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The oscillation of stock price by majority orienting traders with investment position

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Abstract

We consider an interacting particle system for the stock price fluctuation. The change of the stock price with a feedback by the price considering the herding behavior (majority orienting behavior) of traders, gives the van der Pol equation as a deterministic approximation. Considering the investment position of each trader, we introduce the delayed van der Pol equation. The history of investment positions, for example sell or buy, of each trader for a stock makes a memory effect, which is modeled by using the time retardation. The delayed van der Pol equation model seems to be natural and explains typical phenomena, for example triangle pattern, volatility jumps, price jumps and price trends, known for the time series of a stock price.

Keywords: Stock price; Delayed van der Pol equation; Poincare–Bendixson theorem; Behavioral finance

1. Introduction

Using the Ising model, the analysis of the market has been discussed in Refs. [1,2]; besides, modeling the financial market by a certain form of Ising structure of the interactions of agents seems to be successfully achieved in several studies [3,4]. We think that most traders are influenced by rumors, excessively or under excessively react to the information, and like the subjective desirability more than the objective probability [5,6]. The minority traders of a market, who are diffident to their investment position in many cases, are going to follow the decision of the majority. Because they tend to think that the majority of the traders have more accurate information than themselves. A majority orienting model [7] is introduced, which is composed of three elements: the mutation of dealers, the majority rule and the feedback by the price, as basic elements for the change of a stock price in a real market. This model is a ternary interaction model of a finite particle, which makes excursions that are similar to the Ising model [8], assuming a mutation to the other type for each particle. The van der Pol equation is obtained as a deterministic approximation, which seems to explain the oscillation of a stock price.

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Traders make use of information from the history of a stock price in order to gain profits by dealing stocks. We develop the majority orienting model taking into account of the feedback rule considering the history of buying then selling of traders and introduce a delayed van der Pol equation. Our present model makes the majority orienting model more realistic and helps to understand the dynamics for the change of the stock price. Our model seems to give an explanation for the typical phenomena, known for the chart analysis of the time series of a stock price, which is commonly used by traders; for example, triangular pattern volatility jumps, price trends and price jumps, those are perturbed by random noise in a real market.

2. The majority orienting model

The van der Pol equation is obtained from the majority orienting model for the change of a stock price [7]. In the model there are two types of particles in a box plus (+) and minus (−), whose numbers are N_+ and N_-, respectively, with $N = N_+ + N_-$. Let each trader be considered to be a particle in the box and change his position at random by the following step, with three substeps (1), (2) and (3), which are successively applied to the particles in the box. Here a + particle represents a bullish (feeling confident about the future stock price) trader, while a − particle represents a bearish (feeling pessimistic about the future stock price) trader.

(1) *Mutation rule*: One particle out of N particles is chosen at random. It changes its sign to the opposite sign with probability m and does not change with probability $1-m$, $(0 \leqslant m \leqslant 1)$.

(2) *Majority rule*: Three particles are taken at random. If two of the particles taken have the sign + and one has the sign −, the one with − changes its sign to + and the price S increases by 1, while, if two of the particles have the sign − and one has +, the one with + changes to − and the price S decreases by 1. If the three particles have the sign +, no change of sign occurs for the three particles and the price S increases by 3, while, if the three particles have −, no change of sign occurs for the three particles and the price S decreases by 3.

(3) *Feedback rule*: If S is positive, N_+ is decreased by 1 with probability S/N, while, if S is negative, N_+ is increased by 1 with probability $-S/N$. The absolute value of S can be larger than N when m is small. We only discuss the case of $|S| \leqslant N$ in this section. This condition is almost valid when $m \geqslant 0.75$.

Let us represent N_+ and S at step s as $N_+(s)$ and $S(s)$, respectively. Assuming that the duration of a step is τ, and the values of $N_+(s)$, $N_-(s)$ and $S(s)$ are given, we have the following expected values:

$$E\left[\frac{N_+(s+1)-N_+(s)}{\tau N}\right] = m\left\{-\frac{N_+(s)}{N}+\frac{N_-(s)}{N}\right\} + 3\frac{N_+(s)(N_+(s)-1)N_-(s)}{N(N-1)(N-2)} - 3\frac{N_+(s)N_-(s)(N_-(s)-1)}{N(N-1)(N-2)} - v\frac{S(s)}{N} \tag{1}$$

$$E\left[\frac{S(s+1)-S(s)}{\tau N}\right] = 3\frac{N_+(s)(N_+(s)-1)(N_+(s)-2)}{N(N-1)(N-2)} + 3\frac{N_+(s)(N_+(s)-1)N_-(s)}{N(N-1)(N-2)} - 3\frac{N_-(s)(N_-(s)-1)N_+(s)}{N(N-1)(N-2)} - \frac{N_-(s)(N_-(s)-1)(N_-(s)-2)}{N(N-1)(N-2)}. \tag{2}$$

When N is sufficiently large, we obtain the deterministic approximation Eqs. (3) and (4), putting $(N_+(s)-N/2)/N$ as x_t and $S(s)/N$ as y_t, taking an appropriate time scale τ as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}x_t &= -2mx_t + 6x_t\left(\frac{1}{2}+x_t\right)\left(\frac{1}{2}-x_t\right) - vy_t \\ &= -2\left(m-\frac{3}{4}\right)x_t - 6x_t^3 - vy_t,\end{aligned} \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y_t = 6x_t. \tag{4}$$

Assuming the number of the particles N is large enough, we can neglect the random sampling effect of particles, while in the real market the stock price y_t is perturbed by random noise. Hence, we have the following system of stochastic differential equations:

$$\mathrm{d}x_t = \left\{-2(m - \tfrac{3}{4})x_t - 6x_t^3 - vy_t\right\}\mathrm{d}t, \tag{5}$$

$$\mathrm{d}y_t = 6x_t\,\mathrm{d}t + \theta\,\mathrm{d}w_t, \tag{6}$$

where w_t is the standard Brownian motion. This system will be applied to the real stock market by using Kalman filter taking an appropriate state space representation of local linearization [9].

3. Delayed van der Pol equation

Rule 1 in the previous section models the trader's random change of his investment attitude "bullish" or "bearish". Rule 2 models traders' majority orienting behavior. Each trader makes effort to get good data or information in order to predict the stock price. But the information on a stock is heterogeneously distributed among traders in a real market. For example, it is natural to think that a holder of a stock has more accurate information on the stock than a non-holder of the stock. The minority traders of a market, who are diffident to their investment position in many cases, are going to follow the decision of the majority, because they tend to think that the majority of the traders have more accurate information than themselves. The majority orienting behavior of traders is called "herding" and "information cascade" phenomenon is referred to as one of the structural factors of the collapse (crash) of the price often observed in the stock market [10]. Let us consider on the feed back force of rule 3. The term $-vS(s)/N$ of Eq. (1) represents rule 3, which means the stock price gets the feedback force proportional to the excess over the standard price which is assumed to be 0 in the model given in Ref. [7], where the excess takes positive or negative real value. Rule 3 assumes that traders have the consensus of the standard price of a stock and have a tendency to sell and buy to compensate the excess. Even if the standard price exists, it is almost impossible to assume that all traders know it.

It is natural to think that the price, which each trader refers to, should be the price, which calculates the gain-or-loss of the position instead of zero. We assume that the book value of a stock to be the reference price, and that the power of pull back is proportional to the difference of book value and current price. Assume each trader refer the price of a stock of the time at $s-u$ at time s, namely the holding period (investment period) of each trader of a stock to be u. Consider the case of a book value is higher than a current price, a trader with buy-position tends to change his position to "bearish" and going to sell to take a profit. In the case of a book value is lower than a current price, he maintains "bullish". When a trader has the sell position, this relation becomes to reverse. Hence, the stock price receives the pull back pressure caused by the book value of the position at time $s-u$ of each trader. A trader changes their holding position on the basis of the book value at $s-u$. The following rule $3'$ is rewritten as follows:

($3'$) *Feedback rule*: If $S(s)-S(s-u)$ is positive, then N_+ is decreased by 1 with probability $|(S(s)-S(s-u))/N|$, while, if $S(s)-S(s-u)$ is negative, then N_+ is increased by 1 with probability $|(S(s)-S(s-u))/N|$.

Assuming that the holding period (investment period) of each trader stock to be u, $-vS(s)/N$ in Eq. (1) is changed to

$$v\frac{S(s)-S(s-u)}{N}, \tag{7}$$

where $S(s-u)$ is the price t at time $s-u$, which is referred by each trader.

Hence, it seems to be natural to consider a delayed differential equation as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}x_t &= -2mx_t + 6x_t\left(\frac{1}{2}+x_t\right)\left(\frac{1}{2}-x_t\right) - v(y_t - y_{t-u})\\ &= -2\left(m-\frac{3}{4}\right)x_t - 6x_t^3 - v(y_t - y_{t-u}),\end{aligned} \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y_t = 6x_t, \tag{9}$$

where y_{t-u} is the price (the reference price) at $t-u$. From the system, unless y_{t-u} is constant, the argument for the van der Pol equation is not applied. The van der Pol equation of (3) and (4) has the Hopf bifurcation at $m = 0.75$. The orbit and the flow for the equation are illustrated in Fig. 1 of the phase plane, where the sign of y-component of the flow of the van der Pol equation is antisymmetric with respect to the y-axis. The set defined by the equation $(\mathrm{d}/\mathrm{d}t)x_t = 0$ is the curve given by the equation

$$y_t = -\frac{1}{v}\left\{2\left(m-\frac{3}{4}\right)x_t - 6x_t^3\right\}. \tag{10}$$

Eq. (10) has a local maximum and a local minimum when $m<0.75$ (shown by a dotted curve in Fig. 1(a)), which do not exist in $m>0.75$. The sign of x-component of the flow is changed in the curve given by Eq. (10). When $m>0.75$, each orbit is attracted to the fixed point $(0,0)$ drawing a spiral curve and then the price orbit becomes flat. In the case when $m<0.75$, the van der Pol equation has a limit cycle by the Poincare–Bendixson theorem [11]. Here we observe that the behavior of the solution of the delayed van der Pol equation considering the solution of the original van der Pol equation. In order to compare the delayed van der Pol equation with the equation without delay, we take the same values for all parameters other than the delay. We carried out a numerical study, shown in Fig. 2. We assume the price on the time interval $[-u,0]$ is a constant, say α, and take an initial function on the time interval $[-u,0]$ to be constant for the delayed van der Pol equation.

In our present model, the stock price get the feedback force proportional to the excess over the past price to which each trader refers. In $m>0.75$, this system has a limit cycle as well as the system without delay and the orbit tend to expand out of the limit cycle. When $m<0.75$, the solution of this system makes the convergence patterns repeatedly as shown in Fig. 1 or is similar to a limit cycle as shown Fig. 2 depending on parameters, the orbit is not flat contrariwise with the case without delay. It is natural to think that the reference point should be increased gradually with the time because of the demand cost for the risk of the investment. This cost is called "cost of capital" [12]. Let the rate of increase of this cost be g. The reference point price of the trader should be multiplied by e^{ug} by the rate g. Hence Eq. (7) becomes

$$-v\frac{S(s)-S(s-u)\times \mathrm{e}^{ug}}{N}. \tag{11}$$

Hence we have

$$\frac{\mathrm{d}}{\mathrm{d}t}x_t = -2\left(m-\frac{3}{4}\right)x_t - 6x_t^3 - v(y_t - y_{t-u}\times \mathrm{e}^{ug}), \tag{12}$$

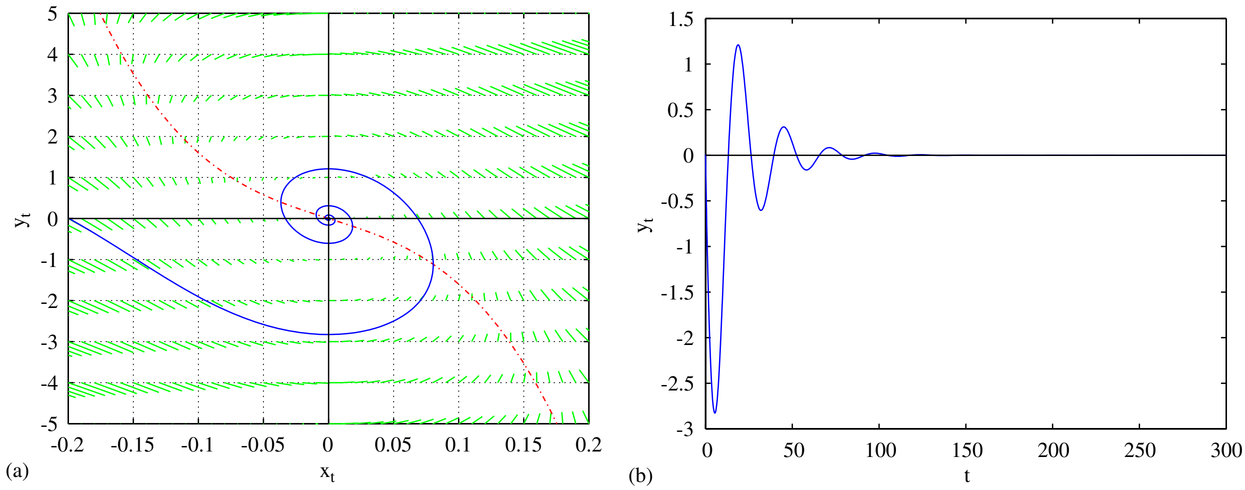

Fig. 1. Numerical simulations of the van der Pol equation by the fourth Runge–Kutta method, $t = 300$, $x_0 = -0.2$, $y_0 = 0$, $m = 0.8$, $v = 0.01$. (a) Phase chart and solution flow, (b) price orbit.

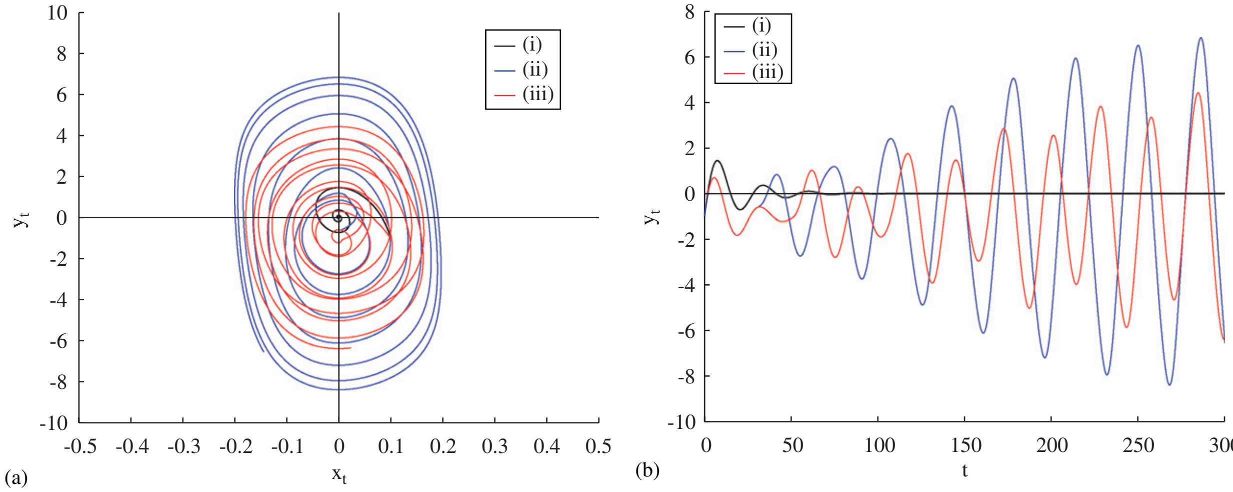

Fig. 2. Numerical simulations of the delayed van der Pol equation and the van der Pol equation by the fourth Runge–Kutta method, $t = 300$, $x_0 = -0.2$, $y_0 = 0$, $m = 0.8$, $v = 0.01$, (i) $u = 0$ (no delay), (ii) $u = 30$, (iii) $u = 50$. (a) Phase chart and orbit at changing delay, (b) price orbit at changing delay.

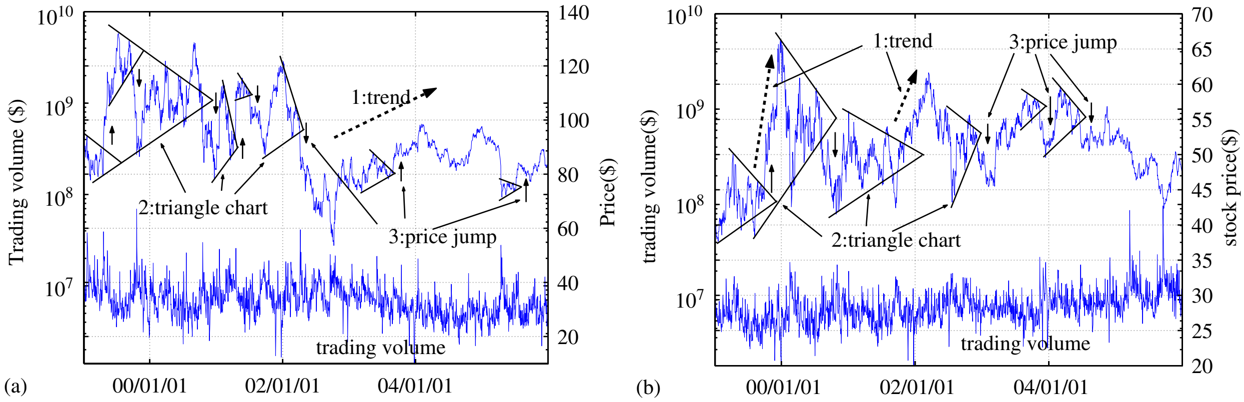

Fig. 3. Change of stock price: January 1999–December 2005. (a) IBM, (b) Wal-Mart.

$$\frac{\mathrm{d}}{\mathrm{d}t} y_t = 6x_t. \tag{13}$$

We show a typical numerical solution of this equation in Fig. 4, where the initial function of $[-u, 0]$ is assumed to be a constant. This corresponds to the case in which each trader made their position when the price fluctuation is small.

4. To understand real stock market

A method is used to predict the future stock price by finding patterns in the time series of a stock price, called chart analysis. We show the time series of stock price (stock price chart) for IBM and Wal-Mart, from January 1999 to December 2005 in Fig. 3. From these charts, we have some patterns about the stock price fluctuation, as follows:

1. The stock price has an upward or downward trend.
2. The stock price has a struggle period. The amplitude of the price is decreased gradually in this period.
3. After a struggle period, the price jumps suddenly.

Pattern 2 makes a wedge-like chart. This form is called "triangle chart" in chart analysis. The chart analysis is an empirical approach, which is not clearly explained theoretically. Our present model with time delay seems to be reasonable to give patterns, which have been observed in chart analysis. From Eq. (8), the set of points $(\mathrm{d}/\mathrm{d}t)x_t = 0$ is given by

$$y_t - y_{t-u} = -\frac{1}{v}\left\{2\left(m - \frac{3}{4}\right)x_t - 6x_t^3\right\}. \tag{14}$$

From Eq. (14), we see that there is a *dynamical hysteresis effect* [11]. The orbit depends on its past history of the price y_{t-u}. This hysteresis effect is summarized as follows.

(A) The case where $m > 0.75$, there is no negative resistance region (the solution flow is attracted to the fixed point), and the amplitude of the price fluctuation decreases gradually because the solution flow is attracted toward the reference point. However, by a big fluctuation of the past price given to the system, the current price is excited again.

(B) The case where $m < 0.75$, the amplitude of the price increases compared with the van der Pol equation without delay because the negative resistance region vibrates on y according to a past price change.

When a past price fluctuation is relatively small, for the case of (A), i.e., $m > 0.75$, the orbit is attracted to the reference point, which is enveloped by a triangle form like a wedge. As it can be observed from Fig. 1(i), the magnitude of flow of the solution flow near $(0, y_{t-u})$ is small. The change of the stock price at $t-u$ makes the sudden increase of magnitude of the solution flow at t. This argument is applied also to the system Eqs. (12) and (13), in which the system gets a continuous external force caused by the demand cost. When an initial function is constant and u is long enough, it makes a limit cycle of the original van der Pol equation. Therefore, although the stock price does not show a trend in the struggle period, the price jumps by the sudden change caused by the change of past price, the price struggles again and the amplitude becomes small gradually. Thereby, the change of a stock price sometimes shows a stairs-like trend given as in Fig. 4.

Since the volatility of the stock price jumps suddenly in the real market, diffusion-jump models may be applied, such as the SVJ model, which is a stochastic volatility model with jumps driven by the Poisson process, the SVCJ model which is a stochastic volatility with correlated jumps driven by a Poisson process,

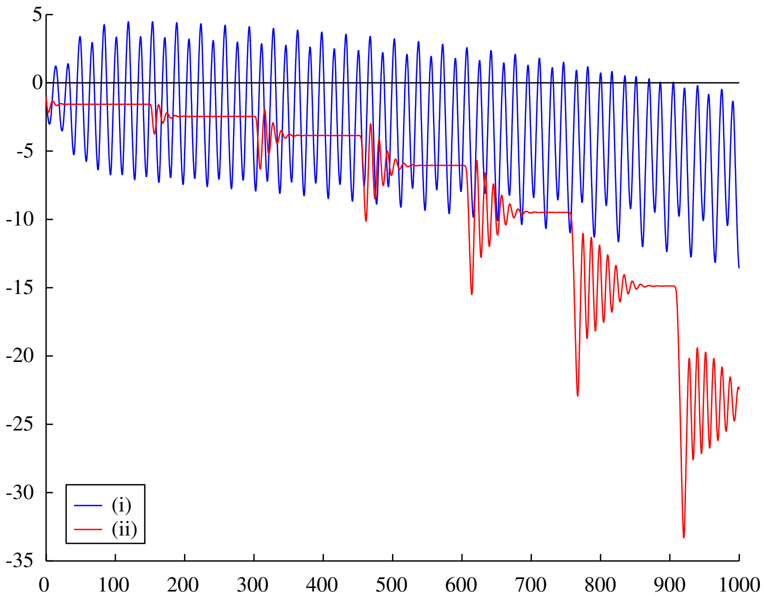

Fig. 4. Numerical simulations of the delayed van der Pol equation considering demand cost by the fourth Runge–Kutta method, $t = 1000$, $x_0 = -0.1$, $y_0 = -1$, (i) $m = 0.7$, $v = 0.02$, $u = 30$, $g = 0.002$, (ii) $m = 0.8$, $v = 0.01$, $u = 150$, $g = 0.003$.

and the SVSCJ model which is a stochastic volatility state-dependent correlated jumps model with the jump frequency depending on volatility [13,14]. Considering the process, our model can explain the volatility jumps and their successive damping.

In addition, the trend depends on the sign of the price of the reference point. The minus sign of the price at the reference point, namely the minus sign of y_{t-u} in Eq. (12) makes a downward trend, while the plus sign of it makes an upward trend.

When u is relatively small with $m<0.75$ as in the case (B), the jump of a stock price tends to be hidden by the amplitude of a struggle. For this reason, a stock price does not show a stair-like trend but a smooth trend (see Fig. 4). In a real market, these two cases may be combined. The parameter m shows the measure of conviction of trader's prediction for the market. A trader does not mutate when his conviction is strong, and mutate easily when it is weak. Stock prices will struggle, in which the buying and the selling almost evenly take place, when the traders think that the future of the corporate performance is unpredictable. When most of traders think that present stock prices are cheap (or expensive), the stock prices will show the trend because they do not hesitate to buy (or to sell) the stock. It is natural to think that conviction of traders is weak in the former case and is strong in the latter case.

The solution flow of the delayed van der Pol equation on a phase plane depends on t not just on x, and y, while it is independent on time for the case of the van der Pol equation, Eqs. (3) and (4). Up to previous argument, as a simple case, the holding period u of a position did not depend on a trader, but assumed that it was fixed. This seems to be a little strong assumption. Let us assume that the holding period of N traders are $u_1, u_2, \ldots, u_N$ respectively. The sum of each pull back power generated from the positions of all traders makes the feedback force as

$$-v\frac{S(s)-(1/N)\sum_{i=1}^{N}S(s-u_i)}{N}. \tag{15}$$

Assume the number of particles N is sufficiently large, and $v(q)=0$ for $q\geqslant s$ and $0\leqslant v(q)$ for $0\leqslant q\leqslant s$ with $\int_{t-s}^{t} v(q)\,\mathrm{d}q=1$, the feedback force is given by

$$-v\left\{y_t-\int_{t-u}^{t} v(q)y_q\,\mathrm{d}q\right\}. \tag{16}$$

The following stochastic equations will be reasonable to analyze real data of time series of a stock price:

$$\mathrm{d}x_t=\left\{-2\left(m-\frac{3}{4}\right)x_t-6x_t^3-v\left(y_t-\int_{t-u}^{t} v(q)y_q\,\mathrm{d}q\right)\right\}\mathrm{d}t, \tag{17}$$

$$\mathrm{d}y_t=6x_t\,\mathrm{d}t+\theta\,\mathrm{d}w_t. \tag{18}$$

It will give a good short time prediction if we take $v(q)$ at time q ($v(q)=V(q)/\int_{t-u}^{t} V(q)\,\mathrm{d}q$, $V(q)$:trading volume), normalized by $\int_{t-u}^{t} v(q)\,\mathrm{d}q=1$. From Eqs. (17) and (18), the reference price is the past price transition weighted average by trading volume. We often observe the phenomenon in which a price struggles, near the price at the time of a big trading volume in the past, which may be explained by our model.

5. Conclusion and remarks

We have introduced the delayed van der Pol equation by considering the effect by a trader's holding position in the van der Pol equation obtained by the majority orienting rule. In this model, the position of each trader makes a memory effect. The system depends on the history of price changes, because each trader calculates his profit and loss with reference to the book value of his holding position. The orbit of the van der Pol equation is asymptotically independent on the initial value at $t=0$. Because of that the delayed van der Pol equation seems to be strongly dependent on the value on the time interval $[-u,0]$, and can generate triangle patterns, price jumps, volatility jumps and price trends which are frequently observed in time series of changes of stock prices. Our present study will show that the delayed van der Pol equation driven by random noise will give a good model for the change of stock price.

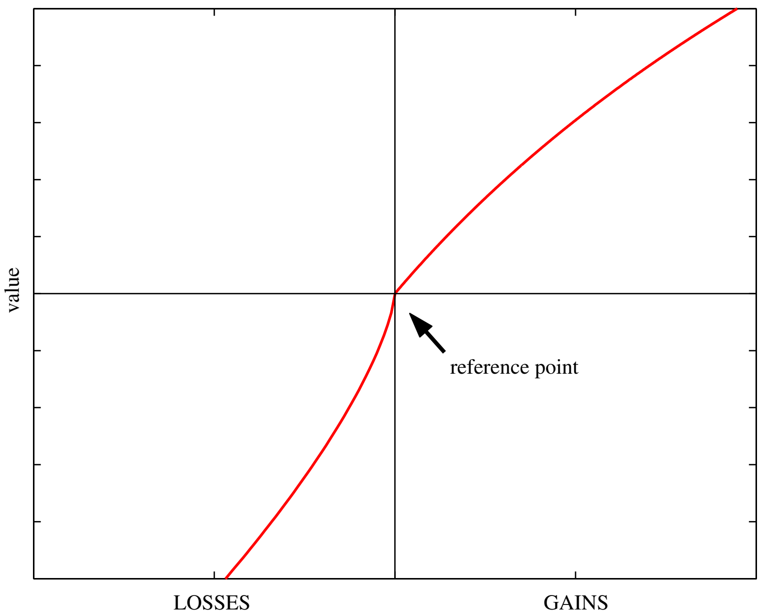

Fig. 5. Value function.

In this model, we assume that the linearity of the feedback power of the price to reference price is proportional to the difference of the price between the book value and the current price. The Prospect theory by Kahneman and Tversky, which is a theory about decision making of the human being in an atmosphere of uncertainty, says that human uses the ‘‘value function’’, as a measure of decision making instead of the linear function, whose conceptual figure of the value function is shown in Fig. 5 [15]. By using the non-linear function, the delayed van der Pol equation shows more intricate orbit, which is an interesting next subjects.

The Prospect theory is one section of the ‘‘Behavioral Finance’’, which is a new finance theory conditioning on the action of the investor who is not necessarily rational under such uncertain environment [5,6]. It includes the above-mentioned ‘‘information cascade’’ in many cases. In ‘‘Efficient Markets Hypothesis’’, which is one of the important hypotheses of the conventional finance theory, each market player is assumed to be ‘‘enough rational’’. If market players are enough rational, the useful information to estimate the future price will not remain in the past price transaction, because they use up the information in order to make profits. Therefore, if the Efficient Markets Hypothesis is true, then the price transition will become completely random noise, and the deterministic dynamics cannot be estimated from the analysis of the past price transaction. However, this assumption is unrealistic as shown by some irrational behaviors in Prospect theory. We think that the market players are ‘‘moderate’’ rational. The news which destabilizes a market occurs at random, we have to use a statistical model. However, we think that a dynamics is applicable in the behaviors of them after the news, which will be estimated with modeling the behavior of market players.

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