TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07)

Thursday January 22, 2015, 13:30–16:30 hrs.

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1. Prove that:

(1) (a) $P \Leftrightarrow Q$ is stronger than $P \Rightarrow Q$,

(2) (b) $(P \wedge Q) \Rightarrow R$ and $(Q \wedge \neg R) \Rightarrow P$ are incomparable.

2. Write the following sentences as a formula of predicate logic:

(1) (a) Not every integer is a multiple of 481.

(1) (b) There are no two natural numbers that are squares and differ five.

3. Determine whether the following formulas hold for all sets A, B and C. If so, give a proof, if not, give a counterexample.

(2) (a) $A^{\mathrm{c}} \subseteq B \cup C \Rightarrow A^{\mathrm{c}} \cap B \subseteq C$

(2) (b) $A^{\mathrm{c}} \cap B \subseteq C \Rightarrow B \subseteq A \cup C$

4. Consider the mapping $f : \mathbb{R} \to \mathbb{R}$ that is defined, for all $x \in \mathbb{R}$, by

$$f(x) = 2x^2 - 3 \ .$$

(1) (a) Determine $f^{\leftarrow}(\{5\})$.

(2) (b) Give the formula that expresses 'f is an injection' and show with a counterexample that f is not an injection.

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5. Define the relation R on $\{0,1,2\} \times \{1,2,3\}$ by

   $$(a,b)R(c,d) \text{ iff } a+b < c+d$$

(1) (a) Prove that R is irreflexive.

(2) (b) Draw a Hasse-diagram of $\langle \{0,1,2\} \times \{1,2,3\}, R \rangle$.

(1) (c) Give the minimal elements of the subset $\{(1,1),(2,2),(1,2),(2,3),(0,2)\}$.

(4) 6. Prove that every integer postage greater than 13 can be formed by using only 3-cent and 8-cent stamps.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.