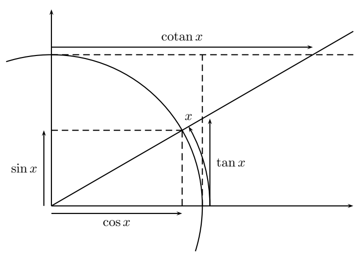
Fiche : Formules de trigonométrie

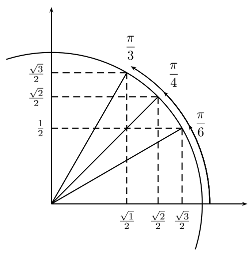
1 Définition



$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

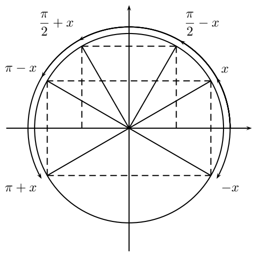
$$1 + \text{cotan}^2 x = \frac{1}{\sin^2 x}$$

2 Valeurs remarquables



x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	indéfini
$\text{cotan}\, x$	indéfini	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

3 Symétries



$$\cos(-x) = \cos x \qquad \cos(\pi + x) = -\cos x \qquad \cos(\pi - x) = -\cos x \qquad \cos\left(\frac{\pi}{2} + x\right) = -\sin x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin(-x) = -\sin x \qquad \sin(\pi + x) = -\sin x \qquad \sin(\pi - x) = \sin x \qquad \sin\left(\frac{\pi}{2} + x\right) = \cos x \qquad \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\tan(-x) = -\tan x \qquad \tan(\pi + x) = \tan x \qquad \tan(\pi - x) = -\tan x \qquad \tan\left(\frac{\pi}{2} + x\right) = -\text{cotan}\, x \qquad \tan\left(\frac{\pi}{2} - x\right) = \text{cotan}\, x$$

4 Addition des arcs

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad \sin(a+b) = \sin a \cos b + \cos a \sin b \qquad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b \qquad \sin(a-b) = \sin a \cos b - \cos a \sin b \qquad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \\ \sin(2x) &= 2\cos x \sin x \end{aligned} \qquad \begin{aligned} \cos^2 x &= \frac{1 + \cos(2x)}{2} \\ \sin^2 x &= \frac{1 - \cos(2x)}{2} \end{aligned}$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2} \qquad \tan p + \tan q = \frac{\sin(p+q)}{\cos p \cos q}$$

$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2} \qquad \sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2} \qquad \tan p - \tan q = \frac{\sin(p-q)}{\cos p \cos q}$$

$$\cos a \cos b = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right) \qquad \sin a \sin b = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right) \qquad \cos a \sin b = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

5 Tangente du demi-angle

En notant $t = \tan\frac{x}{2}$, on a :

$$\cos x = \frac{1-t^2}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2} \qquad \tan x = \frac{2t}{1-t^2}$$