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Ising model, econophysics and analogies

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HIGHLIGHTS

- Methodological analysis of econophysics.
- The use of analogy in econophysics.
- The role of the Ising model in econophysics.

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ABSTRACT

Econophysics emerged in the 1990s by importing statistical physics into economics and finance. Such extension of physics to another context generated a lot of epistemological debates and although it this transfer appears to be internally (disciplinary) justified, there exist a number of works questioning the justification of such extension of physics in economics\finance. This article aims at analysing how econophysicists implicitly promote a Duhemian way of perceiving scientific research by expanding their work into economics.

1. Introduction

In the 1990s physicists turned their attention to economics, and particularly financial economics.[1] The term "econophysics"[2] is a neologism associated with this extension of physics to the study of problems generally considered as falling within the sphere of economics. Mantegna and Stanley [6, p.2] defined econophysics as "a quantitative approach using ideas, models, conceptual and computational methods of statistical physics". Econophysics nowadays refers to a vast literature all around the world [6–13]. Several authors [14–16] showed that econophysics can actually be seen as a new field developed by physicists for physicists.[3] This situation is very interesting since it characterizes the ability of physicists to see, outside of their field, meaningful facts and to suggest explanations for these facts. The context of such disciplinary transfer generates an important question about the analogical reasoning: how can physicists describe economic\financial phenomena by mobilizing their knowledge? Such transfer is directly related to the issue of analogy. By definition, an analogy is a comparison between two objects/systems that have similarities. Analogies play a key role in scientific practices: several authors have emphasized their pedagogical utility [19,20] while others have detailed their heuristic role in the aid of discovery [21,22]. In the context of the development of econophysics, which is characterized by an extension of physics

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1 The influence of physics on the study of financial markets is not new, as witnessed by the work of Bachelier [1] and Black and Scholes [2]. Nevertheless, we cannot yet refer to Black and Scholes' model as econophysics in the term's current meaning, since it was completely integrated into the dominant theoretical current of economics and finance. Econophysics is not an "adapted import" of the methodology used in physics; rather, it is closer to an independent approach developed outside of economics invasion.

2 The movement's official birth announcement came in a 1996 article by Stanley et al. [3] We would point out, however, that Kutner and Grech [4] trace the informal birth of the approach to the paper by Mantegna [5] that studied the evolution of returns on financial markets in terms of Lévy processes.

3 Although, we can mention some exceptions such as Ausloos et al. [17] or Farmer and Lux [18] who proposed an interdisciplinary perspective published in economics journals, the vast majority of the literature dealing with econophysics is still published in physics journals.

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outside of its borders, the issue of scientific analogy became particularly interesting. What do econophysicists see in financial economics that could appear so familiar to them? What are these similarities that paved the way for physicists to export their knowledge to finance? More precisely, I will investigate how econophysicists justify their modelling practices through a formal analogical reasoning in contrast with economists' way of thinking.

The first part of this paper presents how econophysicists formulate their reasoning by presenting a seminal econophysical model [3]. This model can be labelled a work in statistical econophysics, which is still, today, the largest part of the literature in the field [14]. Afterwards, the analogical nature of models will be presented as an essential aspect for econophysicists. Finally, the last part of this article will explain the reasons for why one can observe epistemological gaps between econophysics and economics. This part will explain that what can be seen as an explanation for physicists is simply perceived as a non-justified induction for economists. Beyond offering a methodological analysis of modelling practices used in econophysics, this article also contributes more generally to debates in philosophy of science about the use of analogies in science.

2. Econophysical modelling: a telling example

Statistical physics' main purpose is to explain the macroscopic behaviour of a system and its evolution, in terms of physical laws that govern the motion of the microscopic constituents (atoms, electrons, ions, spins, etc.) that make it up. Statistical physics distinguishes itself from other fields of physics through its methodology, which is based on statistics. This is due to the enormous number of variables on which statistical physicists have to work; for instance, Avogadro's number (6×10^{23}) refers to a gigantic number of equations of motion that have to be solved. This high number of relationships makes a strictly based-equations analysis unworkable, even for a computer. "Quite plainly, this is impossible …[the] subject is so difficult that [physicists] are forced to adopt a radically different approach to that employed in other areas of physics" [23, p. 4]. From this perspective, statistics became a very important tool in physics where particles' behaviour is described through the statistical properties of each particle motion. The methods used in statistical physics are thus essentially dictated by the complexity of the systems, due to the enormous number of constituents. This situation leads statistical physicists to start with statistical information about the motions of the micro constituents' properties of the system in order to statistically infer some macro properties for this system. The statistical approach is so common that "in most situations physicists can forget that the results are statistical at all, and treat them as exact laws of physics" [23, p. 6].[4] This particular integration of statistics into physics occurred in the 1970s as the direct result of this problematic of extremely voluminous data. Jovanovic and Schinckus [16] explained how the progressive computerization of society and economic sphere generated a huge amount of data that began to attract the attention of physicists. The computerization of financial marketplaces and the systematic recording of all transactions have created huge databases that have become attractive for all disciplinary profiles that have a strong background in statistics. This section presents how the first econophysical model emerged and how statistical physics has been gradually extended to economics and finance.[5]

2.1. The magnetic appeal of the Ising model

The idea of the Ising model is to describe microscopically why the system exhibits radical changes in its properties at a critical temperature. This situation was modelled in 1925 by Ernst Ising, who, by uncovering concepts that were not yet developed (universality, renormalization and emergence), correctly demonstrated the phenomenon of magnetization for a system composed of two-state spins.[6]

This model is considered to be the simplest description of a system that has a critical point; it played a central role in the development of research into critical phenomena and it occupies a place of importance in the minds of econophysicists. Briefly, the Ising model consists of discrete variables that represent magnetic moments of atomic spins, which can take one of two states, $+1$ ("up") or -1 ("down"), the two states refer to the direction taken by the spins. There is no way to speed up or slow down the spin of an electron (i.e. its revolution on itself) but its direction can be changed due to particular physical conditions, such as an important change of temperature. The interesting element is that the direction of one spin directly influences the direction of its neighbour spins. This influence can be captured through a function of correlation that measures the extent to which the behaviours of spins are correlated. Interestingly, physicists have observed that the correlation lengths between spins follow a power law. In other words, magnetization (M) evolves as a power law, depending on the level of temperature (t). Statistically speaking, this phenomenon takes the following form,

4 For instance, as Fitzpatrick [23] commented, the familiar equation of state of an ideal gas, $PV = nRT$, is actually a statistical result. In other words, it relates the average pressure (P) and the average volume (V) to the average temperature (T) through the number (n) of particles in the gas. "Actually, it is virtually impossible to measure the pressure, volume, or temperature of a gas to such accuracy, so most people just forget about the fact that the above expression is a statistical result, and treat it as a law of physics interrelating the actual pressure, volume, and temperature of an ideal gas" [23, p. 6].

5 The idea of applying physics method to financial data is not new since some authors worked on this aspect in the 1950s [24]—the word of "phynance" has been used to characterized this trend [25]. Econophysics can be seen as an extension of these works.

6 Ernst Ising (1900–1998) was a German physicist who worked on modelling of ferromagnetism. The Ising model published in *Zeitscrift of Physik* in 1925 is his major contribution to physics—it is quite interesting to mention that although Ising became professor of physics at the Bradley University (Illinois, USA), he never published again after 1935 and he instead focused mainly on teaching activities. For more information on the history of the Ising model, see [26] and for more biographical elements on Ising, see [27].

$$M \sim \left|t^{\beta}\right| \tag{1}$$

where β is called the critical exponents. This statistical characterization is very important because it offers an important tool for analysing the system. Onsager [28] computed the partition function of the Ising model which exhibited a sharp critical point. Since then, grew the idea of universal but not trivial exponents (scaling properties) that characterized power laws. In the Ising model context, this implying that the spin system has the same statistical properties regardless of the scale (microscopic or macroscopic) considered. The scale invariance assumption was not new in physics,[7] but the method allowing the mathematical demonstration of invariance was only established at the end of the 1960s by Kadanoff [29] and Wilson [30] with his renormalization group theory.[8] As a reminder, this theory is "a method for establishing scale invariance under a set of transformation that allows us to investigate changes in a physical system viewed at different distance scales" [33, p. 57]. Before the development of this theoretical framework, universal behaviours (for instance, the fact that the correlation lengths between spins follow a power law) were observed experimentally without theoretical foundation [33]. This theory makes it possible to study mathematically macroscopic regularities that occur as a result of microscopic random interactions without to having study these microscopic interactions. The focus is therefore on the macroscopic level, which is directly observable for physical phenomena. In other words, since the 1970s, due to scale invariance, physicists can infer from the microscopic constituents some key parameters that allow for the capture and description the dynamics of macroscopic behaviours without studying, in detail, what happens at the microscopic level. For these reasons, scale invariance is the foundation of any modern approach of statistical physics that is aimed at understanding the collective behaviour of systems that have a large number of variables that interact with each other. From this perspective, the renormalization group method can then be applied. By performing successive transformations of scales on the original system, one can reduce the number of interacting spins and therefore determine a solution from a finite cluster of spins.

Stanley et al. [3] explicitly referred to the Ising model in their paper where they coined the term "econophysics". After having explained the importance of this Ising model in physics, I come back now to the presentation of this seminal paper. We will see that the statistical characterization (power laws) of the magnetization (correlation lengths between micro entities) is the heart of the explanatory dimension of statistical econophysics.

2.2. What is the link with econophysics?

In terms of the Ising model, the authors used the same analogy to describe the fluctuations of annual growth rates for firms by showing that the dynamics of sales generate the same statistical situation as the one observed for spins movement in the Ising model. Using public data published by American companies, the authors worked on the average annual fluctuations of sales $\sigma(S_0)$ (and number of employees), which they presented as a function of the initial value of sales (initial number of employees) S_0. Observing the evolution of this variable, they noticed that "the remarkable linearity of the $\sigma(S_0)$ vs. S_0 function on a log–log scale over many orders of magnitude may indicate some universal law of economics that is applicable for small companies [...] as for giants of size" [3, p. 311]. This power law discovered by the authors takes the following form (see Fig. 1).

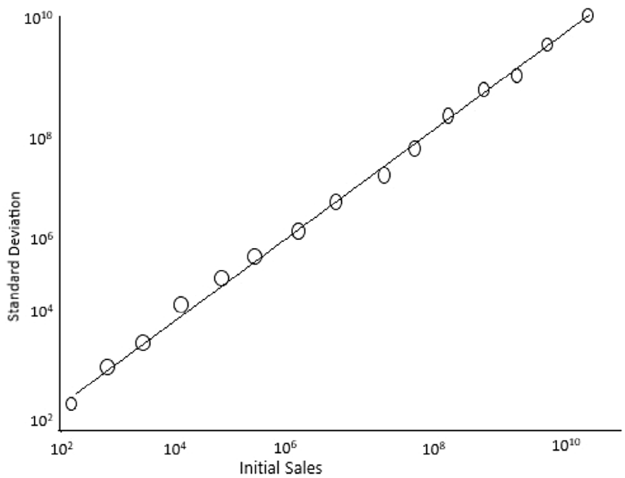

Fig. 1. This diagram shows a power law dependence between the standard deviation of sales and the initial level of sales.
Source: Stanley et al. [3, p. 311].

7 It exists in the work of Euclid and Galileo, for example.

8 Fisher [31] explained that Onsager's work (1944) was a key insight in the development of the renormalization group theory developed by Wilson. For further information about the link between scaling law and the renormalization group theory, see [32] or [31].

This diagram shows a power law dependence between the standard deviation $\sigma(S_0)$ of sales and the initial level of sales (S_0) as expressed in the following relationship:

$$\sigma(S_0) \sim S_0^{\alpha} \tag{2}$$

where α is empirically estimated at 0.82. This power law characterizes the evolution of sales, which increases by following a constant pattern. The authors assumed that this evolution has its origin in the internal structure of each firm. In so doing, they considered that the evolution of sales (or the employee number) results from N independent units, which can be computed as follows:

$$S_0 = \sum_{i=1}^{N} \varepsilon_i \tag{3}$$

where the unit sales ε_i have an average of $\varepsilon = S_0/N$ and an annual variation $u(i)$ independent of S_0. In this context, the annual change in sales can be estimated by:

$$\Delta S = \sum_{i=1}^{N} u(i). \tag{4}$$

Stanley et al. [3] wrote that the evolution of sales for companies can be explained through the same conceptual framework (i.e. Ising model),

> "Each level of the firm hierarchy corresponds to one generation of repeat family and each modification of the head decision by the lower level management corresponds to a mutation. Note that the $\sigma(S_0)$ for firm sales is exactly F (l) for DNA sequences" [3, p.312].

Considering the duality (flying or sitting on the water) of sea birds' behaviour, the authors extended the conceptual framework to the description of sea birds' migration by quantifying their behaviour with the help of an electronic recording device that was placed on the legs of the birds.

It is worth mentioning that the vast majority of statistical econophysicists do not detail the statistical structure of variables as Stanley et al. [3] did. However, this literature often quotes [3] as a seminal paper (see [14] for further information on the importance of this paper for the literature in econophysics). Methodologically speaking, models developed in statistical econophysics are founded on the existence of a power law pattern in which scholars adopt the reasoning proposed by Stanley et al. [3]. In other words, these authors provided the methodological foundations for statistical econophysics by making the Ising model an "exemplar" of the field.

This extension of the Ising model was used for coining the term "econophysics" and it is today a seminal article that founded the scientific justification of the field. It is worth mentioning that although this paper is largely quoted in the econophysics literature, no work questions the scientific justification of the approach. Another seminal article (and probably the first econophysics paper even though the term did not exist yet), by Mantegna [5] is a telling example of the way econophysicists work: the author observed phenomenologically that the anomalous fluctuations of the Milan stock exchange follow a power law with a critical exponent lower than two. Considering the scaling properties of this particular statistical pattern, Mantegna [5] made some recommendations in terms of analysis of the financial markets. However, he did not go beyond the phenomenological description of the financial data ([5] was aware of this aspect since he mentioned it in his discussion section). Interestingly, this paper also contributed to the crystallization of econophysics since it initiated the macro approach that is widely used today by statistical econophysicists. The following section will study in more detail the way econophysicists sustain their methodology and how they justify their analogical extension of the Ising model to a different disciplinary context.

3. Analogy in econophysics

An important literature exists for distinguishing analogy and metaphors in science [34–40]. Roughly speaking, a metaphor refers to a linguistic statement that has been transferred from one domain of application, where it commonly understood, to another domain in which it is unusual; whereas analogical models instead characterize statements that describe relational information through a transfer of a mathematical framework from one domain to another. In other words, metaphor is a simple descriptive comparison between two relevant domains [22], while analogies are more likely to be mathematically formulated since they deal with similar dynamics, relations or processes observed in different domains—from this perspective, the extension of the Ising model by econophysicists in finance can be perceived as an analogical model. Analogies are based on the understanding of something in terms of something else that is well understood and familiar. However, as Bailer-Jones [22, p. 117] explained:

> "Being familiar does not equate with being understood, but familiarity can be a factor in understanding. This is also not to suggest that understanding can be reduce to the use of analogy, but having organized information in one domain

Table 1
Similarities between physical and economic/financial systems.

Physical systems	Economic/financial systems
Known similarities	
Interacting elements	Interacting agents
High number of micro components	High number of individual agents
Micro interactions	Micro behaviours
Observational similarity	
Dynamics following a power law	Dynamics following a power law

(source) of exploration satisfactorily can help to make connections to and achieve the same in another domain (target). The aim is to apply the same pattern assumptions of structural relationship in both source and the target domains".

Analogies have widely been used in physics for different purposes: Podolefsky and Finkelstein [41] explained that analogies play a key role in the physics students' learning process. Jona-Lasinio [42] emphasized the methodological importance of analogies in theoretical physics while other authors [43] or [44] rather detailed the cognitive role of analogies that can generate new ideas through new conceptual parallelisms. There is a large literature extending analogically physics beyond its scope. Physics methods has been applied in biology [45], ecology [46], sociology [47,48] or psycho and political science [49]. All these works generated debates on the significance to use physics approach in a non-physical environment [50,51]. Econophysics is one of these extensions since this field results from the application of physics methods to economics and finance. More precisely, in their seminal article, Stanley et al. [3, p. 316] wrote, "The analogy between economics and critical phenomena [described by the Ising model] is sufficiently strong that a similar story might evolve". It is worth emphasizing that the authors wrote the word "might", showing therefore their deflationary perspective on the use of power law as a form in which the mind can grasp the complex nature of the phenomenon. In this context, the real question is to know if this Ising model (which gives physicists the opportunity to understand the process of magnetization very well) can really help to understand economic/financial phenomena. As mentioned earlier, the Ising model is used by econophysicists as an analogy between the source domain (physical systems) and the target domain (economic/financial systems). This reasoning can be summarized through a tabular representation found in [52].

Hesse [52] suggested to clarify the known similarities and observational ones to better understand the role of analogy in science. In accordance with this suggestion, Table 1 provides a summary of the analogical similarities between physical and economic systems. The horizontal relations are the relations of similarity between the source and target domains while the vertical relations are those between the objects and properties within each domain. What is interesting to emphasize here is the way of listing the horizontal similarities, because econophysicists and economists might agree on these aspects. However, the kind of conclusions one can draw from these characteristics would be totally different: while econophysicists consider that the emergence of a power law is an indication of complexity [53]; economists who use another statistical lens simply do not see this power law. As Bartha [21, p. 6] noticed, the "manner in which we list similarities and differences, the nature of the correspondences between domains: these things are left unspecified [in Hesse's works]". Extending an earlier discussion on an analogy introduced by Keynes [54], Hesse [52] distinguished three kinds of analogies: negative, positive and neutral analogies. The former refers to relations that we know to be different between the two domains; the second one concerns the known (and acceptable) similarities and the latter characterizes what we do not know or what was not known before the association between the source and the target domains. In this sense, a negative analogy between physical systems and economic/financial ones could refer to the fact that in opposition with the former, the latter is composed of micro elements in economic/financial systems that have a human and social consciousness. The horizontal (known) similarities mentioned in the table above illustrate positive analogies, and the observational similarities could be seen as a neutral analogy in a sense that this similarity was neither assumed nor expected in the analogical association of the two domains. On this point, Frigg and Hartmann [55, p. 14] wrote that "neutral analogies play an important role in scientific research because they give rise to questions and suggest new hypotheses". In this occurrence, we can reformulate the situation as in Table 2).

In this analogical reasoning, econophysicists consider that the statement according to which the dynamics of an economic/financial system follows a power law is plausible because of certain known similarities with physical systems that generate this kind of dynamics. Of course such analogical extension requires a particular interpretation of the "plausibility criteria" evoked above. Hesse [52, p. 87] explained that this plausibility must be "acceptable in a scientific sense" and she added that "a tendency to co-occurrence" is an essential requirement for a good analogical association. In the case of econophysicists, they explicitly associate this plausibility with the statistical patterns they observe in economic/financial data. From econophysicists' perspective, the fact that power laws are regularly observed in empirical data and that these patterns can be explained mathematically appear to be an acceptable scientific reason for considering the plausibility evoked above. In other words, for econophysicists, this "co-occurrence" of power law in the source and target domains takes the form of a formal analogy.

Hesse [52] distinguishes between two categories of analogies: formal ones and material ones. When the analogous refers to material entities (material analogy), the association between two domains is mainly based on the sameness or resemblance of common properties. These similarities being observable, the three levels of analogy evoked above are always present, but the negative one appears to be more obvious [56]. For instance, Earth and Mars are both celestial bodies,

Table 2
Analogies between physical and economic/financial systems.

Physical Systems	Economic/financial systems	
Interacting elements	Interacting elements	Positive analogies
High number of micro components	High number of micro components	Positive analogies
Micro interactions	Micro interactions	Positive analogies
Non-human micro entities	Human agents	Negative analogy
Dynamics following a power law		Neutral analogy
Plausibility	Dynamics following a power law	Neutral analogy

spherical, have moons and orbit the sun (positive analogy) but the observability of these common properties also makes obvious their differences: the absence of water/atmosphere on Mars, the distance between these two bodies and the sun, the periodicity of their respective circumvolution, etc. When two systems are related by formal analogy, they are both interpreted through the same mathematical framework. Very often, this kind of analogy concerns a situation in which the dynamics between certain ingredients within one domain are perceived as identical (or comparable) to the relations between elements of another domain [22, p. 57]. According to Mellor [56] and Falkenhainer et al. [57], when the similarity between two domains refers to relations or dynamics, then, although the negative analogy is still present, it is less important, since only the formal evolution of the domain is taken as a formal analogy. In this context, the interrelationships between elements become more important than their material resemblances.

The Ising model has been analogically extended from physics to economic/financial systems because the formal characterization of its dynamics seems to be applicable for describing complex economic/financial dynamics. In other terms, the Ising model is a formal analogy between a ferromagnetic system and an economic/financial system where the neutral analogy takes the form of a mathematical characterization (power law) of what was considered a statistical anomaly (occurrence of large fluctuations) by the existing financial (mainstream) knowledge. Given that, one could legitimately question the explanatory nature of a model that is used to explain several diverse phenomena.

When Stanley et al. [3] extended the Ising model in economics they also implicitly imported the scientific fabric usually associated with this model. Articles dealing with econophysics are mainly published in physics journals and assume that readers have a specific disciplinary background for understanding this type of research. For instance, the Ising model and the renormalization group theory are both well known for all statistical physicists; and these two frameworks are often considered as the theoretical foundations econophysics.[9] For econophysicists, the epistemological justification of their works is quite simple: they use a familiar theoretical framework to describe complex phenomena that exhibits the same key features required to be studied through this frame. In other words, econophysicists did not produce their models out of nowhere: given the specific characteristics (emergence of extreme values in a particular dynamics) that they observe as physicists, they choose what appears for them to be an appropriate model (Ising model\power law) to describe this phenomenon. This approach is justified in two ways: by scientific foundations of this familiar framework and by the empirical adequacy of results (what I previously associated with the co-occurrence of power laws in the physical and financial systems). Such extension of physics to another context is implicitly based on a justification that is internally (disciplinary) warranted but it has been questioned by scholars who are not familiar with physics. There exist a number of works [14,16,58,59] criticizing or questioning the justification of such extension of physics in economics\finance. The rest of this paper will analyse further how econophysicists implicitly promote a Duhemian way of perceiving scientific research by extending their work into economics.

4. Econophysicists as a Duhemian field

Econophysics has been developed by physicists who applied their methods to economic data. In so doing, they went out of their discipline and they cannot avoid facing the judgement of economists willing to protect their "disciplinary territory". Although economists acknowledge the technical knowhow of econophysicists, they are reluctant with such kinds of research simply because they consider that these works do not meet their scientific standards [16]. These disagreements are rooted in a set of communal cognitive values/tools that shape the foundations of scientific justification in both communities.

9 This claim has been confirmed by several personal conversations I had with econophysicists (Eugene Stanley, Marcel Ausloos, Tobias Preis, etc.)

The way econophysicists have applied their knowledge to economics and finance is in line with a Duhemian use of analogy. Pierre Duhem (1861–1916) was a French physicist and philosopher well known for his works on the "Newtonian" (inductive) and the "Cartesian" methods [60]. Although the notion of analogy is not ubiquitous in Duhem's works, he referred to this concept when he wrote about how physics as a field can evolve. More precisely, he explained that "The history of physics shows us that the search for analogies between two distinct categories of phenomena has perhaps been the surest and most fruitful method of all the procedures put in play in the constructions of physical theories" [61, p. 95]. The French physicist illustrated his claim with a study on the Maxwell's analogy between electrical flow and heat, where he considered analogies as a final relationship between phenomena and theoretical treatment of phenomena. Precisely, he wrote:

> "it may happen that the equations in which one of the theories is formulated is algebraically identical to the equation expressing the other [...] [analogies are] intellectual economy, a method of discovery by associating two abstract systems; either one of them already known or both being formulated, they clarify each other" [61, p. 96–97].

This reasoning *per analogiam* is also presented by Duhem as a way of understanding science as a human activity that develops in time and requires transgressions across the borders of the domain under investigation [62]; the development of econophysics seems to result from such a way of defining scientific activity. According to Duhem, scientists are not free in their choice of assumptions or models at a given time. Scientific knowledge, experience and even scientists' common sense are always somewhat related to a specific disciplinary tradition. In this sense, theories of the past act as the "nuclei of the victorious theories of the future" [62, p. 80]. Although Duhem's way of thinking is usually associated with conventionalism, it is worth mentioning that the French physicist was an instrumentalist for who the key role of physics is to be "precise". On this point, Duhem wrote,

> An experiment in physics is the *precise observation* of phenomena accompanied by an *interpretation* of these phenomena; this interpretation substitutes for the concrete data really gathered by observation abstract and symbolic representations which correspond to them *by virtue of the theories admitted by the observer* [61, p.147—my italics].

This quotation is a telling summary of what is happening with the extension of physics to economics. The words in italics actually refer to two different scientific communities (economists and econophysicists) having different "observational categoricals" (i.e. first filtering knowledge). When econophysicists see a power law that they associate with the complexity of the system, economists see abnormal observations challenging the Gaussian framework that shapes the majority of financial knowledge.

The analogical extension of knowledge is always constrained by a particular conceptual framework in which what is observed and how this thing is observed cannot be totally separated [61]. Such a perspective is interesting because it offers a mode of transfer for analogies. Regarding econophysics, I mentioned in the previous section that econophysicists are implementing formal analogies in their modelling practices. However, the justification of this transfer from physics to economics/finance requires a Duhemian analysis in order to understand what happens in the econophysicists' minds. By applying the Ising model and its statistical characterization (i.e. power law) in economics and finance, econophysicists gradually and analogically extended the epistemic domain of this well-known model. What is specifically Duhemian in the formal analogies proposed by econophysicists is the way these scientists conjointly extend (to economic\financial systems) the analogical properties and the theoretical framework justifying these properties. Proposing a formal analogy between economic/financial systems and the Ising model is one thing, but to simultaneously extend analogues and the theoretical framework into financial economics is a further (Duhemian) step. Analogies (and their consequences), like assumptions, cannot be formulated in isolation from the peculiar theoretical frame that supports them. [61] explained that this kind of extension does not pop up from nowhere as the result of scholars' individual arbitrariness, but rather that it results from the gradual development of a logic that belongs to a specific disciplinary tradition. Regarding this aspect, Schäfer [62] wrote:

> "Reasoning by analogy has to start with previous knowledge. It has to rely on ideas that are familiar and have proved to be useful in a particular field of research. These ideas are, then, *per analogiam*, carried over in a new domain. Applying familiar ideas to new domain implies usually modifications in the inherited body of knowledge; every genuine development of science does not only add new materials to former knowledge but does single out certain sections as no longer tenable. New knowledge, if new it is, will negate some part of other if the received knowledge" [62, p. 84].

This Duhemian use of analogy has some epistemological consequences, as Schäfer [62, p. 80] explained:

> "his [Duhem] reconstruction of physics required the strict abolition of explanatory ambition [...] and restriction to the descriptive function of physical theory. According to this, the only appraisal of physical theory that could claim to be rational consisted in the check of empirical adequacy which is restricted to the purely internal context of justification" [62, p. 80].

From this perspective, econophysics is not perceived by econophysicists as a simple analogy, but rather as a justified new way of dealing with financial/economic systems. By dealing with power laws in financial\economic systems, econophysics can be perceived as a particular analogical progress that provides "new final and indisputable propositions to the final and indisputable propositions it [physics] already possesses" [61, p.177]. These propositions take the form of mathematical structures (power laws) that formally and analogically characterize the dynamics of financial\economic systems. In line with a Duhemian way of thinking the role of physics, econophysics offers a classificatory knowledge rather than an explanatory one [63].

This situation explains why econophysicists believe that they could replace (or they are totally indifferent to) the existing economic knowledge. Such a Duhemian way of dealing with an imported analogy as a replacement[10] for existing knowledge will allow me to clarify how econophysicists bring their reasoning into economics and finance. First of all, Duhem acknowledged that a mathematical structure of a model is the core of physics—precisely, he considered that "a physical theory is a system of mathematical propositions, deduced from a small number of principles that aim to represent as simply as completely and exactly as possible a set of experimental laws" [61, p. 9]. In so doing, Duhem emphasized the dominance of the mathematical deductive method in physics. By combining the Duhemian use of analogy in the extension of knowledge and the importance of this deductive reasoning, we can now summarize the analogical reasoning econophysicists have in mind:

> *Statement 1: Complex phenomena are composed by a high number of interacting micro elements that generate a dynamics that can be described by a power law.*
> *Statement 2: Financial markets/economic systems are complex phenomena. Conclusion: Financial markets/economic systems exhibit power laws.*

Beyond the plausibility of the conclusion, which is often justified through the co-occurrence of power laws in physical and economic/financial systems, what is interesting in this reasoning is the association between the power law and the notion of complex phenomenon. This way of associating an observable statement with a scientific fact is quite common in science, as Feyerabend wrote:

> "As soon this method [here the association of power law with critical phenomena] is generally accepted and has been standardized, it is only a question of time until no conscious distinction is drawn between the presence of the criterion and the presence of S itself. The presence of the criterion no longer comes into consideration on its own, but one immediately says without further ado that S itself occurred: S has become directly observable" [64, p. 19].

This way of developing physics is in accordance with Duhem's approach in which only abstract and general principles (experimental law) can guide the scholars' mind in unknown situations.

5. Conclusion

Econophysics emerged in the 1990s by importing statistical physics into economics and finance. This transfer appears to be an extension of physics itself rather than an interdisciplinary attempt to develop knowledge in collaboration with another existing field. The fact that the majority of articles dealing with econophysics are published in physics journals is an indicator of the disciplinary background expected to understand this kind of research. As explained in this paper, the theoretical foundations of econophysics refer to the Ising model and renormalization group theory, which are well known for all statistical physicists. Therefore, for econophysicists, the epistemological justification of their works is quite simple: they used a familiar theoretical framework to describe complex phenomena that all have key features required to be studied through this framework. This approach is then justified in two ways: (1) the familiarity/scientific foundations of the imported framework and, (2) the empirical adequacy of results (the co-occurrence of statistical patterns in physical and economic/financial systems). From a Duhemian perspective, this analogical extension of physics is justified for econophysicists because the internal logic of their field is respected.

This paper defended that Duhem's writings offer an interesting framework for understanding how econophysicists justify their analogical extension of physics into economics/finance. Precisely, I claimed here that, from a physicist's point of view, statistical econophysics can be perceived as an analogical and idealized extension of the Ising model which appears to be theoretically, empirically and logically justified.

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[10] See [56] for further information on this aspect.

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