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Ising type models applied to Geophysics and high frequency market data

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ABSTRACT

The classical Ising model was used to re-create the ferromagnetic phenomenon in statistical mechanics. The model describes the behavior of atoms in a lattice. Each atom may interact only with its neighbors, and has two states called spins. When the atoms polarize their spins, the resulting material exhibits a net magnetization. A similar model has been used before in financial math: the spins correspond to the buy/sell position of a trader and the polarization is equivalent with all the traders in the market wanting to sell. This leads to a market crash. In this work, we present extensions and applications to geophysics and high frequency market data.

1. Introduction

In the past few decades, researchers have analyzed a system composed of two state particles arranged using an N-dimensional lattice or graph, using a system called the Ising model [1]. The Ising model was originally developed to describe ferromagnetism, and is essentially one of the first models to exhibit phase transition for lattice dimensions greater than two. In statistical thermodynamics, when the temperature in the system passes a critical value, the system exhibits a polarization (phase transition) and spontaneous magnetization occurs [2]. The model has various applications in many other fields of science. There are known applications of Ising type model in modeling *social behavior* [3], *biological membranes* [4], *finance* [5], and *neural networks* [6], among many others.

In this paper, we focus on two applications of the Ising model: *geophysics* looking at events preceding a major earthquake and *finance* looking at data sampled with high frequency and preceding a major crash. The approach used is similar to our previous work [5], where we describe the behavior of financial market indices near the crash date. If we consider the earth as a two dimensional lattice of oscillators, where each of them can have phase 0 or π (i.e. oscillators will have two possible states), we may use an Ising type model to analyze the frequency of earthquakes occurring. A major earthquake takes place when all the oscillators have the same phase and nothing is going to happen when half of the oscillators have phase 0 and the other half phase π.

In the second part of this work, we study the high frequency market data leading to the Bear Stearns crash which occurred in mid March 2008. We look at the companies that were affected by this event. Using the Ising model, we will try to predict the critical time when stock prices experience phase transition.

2. Financial approach

In the market scenario [5], we consider a lattice composed of N traders, each of which may be in one of the states {*buy*, *sell*}. This is the financial analogy of the Ising model. The working hypothesis is that, at the critical time, all trader decisions

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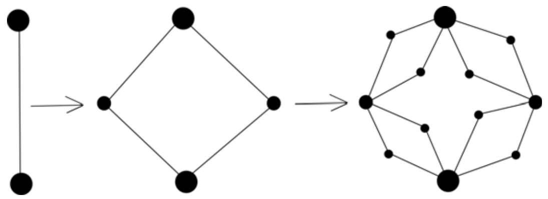

Fig. 1. Construction of a diamond lattice.

are aligned to *sell* a phenomenon which leads to market crashing. For the purpose of market analysis, the following formula for an index price $p(t)$ was assumed [7] and connected to the *diamond Ising model* [8, Section 2.3.2]:

$$\ln p(t) = A + B(t_c - t)^{\beta} + C(t_c - t)^{\beta}\cos(\omega \ln(t_c - t) + \phi), \tag{1}$$

where $A, B, C, \beta, \omega, \phi$ are unknown constants and t_c denotes the time of the crash. The structure of the diamond type lattice may be seen in Fig. 2. This type of lattice is constructed iteratively. The construction starts with two traders which are connected. The connection is represented by a straight line. In the next step, each link is replaced by a diamond and two traders are added at the two unoccupied vertices. At each step, the procedure is repeated for every existing link (see Fig. 1).

According to the model evolution in (1), bottoms of market cycles will occur when the argument of the cosine in the above price evolution will be of the form $(2k+1)\pi$, $(k \in \mathbb{Z})$. That gives us the condition for the time t_N of the Nth market bottom:

$$\omega \ln(t_c - t_N) + \phi = \pi + 2N\pi.$$

Thus the time of the Nth low satisfies:

$$t_N = t_c - \mathrm{e}^{((2N+1)\pi - \phi)/\omega}.$$

Since we want to study the market's cyclical behavior before reaching the crash state, we assume that all minima before the N'th are normal market oscillations, while after the final minima time t_N, the minimum reaches critical levels. The N'th market cycle may be defined in the following way:

$$\begin{aligned} T_N &= t_N - t_{N-1} = \mathrm{e}^{(2\pi N + \pi - \phi)/\omega} - \mathrm{e}^{(2\pi(N-1)+\pi-\phi)/\omega} \\ &= \mathrm{e}^{(\pi-\phi)/\omega}(1 - \mathrm{e}^{-2\pi})\mathrm{e}^{(2\pi/\omega)N}. \end{aligned}$$

Using the substitution $\alpha = \mathrm{e}^{(\pi-\phi)/\omega}(1 - \mathrm{e}^{-2\pi})$, $\xi = 2\pi/\omega$ we have:

$$\ln T_N = \xi N + \ln(\alpha).$$

Thus T_N is an exponential sequence, which depends on the unknown N. But by selecting arbitrary but reasonable N_1, we have:

$$\begin{aligned} \ln T_N &= \xi N + \ln(\alpha) = \xi N_1 + \ln(\alpha) + \xi(N - N_1) = \xi N_1 + \ln(\alpha) + \xi \Delta N \\ &= \xi N_1 + (\ln(\alpha) + \xi \Delta N) = \xi N_1 + \ln(\alpha \mathrm{e}^{\xi \Delta N}) = \xi N_1 + \ln(\alpha_0) \end{aligned} \tag{2}$$

where $\Delta N = N - N_1$ and $\ln(\alpha_0) = \ln(\alpha \mathrm{e}^{\xi \Delta N})$. Since N_1 is known, to obtain ξ and α_0, we can use least square fitting. At this point, we can calculate t_c from the relation:

$$t_c = t_N + \mathrm{e}^{(2\pi N + \pi - \phi)/\omega} = t_N + T_N(1 - \mathrm{e}^{-\xi})^{-1} = t_N + \alpha_0 \mathrm{e}^{\xi N_1}(1 - \mathrm{e}^{-\xi})^{-1}. \tag{3}$$

If we decide to operate with maxima (instead of minima), we chose a different substitution $\alpha = \mathrm{e}^{(-\phi)/\omega}(1 - \mathrm{e}^{-2\pi})$ because the argument of cosine in Eq. (1) attains maximum at: $\omega \ln(t_c - t) + \phi = 2k\pi$.

3. Application to Geophysics

3.1. Motivation

Even though regions of the earth where the earth displacement has been observed are called *fault lines*, they do not normally consist of a single clean fracture. Geologists use the better term *"fault zone"* when referring to regions of complex deformations associated with the fault plane. The San Andreas Fault in the United States of America is one of the largest and best studied such zone. At such sites, it is well known that, a series of small earthquakes may increase the likelihood of a larger event by destabilizing the surrounding zones and producing new sites for possible earthquake epicenters [9]. For this reason, we believe that, using the temporal evolution model (1) resulting from a diamond lattice Ising model as described in the previous section would be particularly useful for modeling earthquake events.

In our approach, for simplicity, we consider the oscillators (earth sites) with the same amplitude and frequency, but two possible phases $\{0, \pi\}$. The result of this process is the temporal evolution of the amplitude of oscillations at a certain point

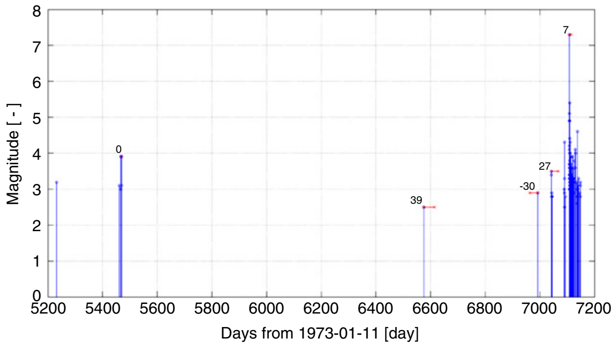

Fig. 2. Earthquake at 1992/28/06, latitude: 34.2°, longitude −116.44°.

on the earth. As in the financial case, we will operate with two equations for predicting the "crash" date, which in this case, is the critical point when some major earthquake occurred. Eqs. (2) and (3) are the equations we work with:

$$\ln T_N = \xi N_1 + \ln(\alpha_0)$$
$$t_c = t_N + T_N(1 - \mathrm{e}^{-\xi})^{-1} = t_N + \alpha_0 \mathrm{e}^{\xi N_1}(1 - \mathrm{e}^{-\xi})^{-1}.$$

3.2. Methodology

The geological data about earthquakes was obtained from the US Geological Survey (USGS) from 1st January 1973 to 9th November 2010. The downloaded data contains information about the date, longitude, latitude, and the magnitude of each recorded earthquake in the region.

The location of the major earthquake chosen defines the area studied. The area chosen cannot be too small (lack of data) or too big (noise from unrelated events). The data is obtained using a *square* centered at the coordinates of the major event. The sides of the square were usually chosen as ±0.1°–0.2° in latitude and ±0.2°–0.4° in longitude. A segment 0.1° of latitude at the equator is ≈11.11 km ≈6.9 miles in length.

The earthquake magnitude is the recorded data used in the analysis. The policy of the USGS regarding the recorded magnitude is the following [10]:

- The magnitude is a dimensionless number between 1 and 12.
- The reported magnitude should be the moment magnitude, if available.
- The least complicated, and probably most accurate, terminology is to just use the term "*magnitude*", and to use the symbol M.

After selecting the data, we identify the maxima as the earthquake prior to the major earthquake date. At this point, we can use (2) *for a least square fitting, and* (3) *to estimate the earthquake date.*

3.3. Results

In the following Figs. 2–4, we display typical results for three specific earthquakes. The figures show the magnitude (height of the bar) and the difference (in days) between prediction and the actual value (plotted using a red line with a number on the top of it).

The data for these figures was measured in the western hemisphere (i.e. −180° in longitude). The entire set of earthquakes analyzed is presented in Table 1. The table displays the date of the major earthquake event, earthquake coordinates, the prediction accuracy using absolute value of days, the time frame considered (i.e. difference in days between the first earthquake we used and the major earthquake date), and finally the relative precision, taking into consideration the time frame considered. All the earthquakes regarded as major events have magnitudes greater than 7, with the exception of the 1994/01/17 earthquake with a magnitude 6.7.

Parameters for Fig. 2 were:

- Latitude: 34.2° ± 0.1°
- Longitude: −116.44° ± 0.2°
- Maximum of the magnitude 7.3 occurred at 1992/28/06.
- Time on the x axis is measured in days from 1973/11/01.
- The interval used was 1637 days.

The prediction was very good, the major earthquake was predicted with precision of 7 days.

Parameters for Fig. 3 were:

- Latitude: 32.13° ± 0.1°
- Longitude: −115.30° ± 0.2°

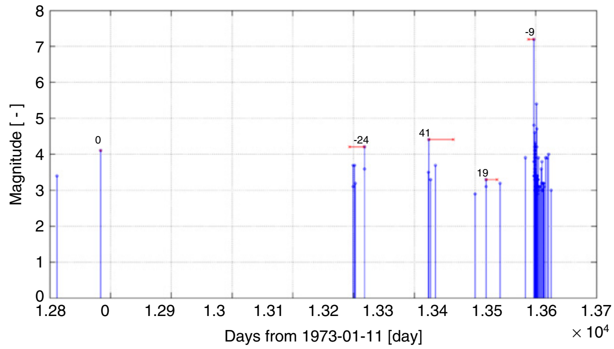

Fig. 3. Earthquake at 2010/04/04, latitude: 32.13°, longitude −115.30°.

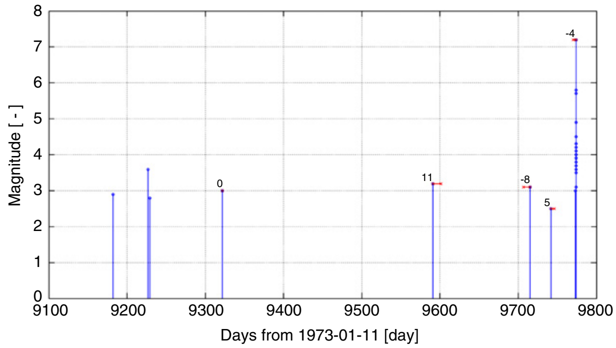

Fig. 4. Earthquake at 1999/10/16, latitude: 34.59°, longitude −116.27°.

Table 1
Results for earthquakes.

Event occurred	Latitude (°)	Longitude (°)	Time frame (days)	Precision (days)	Relative precision : $\frac{\text{Precision}}{\text{Time frame}} \times 10^3$
1974/11/09	−12.50	−77.76	320	3	9.4
1992/04/25	40.37	−124.32	910	35	38.5
1992/06/28 Fig. 2	34.20	−116.44	1637	7	4.3
1994/01/17	34.21	−188.54	1776	46	25.9
1995/07/30	−23.34	−70.29	1616	44	27.2
1995/09/14	16.78	−98.60	359	7	19.5
1999/10/16 Fig. 4	34.59	−116.27	545	4	7.3
2002/11/03	63.52	−147.44	121	2	16.5
2004/07/15	−17.66	−178.76	141	1	7.1
2006/01/02	−19.93	−178.18	504	39	77.4
2010/04/04 Fig. 3	32.13	−115.30	713	9	12.6

- Maximum of the magnitude 7.2 occurred at 2010/04/04.
- Time on the *x* axis is measured in days from 1973/11/01.
- The time interval used was 713 days.

The prediction was also really good 9 days before the actual event, we consider earthquakes at $\approx 1.33 \times 10^4$, 1.34×10^4, 1.35×10^4 as clustered. Thus we took only the biggest value and denote it as a maximum.

Parameters for Fig. 4 were:

- Latitude: 34.59° ± 0.2°
- Longitude: −116.27° ± 0.4°
- Maximum of the magnitude 7.2 occurred at 1999/10/16.
- Time measured in days from 1973/11/01.
- The time interval used was 545 days.

The prediction was very good. The earthquake was predicted with a precision of 4 days.

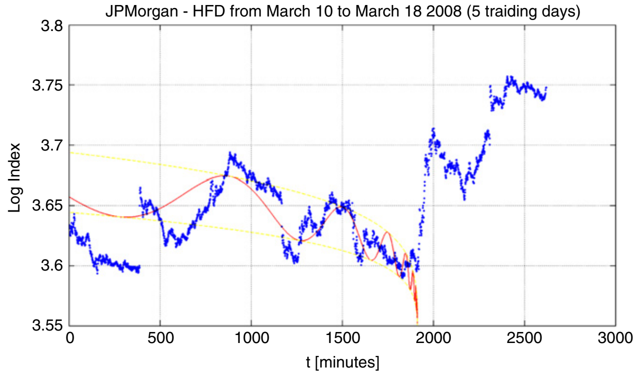

Fig. 5. JPMorgan Chase — The solid line represents the best fit with curve (1).

Table 2
Results for high frequency data.

Stock name	Crash date (min)	Estim. crash date t_c (min)	Precision (min)	β
LBC	1944	2042	98	0.20
City	885	872	13	0.20
JPMorgan	1908	1914	6	0.20
IAG	1984	1948	36	0.05

4. Application of the Ising model to financial data sampled with high frequency

In this section, we study high frequency data corresponding to the collapse of the Bear Stearns in March 2008. The data used consists of the week (five trading days) March 10–March 14, 2008 before the merging announcement over the weekend as well as the two following trading days March 17, 18. On Friday, March 14, 2008 at about 9:14 am, JPMorgan Chase and the Federal Reserves Bank of New York announced an emergency loan to Bear Stearns (of about 29 billion, terms undisclosed) to prevent the firm from becoming insolvent. This bailout was declared to prevent the very likely crash of the market as a result of the fall of one of the biggest investment banks at the time. This measure proved to be insufficient to keep the firm alive and two days later, on Sunday March 16, 2008, Bear Stearns signed a merger agreement with JPMorgan Chase essentially selling the company for $2 a share (price revised on March 24 to $10/share). The same stock traded at $172 in January 2007 and $93 a share in February 2007. Today, this collapse is viewed as the first sign of the risk management meltdown of investment bank industry in September 2008 and the subsequent global financial crisis and recession.

4.1. Methodology

In this part of the article, we study the behavior of stocks representing the financial institutions which should be affected by the impeding crisis. Specifically, we analyze JPMorgan Chase (JPM), IAG (IMGOLD), Lehman Brothers (LBC), City Bank. During the days considered, the price of each stock fluctuated more than 10%. All the stocks (with the exception of LBC) were sampled with the period $T = 1$ min. We use the formula (1), which may also be found in Ref. [5] and its references. Estimating parameters A, B, C was done using the least square estimation method. The parameter β in (1) was chosen using a least square fit of the stock price data.

The minima *(maxima)* were chosen in a similar way with [5]. We picked the minima *(maxima)* preceding the crash date and we used the least square estimation to obtain the crash date t_c from Eq. (3). In the cases of City and IAG, (Figs. 6–8) the asset price decline while in the case of the JPMorgan Chase (Fig. 5), the stock price suffers a major increase. We interpret t_c as the critical date at which stock prices change their future development very rapidly. The least accurate prediction of t_c was for LBC. This could be caused by the smaller data used (6× smaller that all the other stock). During the time period considered, the Lehman Brothers company was thought to be close to bankruptcy as well, and the exchanges suspended trading for long periods during the trading day. The results in terms of accuracy are presented in Table 2.

5. Conclusions

We found that predictions based on the diamond lattice Ising model previously used in finance gave very good results when applied to earthquake phenomena, an area notoriously hard to predict. The method performed better within a smaller time frame; however, the relative precision was scale invariant, (last column in Table 1) typically in the range 1%–3%. According to these results, a model of earth composed of two dimensional lattice of oscillator seems to be well justified and may open a whole new angle of approach to studying the earthquake phenomena.

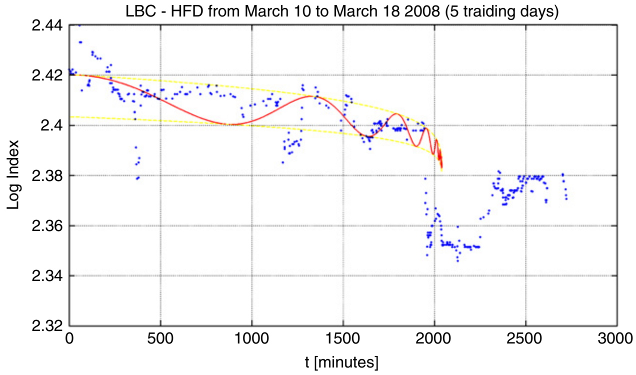

Fig. 6. LBC — The solid line represents the best fit with curve (1). This figure uses market events instead of 1 min records. Difference between two events is ≈6 min.

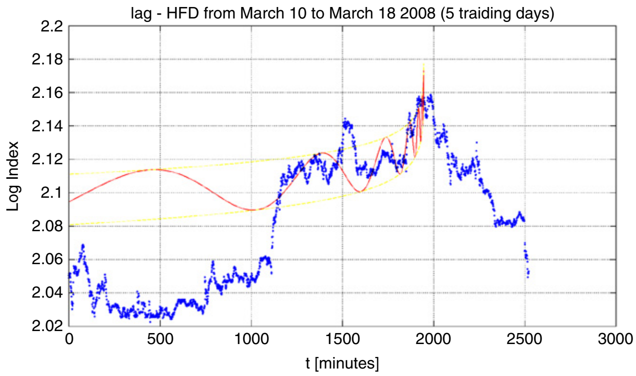

Fig. 7. IAG — The solid line represents the best fit with curve (1).

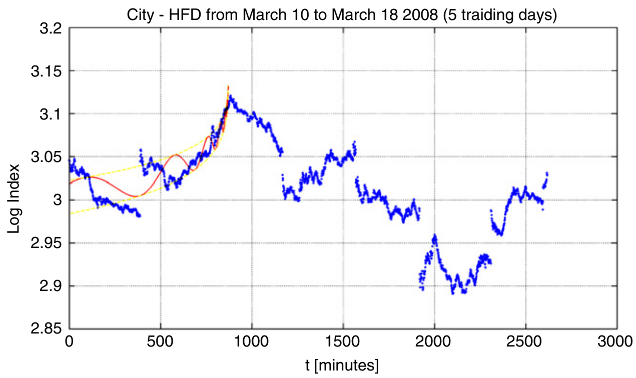

Fig. 8. City — The solid line represents the best fit with curve (1).

From previous studies [5], we knew that the Ising model may be applied to the study of financial indices sampled daily. The current work shows that the model may be applied to high frequency data as well and particularly to individual equity data. We also find that the precision of the estimated critical time is much better when using this type of data (Table 2). The range of prediction is of the order of few minutes and the relative accuracy is less than 1%.

We would like to reiterate that both applications have a connection to the diamond Ising model (see Fig. 2) which was made in Ref. [8]. The common structure in those cases are 2-state (*{buy, sell}*/phase $\{0, \pi\}$) entities (traders/oscillator) aligned in the diamond lattice.

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