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Linking market interaction intensity of 3D Ising type financial model with market volatility

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HIGHLIGHTS

- Utilize the three dimensional Ising dynamic system to develop a financial time series model.
- Interactions among market participants can lead to the herding behavior and price imbalance.
- The real market is consistently operating near the critical point of the system.

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ABSTRACT

Microscopic interaction models in physics have been used to investigate the complex phenomena of economic systems. The simple interactions involved can lead to complex behaviors and help the understanding of mechanisms in the financial market at a systemic level. This article aims to develop a financial time series model through 3D (three-dimensional) Ising dynamic system which is widely used as an interacting spins model to explain the ferromagnetism in physics. Through Monte Carlo simulations of the financial model and numerical analysis for both the simulation return time series and historical return data of Hushen 300 (HS300) index in Chinese stock market, we show that despite its simplicity, this model displays stylized facts similar to that seen in real financial market. We demonstrate a possible underlying link between volatility fluctuations of real stock market and the change in interaction strengths of market participants in the financial model. In particular, our stochastic interaction strength in our model demonstrates that the real market may be consistently operating near the critical point of the system.

1. Introduction

Financial market is difficult to model since it is a highly complex system and influenced by many factors such as policy, technology and environment. Identifying the most critical influence of the price fluctuations (defined as returns) in stock price is very important in understanding the mechanisms underlying the stock market. Econophysics became a quantitative approach using models and concepts coming from physics to deal with complex financial systems whose properties cannot be simply derived or predicted from the knowledge of initial states of these systems [1–13]. Both the microscopic analysis of interacting components [14–19] and the holistic analysis focusing on the macroscopic behavior are essential to better

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understand the current scientific modes of reasoning in this new field [20]. The interactions among the traders can lead to the herding behavior [21,22] and price imbalance [9,23,24], which are caused by sharing the similar views on the financial market among traders resulting in volatility clustering of return statistics. Simple microscopic interaction models such as Ising dynamic model [3,13,25–28], percolation model [8,29] and contact model [30,31] can simulate the stylized facts [32] and complexity of financial market, and those models allow a deeper understanding of mechanisms in the financial market. Based on the interaction behavior of individual market participants, those agent-based approaches have provided a qualitative understanding of price formation mechanisms [4–6]. But due to the insufficient quantitative accuracy, it has not yet been widely accepted by practitioners [1].

In this paper, financial market simulation is investigated using three-dimensional Ising model, which has been used to describe the interaction of spins in ferromagnetism in physics [33,34]. The classical spins are considered as the constituent microscopic magnetic moments that give rise to ferromagnetism. In the three dimensional Ising model, each spin localized on one of the sites of a lattice can point either 'up' or 'down', and the direction can be influenced by its six nearest neighbors. So in this study, we simulate a financial model with each market participant's (or trader's) opinion in the stock market being either '+' or '−', referring to their choice of 'buy' or 'sell' respectively. Its decision at time t can be influenced by the following factors: first, its decision at the previous time step $t-1$; secondly, the sum of its six nearest neighboring traders' decisions at the previous time step $t-1$; thirdly, the market interaction intensity at time t that may be affected by the 'market-temperature', which could be related to day's trading volume, financial news, political change and so on; fourth, the stochastic term of information from the stock market at time t, $\xi(t)$, and its value are either 1 or -1 with equal probability $P\{\xi(t)=1\}=0.5$ and $P\{\xi(t)=-1\}=0.5$. Stock market is composed of multiple agents interacting in such a way as to generate the macro-properties in the financial systems. Trading at the stock market takes place by matching bid and ask prices of traders, who can be either institutions or individual investors. Buy and sell orders at time t are placed in the exchange, and prices are determined through the interplay of supply and demand, so there are some researches about order-driven models for stock market [35,36]. If there is a big disequilibrium between supply and demand in the stock market, the stock price will fluctuate widely [24].

In 3D Ising model, there is an intrinsic critical point for the interaction strength parameter β_c. We show that when $\beta > \beta_c$, there is a convergence in opinions among traders, or herding behaviors that cause large fluctuations and crashes in the market. Connecting the interaction intensity β with the past price volatilities, we illustrate that the β value is consistently near the critical point. We also carry out further statistical analysis on the proposed modeling results with real market data by studying their distribution and long memory properties. The distribution properties are studied in particular, the long memory properties are studied using leptokurtosis [37] and fat-tail exponent [38], while the persistence properties are studied using composite multiscale entropy (CMSE) [39,40] and detrended fluctuation analysis (DFA), and we estimate the parameters of the proposed model that best resembles the empirical statistics.

2. Interacting agent based 3D Ising financial type model

One dimensional and two dimensional Ising models have already been successfully investigated in modeling financial model [3,13]. Though the analytical solution of critical point of 3D Ising model still remains open, the attempt to find the critical point using Monte Carlo simulations provides a rich understanding of the critical behavior of the 3D Ising model. To relate the financial market with 3D Ising model, we consider the Ising model on three-dimensional (3D) integer lattice $\mathbb{Z}^3$, and assume that $\Omega_{N^3}=\{1,2,\dots,N\}\times\{1,2,\dots,N\}\times\{1,2,\dots,N\}$ is the set of N^3 points, called sites on Z^3. We assume that each trader occupies one site of the cubic lattice, and holds either a positive opinion $(+1)$ or a negative opinion on the stock market (-1). Each agent trades based on the opinion. Their opinions switch between the two states according to the interactions rules. The investor's opinion may change through the influence of neighbors or exogenous environment [26,41,42].

The microscopic nearest-neighbor Hamiltonian of the 3D Ising model is defined by Ref. [34]

$$H_{N^3}(\delta_i) = -\sum_{j\in\mathbb{N}(\delta_i)} J_{ij}\delta_i\delta_j - h\delta_i \tag{1}$$

for each $\delta_i \in Z^3$. The nearest-neighbor is defined as follows, for δ_i at a fixed site (x,y,z) the neighborhood is $\mathbb{N}(x,y,z)=\{(x_1,y_1,z_1)\,|\,|x_1-x|+|y_1-y|+|z_1-z|=1\}$. For example, site (i,j,k)'s neighborhood is $\mathbb{N}((i,j,k))=\{(i-1,j,k),(i+1,j,k),(i,j-1,k),(i,j+1,k),(i,j,k-1),(i,j,k+1)\}$. The exchange coupling constant J_{ij} is the strength of internal interaction which indicates the nearest-neighbor exchange strength and we set $J_{ij}=1$ in this paper. h is a real number, which is the strength of an externally applied magnetic field. In our financial model, since we focus on how the interactions among investors influence the fluctuations of the stock prices, we simply consider the financial model without external influence. In this paper, the 3D Ising model with periodic boundary conditions is used throughout. The finite Gibbs state $\mu^{\beta}_{N^3}$ at inverse temperature $\beta=\frac{1}{k_BT}$, (where T is the temperature, k_B is the Boltzmann constant) is a probability measure given by

$$\mu^{\beta}_{N^3}(\delta) = [Z^{\beta}_{N^3}]^{-1}\exp\{-\beta H_{N^3}(\delta)\} \tag{2}$$

where $Z^{\beta}_{N^3}$ is called the partition function and is given by

$$Z^{\beta}_{N^3} = \sum_{\delta\in\Omega_\Lambda} \exp\{-\beta H_{N^3}(\delta)\}. \tag{3}$$

Since the returns at time t is controlled by the imbalance between the demand and supply of stocks, and the excess in total demand and supply moves the price up or down [23], we calculate the total number of positive opinion holders $\mathcal{N}^+$ and negative opinion holders $\mathcal{N}^-$. At the end of each trading day, the difference in the number of buy and sell trades leads to the price fluctuation of the stock market, so the stock price of the model at the time $t(t = 1, 2, \ldots)$ is defined as $S_t = e^{\alpha\xi(t)(\mathcal{N}^+(t)-\mathcal{N}^-(t))/\mathcal{N}} S_{t-1}$. $\xi(t)$ is the positive or negative information from the stock market at time t, and its value is either 1 or -1 with probability $P\{\xi(t) = 1\} = 0.5$ and $P\{\xi(t) = -1\} = 0.5$, independent of the sign of returns. S_0 is the initial price at time $t = 0$, where $\alpha > 0$ is the depth parameter of the market and $\mathcal{N}$ is the total number of traders, which is $N \times N \times N$ in this paper. Then we have

$$S_t = S_0 \exp\left\{\sum_{k=1}^{t} \frac{\alpha\xi(t)(|\mathcal{N}^+(t) - \mathcal{N}^-(t)|)}{\mathcal{N}}\right\}. \tag{4}$$

The formula of the single-period stock logarithmic price return from $t - 1$ to t is given by

$$r(t) = \ln S_t - \ln S_{t-1}. \tag{5}$$

In order to better understand the effect of market interaction intensity, we set the depth parameter $\alpha = 1$. Although simple, this financial model contains the some essential features of traders: stochasticity and interaction.

3. Monte Carlo simulation of 3D Ising financial market

In this section, we simulate the 3D Ising financial model for both constant β and stochastic β, and compare our results with the real stock market's historical return time series. We choose the data Hushen 300 index (HS300) from Chinese stock market, which is the weighted sum of the market capitalizations of 300 companies representative of the Chinese economy. We analyze the fluctuations—measured by the daily logarithm returns of the closing price that cover more than 10 years from January 2005 to September 2015. The total number of data points in this period is 2609. The simulation steps [43] are as follows:

Step 1. Randomly set the opinion of each investor in the stock market (at each site of $N \times N \times N$ cubic lattice) of randomly oriented opinion of buying $(+1)$ or selling (-1) their stock with probabilities $p_1 = p_{-1} = 0.5$ at the beginning of each day.

Step 2. Determine whether each investor should change its opinion about the stock market using the following criteria: check the influence from its neighbors, whether $\frac{\exp\{-\beta(\delta_{(x,y,z)}\sum \delta_{\mathbb{N}(x,y,z)})\}^2}{1+\exp\{-\beta(\delta_{(x,y,z)}\sum \delta_{\mathbb{N}(x,y,z)})\}^2}$ is greater than a random number between 0 and 1. If this condition is satisfied, the investor would flip its opinion; otherwise, it would keep its previous decision. After all of the traders on the N^3 lattice have decided on their trading decision, we move on to the next step.

Step 3. Execute step 2 for m times. Each time interval represents one trading minute. During that time, each trader is given one chance to change its opinion state. 240 steps constitute one trading day, corresponding to 4 h in Chinese market, i.e. $m = 240$. The market interaction intensity is fixed for each day.

Step 4. Randomly choose $\xi(t)$ from $+1$ or -1 with probability $P\{\xi(t) = 1\} = 0.5$ and $P\{\xi(t) = -1\} = 0.5$ for each day.

Step 5. After each day, we calculate new lattice configuration value of all the opinions of investors. Since we suppose each trader has the same unit of stock being traded, the return time series are constructed from Eqs. (4) and (5) based on the trading actions of interacting agents.

3.1. Simulation of 3D financial market with constant market interaction intensity β

The 3D Ising model exhibits phase transition, i.e., there is a critical point $\beta_c > 0$. At the equilibrium state, as β approaches the critical inverse temperature β_c from below, spin fluctuations are present at all scales of length. But for $\beta > \beta_c$, as β increases, the interaction among neighbors begins to extend. These changes take the form of spin fluctuations, which are exhibited by most spins pointing in the same direction and there are two distinct pure phases, so most of the investors are short-term investors and they change their opinion quickly after being influenced by their neighbors. If this situation lasts for a long time, it will lead to the crashes and crises of the financial markets. Ref. [44], found the empirical fact that large price swings occur when the preponderance of trades have the same buy or sell decision, and most agents sell their stocks during the spreading panic of market crashes and buy their stocks during the stock market boom period.

Fig. 1 displays a sketch of configurations with $N = 6$, $m = 240$ and $day = 1$ for $\beta < \beta_c$ and $\beta > \beta_c$ in Fig. 1(a) and (b) respectively. When $\beta = 0.19$, $\beta < \beta_c$, there is a balance between supply and demand, but when $\beta = 0.3$, $\beta > \beta_c$, the buyers are dominating the population. More general results are presented in Fig. 2. We find that as the market interaction intensity β increases from 0.01 to 0.35, the range of imbalance between supply and demand and its volatility become larger, and this is more pronounced near the critical point β_c of 3D Ising system. The range is smaller when the market interaction

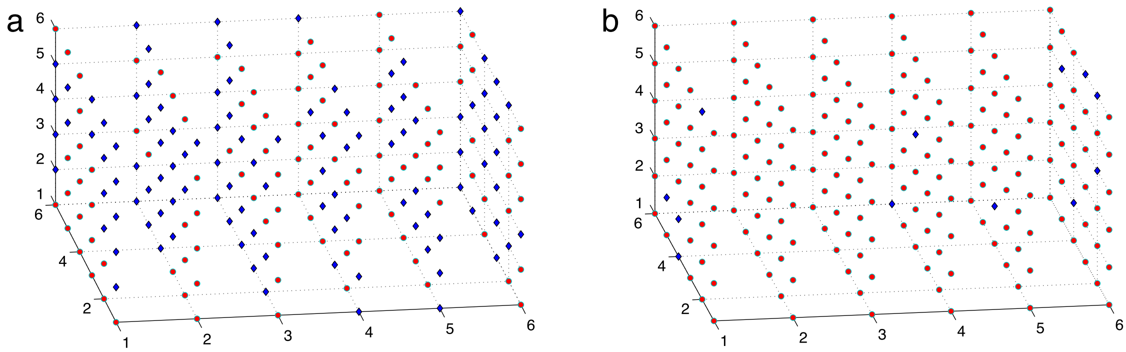

Fig. 1. Traders with their trade opinion under constant market interaction intensity β with $N = 6$, $m = 240$, $day = 1$. The red circles represent buyers while the blue diamonds represent sellers. (a) A configuration in the set $\beta = 0.19$, $\beta < \beta_c$. (b) A configuration in the set $\beta = 0.3$, $\beta > \beta_c$.

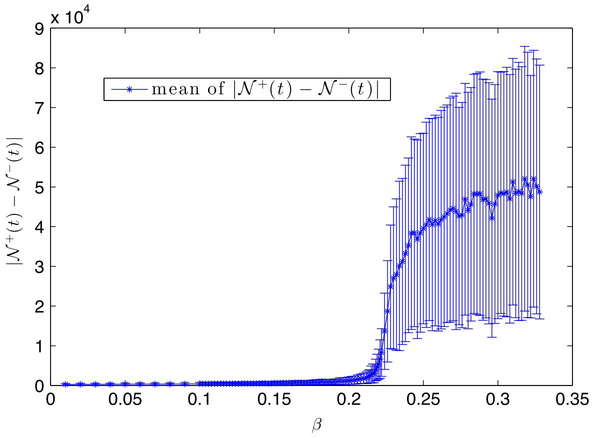

Fig. 2. The error bar of imbalance between demand and supply of stocks plotted against the market interaction intensity β. The bars are the standard errors. The financial model is simulated for 400 times and the mean and standard deviation are obtained for each β, where $N = 50$, $m = 240$, $day = 1$.

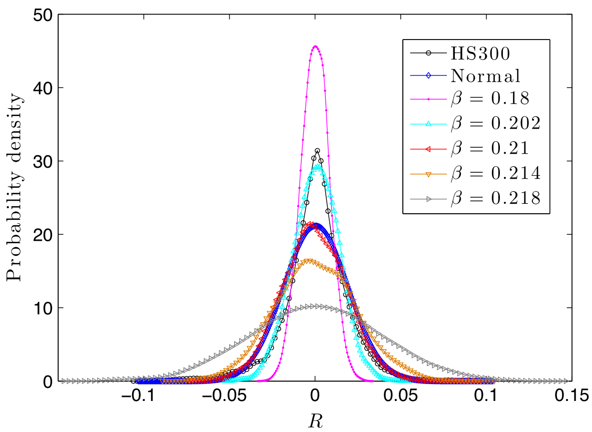

Fig. 3. The probability density function (PDF) of logarithmic returns of HS300 Index, the normal distribution with the same mean and standard deviation as HS300 returns, and the financial model with constant market interaction intensity $\beta = 0.18$, $\beta = 0.202$, $\beta = 0.21$, $\beta = 0.214$, $\beta = 0.218$, where $N = 50$, $m = 240$, $day = 2609$.

intensity β is far smaller or bigger than β_c. Furthermore, we obtained the probability density of return time series of the financial model with $\beta = 0.18$, $\beta = 0.202$, $\beta = 0.21$, $\beta = 0.214$, $\beta = 0.218$ and also give HS300 returns and the normal distribution with the same mean and standard deviation as HS300 returns for comparison between simulation and empirical data. From Fig. 3, we observe that as the market interaction intensity increases, the peak becomes lower and the tail becomes fatter. Though when $\beta = 0.21$, its distribution fits the normal distribution well, but still far away from HS300 of real stock market.

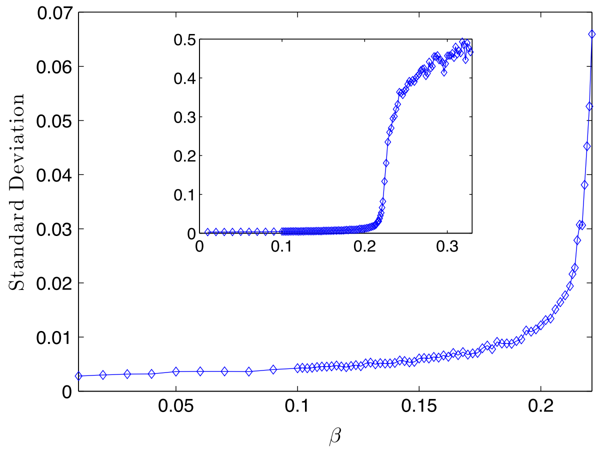

Fig. 4. The standard deviations of return time series change with the market influence intensity β. The Ising financial model is simulated for 400 times, and the standard deviation is obtained for each β, where $N = 50$, $m = 240$, $day = 1$.

3.2. Simulation of 3D Ising type financial market with stochastic market interaction intensity β

In this subsection, we simulate the financial market with stochastic market interaction intensity β which is an influencing factor of market volatility. We have support for this assumption for the following reasons. First, in empirical results, it has been shown that larger previous volatility in the last few days brings traders' opinions closer to each other, resulting in large subsequent price fluctuation [1]. Secondly, the daily price change is not deterministic, but heavily depends on the number of spin values, which are affected by the interaction intensity in financial market. Thirdly, there is volatility clustering, described as big fluctuation often followed by a series of heavy fluctuations and small change of price often occurs after a small fluctuation in the stock market. Fourthly, the standard deviation of simulating returns is related to the market interaction intensity. Fig. 4 shows that as the market interaction influence β increases, the standard deviation of financial market return time series is increasing. When β is below 0.2, it increases slowly; and when β is around its critical point $\beta_c = 0.2217$ [33], for example from 0.21 to 0.24, it increases drastically; it becomes slow again after $\beta > 0.24$. So we connect the intensity of interaction among the market investors $\beta(t)$ with the previous Δt days volatility of real stock market. In this paper, we choose Δt as 3 days, 5 days, 10 days, 15 days and 20 days. When simulating the financial market, we use the following mechanism to determine the interaction strength at time t.

$$\beta(t) \sim \begin{cases} U(0.000, 0.090), & 0 < \sigma_{t-\Delta t,t-1} \leq 0.004 \\ U(0.090, 0.192), & 0.004 < \sigma_{t-\Delta t,t-1} \leq 0.01 \\ U(0.192, 0.212), & 0.01 < \sigma_{t-\Delta t,t-1} \leq 0.02 \\ U(0.212, 0.215), & 0.02 < \sigma_{t-\Delta t,t-1} \leq 0.03 \\ U(0.215, 0.218), & 0.03 < \sigma_{t-\Delta t,t-1} \leq 0.04 \\ U(0.218, 0.219), & 0.04 < \sigma_{t-\Delta t,t-1} \leq 0.05 \\ U(0.219, 0.220), & 0.05 < \sigma_{t-\Delta t,t-1} \leq 0.06 \\ U(0.220, 0.221), & 0.06 < \sigma_{t-\Delta t,t-1} \leq 0.07 \\ U(0.221, 0.222), & 0.07 < \sigma_{t-\Delta t,t-1} \leq 0.08 \end{cases}$$

where $\sigma_{t-\Delta t,t-1}$ is the last Δt days volatility of real stock market, given by $\sigma_{t-\Delta t,t-1} = \sqrt{\frac{1}{\Delta t}\sum_{i=1}^{\Delta t}(r_{t-i} - \bar{r})^2}$, and $\bar{r} = \frac{1}{\Delta t}\sum_{i=1}^{\Delta t} r_{t-i}$.

Fig. 5 plots the probability density and Fig. 6 plots the return time series for both simulation and real stock market. By comparing the market interaction intensity with the volatility of real stock market, the simulation returns and its probability are much more similar to the real stock market. In particular, Fig. 5 presents the sharp peak and heavy tail as in real stock market, especially when $\Delta t = 10$, $\Delta t = 15$ and $\Delta t = 20$. More detailed comparison will be provided in Section 4.

4. Stylized facts of 3D Ising type financial market and real stock market

Stylized facts [11,32], which are exhibited in financial return time series but cannot be found in equilibrium models, include leptokurtosis, volatility clustering and multi-fractal properties [27]. In this section, we investigate the stylized facts of the financial model with constant market interaction intensity, stochastic market interaction intensity and HS300 index from Chinese market. Such analysis could give us a better understanding of the features of both our financial model and the real stock market.

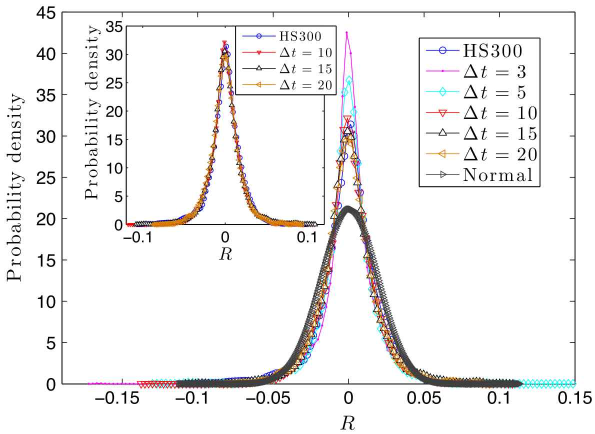

Fig. 5. The probability density function (PDF) of logarithmic returns of HS300 Index, the normal distribution with the same mean and standard deviation as HS300 returns, and the financial model with stochastic market interaction intensity defined on average volatilities of previous time steps for $\Delta t = 3$, $\Delta t = 5$, $\Delta t = 10$, $\Delta t = 15$, $\Delta t = 20$. Here $N = 50$ and $m = 240$. The inset is the PDF of logarithmic returns of HS300 Index, and simulation with $\Delta t = 10$, $\Delta t = 15$ and $\Delta t = 20$.

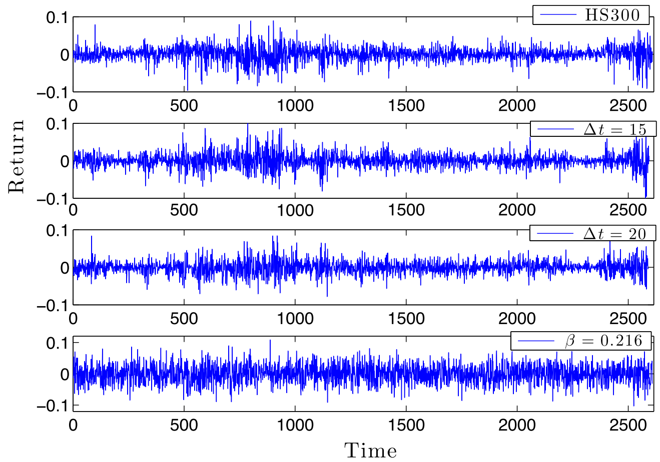

Fig. 6. The return time series of HS300, stochastic markets interaction intensity with $\Delta t = 15$, $\Delta t = 20$ and constant market interaction intensity $\beta = 0.216$, where $N = 50$ and $m = 240$.

4.1. Statistical properties of financial model and real stock market

The statistical properties include mean, standard deviation, maximum, minimum, kurtosis and skewness. Mandelbrot [37] pointed out the insufficiency of the normal distribution for modeling the marginal distribution of asset returns and their heavy-tailed characteristic. Since then the "fat-tail" phenomenon, which is a non-Gaussian characteristic of the distribution of price fluctuations has been repeatedly observed in various market data, see Refs. [25,27]. One way to quantify the deviation from the normal distribution is the kurtosis and skewness of the distribution defined as

$$kurtosis = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^4}{(n-1)\sigma^4}, \qquad skewness = \frac{\sum_{t=1}^{n}(r_t - \bar{r})^3}{(n-1)\sigma^3}$$

where r_t denotes the return of tth trading day, $\bar{r}$ is the mean of r, n is the total number of the data points, and σ is the corresponding standard deviation. It is known that the kurtosis of the Gaussian distribution is 3 and the skewness of standard normal distribution is 0. The kurtosis shows the centrality of data and the skewness shows the symmetry of the data. Higher kurtosis is associated with more frequent tail data. The kurtosis of the real markets is usually larger than 3 and skewness is away from 0 for empirical stock price data. The statistical properties for constant market interaction intensity and HS300 returns are displayed in Table 1. When $\beta < \beta_c$, the value of kurtosis is similar to normal distribution, but when $\beta > \beta_c$, the kurtosis is smaller than normal distribution due to the lack of returns in the center of probability density. $\max\{|\max|, |\min|\}$ of returns are increasing as β increases, which means the increasing market interaction intensity leads to the collective behavior of the interacting individuals. In Table 2, $\bar{\beta}$ is the average of stochastic β, and $\hat{\beta}$ is the median

Table 1
Statistical properties of constant market interaction intensity and real stock market.

Stock	*Mean*	*std*	*max*	*min*	*Kurtosis*	*Skewness*
$\beta = 0.16$	−0.0001	0.0066	0.0244	−0.0248	3.0488	−0.0236
$\beta = 0.18$	0.0000	0.0083	0.0282	−0.0281	2.9355	−0.0176
$\beta = 0.2$	−0.0001	0.0125	0.0421	−0.0370	2.8906	0.0418
$\beta = 0.202$	0.0005	0.0133	0.0455	−0.0439	2.9720	−0.0820
$\beta = 0.204$	0.0001	0.0141	0.0488	−0.0504	3.0732	0.0186
$\beta = 0.206$	0.0001	0.0156	0.0537	−0.0516	2.9449	0.0336
$\beta = 0.208$	−0.0003	0.0160	0.0552	−0.0641	3.1412	0.0170
$\beta = 0.21$	−0.0005	0.0187	0.0834	−0.0615	3.1271	0.0560
$\beta = 0.212$	0.0006	0.0203	0.0658	−0.0700	2.8976	−0.0065
$\beta = 0.214$	0.0004	0.0237	0.0891	−0.0736	2.9882	0.0722
$\beta = 0.216$	−0.0008	0.0286	0.1083	−0.1032	3.0614	−0.0110
$\beta = 0.218$	0.0003	0.0380	0.1212	−0.1217	2.9429	−0.0258
$\beta = 0.22$	−0.0003	0.0557	0.1782	−0.2142	3.0449	−0.0338
$\beta = 0.23$	−0.0023	0.2583	0.5520	−0.5555	2.1500	−0.0114
HS300	0.0005	0.0189	0.0893	−0.0970	6.1115	−0.4638

Table 2
Statistical properties of stochastic market interaction intensity and real stock market.

Stock	*Length*	*Mean*	*std*	*max*	*min*	*Kurtosis*	*Skewness*	$\bar{\beta}$	$\hat{\beta}$
$\Delta t = 3$	2607	−0.0001	0.0172	0.0897	−0.1656	11.1027	−0.9675	0.1739	0.1982
$\Delta t = 5$	2605	0.0001	0.0176	0.1559	−0.1225	10.5511	−0.1002	0.1857	0.2010
$\Delta t = 10$	2600	0.0003	0.0178	0.0997	−0.1287	6.6292	0.0428	0.1943	0.2037
$\Delta t = 15$	2595	0.0003	0.0187	0.0998	−0.0991	6.1824	0.2219	0.1964	0.2043
$\Delta t = 20$	2590	−0.0003	0.0176	0.0840	−0.0780	4.9631	0.1047	0.1976	0.2044
HS300	2609	0.0005	0.0189	0.0893	−0.0970	6.1115	−0.4638		

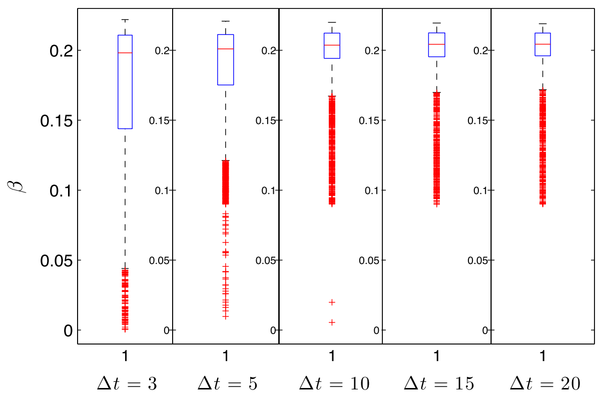

Fig. 7. The box plot of stochastic market interaction intensity β with $\Delta t = 3$, $\Delta t = 5$, $\Delta t = 10$, $\Delta t = 15$, $\Delta t = 20$.

of stochastic β. For the simulation data with $\Delta t = 10$ and $\Delta t = 15$, the mean and median of stochastic β are much closer to critical point of Ising model and become stable as Δt increases, this phenomenon is illustrated in Fig. 7, the box plot of dispersion of our dataset β. Table 2 shows that increasing time length Δt decreases the kurtosis and increases $\max\{|\max|, |\min|\}$ of returns. The results of $\max\{|\max|, |\min|\}$, standard deviation and kurtosis with $\Delta t = 15$ are closer to HS300.

Since the simulation data with $\Delta t = 15$ is the closest to empirical data, and the effective β value is very close to the critical value β_c, we see that the real market behaves near the critical point of the system.

4.2. Power law and volatility clustering

Power-law distributions occur frequently in natural and social fields. A few notable examples are Pareto's law for income distributions, behavior near a second-order phase transition and Zipf's law. So the tail distributions of market returns can be defined as [44,45]

$$P(|r(t)| > x) \sim \frac{1}{x^{\alpha}}.$$

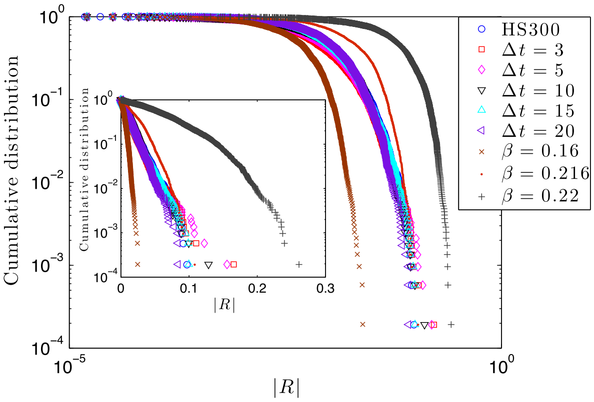

Fig. 8. The cumulative distribution of absolute returns for the HS300, simulation with stochastic market interaction intensity $\Delta t = 3$, $\Delta t = 5$, $\Delta t = 10$, $\Delta t = 15$, $\Delta t = 20$ and constant market interaction intensity $\beta = 0.16$, $\beta = 0.216$ and $\beta = 0.22$.

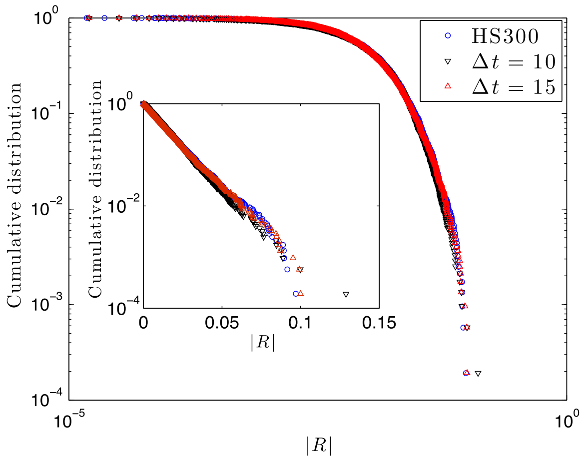

Fig. 9. The cumulative distribution of absolute returns for simulation with $\Delta t = 10$, $\Delta t = 15$ and the HS300. The simulation reproduces the return distribution well.

Table 3
Power law and long memory behavior of returns.

Stock	$s = 10\%$	$s = 7\%$	$s = 5\%$	$s = 3\%$	$s = 1\%$	H
HS300	−3.4727	−3.7655	−4.1756	−4.6462	−8.3455	0.5322
$\Delta t = 10$	−3.5109	−3.7805	−4.0728	−4.4020	−4.9792	0.4832
$\Delta t = 15$	−3.4303	−3.7714	−4.2388	−4.8734	−6.2668	0.5049
$\Delta t = 20$	−3.9880	−4.4580	−4.8783	−5.4209	−6.8330	0.4379

The main feature of this function is its invariance of scale. In other words, the shape of the function is preserved under re-scaling. Power-law distributions show no typical scale or size, and in some cases they are related to fractals, which also lack typical scales. In Fig. 8, the log–log plot and the semi-log plot of cumulative distributions of simulated returns for constant market interaction intensity $\beta = 0.16$, $\beta = 0.216$ and $\beta = 0.22$ and stochastic market interaction intensity $\Delta t = 3$, $\Delta t = 5$, $\Delta t = 10$, $\Delta t = 15$, $\Delta t = 20$, and the corresponding plots of returns of HS300 are shown. As the constant market interaction intensity β increases, the tail gets fatter. As the memory Δt increases, the tail gets thinner and converges around $\Delta t = 20$. Fig. 9 gives the log–log plot and the semi-log plot of cumulative distributions of simulated returns for HS300 and stochastic market interaction intensity $\Delta t = 10$, $\Delta t = 15$, those three curves match well. In Table 3, the values of exponents α are given for returns both from HS300 and simulation data which are much closer to the real stock market with $\Delta t = 10$, $\Delta t = 15$ and $\Delta t = 20$. The tail exponents α are measured for $s = 10\%$, $s = 7\%$, $s = 5\%$, $s = 3\%$ and $s = 1\%$ of the tail region respectively. As Δt increases, there is a tendency to have fatter tail. For fixed Δt, as the tail of region s becomes smaller, the tail also gets fatter. These empirical results show that the price model with stochastic market interaction intensity is in accordance with the real market to some degree, especially when $\Delta t = 15$.

In this subsection, we also explore the volatility clustering property of the absolute returns and squared returns of HS300 and simulation data with $\Delta t = 15$. Fig. 10(a) shows the non-persistence behavior of raw returns. In Fig. 10(c)(e), the absolute

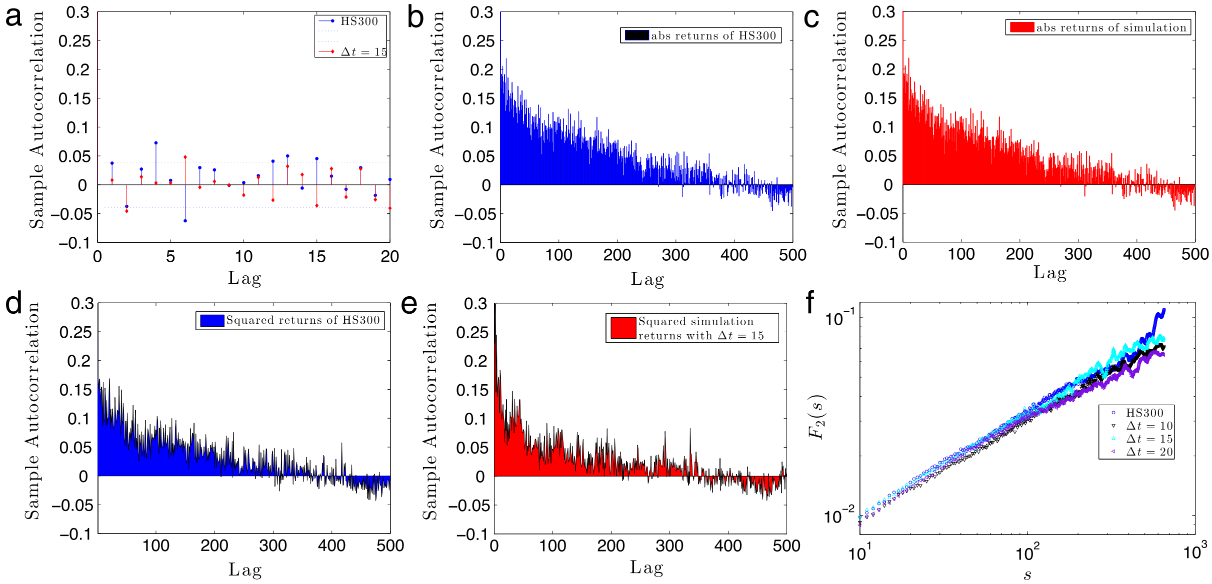

Fig. 10. The volatility clustering behavior of absolute returns and squared returns. (a) The autocorrelation of returns for HS300 and simulation with $\Delta t = 15$. (b) The autocorrelation of absolute returns for HS300. (c) The autocorrelation of absolute returns for simulation with $\Delta t = 15$. (d) The autocorrelation of squared returns for HS300. (e) The autocorrelation of squared returns for simulation with $\Delta t = 15$. (f) The log–log plot of DFA result on $F_2(s)$ versus s for the real stock markets and the simulation data with $\Delta t = 10$, $\Delta t = 15$ and $\Delta t = 20$.

returns and squared returns of simulation data with $\Delta t = 15$ show the similar behavior of decay pattern with increasing time lag. Those characteristics are similar in real stock market presented in Fig. 10(b)(d). To further investigate the stylized facts in the proposed model, we use the DFA method to study the long memory behavior of financial returns of the model and the empirical data. DFA method [27,46] can determine the Hurst index of time series. If the Hurst index is larger than 0.5, the fluctuations are persistent; it is anti-persistent if the Hurst index is smaller than 0.5. DFA-3 is used to investigate the long-range correlations in the return series in the following analysis. In Table 3, H represents the Hurst index, and the values are 0.5322, 0.4832, 0.5049 and 0.4379 for HS300, the simulation data with $\Delta t = 10$, $\Delta t = 15$ and $\Delta t = 20$ respectively. The results indicate that HS300 and the simulation data with $\Delta t = 15$ show very similar persistence in return volatilities. Fig. 10(f) illustrates the results of this phenomenon.

4.3. Composite multiscale entropy (CMSE)

Measuring the complexity of nonlinear dynamic process like financial time series is one of the most significant aspects to explore the nonlinear nature of financial markets. Multiscale Entropy (MSE) approach was developed to quantify complexity of financial market [39,47] and also in other areas such as Physiological time series [48,49]. If the entropy is smaller, it means the time series is more regular and more predictable; if it is greater, it means more randomness in the time series. Recently, Composite Multiscale Entropy [39,40] (CMSE) is examined to be more stable in measuring complexity than MSE, especially in short time series. So in this section, we use CMSE methods to analyze the complexity of the return time series $R(t)$ and squared return time series $(R(t))^2$ of HS300 index and simulation data.

First, we introduce the methods of CMSE.

(i) For a given time series $\mathbf{x} = \{x_1, x_2, \ldots, x_N\}$ (N is the length of $\mathbf{x}$), we get the kth coarse-grained time series $\mathbf{y}_k^{(\tau)} = \{y_{k,1}^{\tau}, y_{k,2}^{\tau}, \ldots, y_{k,p}^{\tau}\}$ of time scale factor τ by dividing the original time series into non-overlapping windows of length τ and averaging data points inside each window:

$$y_{kj}^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_i, \quad 1 \le j \le \frac{N}{\tau}, \ 1 \le k \le \tau.$$

(ii) The sample entropy (SampEn) is calculated for each coarse-grained time series within a tolerance $\epsilon = 0.15$ and $m = 2$. It is defined as the negative logarithm of the conditional probability that two sequences of m consecutive data points which are similar to each other will remain similar when the next point is included. So it is computed as

$$SampEn(\mathbf{y}_k^{(\tau)}, m, \epsilon) = -\log\left(\frac{n_{k,\tau}^{m}}{n_{k,\tau}^{m+1}}\right).$$

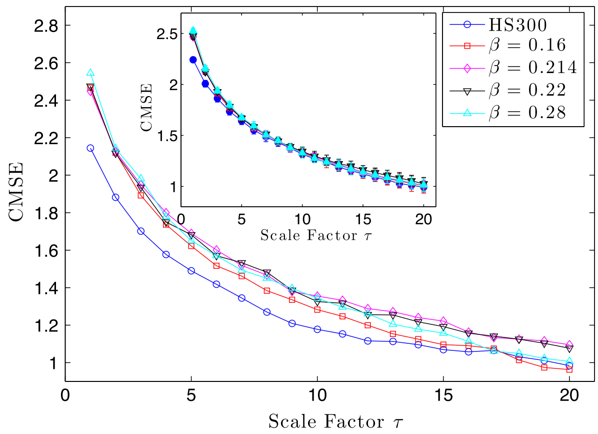

Fig. 11. The CMSE of returns of HS300 and constant market interaction intensity. The smaller figure inside is the CMSE of corresponding shuffled return time series with error bar. The bars are the standard errors.

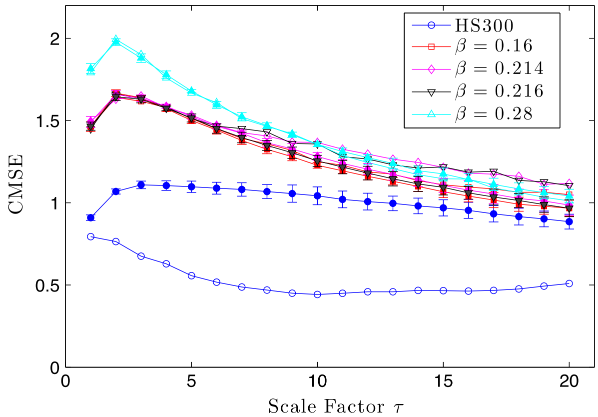

Fig. 12. The CMSE of squared returns of HS300 and constant market interaction intensity. The solid ones are the CMSE of corresponding shuffled squared return time series with error bar. The bars are the standard errors.

(iii) Then the composite sample entropies are given by the CMSE algorithm as in Ref. [39].

$$CMSE(\mathbf{x}, \tau, m, \epsilon) = \frac{1}{\tau} SampEn(\mathbf{y}_k^{(\tau)}, \tau, m, \epsilon).$$

In Fig. 11, for the constant market interaction intensity, the trends in the complexity of model results and the HS300 returns are similar: as the scale factor τ increases, complexity decreases. But the complexity of simulation data is larger than HS300 returns, which means it is more disordered than the real stock market. For the shuffled time series (shuffle their order for 10 times), the complexity of both real stock market and simulations is almost the same, which means the complexity may be caused by the memory of the returns. For the complexity of squared shown in Fig. 12, the complexity of simulation stays almost the same for both original ones and shuffled ones, which are both bigger than the real stock market. To investigate the effect of Δt on the complexity measure, we test the CMSE on a set of returns and squared returns and shuffled them from stochastic market interaction intensity of different Δt in Figs. 13 and 14. Only the complexity of shuffled squared returns has an increasing trend as the time scale Δt increases. The shapes of figures are fitted with the real stock market well.

5. Conclusion

In this work, the three dimensional Ising model is applied to develop a model of stock price fluctuation. We simulate the underlying mechanisms behind fluctuation and volatility in financial market through nearest neighbor interactions in this configuration. The statistical resemblance with real data indicates that our model is a plausible qualitative explanation for the stylized fact and complexity found in the simulations is in reasonable agreement with the complex stock market, showing that despite its simplicity the discrete model of interaction heterogeneity spins analyzed in this work displays behavior similar to that seen in nature. This is the first time using 3D Ising model to simulate financial market, though the dynamics of the financial model is not drastically different from one dimensional and two dimensional Ising model, this

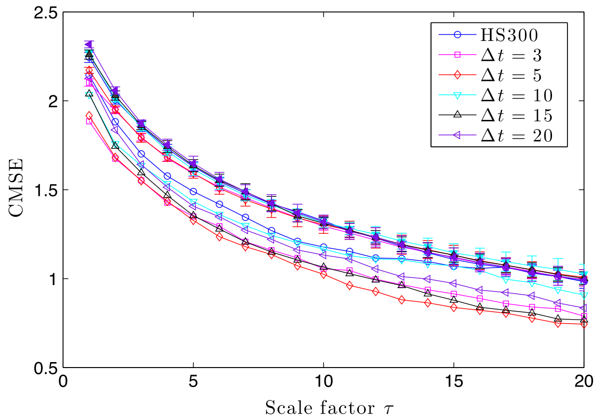

Fig. 13. The CMSE of returns of HS300 and stochastic market interaction intensity. The solid ones are the CMSE of corresponding shuffled return time series with error bar. The bars are the standard errors.

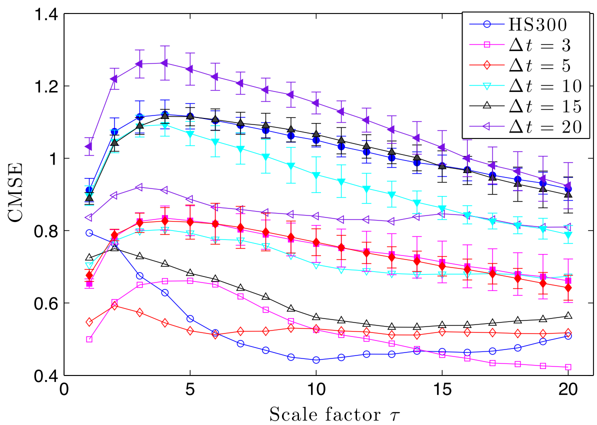

Fig. 14. The CMSE of squared returns of HS300 and stochastic market interaction intensity. The solid ones are the CMSE of corresponding shuffled squared return time series with error bar. The bars are the standard errors.

extends the validity of the Ising model to higher dimensions when applied to the context of financial market. We find that the critical phenomenon plays an important role in fluctuation behavior and complexity dynamics in financial modeling. The most intriguing finding in this work is the implication that the financial market is constantly operating around the critical point. As our model of stochastic interaction strength β has shown, the empirical statistics resembles most closely that of the 3D Ising model when β is fluctuating narrowly around the critical point β_c. This resonates with the view that the critical behavior observed in financial market is indeed a manifestation of the system being near the critical point.

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