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Mechanisms of investors' bounded rationality and market herding effect by the stochastic Ising financial model

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ABSTRACT

The widespread herding effect prevalent in China's stock market is caused by investors' bounded rationality behavior. However, the underlying mechanisms are not fully understood. This article aims to use the stochastic Ising financial model, which explains this problem from the perspective of Econophysics. The probability density function and power-law distribution of return data simulated by the stochastic Ising financial model show sharp peaks and fat tails, similar to the stylized facts of real Chinese stock market data. Further, we use this financial model to simulate herding behavior in the market. The results show that when the degree of investor irrationality (represented by β) increases, the Herd Effect Index (HERDI) also rises. When β exceeds a critical point, the HERDI index fluctuates more violently. Through empirical analysis, we find that there is an interactive relationship between investors' bounded rational behavior and market herding effects, and this model can effectively explain these two phenomena.

1. Introduction

The continuous development of financial markets has led to the study of investor behavior and market phenomena becoming an important topic in the field of finance. Bounded rationality of investors and herding effect in the market are two common market phenomena that have important implications for the stability and volatility of financial markets. Therefore, studying the mechanisms of these two phenomena is of great significance for understanding the operating mechanism of financial markets.

In recent years, herd behavior caused by bounded rational investors has continuously emerged in the stock market. Moderate herding behavior can enhance market liquidity, but excessive herding behavior can cause bubbles or collapse, devouring personal wealth and triggering the collapse of the entire financial market [1–3]. Many papers have studied the measurement of herd behavior, but the mechanism of its formation is not fully understood.

Econophysics models can use dynamic methods to simulate the herd effect formed by investors imitating each other due to bounded rationality [4–6]. The financial market is a highly complex system with numerous participants communicating and interacting. The exchange of information and mutual influence among investors create highly complex dynamics [7,8]. Econophysics models mainly include Ising model [9–14], percolation model [15], voting model [16–18] and Potts model [19]. Schinckus believes that from the perspective of physicists, econophysics can be seen as an idealized extension of the Ising model, as the model is theoretically, empirically, and logically sound [20].

Since the early 1980s, people have attached increasing importance to the various financial anomalies that continue to appear in the market, and traditional financial theory has faced great challenges. As a new financial theory that explains the abnormal behavior

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of financial markets, behavioral finance emerged [21–23]. Due to the herd mentality, investors' irrational imitation of other people's investment decisions in incomplete information will lead to relatively low returns in the trading market, and the overall market trading prices are also greatly affected. Tests for herding effect can be divided into two categories. The first research method is based on covariant behavior. Among these, the LSV model is the most representative model [24]. LSV model was initially proposed by Lakonishok, Shleifer, and Vishny, to study the herding behavior of 769 American pension funds from 1985 to 1989. They calculated the proportion of net buying funds of individual stocks in the total number of funds trading the stock, and then characterized the imbalance of trading power between buyers and sellers and measured the herding behavior of institutional investors. Although the investigation found no herd behavior within the tested fund groups, minor herding behavior was observed among a few smaller stock transactions. LSV model only considers the number of transactions between buyers and sellers but does not consider the dimension of volume and turnover, which cannot sensitively capture the low degree of herd behavior. A subsequent revision to the LSV model was introduced by Wermers as the PCM model, accounting for the disparate buying and selling directions [25]. The new model tracked changes in holding ratios across a periodic range as the measure of herding behavior. However, the disparity of results between the LSV and the updated PCM model was considerably slight, because there were still great limitations in distinguishing true and misleading herd behavior.

The second category is based on the degree of dispersion. Christie and Huang used the cross-section dispersion of stock returns (CSSD) to measure the difference in investment behavior between investors in the market, which is referred to as the CH model [26]. If there is obvious herd behavior, that is, the investment behavior of most investors is consistent, the degree of CSSD should be low, while when the degree of herd behavior is low, the investment behavior of investors is not consistent, and the degree of CSSD is greater. However, when the CH model is faced with a lower degree of herding behavior, the measurement indicators are not statistically significant. This implies that the original assumption that there is no herding behavior in the market when the indicators are not significant cannot be ruled out. Consequently, the level of herding behavior in the market will be underestimated. In light of this, Chang, Cheng, and Khorana designed the CCK model [27], which is based on the CH model. The CCK model converts the discrete indicators of individual stock returns and market returns in the CH model into the cross-sectional absolute deviation of stock returns (CSAD), so as to measure the consistency of investor behavior. In addition, the capital asset pricing model (CAPM) is used to deduce and evolve, and the market rate of return is combined with the absolute difference of return. The model proposes that when there is no herd behavior in the market, there is a linear positive correlation between the absolute difference of returns and the market rate of return, but not with the quadratic term of the market rate of return. In the case of slight herding behavior, the quadratic term of market rate of return has a negative impact on the absolute difference of returns. Conversely, in instances of serious herding behavior, the absolute difference of returns exhibits a linear negative correlation with the market rate of return, in addition to a negative correlation with the quadratic term of the market rate of return.

Following similar methods, literature [28–31] use the above methods to test the existence of herding in the stock market. Zhang constructed a new herd effect index as HERDI to measure the dynamic change process of herding effect [32]. From the existing research, studies on herd effect mostly focus on the existence and level of herd effect in a static interval with LSV, CSAD, and other models. However, there is relatively little dynamic quantitative research on herd behavior and its internal formation mechanisms. Therefore, we use the Ising model to fill this research gap.

The Ising model holds an important position in financial research, which can simulate a closed system in which each agent can make social decisions [33]. The dynamic update rules of the Ising model can be used to describe the formation of bounded rational agent decisions [34]. Because Ising model has phase transition and a mechanism of "eliminating instability" [35], it can lead to collective imitation, which can lead to financial foam and subsequent collapse [36]. The spin model proposed by Bornholdt is an adapted version of the underlying Ising model, which proposes a financial market model in parallel with the ferromagnetic model [37]. A closely related model by Cond and Bouchaud, also inspired by the original Ising framework, explains the link between the heavy tail observed in the empirical distribution of financial returns and the herding behavior of financial market participants [38]. Lima established a stock price model using a two-dimensional classical Ising model, which can test the explanatory power of rational and irrational entities relative to financial market characteristics within the same framework [9]. The price model established by agent-based Ising dynamic system is used to study the dynamic characteristics of return series [38], such as volatility clustering [39,40], power law distribution, multifractal behavior [41,42] and asset price changes [43,44].

These models are based on the view that financial markets can be seen as a highly volatile complex system of interacting agents that interact in very complex ways to explain some of the stylized facts found in financial time series. If the financial market is regarded as a social network, its nodes correspond to individual investors, who interact like spins in the lattice, resulting in macro phenomena of trend (herding behavior) and reversal (reverse behavior) [10]. In this paper, we constructed the HERDI index based on the traditional CSAD model. Dynamically analyze the herd level in the market. Further, use the stochastic Ising financial model to simulate the interaction between investors in the market and the HERDI index under different market intensity parameters β, aiming to explain the root of herd behavior from the perspective of market strength parameters in the random Ising model.

The paper is organized as follows: Section 2 provides the construction process of the detailed stochastic Ising model and stochastic Ising financial model, as well as the construction of the HERDI index, and presents descriptive statistical analysis of the data used. Section 3 provides the result analysis of the HERDI index constructed and analyzes the statistical characteristics of the stochastic Ising financial model. Section 4 combines the stochastic Ising financial model with the HERDI, showcasing the HERDI index simulated by the stochastic Ising financial model and the tail situation of the real market. Section 5 summarizes the entire paper and suggests future research directions.

2. Methodology and data

2.1. Stochastic Ising model

Consider the stochastic Ising model on two-dimensional integer lattice Z^2, and assume that $S = \{1,2,\dots,N\} \times \{1,2,\dots,N\} \times \{1,2,\dots,N\}$ is the set of N^2 points, called sites on Z^2. The spin of the stochastic Ising model can point up (spin value $\sigma = +1$) or point down (spin value $\sigma = -1$), and it flips between the two orientations. So we can assume on each site stands an investor, who holds either a positive opinion on the stock market (+1) or a negative opinion on the stock market (−1) and trades based on the opinion. The investor's attitude may change by the influence of neighbors or outside environment. We consider an stochastic Ising model with the following system of Hamiltonian, for each $\sigma \in Z^2$,

$$H_{N^2,h}(\sigma_x) = -\sum_{x,y\in\mathbb{N}} J\sigma_x\sigma_y - h\sum_{x\in N^2}\sigma_x \tag{1}$$

where h is a real number, which is the strength of an externally applied magnetic field. The exchange coupling constant, J is the strength of internal interaction which indicates the nearest-neighbor exchange strength. For a fixed site x, define a neighborhood $N(x) = \{(x,y)| \parallel x-y \parallel = 1\}$. where $\parallel\cdot\parallel$ is a Euclidean distance. The finite Gibbs state μ_Λ^h at inverse temperature $\beta = \frac{1}{k_B T}$ (T is the temperature and k is a constant) is a probability measure given by:

$$\mu_{\Lambda,h}^{\beta}(\sigma) = [Z_{\Lambda,h}^{\beta}]^{-1}\exp\{-\beta H_{\Lambda,h}(\sigma)\} \tag{2}$$

where $Z_{\Lambda,h}^{\beta}$ is called the partition function and is given by:

$$Z_{\Lambda,h}^{\beta} = \sum_{\sigma\in\Omega_\Lambda} exp\{-\beta H_{\Lambda,h}(\sigma)\} \tag{3}$$

For this stochastic Ising model, the energy effect dominates at sufficiently low temperatures. The two-dimensional stochastic Ising model has a critical point $\beta_c \approx 0.45$. If $\beta > \beta_c$, as the value of β increases, most spins flip to the same direction. If $\beta < \beta_c$, it is in a chaotic state.

2.2. Stochastic Ising financial model

In this model, we assume that the spin body on each lattice point is an investor. On each different trading day, investors on the grid will be influenced by their surrounding "neighbors". We randomize the intensity of this influence β within a certain range, which better aligns the model with the complex system of the stock market. In the stochastic Ising financial model, the external environment can also affect the intensity of investors' impact. In order to simplify the model and focus more on the impact of investor interactions on stock market price fluctuations, we set $h = 0$. Because financial price models heavily rely on the number of spin values that change over time and market strength, we define the intensity of interaction between market investors as $\beta = \lambda\eta$, η is a uniformly distributed random variable in the interval $[0,1]$, λ is a strength parameter.

Here we present an independent market and assume that there are n^2 traders in the market, and investors on each grid point can buy and sell one unit of stock at each time point t_c. If the number of people buying stocks at this time is greater than the number of people selling stocks, it proves that most investors in the market hold an optimistic attitude towards this stock, which will drive the stock price up. On the contrary, if the number of people selling stocks at this time is greater than the number of people buying stocks, it proves that most investors in the market hold a negative attitude towards this stock, which will lead to a decline in stock prices. $x^+(t)$ investors who choose to buy stocks and $x^-(t)$ investors who choose to sell stocks jointly determine the price of the stock. $x_{i,j}(t)$ represents the investment attitude of investors at lattice points $(1 \le i \le n, 1 \le j \le n)$, then $x(t) = (x_{11}(t),\dots,x_{1n}(t),\dots,x_{nn}(t))$ represents the set of investment attitudes of n^2 investors. The investment space configuration for all n^2 investors from time 1 to t is:

$$\chi = \{x = (x(1),\dots,x(t))\} \tag{4}$$

For a given configuration $x \in \chi$ and trading time on day t

$$N(x(t)) = x^+(t) - x^-(t) \tag{5}$$

We use $\xi_t(x)$ represents the random message variable that arrives on the t trading day, $\xi_t = 1$ represents good news, $\xi_t = -1$ represents bad news, with probabilities of p_1, p_{-1}, where $p_1 + p_{-1} = 1$. These investors will make buy and sell trading decisions with corresponding probabilities. $\xi_t(x)|N(x(t))| > 0$ indicates that the number of buying traders is greater than the number of selling traders, driving the stock price up. Otherwise, the price of stocks will decrease or remain unchanged. The price of the model at time t ($t = 1,2,\dots$) is

$$S_t = e^{\gamma\xi_t(x)|N(x(t))|/n^2} S_{t-1} \tag{6}$$

In the model $\gamma > 0$ represents the market depth parameter, aiming to set the return rate at [−10%, +10%], which is more in line with the situation in the Chinese stock market. S_0 represents the initial price of the stock when $t = 0$. Therefore, it is obtained that

$$S_t = S_0\exp\{\gamma\sum_{k=1}^{t}\frac{\xi_t(x)|N(x(t))|}{n^2}\} \tag{7}$$

The logarithmic return of stocks from $t-1$ to t is

$$r(t) = \ln S_t - \ln S_{t-1} \tag{8}$$

There are several advantages to taking logarithmic returns here: (1) The logarithmic return obtained by taking values within all real number categories facilitates data processing. (2) Due to the limitations of stock market fluctuations, except for the day of new stock issuance, the daily change in stock prices usually does not exceed $\pm 10\%$. In this region, the logarithmic return is approximately equal to the percentage return. Therefore, we adopt the logarithmic return to facilitate data testing. (3) The logarithmic rate of return can be accumulated and used to calculate cumulative returns, as well as to intuitively calculate the continuous compound interest returns over a certain period of time. (4) Ensure that negative prices do not occur during simulation.

2.3. Construction of HERDI index

When market prices fluctuate significantly, investors often suppress their private information and form herd behavior around consensus information in the market. This situation will lead to an accelerated decrease in the dispersion of returns, resulting in a nonlinear relationship between the dispersion index and the average market return. The mainstream method uses the CSAD model to detect the herd effect [27]. According to the definition of the CSAD model:

$$CSAD_t = \gamma_0 + \gamma_1 |R_{m,t}| + \gamma_2 R_{m,t}^2 + \varepsilon_t \tag{9}$$

When the regression coefficient γ_2 of the quadratic term in the CSAD model is significantly negative, it means that there is a herd behavior in the market. However, this model cannot dynamically reflect changes in market herding behavior over a period of time. From CSAD model, it can be seen that if a specific period frequency is used as the window width, multiple quadratic coefficients of a specific frequency can be estimated, forming a time series of quadratic coefficients γ_2 of a specific frequency. The time series of γ_2 actually reflects the nonlinear relationship between the cross-sectional dispersion index and the average market return, which can serve as the observation basis for the dynamic changes of the market herd effect.

Thus, we define the Herd Effect Index as HERDI [32]. Considering that the stock market index and volatility are both positive and the γ_2 sequence has both positive and negative values, let γ_2^t be the quadratic coefficient of the CSAD model in the t-th period, Γ be the set of γ_2 in the m-period sample, $MAX(\Gamma)$ be the maximum value function, and $MEDIAN(\Gamma)$ be the median function. The standardization process is as follows:

$$HERDI_t = \frac{MAX(\Gamma) - \gamma_2^t}{MEDIAN(\Gamma)} \tag{10}$$

The standardization process uses dimensional processing to project the original γ_2 sequence into a positive range. A larger HERDI indicates a greater degree of herd behavior, while a smaller HERDI indicates weak or non-exist herd behavior. Compared to prior methodologies used for measuring herd behavior, the HERDI index wholly retains the quadratic term coefficient information from the CSAD method, constructing a time series for the herd behavior proxy variable. On one hand, it maintains the CSAD method's superior ability and enhanced sensitivity in capturing the overarching herd behavior in the market. On the other hand, it fully reflects the fluctuations in the nonlinear relationship between metrics like CSAD's dispersion index for cross-sectional returns and the average market portfolio returns. The HERDI index adopts a dynamic approach to evaluate herding behavior, which can capture real-time changes in herding effects under market conditions. This is in stark contrast to the static or lagged measurements typically provided by models such as CSAD. It vividly presents the dynamic process of market herd behavior's magnitude, showing its repeating changes. Moreover, HERDI has a certain warning mechanism when combined with the Ising financial model. The sensitivity of the HERDI model to the critical state of the market strength parameter β of the Ising model makes it a powerful warning system.

2.4. Data

The data include the daily yield of the SSE Index from January 1, 2005 to December 31, 2022, totaling 4374 data. This period covers many major stock market events: such as the 2005 split share structure reform, the 2008 financial crisis and the 2015 stock market disaster. In the part of herding index construction, we also selected the full sample data of a shares in Shanghai Stock Exchange in the same period. In terms of data screening, we only retained the daily closing price of all stocks in the sample range that are trading every year as a sample, and excluded the stocks whose data are missing for more than 20 days in one year. All the selected data come from CSMAR database.

In Table 1, we analyzed the statistical characteristics of cross-sectional yield difference CSAD and monthly γ_2 calculated from the SSE Index yield from January 1, 2005 to December 31, 2022. From Table 1, we can see that the kurtosis of the yield series of the three sets of data are all greater than 3, showing a phenomenon of sharp peaks and thick tails. From the skewness values, they are all biased distributions, where monthly γ_2 shows a right skewed distribution, which means that the distribution has a longer tail on the right side and the main body of the distribution is concentrated on the left side. This indicates that the probability of negative values appearing in the data is higher than the probability of positive values appearing, but the probability of a large positive return rate appearing is higher than that of a large negative return rate appearing. And all three sets of data have passed the ADF test, and there is no unit root in the test sequence, which is a stationary time series.

Table 1
Descriptive statistics of data samples.

Variables	Observations	Mean	Std	Max	Min	Kurtosis	Skewness	ADF test
CSAD	4374	0.019	0.006	0.080	0.007	12.684	2.168	−8.699
$monthly\gamma_2$	216	−1.411	29.416	139.759	−135.598	10.866	0.067	−17.266
HERDI	216	16.033	3.341	31.273	0.000	10.866	0.067	−2.324

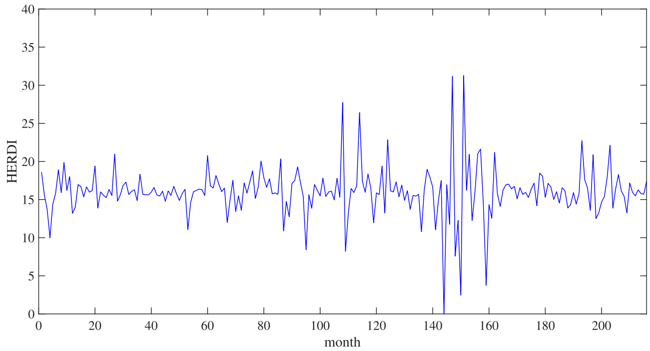

Fig. 1. Trend chart of HERDI index for 216 months.

3. Results

3.1. The trend of HERDI index

Fig. 1 shows the HERDI index from January 1, 2005 to December 31, 2022. The increase of HERDI index represents the increase of herding behavior in the market. We make a difference between each data point and the skew center of the group of HERDI index data to get Fig. 2. The difference can reveal the degree of deviation of each data point from the skew center, and help us judge which time the HERDI index value has increased significantly. We used the percentile method to determine the 95% confidence interval. Quantiles of 2.5% and 97.5% were found as the lower and upper limits of the interval. Among them, we are more concerned about the upper limit of the threshold (6.74). A large positive difference indicates that herding behavior is more intense than skew centers in a specific market state. The red line in Fig. 2 is the upper threshold limit (6.74) of the 95% confidence interval, and the HERDI index and time exceeding the upper threshold limit are as follows: in December 2013, the HERDI index was 27.73; In June 2014, the HERDI index was 26.43; In April 2015, the HERDI index was 22.84; In March 2017, the HERDI index was 31.16; In July 2017, the HERDI index was 31.27; In January 2021, the HERDI index was 22.72; In addition, in November 2021, the HERDI index was 22.11, which was also very close to the upper threshold.

At these moments, market sentiment is in a frenzy, and the irrationality of investors is increasing. Market participants have begun to have relatively clear and consistent expectations, tend to follow the behavior of the market or groups more consistently, and the degree of diversification of stock market transactions has declined significantly. At this point, a higher HERDI index may indicate an increase in potential market risks. Because excessive herding will lead to market prices deviating from their intrinsic value, increasing the risk of market bubbles or crashes.

3.2. The application of stochastic Ising model in financial markets

3.2.1. Statistical characteristics of price return series simulated by stochastic Ising financial model

In this section, we use constant β and random β to simulate a two-dimensional stochastic Ising financial model and compare our results with the historical return time series of the real stock market on the Shanghai Composite Index. At random β in the simulation, we set different values for the intensity of the impact on each trading day. When the intensity of the impact is high, investors at that grid point are more likely to change their thoughts due to the attitudes of their surrounding "neighbors", forming a consistent investment decision, causing the stock price to move in a specific direction. On the contrary, if the intensity of impact is low, investors will be more firm in their ideas and less susceptible to the influence of others, making stock prices less likely to fluctuate.

We analyzed the logarithmic return time series of the closing price from January 1, 2005 to December 31, 2022. The specific simulation steps are as follows: first, randomly set the opinions of each investor in the stock market (at each grid point in N^2), and set the probability of buying (+1) or selling (−1) their stocks at the beginning of each day as $p_1 = p_{-1} = 0.5$. The intensity of market interaction is fixed daily. Each time interval represents a trading minute. During this period, each trader has an opportunity

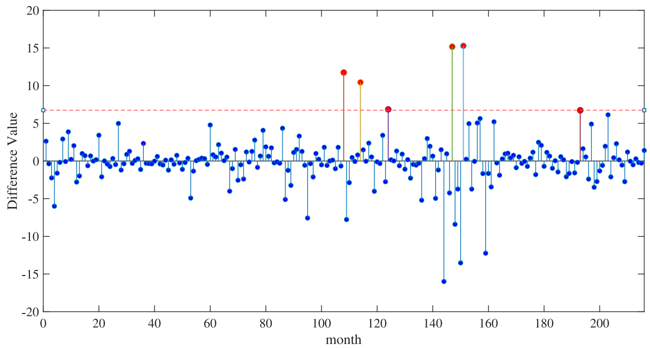

Fig. 2. Deviation of HERDI index from skew Center.

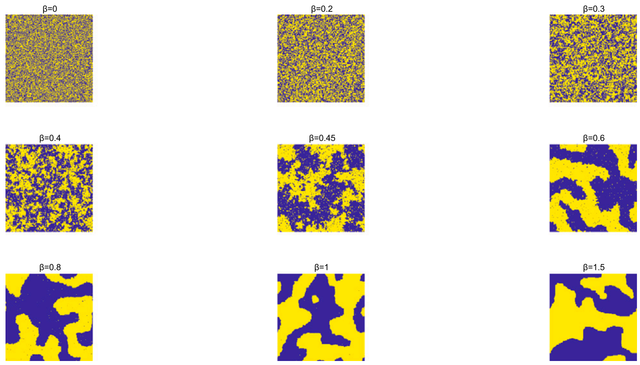

Fig. 3. The Role of β in the stochastic Ising Mode. β = 0, 0.2, 0.3, 0.4, 0.45, 0.6, 0.8, 1, 1.5, respectively.

to change their opinion state. Each trading day consists of 240 steps, corresponding to 4 h in the Chinese market, which is m = 240. After each day, we calculate the new lattice configuration values for all investor opinions. Since we assume that each trader has the same stock trading unit, the return time series is constructed by Eqs. (7) and (8).

Fig. 3 shows the respective $\beta < \beta_c$ and $\beta > \beta_c$ simulation diagrams at N = 200, m = 240, and t = 1, respectively. When $\beta = 0$, 0.2, 0.3, or 0.4, $\beta < \beta_c$, the number of buyers and sellers is relatively balanced, and there is no situation where particles of the same color gather together in a large area; When $\beta = 0.45$, $\beta = \beta_c$, a phase transition occurs; When $\beta = 0.6$, 0.8, 1 or 1.5, $\beta > \beta_c$, it is obvious that there is a situation where particles dominate, with either black or white grids with many identical particles gathered together. As β increases, the aggregation effect is becoming increasingly evident. Therefore, under different interaction intensities, the changes of different particles in the lattice are different.

The more general results are shown in Fig. 4. The results indicate that as the intensity of market interactions increases β from 0.01 to 0.45, the range of supply and demand imbalance and its volatility increase, and it is more obvious near at the critical point of the two-dimensional stochastic Ising system β_c. When the intensity of market interaction β is far less than or greater than β_c, the bar range is relatively small.

Furthermore, we presented probability density plots of simulations under different β conditions, which more intuitively reflected the peak distribution of returns under different β. From Fig. 5, it can be seen that as the intensity of market interaction β increases, the peak decreases. We found from the tail curve of probability density that as the absolute value of the return increases, its tail curve begins to cross the normal distribution curve outward, reflecting the nature of a thick tail.

We show the power law distribution [45,46] of simulation results under different intensity parameters in Fig. 6. During the simulation, it can be seen that as β increases, the fluctuations gradually increase, and the tail of probability density becomes thicker and thicker.

4. Linking stochastic Ising financial model and herd effect index

4.1. Herd behavior simulated by stochastic Ising financial model

The stochastic Ising financial model and the herd effect actually have a very similar fundamental driving force at the micro level: there is mutual influence between individual behavior and the behavior of other individuals in the surrounding environment. They all focused on how to generate collective and collective behavior patterns from a large number of individual, individual,

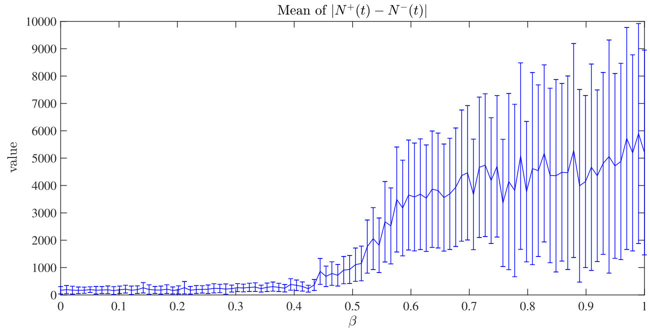

Fig. 4. The error column of stock supply and demand imbalance corresponds to the intensity of market interaction β. Perform 400 simulations on the stochastic Ising financial model to obtain the mean and standard deviation of each β, where N = 200, m = 240, and day = 1.

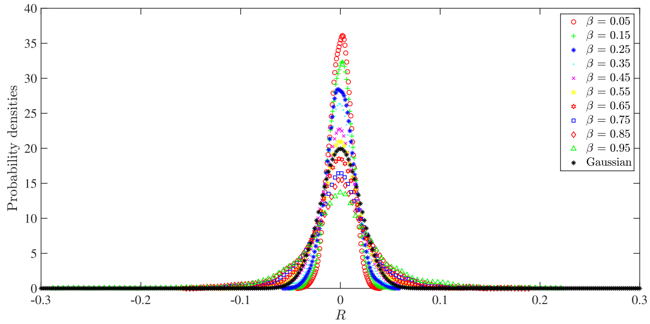

Fig. 5. Simulated probability density function. As the absolute value of the yield increases, the probability density tail curve begins to cross the normal distribution curve outward, reflecting the nature of a thick tail.

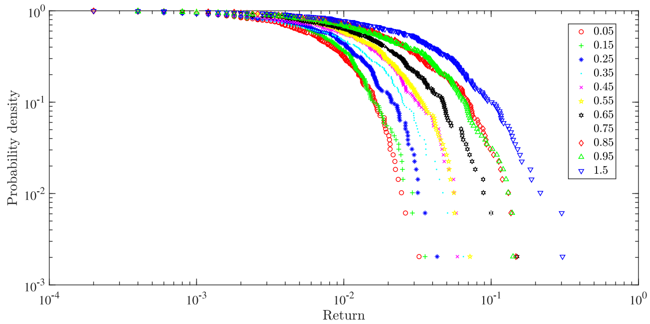

Fig. 6. Distribution of power law returns simulated by a stochastic financial model.

and independent decision-making behaviors, exploring the complex interaction between individual and group behavior. The HERDI index is a quantitative indicator that describes the intensity of herd behavior in the market. It usually measures the existence of group behavior by calculating the difference between individual behavior and group average behavior.

We assume that the stochastic Ising financial model simulates 1000 times in each day, converting the logarithmic return data of each simulation into a common return as the return R_i of an individual stock. The average of all 1000 simulated return data is used as the market return R_m, and according to the calculation method of $CSAD$, $CSAD_t = \frac{1}{n}\sum_{i=1}^{n}|R_{it} - R_{mt}|$, simulating the data of $CSAD_t$ under day = $[0, 490]$. Select 20 days as a window period, perform quadratic regression on $CSAD_t$ and R_{mt} according to Eq. (9), estimate 24 sets of quadratic coefficients, and standardize the time series of quadratic coefficients according to Eq. (10).

In the stochastic Ising financial model, the β parameter describes the strength of the interaction between market participants, and the increase or decrease in this influence directly affects the high and low changes of the HERDI index, thereby reflecting the degree of herd behavior in the market. The stochastic Ising financial system will undergo a phase transition near the critical point, similarly, financial markets may also experience a sudden transition from a disordered state to herd behavior under specific conditions. By observing the HERDI index, one can indirectly perceive the changes in β coefficients in the stochastic Ising financial model, thereby understanding the collective characteristics of market behavior.

Table 2
Descriptive statistics of simulated HERDI index.

β	Mean	Std	Max	Min	Kurtosis	Skewness
0.15	18.038	0.944	19.823	16.464	2.038	−0.013
0.25	18.480	0.618	19.709	17.328	2.779	0.481
0.35	18.783	0.536	19.920	17.885	2.313	0.190
0.45	26.122	1.272	28.351	24.056	2.038	0.048
0.65	29.546	1.664	33.583	26.772	2.806	0.454
0.85	31.763	1.641	34.764	28.000	2.664	−0.269

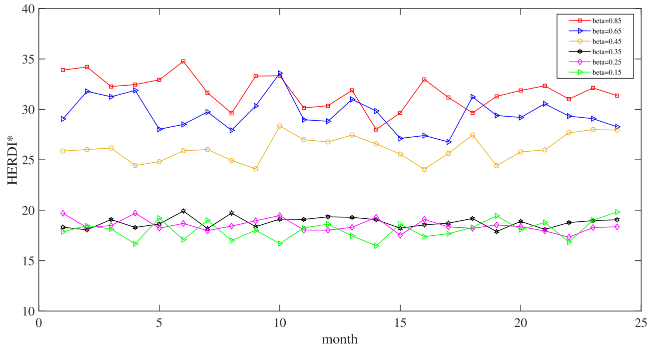

Fig. 7. $HERDI$ index simulated by stochastic Ising financial model under different β conditions.

Table 3
Mann Whitney U test results under different β.

β	0.85	0.65	0.45	0.35	0.25	0.15
0.85	1.000					
0.65	0.000	1.000				
0.45	0.000	0.000	1.000			
0.35	0.000	0.000	0.000	1.000		
0.25	0.000	0.000	0.000	0.074	1.000	
0.15	0.000	0.000	0.000	0.004	0.110	1.000

The results are shown in Fig. 7. Fig. 7 shows the HERDI index simulated by the stochastic Ising financial model under different β conditions. A descriptive statistical analysis of the simulated HERDI index under different β is shown in Table 2. Overall, with the increase of β, the simulated HERDI index also rose, indicating a positive correlation between the simulated HERDI index and β. The stronger the interaction between market participants, the more obvious the herding behavior in the market. Next, we use the Mann–Whitney U test to test the difference of the simulated HERDI index under different β in each two groups. Mann–Whitney U test is a non-parametric test method used to judge whether two independent samples come from the same distribution, especially when the samples do not meet the normal distribution or the sample size is small, Mann–Whitney U test can provide more reasonable test results. We performed pairwise tests on six sets of simulated HERDI data and showed the p-value in Table 3. The p-value reflects the probability that observed sample differences (or more extreme differences) will occur if the original hypothesis is true. The original hypothesis H0 is "two samples from the same population", and if the p-value is less than the set significance level (here we set the significance level at 0.05), the original hypothesis can be rejected, indicating a significant difference between the two sets of data.

From Table 3, it can be seen that the p-value of the pairwise test when β is greater than 0.45 (β = 0.85, 0.65, 0.45) and β is less than 0.45 (β = 0.15, 0.25, 0.35) is less than 0.05, which proves that when β is greater than 0.45, the HERDI index simulated by stochastic Ising financial model is significantly different from that when β is less than 0.45. When β was greater than 0.45, the p-value between each two groups was less than 0.05, and the simulated HERDI data were significantly different. It shows that after β reaches the critical point, the impact of interaction intensity on the behavior dynamics of market participants intensifies, and the synchronization of investor behavior in the market strengthens, which promotes the formation of market trends. When the market generally imitates behavior in a certain direction, such as large-scale buying or selling, this collective behavior leads to price deviation from basic value and even triggers market bubbles or collapses.

4.2. The tail distribution of the real market

Fig. 8 shows that during the period of bubbles generation and collapse (2013–2015,2016–2018), β in stochastic Ising's financial model is 0.45 higher than the critical point, and the tail curve of the actual market changes within the range of β = [0.45,0.85]. At the same time, the HERDI index based on market data has also significantly increased. It is proven that the increase in β in the stochastic Ising financial model leads to a gradual increase in the intensity of mutual influence among investors. Investors are easily

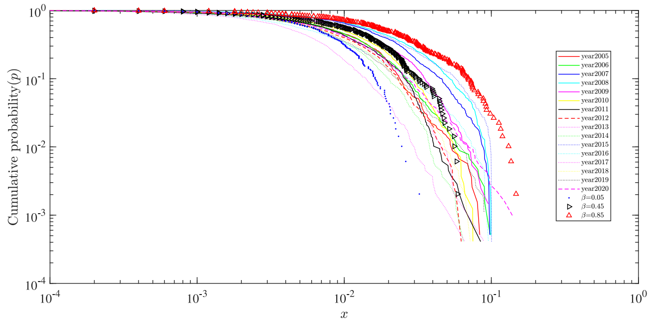

Fig. 8. Scope of β in stochastic Ising financial model. From 2013 to 2015, 2016 to 2018, the tail curve of the actual market was on the right side of the β = 0.45 tail simulated by the stochastic Ising model.

influenced by their surrounding "neighbors" and change their investment attitudes, starting to actively trade with others or market fluctuations, which leads to the emergence of herd behavior.

5. Conclusion

This paper applies the new Herd Effect Index (HERDI) to dynamically analyze the size and degree of market herding effect. The constructed HERDI index provides a dynamic regulatory perspective for regulatory agencies, helping them understand and quantitatively analyze market conditions from a dynamic perspective. At the same time, it further supplements and improves investors' risk management strategies. Moreover, based on the mutual influence between investors in the stock market, a financial market model was established using a two-dimensional stochastic Ising model. We simulated probability density plots under different interaction intensity β and discussed how market interaction intensity β affects the tail power-law distribution of stochastic Ising's financial price model. We found that as market intensity increases, the tail of probability density becomes fatter.

Finally, we used the stochastic Ising financial model to simulate the HERDI index and found that the larger the market intensity β, the larger the simulated HERDI index, and the greater the herd behavior in the market. The simulation results were compared with the real tail of the Chinese stock market. It is found that for the general stock market, the market intensity β is below the critical point of 0.45, but if there is a bubble and collapse, the market intensity β is greater than 0.45.

The stochastic Ising financial model captures and simulates herding behavior in financial markets through the parameter β, which is quantified using the HERDI index. The larger the β value, the more significant the herding effect, and the larger the HERDI index, the more likely the irrational behavior in the market is to be amplified. By adjusting the β value in the Ising model, researchers can simulate the impact of different levels of herding on the financial market, so as to deeply understand the market dynamics and its potential risks to financial stability. The stochastic Ising financial model provides a physical framework for analyzing and understanding herd behavior in the market, while HERDI provides an empirical measurement method for this behavior.

This finding can also be used to warn of market risk, which is reflected in the intensification of volatility in the HERDI index, especially when the β coefficient is close to the critical value, and can be regarded as an early warning signal of intensified collective market behavior and increased risk. In this case, the collective behavior of market participants tends to be consistent, and the market may be more vulnerable to information or events, resulting in sharp changes in prices. At this time, investors and market regulators can use this early warning signal to adjust strategies and measures. For investors, this may mean increasing the diversification of portfolios and adjusting their exposure to reduce the risk of potential collective behavior when the β coefficient is close to the threshold. For regulators, it may indicate the need to further monitor market sentiment and behavior to take timely measures to prevent possible market crashes. At the same time, as an indicator of market sentiment, HERDI's sharp fluctuations point out that market sentiment may be changing sharply.

The enlightenment of this paper for the follow-up research direction is to consider whether it is possible to use the market intensity β in the stochastic Ising financial model to construct an index to measure the herding effect, give different β values to simulate the degree of herding behavior that will occur in the market, and timely control the formation and accumulation of bubble. We can also consider building β through text information such as stock comments, which can analyze and predict the state of investors in the market in real time, provide trading opportunities for short-term traders, and remind long-term investors that the market may face important turning points.

CRediT authorship contribution statement

Yun Lan: Writing – original draft, Visualization, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Wen Fang:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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References

[1] M. Zhu, Informative fund size, managerial skill, and investor rationality, J. Financ. Econ. 130 (01) (2018) 114–134.
[2] V.N.C. Mushinada, Are individual investors irrational or adaptive to market dynamics?, J. Behav. Exp. Financ. 25 (2020) 100243.
[3] M. Krishnadas, K. Harikrishnan, G. Ambika, Recurrence measures and transitions in stock market dynamics, Phys. A 608 (2022) 128240.
[4] H.T. Diep, M. Kaufman, S. Kaufman, Dynamics of two-group conflicts: A statistical physics model, Phys. A 469 (2017) 183–199.
[5] M. Kaufman, S. Kaufman, H.T. Diep, Scenarios of social conflict dynamics on duplex networks, J. Policy Complex Syst. 3 (02) (2017) 3–13.
[6] M. Kaufman, H.T. Diep, S. Kaufman, Sociophysics of intractable conflicts: Three-group dynamics, Phys. A 517 (2019) 175–187.
[7] C.Y. Chen, D.R. Tan, H.M. Qin, B.H. Wang, J.X. Gao, Analysis of systemic risk from the perspective of complex networks: overview and outlook, Control Theory Appl. 39 (12) (2022) 2202–2218.
[8] Y.F. Lu, Z.Y. Zheng, K. Tian, Y. Zhang, A novel agent model of heterogeneous risk based on temporal interaction network for stock price simulation, Phys. A 625 (2023) 128981.
[9] L. Lima, Modeling of the financial market using the two-dimensional anisotropic Ising model, Phys. A 482 (2017) 544–551.
[10] W.X. Zhou, D. Sornette, Self-organizing Ising model of financial markets, Eur. Phys. J. B 55 (2007) 175–181.
[11] K. Sznajd-Weron, R. Weron, A simple model of price formation, Internat. J. Modern Phys. C 13 (01) (2002) 115–123.
[12] E. Callen, D. Shapero, A theory of social imitation, Phys. Today 27 (07) (1974) 23–28.
[13] E.W. Montroll, W.W. Badger, Introduction to Quantitative Aspects of Social Phenomena, Gordon and Breach, New York, 1974.
[14] A. Orléan, Bayesian interactions and collective dynamics of opinion: Herd behavior and mimetic contagion, J. Econ. Behav. Organ. 28 (02) (1995) 257–274.
[15] H. Niu, J. Wang, Volatility clustering and long memory of financial time series and financial price model, Digit. Signal Process. 23 (02) (2013) 489–498.
[16] M.J. De Oliveira, Isotropic majority-vote model on a square lattice, J. Stat. Phys. 66 (1992) 273–281.
[17] D.F. Melo, L.F. Pereira, F. Moreira, The phase diagram and critical behavior of the three-state majority-vote model, J. Stat. Mech. Theory Exp. 2010 (11) (2010) P11032.
[18] A.L.M. Vilela, B.J. Zubillaga, C. Wang, M. Wang, R. Du, H.E. Stanley, Three-state majority-vote model on barabási-albert and cubic networks and the unitary relation for critical exponents, Sci. Rep. 10 (2019) 8255.
[19] S. Bornholdt, A q-spin potts model of markets: Gain-loss asymmetry in stock indices as an emergent phenomenon, Phys. A 588 (2022) 126565.
[20] C. Schinckus, Ising model, econophysics and analogies, Phys. A 508 (2018) 95–103.
[21] R.J. Bloomfield, W.B. Tayler, F. Zhou, Momentum, reversal, and uninformed traders in laboratory markets, J. Finance 64 (06) (2009) 2535–2558.
[22] N. Barberis, A. Shleifer, R. Vishny, A model of investor sentiment, J. Financ. Econ. 49 (03) (1998) 307–343.
[23] T. Takahashi, T. Hadzibeganovic, S. Cannas, T. Makino, H. Fukui, S. Kitayama, Cultural neuroeconomics of intertemporal choice, Neuroendocrinol. Lett. 30 (2009) 185–191.
[24] J. Lakonishok, A. Shleifer, R.W. Vishny, The impact of institutional trading on stock prices, J. Financ. Econ. 32 (01) (1992) 23–43.
[25] R. Wermers, Mutual fund herding and the impact on stock prices, J. Finance 54 (02) (1999) 581–622.
[26] W.G. Christie, R.D. Huang, Following the pied piper: do individual returns herd around the market, Financ. Anal. J. 51 (04) (1995) 31–37.
[27] E.C. Chang, J.W. Cheng, A. Khorana, An examination of herd behavior in equity markets: An international perspective, J. Bank. Financ. 24 (10) (2000) 1651–1679.
[28] W.R. Yang, M.C. Chuang, Do investors herd in a volatile market? Evidence of dynamic herding in Taiwan, China, and US stock markets, Financ. Res. Lett. 52 (2023) 1003364.
[29] S.P. Da Gama, M.C. Klotzle, A.C.F. Pinto, L.L. Gomes, Herding behavior and contagion in the cryptocurrency market, J. Behav. Exp. Financ. 22 (2019) 41–50.
[30] D. Zheng, H. Li, X. Zhu, Herding behavior in institutional investors: Evidence from China's stock market, J. Multinatl. Financ. Manag. 32 (2015) 59–76.
[31] Q. Zhou, S.L. Sun, Q. Liu, The capital flow of stock market studies based on epidemic model with double delays, Physica A 526 (2019) 120733.
[32] Y.F. Zhang, L.K. Lei, Y. Wei, A new measurement index of herd effect and its predictive effect on stock market volatility in China, Syst. Eng. Theory Pract. 40 (11) (2020) 2810–2824.
[33] D. Cividino, R. Westphal, D. Sornette, Multi-Asset Financial Bubbles in an Agent-Based Model with Noise Traders' Herding Described by an N-Vector Ising Model, Soc. Sci. Elec. Pub., 2023.
[34] B.M. Roehner, D. Sornette, Thermometers of speculative frenzy, Eur. Phys. B 16 (04) (2000) 729–739.
[35] D. Sornette, Sweeping of an instability: an alternative to self-organized criticality to get powerlaws without parameter tuning, J. Phys. I 4 (02) (1994) 209–221.
[36] T. Kaizoji, S. Bornholdt, Y. Fujiwara, Dynamics of price and trading volume in a spin model of stock markets with heterogeneous agents, J. Phys. A 316 (1–4) (2002) 441–452.
[37] S. Bornholdt, Expectation bubbles in a spin model of markets: Intermittency from frustration across scales, Internat. J. Modern Phys. C 12 (05) (2001) 667–674.
[38] W. Fang, J. Wang, Statistical properties and multifractal behaviors of market returns by Ising dynamic systems, Internat. J. Modern Phys. C 23 (03) (2012) 1250023.
[39] D. Sornette, Physics and Financial Economics (1776-2014): Puzzles, Ising and Agent-Based Models, Swiss Finance Institute Research Paper Series, 2014.
[40] W. Fang, J. Ke, J. Wang, L. Feng, Linking market interaction intensity of 3D Ising type financial model with market volatility, Phys. A 461 (2016) 531–542.
[41] J. Kukacka, L. Kristoufek, Do complex financial models really lead to complex dynamics? Agent-based models and multifractality, J. Econ. Dyn. Control 113 (2020) 103855.
[42] J. Kukacka, L. Kristoufek, Does parameterization affect the complexity of agent-based models, J. Econ. Behav. Organ. 192 (2021) 324–356.
[43] R. Westphal, D. Sornette, Market impact and performance of arbitrageurs of financial bubbles in an agent-based model, J. Econ. Behav. Organ. 171 (2020) 1–23.
[44] H.T. Diep, G. Desgranges, Dynamics of the price behavior in stock markets: A statistical physics approach, Phys. A 570 (2021) 125813.
[45] X. Gabaix, P. Gopikrishnan, V. Plerou, H.E. Stanley, A theory of power-law distributions in financial market fluctuations, Nature 423 (2003) 267–270.
[46] A. Clauset, C.R. Shalizi, M.E.J. Newman, Power-law distributions in empirical data, SIAM Rev. 51 (2009) 661–703.