TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Examination Logic & Set Theory (2IT61)

Monday April 8, 2013, 14:00–17:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(1) 1. Determine $\mathcal{P}(\{0,1\} \times \{2\})$.

(2) 2. Prove with a *calculation* (i.e., using the formal system based on standard equivalences and weakenings described *Part I* of the book) that the abstract proposition

$$(P \wedge R) \Rightarrow \neg(Q \wedge \neg(P \wedge R))$$

is a tautology.

3. Let A be the set of all airplanes, and let P be the set of all pilots. Furthermore, let N be a predicate on $P \times A$, and let F be a predicate on A, with the following interpretations:

$N(p, a)$ means 'p navigates a', and
$F(a)$ means 'a is flying'.

Give formulas of predicate logic that express the following statements:

(1) (a) If every pilot navigates an airplane, then every airplane is flying.

(1) (b) Every flying airplane is navigated by exactly one pilot.

(2) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\neg\exists_x[P(x) \vee Q(x)] \Rightarrow \forall_y[\neg Q(y)]$$

is a tautology.

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(3) 5. Check, for each of the following formulas, whether it holds for all sets A, B, and C. If so, then give a proof; if not, then give a counterexample.

(a) $A \cap B \subseteq C \Rightarrow B \setminus A \subseteq B \setminus C$;

(b) $A \cap B \subseteq C \Rightarrow B \setminus C \subseteq B \setminus A$.

(3) 6. Define the binary relation S on $\mathbb{R}$ for all $x, y \in \mathbb{R}$ by

$$x \; S \; y \text{ if, and only if, there exists } a \in \mathbb{R}^+ \text{ such that } y = ax \ .$$

(Note: $\mathbb{R}^+ = \{x \in \mathbb{R} | x > 0\}$.)

Prove that S is an equivalence relation on $\mathbb{R}$.

7. Let A and B be sets, and let $F : A \to B$ and $G : B \to A$ be mappings such that the formula $\forall_y [y \in B : F(G(y)) = y]$ holds.

(2) (a) Prove that F is a surjection.

(2) (b) Show with a counterexample that F is not necessarily an injection.

(3) 8. The sequence $a_0, a_1, a_2, \ldots$ of natural numbers is inductively defined by:

$$\begin{aligned} a_0 &:= 3 \ , \\ a_1 &:= 6 \ , \text{ and} \\ a_{i+2} &:= a_{i+1} + 2 \cdot a_i \qquad (i \in \mathbb{N}) \ . \end{aligned}$$

Prove that $a_n = 3 \cdot 2^n$ for all $n \in \mathbb{N}$.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.