TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Examination Logic & Set Theory (2IT61)

Monday January 21, 2013, 14:00–17:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(1) 1. Show that the following abstract proposition is a contingency (i.e., not a tautology and not a contradiction):

$$((a \wedge b) \Leftrightarrow (\neg c \vee b)) \wedge \neg(a \Rightarrow c) \;\; .$$

(2) 2. Prove with a *calculation* (i.e., using the formal system based on standard equivalences and weakenings described *Part I* of the book) that the abstract propositions

$$P \Rightarrow ((Q \Rightarrow R) \wedge (Q \vee R)) \text{ and } (\neg P \Rightarrow Q) \Rightarrow R$$

are *comparable* (i.e., the left-hand side formula is stronger than the right-hand side formula, or the right-hand side formula is stronger than the left-hand side formula).

(2) 3. Determine whether the formula

$$\forall_x[x \in \mathbb{Z} : \exists_y[y \in \mathbb{Z} : 2x - y = 3]]$$

is true or false, and give arguments for your answer.

(2) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists_x \forall_y [P(x) \Rightarrow Q(y)] \;\Rightarrow\; (\neg\forall_u[P(u)] \vee \exists_v[Q(v)])$$

is a tautology.

(2) 5. Check whether the formula

$$A \cap B \subseteq C \;\Rightarrow\; A \backslash C \subseteq B^{\mathrm{C}}$$

holds for all sets A, B and C. If so, then give a proof; if not, then give a counterexample.

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(2) 6. (a) Show with a counterexample that the formula

$$\forall_x[x \in A : G_1(F(x)) = G_2(F(x))] \Rightarrow \forall_y[y \in B : G_1(y) = G_2(y)]$$

does not hold for all sets A, B and C, and for all mappings $F : A \to B$, $G_1 : B \to C$ and $G_2 : B \to C$.

(2) (b) Let A, B and C be sets and let $F : A \to B$, $G_1 : B \to C$ and $G_2 : B \to C$ be mappings.
Prove: if F is a *surjection*, then the formula

$$\forall_x[x \in A : G_1(F(x)) = G_2(F(x))] \Rightarrow \forall_y[y \in B : G_1(y) = G_2(y)]$$

does hold.

(4) 7. Prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for every natural number n.

8. Define the binary relation R on $\mathbb{N} \times \mathbb{N}$ for all $k, \ell, m, n \in \mathbb{N}$ by

$$(k, \ell)\ R\ (m, n) \text{ if, and only if, } k \leq m \wedge (k = m \Rightarrow \ell \leq n)\ .$$

(2) (a) List the three properties the relation R should satisfy for being a *reflexive ordering* together with their defining formulas, and prove one of these properties. (You may choose yourself which of the three properties you prefer to prove.)

(1) (b) Draw a Hasse diagram of $\langle \{0,1,2\} \times \{0,1,2\}, R \rangle$.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.