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The stock market learned as Ising model

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Abstract. The collective behaviors of the financial systems resemble the physics theory of critical phenomena. Here we treat the stock market as the famous phase transition model: Ising model and the model parameters are learned from the real stock return time series. A comparative analysis between three world major stock markets have been given.

1. Introduction

The research about the collective behaviors of the complex financial system are extremely attractive. There are diverse methods been put forward to explore the complexity of financial systems [1]. The applications of those methods from statistical physics have provided us many new insights about financial systems [2–7]. The analogy between financial system, e.g. stock market, and real physical system such as spin system is a very interesting topic not only for physicists but also for financial industry.

The stock price are formed through the exchanges among many market participants. Thus the change in prices are correlated due to endogenous and exogenous events shared by many stocks. The inverse Ising problems offered us very flexible tools to infer the interaction patterns from the empirical data [8]. In ref. [9], three different inference methods have been used to obtain the interaction strength of stocks from the traded volumes. The authors show that the traded volumes are informative enough for the inference of collective behaviors. In refs. [10–12], Bury mapped the stock interactions to a spin model with pairwise interactions. Some cluster structure based on the coupling matrix have been uncovered with very clear economic meaning. Although extensive studies have been proceeded with the presence of large amount of financial data [9–14]. A comparative analysis between different countries is still lacking.

Here we try to infer the interaction strength of the stock markets from empirical data by recognizing the market as a spin glass model. We use the stock return time series from three markets for different countries. The statistical properties of the coupling matrix have been analyzed from which some common patterns have been revealed.

The paper is organized as follows. In Sec. 2 we introduce the methodologies used in this paper. In Sec. 3 we present the main empirical results. The last section provides our conclusion and discussion.

2. Data and Methodology

2.1. Data

Our datasets are consist of the daily adjusted returns time series of the constitute stocks from three major stock indexes in the world: S&P 500 (the US), FTSE 350 (the UK) and SSE 380 (China). After removing those stocks with relative small sample sizes, we still have 401, 264, and 295 stocks for the three markets respectively. We choose those indexes to make the system size as large as possible. In the S&P 500 dataset, each stock has 4025 daily records from 4 January 1999 to 31 December 2014. The FTSE 350 stocks include 3000 daily returns in the period between 10 October 2005 and 26 April 2017. The SSE 380 stocks consist of 2700 daily returns from 21 May 2004 to 19 November 2014.

In this paper, we use the logarithm return $r_i(t)$ which is defined as

$$r_i(t) = \ln(p_i(t+1)) - \ln(p_i(t)). \tag{1}$$

$p_i(t)$ is the adjusted closure price of stock i at time t. Then the return sequence are binarized as the state of each stock or spin:

$$s_i(t) = 1, r_i(t) \geq 0; s_i(t) = -1, r_i(t) < 0. \tag{2}$$

Here $s_i(t)$ is the state of stock i at time t. In our empirical analysis, $r_i(t) = 0$ is very rare. We then use the state sequence of those stocks to infer the Ising model interaction strengths and external fields.

2.2. Definition of the problem

As we mentioned in the previous context, we make an analogy between the financial system and Ising model. The binarized states of each stocks in three stock markets have been directly mapped to a two state Ising spin model from which the two point cross-correlation should be reconstructed during the inference process.

We consider the Ising model with N binary spin variables $\{s_i = \pm 1\}, i = 1, \ldots, N$. The pairwise couplings J_{ij} determine the interactions between spins. The Hamiltonian of the system with a spin configuration $\mathbf{s} = \{s_i\}$ is specified by

$$H_{\mathbf{J},\mathbf{h}}(\mathbf{s}) = -\sum_{i<j} J_{ij}s_is_j - \sum_i h_is_i. \tag{3}$$

We then introduce two methods borrowing from inverse problem researches which can help us reconstruct the interaction strength and external fields of Ising model.

2.3. naive mean field   https://youtu.be/40oxMhlQMiw

The exact learning is general slow and can be substituted by the approximate inferences methods which are based on expansion of the free energy of a system for small fluctuations around its mean value. The equation of the coupling strength J can be derived form the second order derivation of free energy [8]:

$$\mathbf{J}^{nMF} = \mathbf{A}^{-1} - \mathbf{C}^{-1}. \tag{4}$$

Similarity, we can also get the external field of the system which is written as follows:

$$h_i^{nMF} = \operatorname{artanh}\langle s_i \rangle - \sum_{j=1}^N J_{ij}^{nMF}\langle s_i \rangle. \tag{5}$$

Here $\mathbf{A}$ is a diag matrix with elements $A_{ii} = 1 - \langle s_i \rangle^2$ and $\mathbf{C}$ is the covariance of the state sequences of the system. This is called the naive mean field method(nMF).

2.4. TAP approximation

The second-order correction to the nMF approximation requires solving Thouless-Anderson-Palmer (TAP) equations. In 1977, Thouless, Anderson, and Palmer (TAP) added a term to the Gibbs free energy [8]. Thus we have

$$(\mathbf{C}^{-1})_{ij} = \quad -J_{ij}^{TAP} - 2(J_{ij}^{TAP})^2 \langle s_i \rangle \langle s_j \rangle, \tag{6}$$

$$h_i^{TAP} = \quad h_i^{nMF} - \langle s_i \rangle \sum_{j=1}^{N} (J_{ij}^{TAP})^2 (1 - \langle s_j \rangle^2). \tag{7}$$

Then we can get the coupling strength of the system as follows:

$$J_{ij}^{TAP} = \frac{-2(\mathbf{C}^{-1})_{ij}}{1 + \sqrt{1 - 8(\mathbf{C}^{-1})_{ij} s_i s_j}}. \tag{8}$$

From Eq(4) and (8), we know the the coupling strength depends on the inverse of covariance matrix **C**. The only way to guarantee the invertibility of a covariance matrix is to make the length of the time series as long as possible. Here the lengths of the time series are significantly larger than the system size. Thus the covariance **C** is invertible.

3. Results

Here in Fig. 1 we have calculated the distributions of the coupling strength **J** for three markets. The black lines are the coupling strength distributions of the three markets and for comparison purpose, we also simulate the shuffled return time series and the red lines denoted the coupling strength distributions of three markets for the shuffled case. The shuffling procedure is proceed by randomly exchange the stock return time series. Thus we want to remove the effect of cross-correlations related to the collective behaviors. It's obvious that the coupling strength of three markets largely deviated from the shuffled case, which can not be recognized as Edwards-Anderson model with Gaussian distributed coupling strengths. The heterogeneity coupling strengths give an hint about the criticality properties of the stock markets which is common for three markets.

Table 1: The first four statistical moments for the distributions of interaction strength J_{ij} for both mean field and TAP methods. The first value in each cell is the statistical moments of J_{ij} for mean field method and the second value for TAP method.

	mean	variance	skewness	kurtosis
SP 500	0.0045;0.0045	0.0015;0.0015	1.95;1.94	23.19;23.16
FTSE 350	0.0052;0.0052	0.0016;0.0016	2.62;2.62	32.93;32.83
SSE 380	0.0054;0.0054	0.0020;0.0020	1.07;1.06	8.86;8.83

In table1, we present the first four order statistical moments for the distribution of the interaction strength J_{ij}. The skewness of SP 500 and FTSE 350 markets are twice as much as the value of SSE 380. This means the interaction strengths of SP 500 and FTSE 380 are right skewed compare with SSE 380. The kurtosis of SP 500 and FTSE 350 are extremely larger than that of SSE 380. Thus we know the interaction strength distribution of SP 500 and FTSE 350 are very heterogeneous and heavy tailed.

Fig.2 shows the reconstructed external field h for three markets. The distributions have their own identities. The external field for US market has relative wider and flatten distribution. But
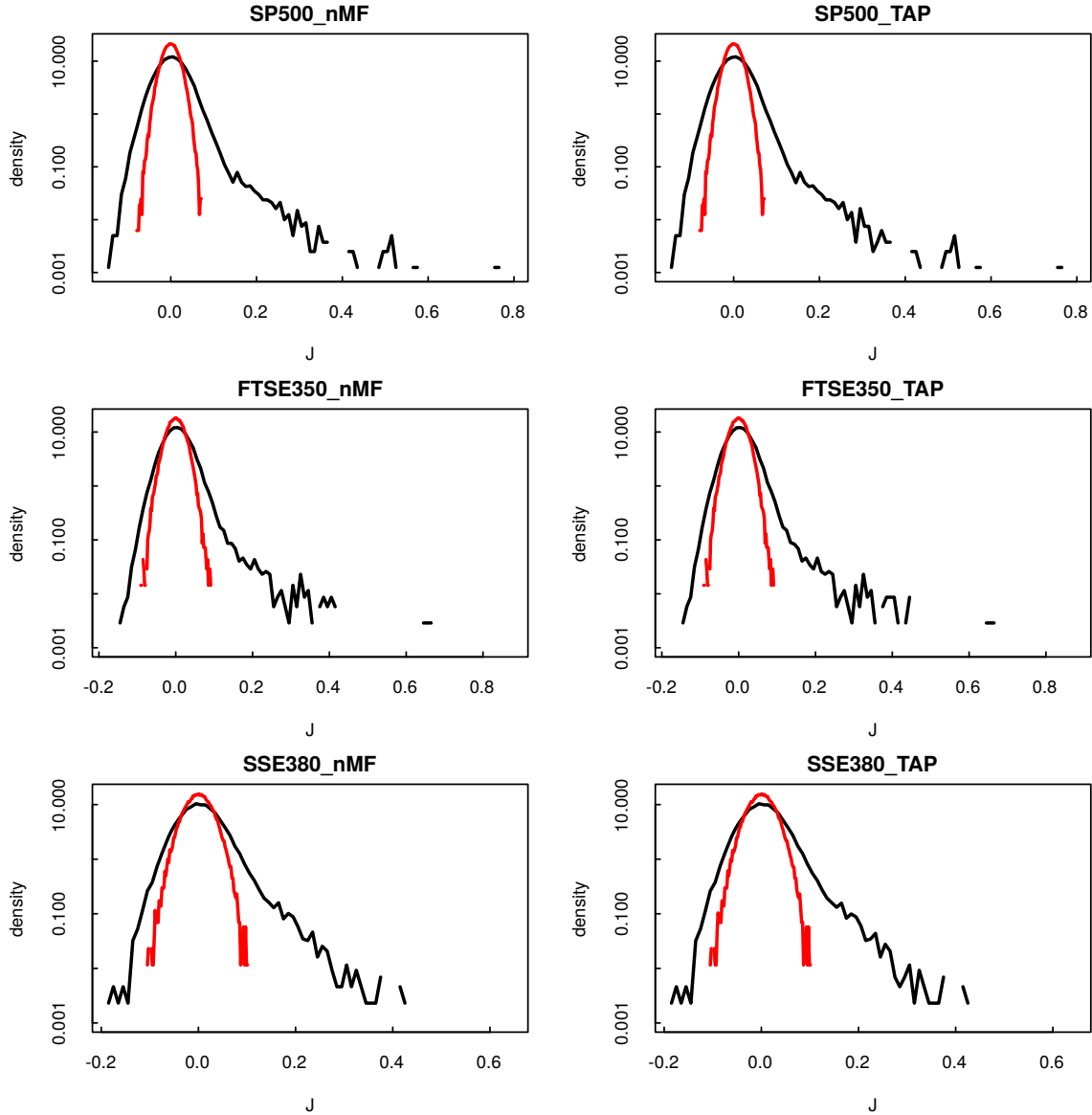
Figure 1: The interaction strength distributions of the inferred Ising spin system for three different markets. We use both mean field and TAP approximation methods. The left column is the results for three stock markets with mean field method and the right column is the result with TAP approximation method.

the external field for the UK market is more peaked. And the fluctuation of the distribution of China market is relatively larger than other two western markets. This maybe interpreted as the immaturity of China market.

Here in table2 we give the first four order statistical moments for the distributions of external field h_i for both mean field and TAP methods. From the statistical moments we know that the external fields recovery form empirical data are very close to white noise with very small mean, variance and skewness. Also the kurtosis are also very close to the value of Gaussian distribution with kurtosis 3.

The discussion about the market criticality is always a very attractive topic among
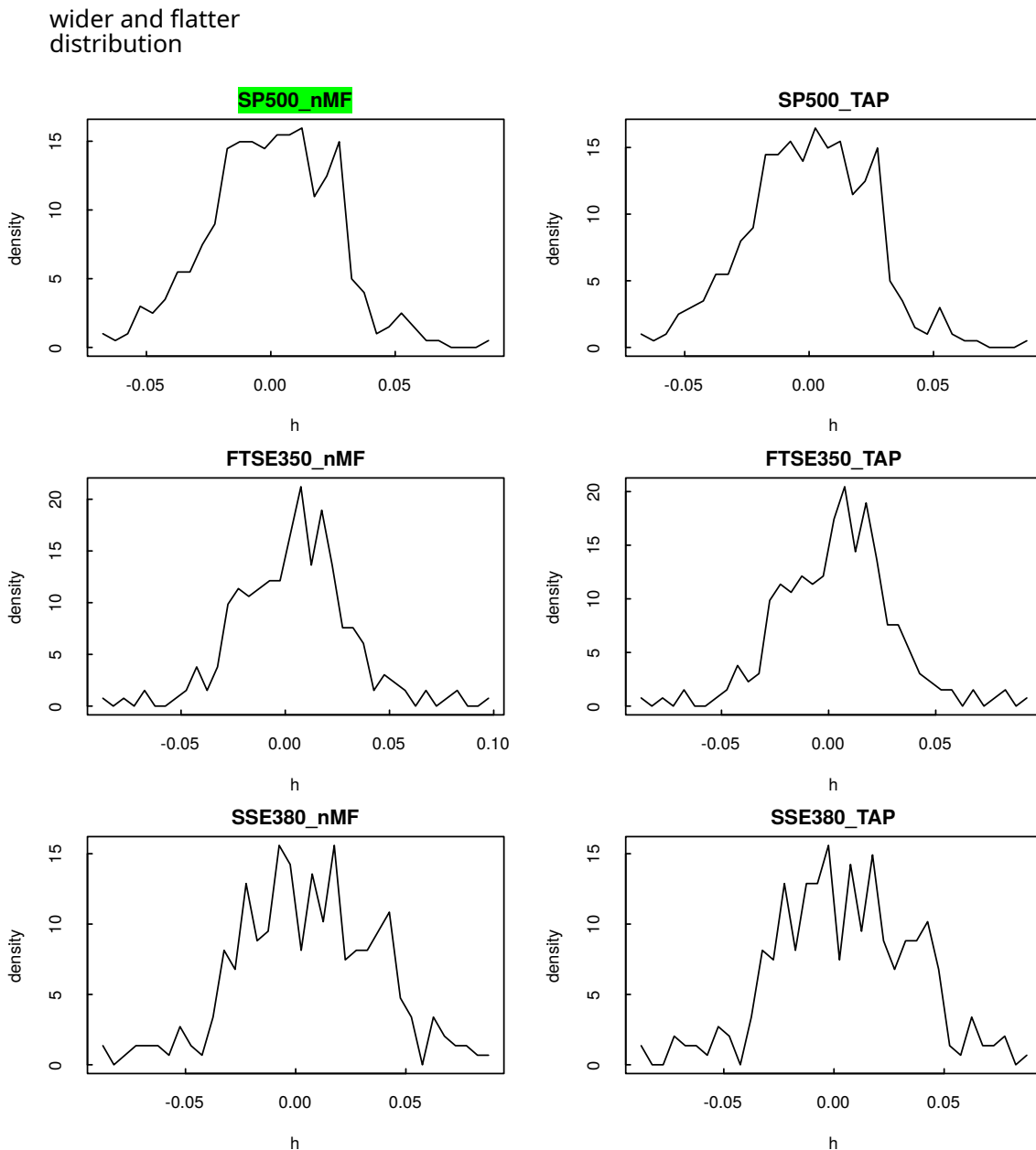
wider and flatter
distribution

Figure 2: The external field distributions for three markets inferred with both mean field and TAP approximation methods. The left column is the results for three stock markets with mean field method and the right column is the result with TAP approximation method.

Table 2: The first four statistical moments for the distribution of external field h_i for both mean field and TAP methods. The first value in each cell is the statistical moments of h_i for mean field method and the second value for TAP method.

	mean	variance	skewness	kurtosis
SP 500	0.0001;-0.00013	0.00061;0.00060	-0.0097;-0.0082	3.10;3.10
FTSE 350	0.0039;0.0037	0.00070;0.00069	0.053;0.055	4.18;4.17
SSE 380	0.0053;0.0047	0.0010;0.00098	-0.12;-0.11	3.09;3.10

econophysics community. Here the heterogeneous interaction strengths can be seen as a characteristic of criticality which definitely requires further investigation and validation from multiple perspectives. But the fingerprint provided here shows universal criticality of the world wide stock markets.

4. Conclusion

In conclusion, we use the binarized approach to transform the return time series of three major stock markets into Ising spin states with ± 1. The analogy between Ising model and stock market has revealed the non-Gaussian properties of the stock market interactions. The coupling strength distributions and the external field distributions have shown market specific fingerprints. The detailed comparison analysis among different stock markets from the view point of critical pairwise interaction model are very attractive and more attention about this topic is called for. A detailed comparison analysis based on Monte Carlo simulation will be given in the future research and the detailed analysis about the interaction matrix dynamics based on eigenvector dynamics and complex networks should also be interesting directions [3, 15].

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