

Examination cover sheet

(to be completed by the examiner)

Course name: Logic and Set Theory (final examination)

Course code: 2IT61/2IHT10

Date: April 12, 2016

Start time: 18:00

End time : 21:00

Number of pages: 2

Number of questions: 8

Maximum number of points/distribution of points over questions: 20 (the number between parentheses in front of a problem indicates how many points you score with a correct answer to it)

Method of determining final grade: the grade for this examination will be determined by dividing the total number of scored points by 2

Answering style: open questions

Exam inspection: a review session will be organised

Other remarks: A partially correct answer is sometimes awarded with a fraction of the points.

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

Final examination Logic & Set Theory (2IT61/2IHT10)

Tuesday April 12, 2016, 18:00–21:00 hrs.

- (2) 1. Let φ be the formula

$$(a \Rightarrow \neg b) \vee (\neg c \wedge d) \vee ((d \wedge e) \Leftrightarrow \mathbf{False}) .$$

Show that φ is contingent (i.e., neither a tautology, nor a contradiction).

2. Let \mathbf{H} be the set of all *humans*, let \mathbf{W} be the set of all *weekly tests* and let \mathbf{F} be the set of all *final examinations*. Furthermore, let Adam denote a particular human, i.e., $\text{Adam} \in \mathbf{H}$, and let $S \subseteq \mathbf{H}$ and $P \subseteq \mathbf{H} \times (\mathbf{W} \cup \mathbf{F})$ be the predicates with the following interpretation

$$\begin{aligned} S(x) &\text{ means '}x \text{ is a student' ,} \\ P(x, y) &\text{ means '}x \text{ has passed } y\text{' .} \end{aligned}$$

Give formulas of predicate logic that express the following statements:

- (1) (a) Adam has passed all weekly tests but no final examination.
(1) (b) If a student has passed all weekly tests, then this student has also passed a final examination.
- (3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists x \forall y [P(x, y) \Rightarrow Q(y)] \Rightarrow \forall z [\neg Q(z) : \exists u [\neg P(u, z)]]$$

is a tautology.

- (2) 4. Does the equivalence

$$\forall x [x \in \mathbb{N} : \exists y [y \in \mathbb{N} : P(x, y)]] \stackrel{val}{=} \exists y [y \in \mathbb{N} : \forall x [x \in \mathbb{N} : P(x, y)]]$$

hold for all binary predicates P on natural numbers? (Give a proof or a counterexample.)

- (2) 5. Check whether the formula

$$A \times A \subseteq B \times C \Rightarrow (A \setminus B) \cap C = \emptyset$$

holds for all sets A , B and C . If so, then give a proof; if not, then give a counterexample.

Final examination Logic & Set Theory (2IT61/2IHT10)

Tuesday April 12, 2016, 18:00–21:00 hrs.

6. Consider the mapping (function) $f : \mathbb{R} \rightarrow (\mathbb{R}^+ \cup \{0\})$ defined for all $x \in \mathbb{R}$ by

$$f(x) = x^2 - 6x + 9 .$$

- (1) (a) Show that f is not an injection.
 - (1) (b) Let $X \subseteq \mathbb{R}$. Give the formula for ‘ $f : X \rightarrow (\mathbb{R}^+ \cup \{0\})$ is a bi-jection’.
 - (1) (c) Give an $X \subseteq \mathbb{R}$ such that $f : X \rightarrow (\mathbb{R}^+ \cup \{0\})$ is a bi-jection.
- (3) 7. Prove that $7^n - 1$ is divisible by 6 for all $n \geq 1$.
- (3) 8. We define a binary relation R on \mathbb{N}^+ , the set of all positive natural numbers, for all $x, y \in \mathbb{N}^+$ by

$$x R y \text{ if, and only if, there exist } m, n \in \mathbb{N}^+ \text{ such that } x^m = y^n.$$

Prove that R is an equivalence relation.