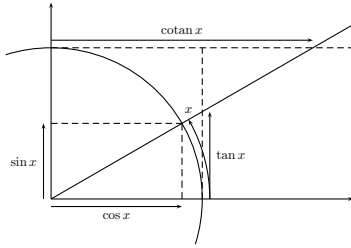


# FICHE : FORMULES DE TRIGONOMETRIE

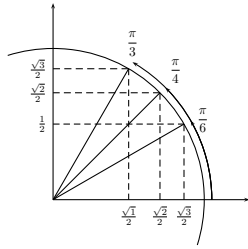
## 1 Définition



$$\cos^2 x + \sin^2 x = 1 \quad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

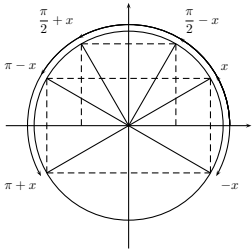
$$1 + \cotan^2 x = \frac{1}{\sin^2 x}$$

## 2 Valeurs remarquables



$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	indéfini
$\cotan x$	indéfini	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

## 3 Symétries



$$\begin{aligned} \cos(-x) &= \cos x & \cos(\pi+x) &= -\cos x & \cos(\pi-x) &= -\cos x & \cos\left(\frac{\pi}{2}+x\right) &= -\sin x & \cos\left(\frac{\pi}{2}-x\right) &= \sin x \\ \sin(-x) &= -\sin x & \sin(\pi+x) &= \sin x & \sin(\pi-x) &= \sin x & \sin\left(\frac{\pi}{2}+x\right) &= \cos x & \sin\left(\frac{\pi}{2}-x\right) &= \cos x \\ \tan(-x) &= -\tan x & \tan(\pi+x) &= \tan x & \tan(\pi-x) &= -\tan x & \tan\left(\frac{\pi}{2}+x\right) &= -\cotan x & \tan\left(\frac{\pi}{2}-x\right) &= \cotan x \end{aligned}$$

## 4 Addition des arcs

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \\ \sin(2x) &= 2 \cos x \sin x \end{aligned}$$

$$\begin{aligned} \cos^2 x &= \frac{1 + \cos(2x)}{2} \\ \sin^2 x &= \frac{1 - \cos(2x)}{2} \end{aligned}$$

$$\begin{aligned} \cos p + \cos q &= 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2} \\ \cos p - \cos q &= -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2} \end{aligned}$$

$$\begin{aligned} \sin p + \sin q &= 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2} \\ \sin p - \sin q &= 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2} \end{aligned}$$

$$\begin{aligned} \tan p + \tan q &= \frac{\sin(p+q)}{\cos p \cos q} \\ \tan p - \tan q &= \frac{\sin(p-q)}{\cos p \cos q} \end{aligned}$$

$$\begin{aligned} \cos a \cos b &= \frac{1}{2} (\cos(a+b) + \cos(a-b)) & \sin a \sin b &= \frac{1}{2} (\cos(a-b) - \cos(a+b)) & \cos a \sin b &= \frac{1}{2} (\sin(a+b) - \sin(a-b)) \end{aligned}$$

## 5 Tangente du demi-angle

En notant  $t = \tan \frac{x}{2}$ , on a :

$$\cos x = \frac{1-t^2}{1+t^2} \quad \sin x = \frac{2t}{1+t^2} \quad \tan x = \frac{2t}{1-t^2}$$