

# **Analysis of Rubik's Cube Solving Algorithm**

A Synopsis Submitted  
in Partial Fulfillment of the Requirements  
for the Course of  
**Minor Project - I**  
In  
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**Artificial Intelligence & Machine Learning**  
Under

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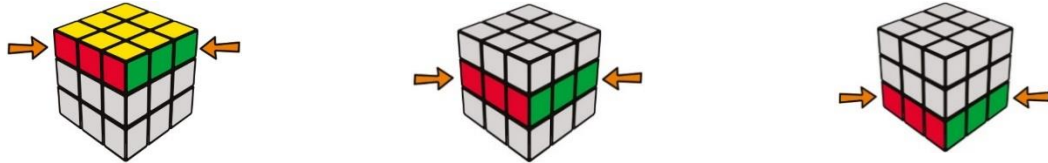


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# Synopsis

## 1. Introduction

The Rubik's Cube or Magic Cube is a 3 – dimensional combination puzzle which was invented by a Hungarian sculptor and professor of architecture named Ernő Rubik in the year 1974. The main motive for developing a cube by the professor was to explain 3D geometry to his students.

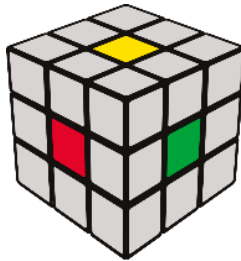


A 3x3x3 Rubik's cube has six faces with eight corners, twelve edges and six center pieces. Each of the faces have one of these six colors – white, yellow, green, blue, red and orange.

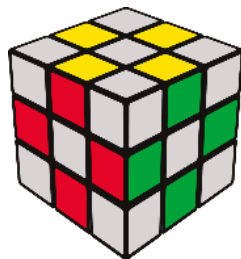
There are three layers – Top, Middle and Bottom layer.

**Centers** - Each of the six faces are defined by their own centers, i.e., the center piece of each side remains intact, throughout the process of solving the cube. Centers can't move.

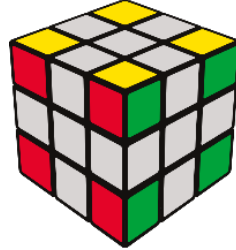
In a typical cube, white center is opposite of yellow, blue is opposite the green center and hence, red is opposite of orange center piece.



**Edges** – There are 12 edges in a cube with each edge piece having two colored tiles.



**Corner** – There are 8 corner pieces with each corner piece having three colored tiles.



A solved Rubik's cube is scrambled by twisting and turning its faces. Each face is represented by a letter.

R – Right face

L – Left face

U – Upward face

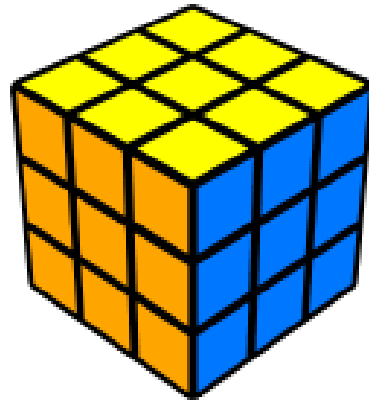
D – Downward face

F – Front face

B – Back face



Scrambled Rubik's Cube



Solved Rubik's Cube

There are  $8!$  ways to arrange the corner cubes, i.e., 40,320 ways while there are  $12! / 2$  (239,500,800) ways to arrange the edges. 11 edges can be flipped independently, with the flip of the twelfth depending on the previous ones, giving  $2^{11}$  (2,048) possibilities, which is approximately 43 quintillion. Basically, if one had one standard-sized Rubik's Cube for each permutation, one could cover the Earth's surface 275 times, or stack them in a tower 261 light-years high

There are a number of algorithms to solve a  $3 \times 3 \times 3$  Rubik's cube. This project mainly aims at analyzing various algorithms, thereby comparing and computing their time and space complexities. The algorithms would be –

1. Thistlethwaite's Algorithm
2. Kociemba's Algorithm
3. Korf's Algorithm
4. Tomas Rokicki's Algorithm

## 2. Motivation

Rubik's cube was invented with the purpose of teaching three-dimensional geometry to students, but over time it has not only motivated mathematicians & computer scientists to find the best & efficient solution to the cube, but also to find the best algorithm to scramble it properly. A staggering 43,252,003,274,489,856,000<sup>[1]</sup> possible states can be obtained by applying Markov-chain <sup>[1]</sup> Scientists around the world have often used Rubik's cube as a case study for developing new and better solving, scrambling & permutation algorithms.

The reason for choosing Rubik's cube is because it is very important in the field of Mathematics and Artificial Intelligence. Rubik's cube can be used to form complex counting equations while studying permutations & combinations. It is also popularly used for testing, training, and developing new or improved existing Artificial Intelligence models & algorithms.

## 3. Related work

Rubik's cube is a very popular mechanical puzzle which has attracted attention around the world because of its unique characteristics <sup>[8]</sup>. It is widely used for scientific research and development. Many scholars have worked on this to find an optimal solution to solve a Rubik's cube & many are still working to optimize it more using different techniques. Some people are working on famous Rubik's cube solving algorithm & optimizing them & some are working on mathematical approaches to develop a solution of their own.

The four most important approaches are the work of Thistlethwaite, Kociemba, Rokicki & Korf. Their advanced algorithms are based on group theory concepts and apply advanced concepts such as symmetry cancelation and dedicated traversal methods (e.g., Iterative Deep Searching combined with A\*) <sup>[6]</sup>.

Thistlethwaite's Algorithm (TWA) works by dividing the problem into 4 sub-problems - specifically subgroups and subsequently solving those sequences until it is solved <sup>[6]</sup>.

Kociemba's Algorithm takes the idea of dividing the problem into subgroups from Thistlethwaite but reduces the number of needed subgroups to only 2. This method uses an advanced implementation of IDA\*, generating small maps, calculating, and removing symmetries from the search tree and tends to solve the Cube close to the shortest number of moves possible <sup>[6]</sup>.

Rokicki realized that the initial parts of the pathways computed by Kociemba's Algorithm are solutions to a large set of related configurations. He exploits this property by dividing the problem into 2 billion cosets, each containing around 20 billion related configurations. With this method he was able to push the up-per bound to 22 moves sufficing to solve the Cube from any initial scrambled configuration <sup>[6]</sup>.

Korf's algorithm can solve any scrambled cube in 20 moves or fewer. It works by searching for solutions using IDA\* [9].

There are many research papers in famous international journals which focus on study & development of an optimized algorithm for solving it & to find the God's Number for the cube. God's number is the maximum number of moves required to solve any of the 43 quintillion permutations combinations of the cube <sup>[10]</sup>.

In today's world many AI & Robotics enthusiasts are working to solve the Rubik's Cube using deep neural networks & implementing it using a robotic arm [7]. Various Scientists are using Rubik's Cube as a model to study crystal electron diffraction & other scientific issues. Research Papers & Articles of some famous algorithms have been listed below –

<https://tomas.roicki.com/rubik25.pdf>

[https://www.researchgate.net/publication/226585368\\_Twenty-Two\\_Moves\\_Suffice\\_for\\_Rubik's\\_CubeR](https://www.researchgate.net/publication/226585368_Twenty-Two_Moves_Suffice_for_Rubik's_CubeR)

<http://www.youngscientist.com.au/wp-content/uploads/2015/02/Physics-10-12-Justin-Marcellienus-report.pdf>

<https://www.cs.princeton.edu/courses/archive/fall06/cos402/papers/korfrubik.pdf>

<https://www.nature.com/articles/s42256-019-0070-z>

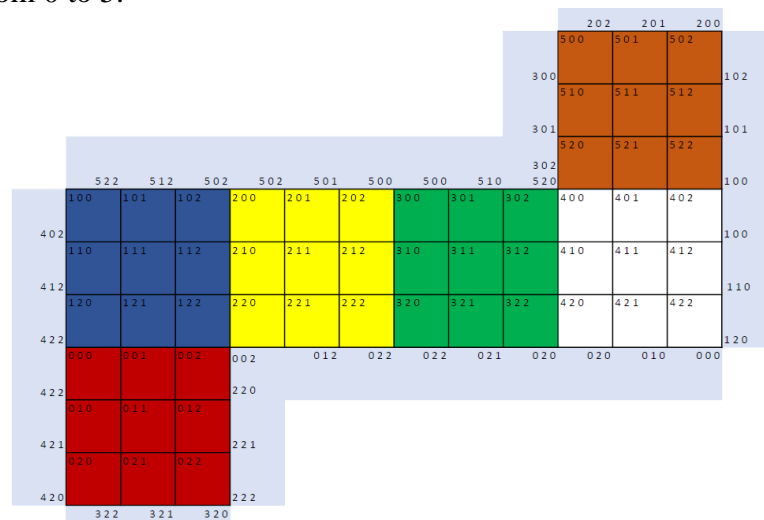
## 4. Proposed Method

Programming Language: C++

### Solving Algorithms: Thistlethwaite's, Kociemba's, Korf's, Tomas Rokicki's Algorithm

### Scrambling Algorithm: Markov-chain Algorithm

Representation of Rubik's cube: The cube will be represented in three dimensional array as A[6][3][3]. All the colors will be represented in 6 integers ranging from 0 to 5.



### Environment Specifications:

<u>Type</u>	<u>Name</u>
Operating System	Windows 10 V-21H1 B-19042.1110 64bit
Memory	8 GB 2400 MHz DDR4
Storage	250 GB SATA3 2.4 SSD
C++	Version ISO/IEC 14882:2020 OR C++20
IDE	Visual Studio 2019 V16.11

Parameters: Parameters for evaluation of the algorithms are

1. No. of Moves
2. Time Complexity
3. Space Complexity
4. Line of Code

### **5. Methodology**

In this project, working of all the four algorithms - Thistlethwaite's, Kociemba's, Korf's and Tomas Rokicki's, would be studied and analyzed, followed by coding of the algorithms. For the purpose of scrambling an unsolved cube, Markov-chain Algorithm would be studied and used. After merging the four algorithms and the scrambling algorithm in a single file, the four algorithms would be analyzed on the basis of –

1. The number of steps each algorithm takes to solve the scrambled cube
2. Time and space complexity of each algorithm

Lastly, a project report would be prepared consisting of all the details of the project and the observed results.

### **6. Plan of work**

<b>Date</b>	<b>Expected Work</b>
1 <sup>st</sup> September 2021 – 15 <sup>th</sup> September 2021	<ul style="list-style-type: none"><li>• Detailed study and analysis of all the four algorithms that need to be implemented.</li></ul>
16 <sup>th</sup> September 2021 – 30 <sup>th</sup> September 2021	<ul style="list-style-type: none"><li>• Beginning the coding part</li><li>• Designing a method to shuffle the Rubik's Cube</li><li>• Starting implementing the four main algorithms</li></ul>
1 <sup>st</sup> October 2021 – 20 <sup>th</sup> October 2021	<ul style="list-style-type: none"><li>• Completion of the code for all the algorithms</li><li>• Merging all the algorithms into a single file</li><li>• Starting designing an algorithm to solve the Rubik's cube</li></ul>
21 <sup>st</sup> October 2021 – 10 <sup>th</sup> November 2021	<ul style="list-style-type: none"><li>• Analyzation of results of all the algorithms</li><li>• Completion of the project</li><li>• Preparation of Project Report</li></ul>

## 7. References

- [1] <https://theconversation.com/how-hard-is-it-to-scramble-rubiks-cube-129916>
- [2] [https://www.researchgate.net/publication/220867821\\_An\\_Evolutionary\\_Approach\\_for\\_Solving\\_the\\_Rubik's\\_Cube\\_Incorporating\\_Exact\\_Methods](https://www.researchgate.net/publication/220867821_An_Evolutionary_Approach_for_Solving_the_Rubik's_Cube_Incorporating_Exact_Methods)
- [3] <https://d4mucfpksyvv.cloudfront.net/papers/solving-rubiks-cube.pdf>
- [4] <https://cjme.springeropen.com/articles/10.1186/s10033-018-0269-7>
- [5] <https://personal.math.ubc.ca/~cass/courses/m308/projects/rtran/rtran.pdf>
- [6] [https://www.researchgate.net/publication/220867821\\_An\\_Evolutionary\\_Approach\\_for\\_Solving\\_the\\_Rubik's\\_Cube\\_Incorporating\\_Exact\\_Methods](https://www.researchgate.net/publication/220867821_An_Evolutionary_Approach_for_Solving_the_Rubik's_Cube_Incorporating_Exact_Methods)
- [7] <https://d4mucfpksyvv.cloudfront.net/papers/solving-rubiks-cube.pdf>
- [8] <https://cjme.springeropen.com/articles/10.1186/s10033-018-0269-7>
- [9] <https://medium.com/@benjamin.botto/implementing-an-optimal-rubiks-cube-solver-using-korf-s-algorithm-bf750b332cf9#:~:text=on%20my%20GitHub,-.Korf's%20Optimal%20Algorithm,is%20a%20tree%2Dtraversal%20algorithm.>
- [10] <https://www.cubelelo.com/blogs/cubing/how-to-solve-rubiks-cube-in-20-moves>