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Correlation analysis of the damage spreading problem in a 2-dimensional Ising model

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Abstract

We have studied the time dependence of the damage-spreading in a 2-dimensional Ising Model using time series analysis. We have found that the signal has a Hurst exponent 0.5 in high temperatures, and it approaches 1 near the temperature T_{D} where the damage goes to zero. We have also measured the correlation of the signal and found that it is strongly correlated near T_{D} and describes a persistent fractional Brownian motion (FBM). Therefore the Hamming distance in the Ising model can be used to generate FBM near T_{D}. We suggest that the system presents a $1/f$ spectrum in the transition temperature T_{D}.

It has been shown that damage spreading is a useful tool in information theory [1–3], Biology [4] and Physics problems [5]. A good review of the subject can be found in [6]. Since one of the first works in physics [7], much effort has been devoted to the question, whether the transition found for the spreading of damage coincides with the magnetic or any other dynamical transition already known in the literature. Above the transition temperature T_{D}, in Glauber dynamics, the damage spreads and the fraction of damaged sites is finite; below T_{D} it is zero. It turned out that T_{D} depends on the system size and the Monte Carlo update procedure. It has been shown that T_{D} for the heat-bath dynamics coincides with the Curie temperature T_{C} ($T_{\mathrm{C}} = 2.269\ldots J/k_{\mathrm{B}}$) [8], whereas for Glauber and Metropolis dynamics T_{D} is smaller than T_{C} [9–11]. For the three dimensional Ising Model with Glauber and Metropolis dynamics T_{D} is about 4% smaller [9,10].

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It is the goal of this paper to study the time dependence of the damage in temperatures above T_D using methods of time series analysis. We determine the temperature dependence of the r.m.s. fluctuations and compare it with the magnetic properties of the Ising model above T_C.

We consider the time evolution of two Ising systems which differ initially only in the orientation of one spin. The time development is calculated in Glauber dynamics. We first simulate a system until it is near equilibrium. Then, a replica is made of this configuration. We create the initial damage by flipping a single spin in the center of the replica. We evolve both the original system A and the damaged replica B using identical dynamics and the same random numbers (same Monte Carlo rules). After some time t the initial single damage can spread into a large region depending on the temperature of the system, where the orientation of the spins $S_i(t) = \pm 1$ in the replica differs from the corresponding spins in the original system. It is this region that is called the damage. The fraction of sites which have different spin orientation in the two configurations (the normalized damage) is calculated by

$$D(t) = \frac{1}{2N} \sum_{i=1}^{N} |S_i^A(t) - S_i^B(t)| ,$$

where N is the total number of sites.

Specifically the simulation is done in a ferromagnetic square lattice of 40×40 spins. The pair of spins has the exchange energy $-JS_iS_j$ $(J > 0)$, which is $-J$ or $+J$ if the spins align parallel or antiparallel, respectively. In Glauber dynamics we flip spins with probability $\exp(-\Delta/k_B T)/[\exp(\Delta/k_B T) + \exp(-\Delta/k_B T)]$, 2Δ being the energy difference between the original and the flipped configuration. We first set up a lattice randomly and then let it evolve for 800 MCS (Monte Carlo steps per spin) to permit this original system A to get near thermal equilibrium. We then make a copy B of the original system and introduce the one spin damage in this copy. We use the same dynamics and same random numbers on both lattices. We let the two systems (and hence the damage) evolve for additionally 800 MCS, in order to let the damage evolve into a stationary state. At this stage we begin recording the damage that fluctuates in time for each MCS. We have measured the damage for 32 600 steps. We do not average over many replicas as it is usually done [12] as we are interested in the time evolution rather than static properties.

Fig. 1a–c show the unnormalized damage fluctuations $D(t)$ for high $(T = 4J/k_B)$, intermediate $(T = 2.6\,J/k_B)$ and low temperature $(T = 2.4\,J/k_B)$ all higher than T_C. The low temperature pattern is more persistent than that of the higher one. These time series are similar to curves obtained in other systems including biological systems, economics, etc. [13,14]. We use the rms fluctuation method to characterize these curves quantitatively [13,14]. The properties in Fig. 1 are not local trends, but representative for the total curve. We emphasize that each of the three figures 1a, 1b and 1c has separately, the same fluctuation whichever time range we choose from each corresponding data for arbitrary shifts of the time interval.

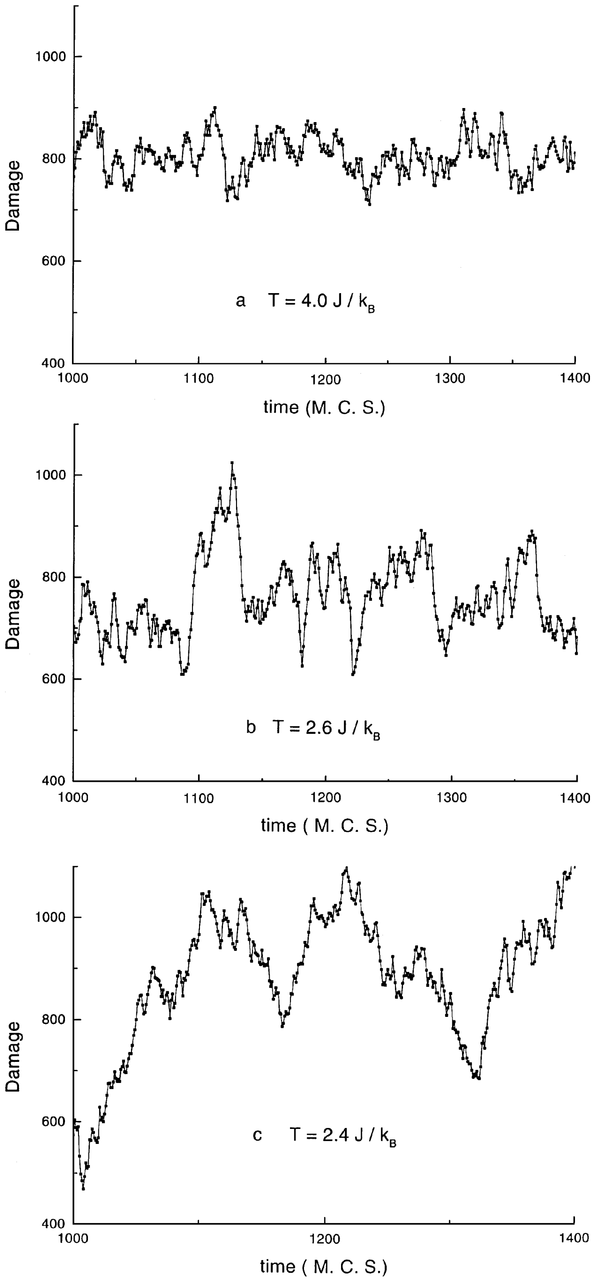

Fig. 1. The damage against Monte Carlo step is plotted within a range of 1000 to 1400 steps from 32 600 steps, where the initial 800 are not included for analysis. (a), (b) and (c) are calculated at the temperatures of $T = 4.0J/k_B$, $2.6J/k_B$ and $2.4J/k_B$, respectively. The lower temperature curve has more persistence which reflects stronger correlation in the system. The lower the temperature, we have found more persistence for longer period. This fact is related to the critical slowing down of the Ising model near T_C.

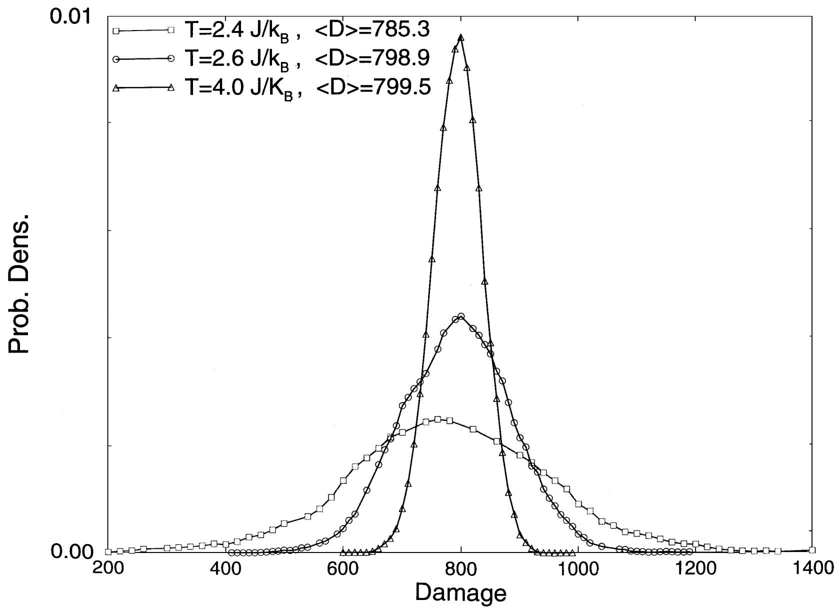

Fig. 2. The probability density of the damage $D(t)$ for three different temperatures. For $T = 2.4J/k_B$ the average damage is equal to 785, for $T = 2.6J/k_B$ the average damage is equal to 798.9, for $T = 4J/k_B$ the average damage is equal to 799.5.

Fig. 2 shows the probability density of the damage size for each temperature of Fig. 1. The curves are all normalized to area $A = 1$. For the case of high temperatures, $T = 1.8T_C$ ($T \cong 4J/k_B$) one gets a Gaussian distribution centered around a damage of 50% of the lattice, as expected for uncorrelated spins. When the temperature is decreased towards the critical temperature of the Ising model the distribution becomes much wider. A plausible explanation for this is, that the time evolution becomes more strongly correlated. At the same time a weak shift of the maximum towards smaller damages appears, which reflects the fact that the damage vanishes at low temperatures due to ferromagnetic alignment of the spins.

We define the mean fluctuation function

$$s(p) = \left\langle [d(t+p) - d(t)]^2 \right\rangle^{1/2} \propto p^{\mathrm{H}},$$

where d is the integration of the unnormalized damage $d(t) = \sum_{\tau=0}^{t} D(\tau)$.

In Fig. 3 the damage fluctuation function $s(p)$ is plotted against time lag p (in MCS/spin). We see that the Hurst exponent H (often also called α) [15–19] is different for high and low temperature. For high T, H is close to 0.5 ($H = 0.48$) and for low T (T near the Curie temperature) H is close to 1 ($H = 0.95$). The system has more persistence at low temperature (H near 1 region in Fig. 3) than at high temperature, where it globally shows random fluctuations behavior ($H = 0.5$).

The case of $H = 0.5$ corresponds to a random walk or Brownian motion. This is equivalent to say that the system in high temperature is a stochastic process with the

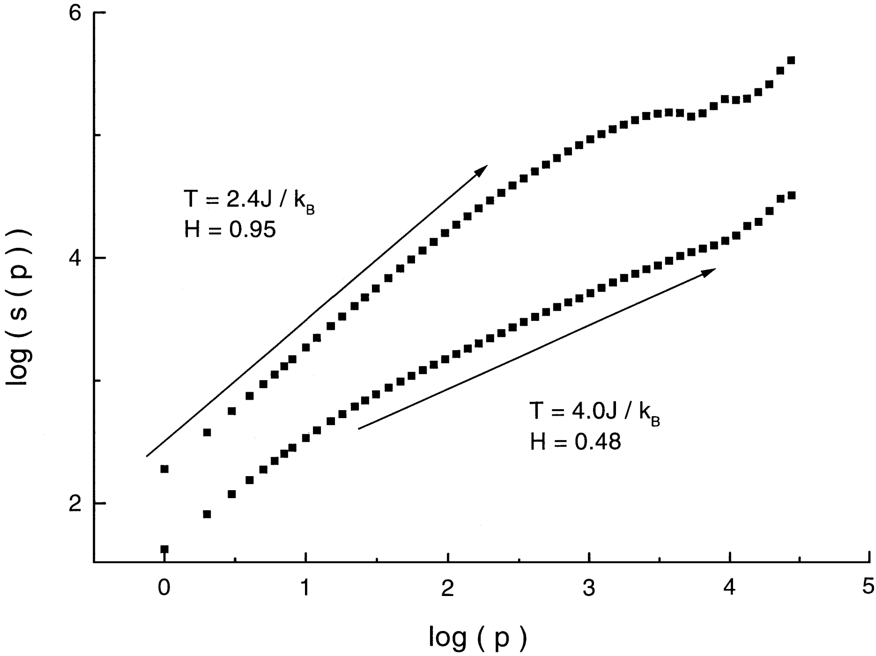

Fig. 3. Log–log plot of the fluctuation of the Hamming distance versus time (MCS). The two lines with slopes $H=0.95$ and $H=0.5$ correspond to $1/f$ noise and Brownian noise, respectively. The results are related to the correlation between spins (long-range order) in low temperatures near T_C ($T=2.4J/k_B$) and uncoupled spins (no correlation) for temperatures high above T_C ($T=4J/k_B$).

spins uncorrelated, so the damage is uncorrelated within the time period p. We also repeated all these calculations using M.F.F and the results were the same.

This result means that in low temperatures the spins are strongly correlated, which is the correct property of the Ising spin model in low temperatures near T_C. Thus H serves as the indicator of the presence of correlation. We say that the system is persistent.

The data set from low temperature indicates the long range correlation in $D(t)$ as seen from Fig. 1c, a positive increment is likely to be found following another positive increment. This is in accordance with the Ising Model in which the spins up tend to align spins up and spins down tend to align spins down for the ferromagnetic system which we have simulated.

These results for the Hurst exponent can be used to calculate the correlation of the increments of d (D unnormalized) $C=2^{(2H-1)}-1$ [13]. We obtain that for high temperature by using $H=0.5$, $C=0$ for all t as it is necessary for an independent random process. For H near 1, C is not zero, but near 1 for low temperature. This means that the system has strong correlation. This feature of Fractional Brownian motion leads to persistance. It means that an increasing trend in the past leads to an increasing trend in the future when $0.5<H<1$. Also, a decreasing trend in the past leads to an average decrease in the future. In other words: The increments are correlated such that positive values of $\Delta D(t) \equiv D(t) - \langle D(t)\rangle$ (where $\langle D(t)\rangle$ is the average damage) are likely to be close (in time) to each other, and the same is true for negative values of $\Delta D(t)$.

In this work we have studied the scale-invariant property of the Hamming distance problem in the 2-dimensional ferromagnetic Ising model. We have found that the successive increments in the Monte Carlo intervals display scale invariant, long-range correlations in low temperature. This result shows that the Hamming distance problem in this system has at least qualitatively, the same features of the Ising model, in both high and low temperatures above T_C. We also point out that the present model gives a very good method to generate fractional Brownian motion with $(0.5<H<1)$. Most methods developed to generate FBM can produce only finite series [13], but the present method can produce very long time series of correlated data and once the spins become correlated in low temperatures the phenomenon continues forever (unless the temperature is changed). It appears that the Hamming distance has fractal properties near T_D and has the same features as the critical phenomena in the Ising model, where the system is fractal at T_C where the response functions such as susceptibility and specific heat can be described by power laws, as well as the magnetization and correlation length [20]. It also seems that near T_D the system presents a $1/f$ noise behavior (H near 1). This means, according to [21], that the $1/f$ noise can be identified with the dynamics of the critical state. We conjecture (work in progress) that like the Hamming distance, the magnetization of the Ising model may present a $1/f$ noise near T_C [22].

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