Suites récurrentes, listes

```
In [1]: import numpy as np
import matplotlib.pyplot as plt

# Discretisation pour l'affichage de graphes de fonctions
m = 300
```

Suites récurrentes

```
In [2]: def u(f, alpha, n):
              ans = alpha
              for k in range(n):
                  ans = f(ans)
              return ans
In [3]: def f0(x):
              return np.sqrt(1 + x)
In [4]: u(f0, 0, 2)
Out[4]: 1.4142135623730951
In [5]: | def listes_graphe(f, x_min, x_max, m):
              x_{graphe} = [x_{min} + (k / m) * (x_{max} - x_{min})  for k  in range(m + 1)]
              y \text{ graphe} = [f(x) \text{ for } x \text{ in } x \text{ graphe}]
              return x_graphe, y_graphe
In [6]: x, y = listes_graphe(np.sin, 0, np.pi / 2, m)
In [7]: plt.plot(x, y)
Out[7]: [<matplotlib.lines.Line2D at 0x7f78991bcb90>]
          1.0
          0.8
          0.6
          0.4
          0.2
          0.0
              0.0
```

```
In [8]:
          def liste_escalier(f, alpha, n):
              u = alpha
              x_escalier = [u]
              y escalier = [0.0]
              for k in range(n):
                  x_escalier.append(u)
                  u = f(u)
                  x escalier.append(u)
                  y_escalier.append(u)
                  y_escalier.append(u)
              return x escalier, y escalier
In [9]: def escalier(f, alpha, n, x_min, x_max, m):
              x_bissectrice = [x_min, x_max]
              y_bissectrice = [x_min, x_max]
              plt.plot(x_bissectrice, y_bissectrice, color='black')
              x_f, y_f = listes_graphe(f, x_min, x_max, m)
              plt.plot(x_f, y_f, color='red')
              x_escalier, y_escalier = liste_escalier(f, alpha, n)
              plt.plot(x_escalier, y_escalier, '--', color='blue')
In [10]: def f0(x):
              return np.sqrt(1 + x)
          escalier(f0, 3.0, 10, 0.0, 3.0, m)
           3.0
           2.5
           2.0
          1.5
           1.0
           0.5
           0.0
               0.0
                     0.5
                            1.0
                                  1.5
                                         2.0
                                               2.5
                                                      3.0
In [11]: escalier(np.cos, 0.0, 10, 0.0, np.pi / 2, m)
           1.6
           1.4
          1.2
           1.0
           0.8
           0.6
           0.4
           0.2
           0.0
               0.0
                    0.2
                         0.4
                                   0.8
                                       1.0
                                            1.2
                                                      1.6
```

```
In [12]: a = 1 / 2
          def f(x):
              return a * (1 + a**2) / (1 + x**2)
          escalier(f, 0.0, 10, 0.0, 1, m)
          1.0
          0.8
           0.6
          0.4
          0.2
           0.0
                      0.2
               0.0
                              0.4
                                      0.6
                                              0.8
                                                      1.0
In [13]: a = 2
          escalier(lambda x: a * (1 + a**2) / (1 + x**2), 1.01 * a, 100, 0.0, 10, m)
          10
           6
           4
           2
In [14]: a = 1
          escalier(lambda x: a * x * (1 - x), 1 / np.pi, 10, 0.0, 1.0, m)
          1.0
          0.8
          0.6
          0.4
          0.2
```

0.2

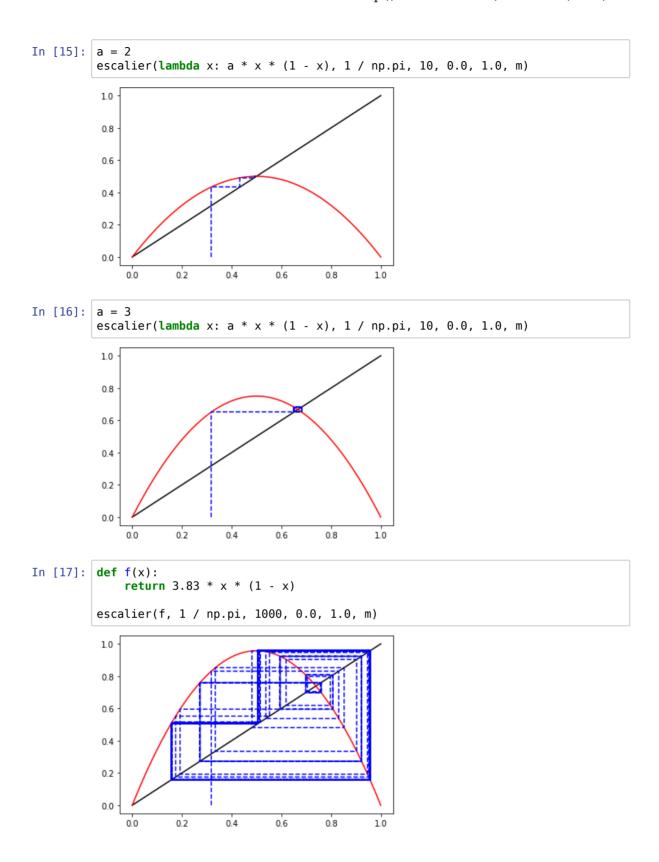
0.0

0.4

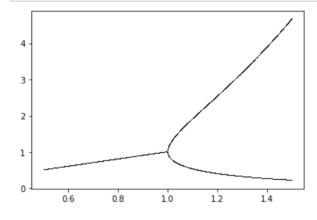
0.6

1.0

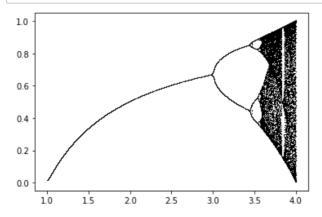
0.8



```
In [19]: bifurcation(f, 1 / np.pi, 1/2, 1.5, 1000, 1000, 1010)
```



```
In [20]: def f(a, x):
    return a * x * (1 - x)
bifurcation(f, 1 / np.pi, 1, 4, 10000, 100, 110)
```



Bloc de somme maximale

On montre que la complexite temporelle de cette fonction est en O(n^3).

```
In [22]: somme_max([5, 3, -3, 2])
Out[22]: 8
In [23]: somme_max([-2, -3])
Out[23]: 0
In [24]: somme_max([1, -2, 5, -1, 7, -1])
Out[24]: 11
In [25]: somme_max([4, -1, 2, 3, -1, 2])
Out[25]: 9
```

Il y avait une erreur d'enonce. Il est corrige dans le nouveau document disponible sur le site.

```
In [26]: def somme_max_lin(t):
              n = len(t)
              v = [0 \text{ for } k \text{ in } range(n + 1)]
              for j in range(1, n + 1):
                  value = v[j - 1] + t[j - 1]
                  if value >= 0:
                       v[j] = value
              ans = 0
              for k in range(n + 1):
                  if v[k] > ans:
                      ans = v[k]
              return ans
In [27]: somme_max_lin([5, 3, -3, 2])
Out[27]: 8
In [28]: somme_max_lin([-2, -3])
Out[28]: 0
In [29]: somme_max_lin([1, -2, 5, -1, 7, -1])
Out[29]: 11
In [30]: somme_max_lin([4, -1, 2, 3, -1, 2])
Out[30]: 9
```

```
In [31]: def somme_max_lin(t):
             n = len(t)
             v = 0
             ans = 0
             for j in range(n):
                 new_v = v + t[j]
                 if new_v >= 0:
                     v = new_v
                 else:
                     v = 0
                 if v > ans:
                     ans = v
             return ans
In [32]: somme_max_lin([5, 3, -3, 2])
Out[32]: 8
In [33]: | somme_max_lin([-2, -3])
Out[33]: 0
In [34]: | somme_max_lin([1, -2, 5, -1, 7, -1])
Out[34]: 11
In [35]: somme_max_lin([4, -1, 2, 3, -1, 2])
Out[35]: 9
```