We could estimate the Pareto exponent by solving any one of these equations for a; in lab we used

$$a = 1 - \frac{\log 10}{\log\left(\frac{P99}{P99.9}\right)}$$

Because of measurement error and sampling noise, we can't find find one value of a which will work for all three equations. Generally, trying to make all three equations come close to balancing gives a better estimate of a than just solving one of them. (This is analogous to finding the slope and intercept of a regression line by trying to come close to all the points in a scatterplot, and not just running a line through two of them.)

```
wtid <- read.csv("wtid-report.csv", as.is = TRUE)
wtid <- wtid[, c("Year", "P99.income.threshold", "P99.5.income.threshold", "P99.9.income.threshold")]
names(wtid) <- c("Year", "P99", "P99.5", "P99.9")
```

i. We estimate a by minimizing

$$\left(\left(\frac{P99}{P99.9}\right)^{-a+1} - 10\right)^2 + \left(\left(\frac{P99.5}{P99.9}\right)^{-a+1} - 5\right)^2 + \left(\left(\frac{P99}{P99.5}\right)^{-a+1} - 2\right)^2.$$

Write a function, `percentile_ratio_discrepancies`, which takes as inputs `P99`, `P99.5`, `P99.9` and a, and returns the value of the expression above. Check that when `P99=1e6`, `P99.5=2e6`, `P99.9=1e7` and $a = 2$, your function returns 0.

```
percentile_ratio_discrepancies <- function(a, p99, p995, p999) {
  return(((p99/p999)^(-a+1) - 10)^2 + ((p995/p999)^(-a+1) - 5)^2 + ((p99/p995)^(-a+1) - 2)^2)
}
percentile_ratio_discrepancies(2, 1e6, 2e6, 1e7)
```

```
## [1] 0
```

ii. Write a function, `exponent.multi_ratios_est`, which takes as inputs `P99`, `P99.5`, `P99.9`, and estimates a. It should minimize your `percentile_ratio_discrepancies` function. The starting value for the minimization should come from (). There are many ways to do the minimization, one is using `nlm()` as in HW4. Check that when `P99=1e6`, `P99.5=2e6`, and `P99.9=1e7`, your function returns an a of 2.

```
exponent.multi_ratios_est <- function(p99, p995, p999) {
  a0  <- 1 - (log(10))/(log(p99/p999))
  min <- nlm(percentile_ratio_discrepancies, a0, p99, p995, p999)$estimate
  return(min)
}
exponent.multi_ratios_est(1e6, 2e6, 1e7)
```

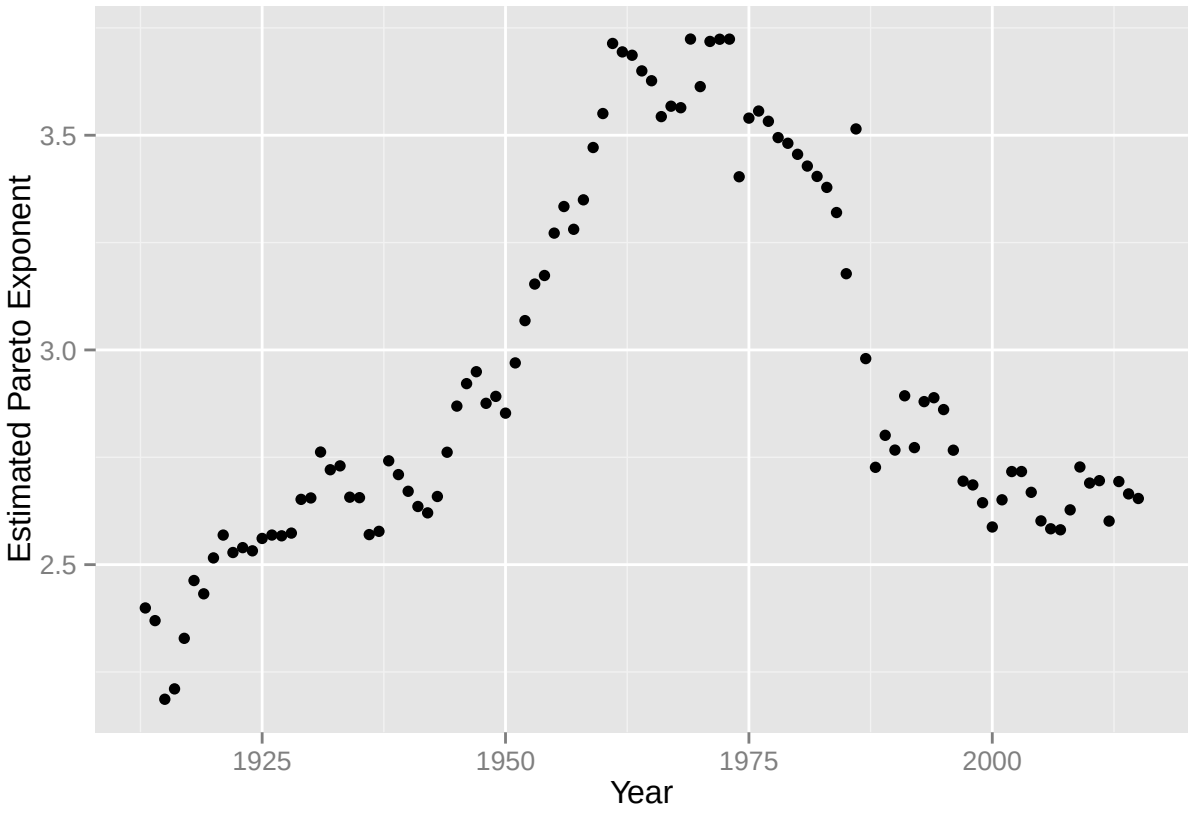
```
## [1] 2
```

iii. Write a function which uses `exponent.multi_ratios_est` to estimate a for the US for every year from 1913 to 2012. (There are many ways you could do this, including loops.) Plot the estimates using `ggplot`; make sure the labels of the plot are appropriate.

```
multi_ratios_allyears <- function(country_data) {

  # country_data should be a dataframe with the variables "Year", "P99", "P99.5", and "P99.9" where eac

  n <- nrow(country_data)
  estimates <- rep(NA, n)
  names(estimates) <- country_data$Year
  for (i in 1:n) {
    if (all(is.na(country_data[i, c("P99", "P99.5", "P99.9")]))) {
      estimates[i] <- NA
    } else {
      estimates[i] <- exponent.multi_ratios_est(country_data$P99[i], country_data$P99.5[i], country_data
    }
  }
  return(estimates)
}
USestimates <- multi_ratios_allyears(wtid)

library(ggplot2)
```

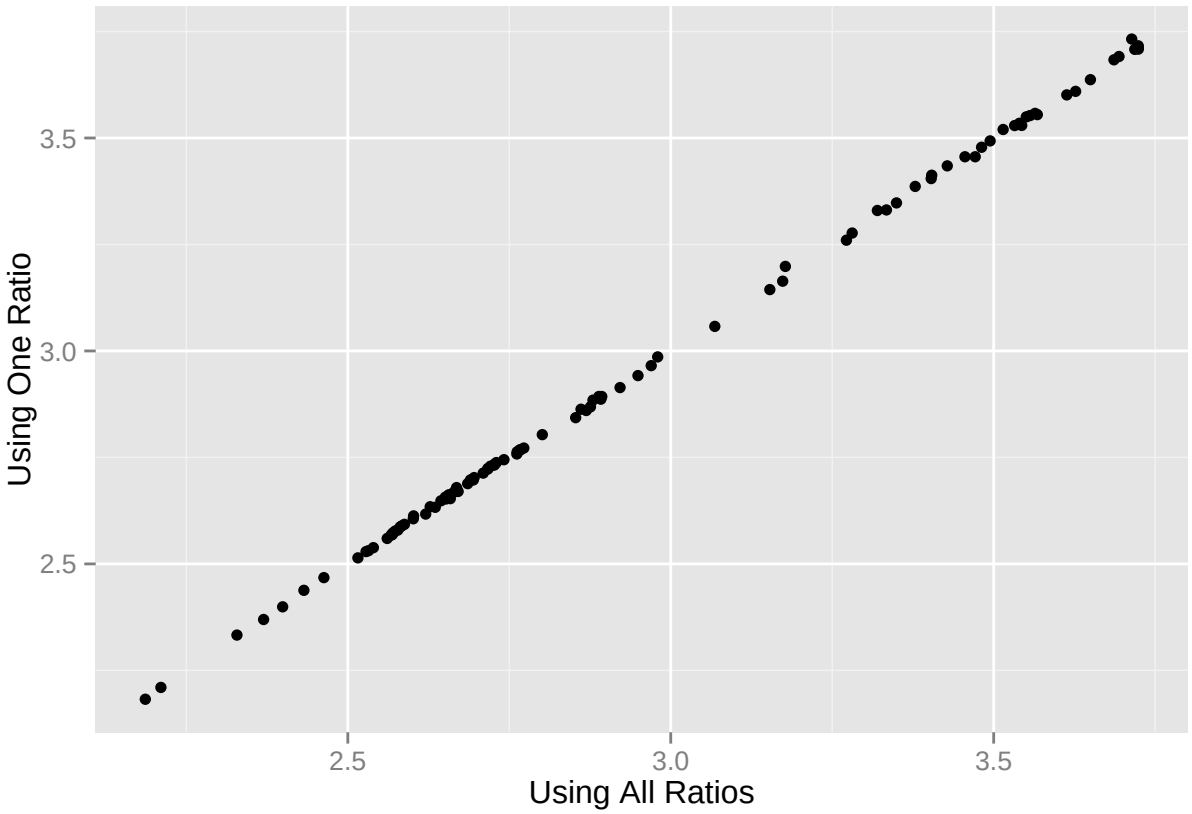
```
## Warning: package 'ggplot2' was built under R version 3.1.3
```

```
ggplot(data = wtid) +
  geom_point(mapping = aes(x = Year, y = USestimates)) +
  labs(main = "Pareto Exponent Estimates Over the Years", x = "Year", y = "Estimated Pareto Exponent")
```



iv. Use () to estimate a for the US for every year. Make a scatter-plot of these estimates against those from problem (iii) using `ggplot`. If they are identical or completely independent, something is wrong with at least one part of your code. Otherwise, can you say anything about how the two estimates compare?

```
single_ratio  <- 1 - (log(10))/(log(wtid$P99/wtid$P99.9))
names(single_ratio) <- wtid$Year

ggplot(data = wtid) +
  geom_point(mapping = aes(x = USestimates, y = single_ratio)) +
  labs(main = "Pareto Exponent Estimates Two Ways", x = "Using All Ratios", y = "Using One Ratio")
```



v. Go to the World Top Incomes Database and obtain data files with `P99`, `P99.5`, and `P99.9` for Canada, China, Colombia, Germany, Italy, Japan, South Africa, and Sweden. These are variables under the 'Average income/Fractiles income level' variable selection and they are labeled, for example, as 'P99 income threshold'. (Note there are subdivisions '-LAD', '-married couples & single adults', '-adults' but we'd like the original value). The WTID website also has data on the average income per "tax unit" (roughly, household) for the US and the other countries. While you're at it, obtain this information from the website as well. This is under `Average fiscal income per tax unit' variable selection`.

Use your function from problem (iii) to estimate a over time for each of them. Note that the size of the dataset is different for each of these countries, and there may be some NA values.

Note: WTID exports data files in xls format, which we haven't studied how to import into `R`. There are two strategies to do this. (1) Open the files in Excel and then save them as csv files (go to FILE -> Save As and choose csv), after which we can read them in to `R` in the usual ways. Otherwise you can use `read.xls()` from the `gdata` package which tries very hard to work like `read.csv()`.

Hint: You may find it helpful to create a separate dataframe for each country, but make sure they all have the same column names. You could also keep all the countries in a single dataframe with a `Country` variable.

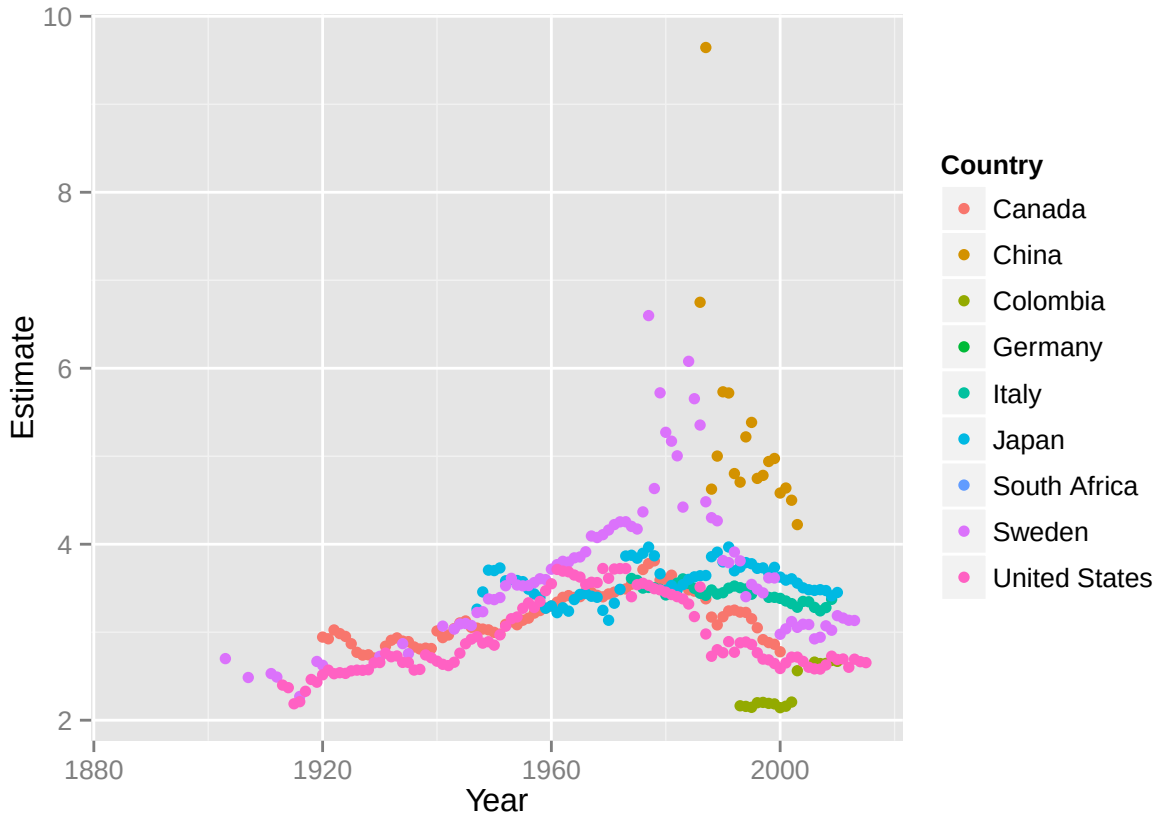
```
wtid2 <- read.csv("wtid-homework.csv", as.is = TRUE)
UStax <- read.csv("UStax.csv", as.is = TRUE)
names(UStax) <- c("Country", "Year", "AverageIncome")

wtid  <- merge(UStax, wtid)
names(wtid2) <- c("Country", "Year", "AverageIncome", "P99", "P99.5", "P99.9")

country_num <- length(unique(wtid2$Country))
```

```
for(i in 1:country_num) {
  these_rows <- wtid2$Country == unique(wtid2$Country)[i]
  this_data  <- wtid2[these_rows, c("Year", "P99", "P99.5", "P99.9")]
  estimates  <- multi_ratios_allyears(this_data)
  wtid2$Estimate[these_rows] <- estimates
}
```

vi. Plot your estimates of a over time for all the countries using `ggplot`. Note that the years covered by the data are different for each country. You may either make multiple plots, or put all the series into one plot. Either way, make sure that the plots are clearly labeled.

```
wtid <- cbind(wtid, USestimates)
names(wtid)[7] <- "Estimate"
wtid2 <- rbind(wtid2, wtid)

ggplot(data = wtid2) +
  geom_point(mapping = aes(x = Year, y = Estimate, color = Country))
```

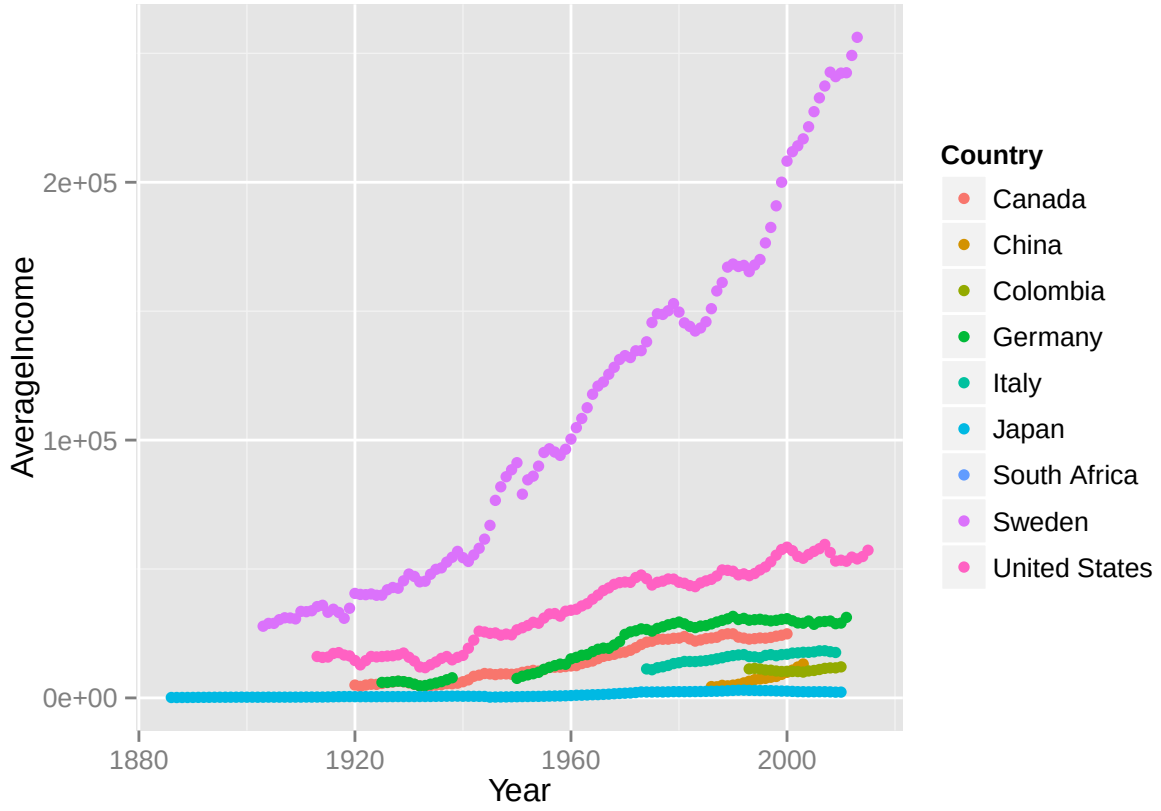
```
## Warning: Removed 291 rows containing missing values (geom_point).
```



vii. Plot the series of average income per "tax unit" for the US and the countries against time in `ggplot`. Hint: You may find it helpful for all this information to be in the same data frames.

```
ggplot(data = wtid2) +
  geom_point(mapping = aes(x = Year, y = AverageIncome, color = Country))
```

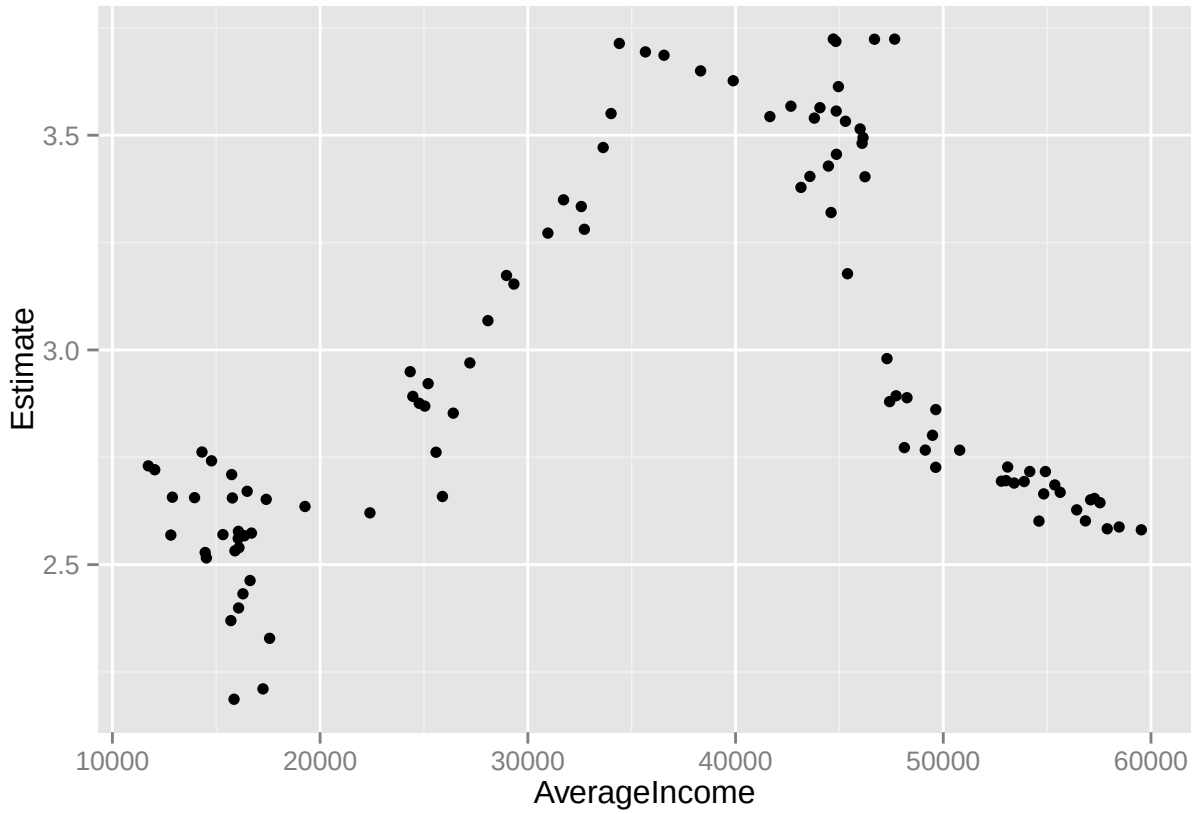
```
## Warning: Removed 121 rows containing missing values (geom_point).
```



viii. The most influential hypothesis about how inequality is linked to economic growth is the "U-curve" hypothesis proposed by the great economist Simon Kuznets in the 1950s. According tho this idea, inequality rises during the early, industrializing phases of economic growth, but then declines as growth continues.

Make a scatter-plot of your estimated exponents for the US against the average income for the US in `ggplot`. Qualitatively, can you say anything about the Kuznets curve? (Remember that smaller exponents indicate more income inequality.)

```
ggplot(data = wtid2[wtid2$Country == "United States", ]) +
  geom_point(mapping = aes(x = AverageIncome, y = Estimate))
```



Our scatterplot doesn't seem to support this hypothesis. Smaller a coefficients mean more inequality, so our scatterplot shows inequality high, then decreasing for a while but now rising again – the opposite of the "U-curve" hypothesis.

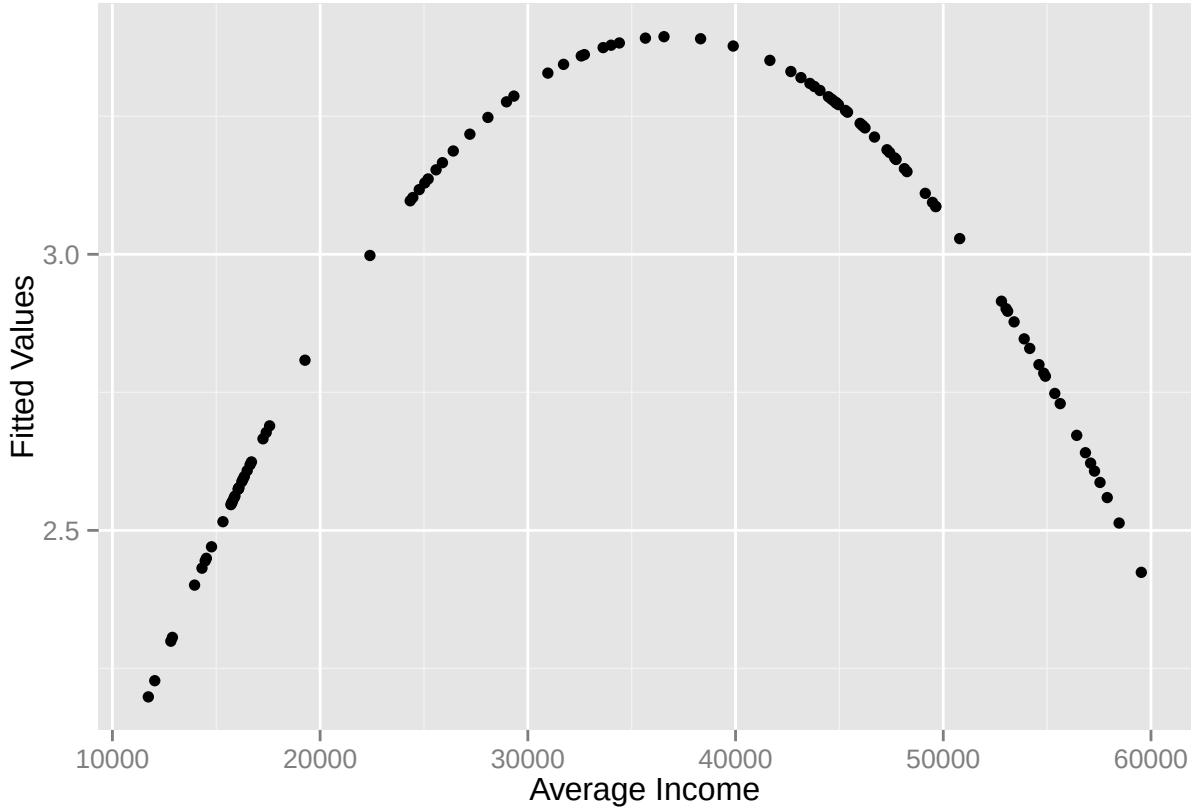
ix. For a more quantitative check on the Kuznets hypothesis, use `lm()` to regress your estimated exponents on the average income for the US, including a quadratic term for income. Are the coefficients you get consistent with the hypothesis? Hint: $lm(y \tilde{} x + I(x^2))$ will regress y on both x and x^2.

```
lm0 <- lm(Estimate ~ AverageIncome + I(AverageIncome^2), data = wtid2[wtid2$Country == "United States",
summary(lm0)
```

```
##
## Call:
## lm(formula = Estimate ~ AverageIncome + I(AverageIncome^2), data = wtid2[wtid2$Country ==
##     "United States", ])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.50724 -0.18364 -0.02531  0.18689  0.54918
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)         8.230e-01  1.515e-01   5.432 3.93e-07 ***
## AverageIncome       1.394e-04  1.015e-05  13.740  < 2e-16 ***
## I(AverageIncome^2) -1.891e-09  1.451e-10 -13.027  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.2466 on 100 degrees of freedom
## Multiple R-squared:  0.6679, Adjusted R-squared:  0.6612
## F-statistic: 100.5 on 2 and 100 DF,  p-value: < 2.2e-16
```

```
ggplot(data = wtid2[wtid2$Country == "United States", ]) +
  geom_point(mapping = aes(x = AverageIncome, y = fitted(lm0))) +
  labs(main = "United States", x = "Average Income", y = "Fitted Values")
```



Our coefficients are not consistent with the hypothesis. Since the coefficient in front of the quadratic term is negative, the model is a parabola opening downwards, not upwards, like a 'U' would.

x. Do a separate quadratic regression for each country. Which ones have estimates compatible with the hypothesis? Hint: Write a function to fit the model to the data for an arbitrary country.

```
reg_fit <- function(data, country) {
  # data is a data frame with the variables Country, Estimate, and Average Income.  This function retur

  if (all(is.na(data$Estimate[data$Country == country]))) {
    return(rep(NA, 3))
  } else {
    lm0 <- lm(Estimate ~ AverageIncome + I(AverageIncome^2), data = data[data$Country == country, ])
    summary(lm0)
    return(lm0$coef)
  }
}
```

```r
country_num <- length(unique(wtid2$Country))

coefs <- matrix(NA, ncol = 3, nrow = country_num)
colnames(coefs) <- c("Intercept", "AverageIncome", "AverageIncome2")
rownames(coefs) <- unique(wtid2$Country)

for (i in 1:country_num) {
  coefs[i, ] <- reg_fit(wtid2, unique(wtid2$Country)[i])
}
coefs
```

```
##                Intercept AverageIncome AverageIncome2
## Canada         2.2660536  1.240966e-04  -3.360837e-09
## China         10.3978092 -1.126763e-03   5.257536e-08
## Colombia      34.6124009 -6.095234e-03   2.867133e-07
## Germany               NA            NA             NA
## Italy          2.5824163  1.594300e-04  -6.591048e-09
## Japan          3.7291069 -5.136191e-04   1.889447e-07
## South Africa          NA            NA             NA
## Sweden         1.0123533  4.414454e-05  -1.496762e-10
## United States  0.8230049  1.394435e-04  -1.890556e-09
```

From the coefficients output, the relationship between the coefficient estimates and the average income is modeled by a "U" shape for China, Colombia, and Japan, but not for the others.

(If we were doing a more rigorous check of the Kuznet hypothesis, we would want to control for other factors, and not just assume that a quadratic was the right functional form for the curve.)