TU/e Technische Universiteit Eindhoven University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Logic and Set Theory (final examination) Course code: 2IT61/2IHT10

Date: April 12, 2016

Start time: 18:00 End time : 21:00

Number of pages: 2

Number of questions: 8

Maximum number of points/distribution of points over questions:20 (the number between parentheses in front of a problem indicates how many points you score with a correct answer to it)

Method of determining final grade: the grade for this examination will be determined by dividing the total number of scored points by 2

Answering style: open questions

Exam inspection: a review session will be organised

Other remarks: A partially correct answer is sometimes awarded with a fraction of the points.

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

Final examination Logic & Set Theory (2IT61/2IHT10)

Tuesday April 12, 2016, 18:00–21:00 hrs.

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(2) 1. Let φ be the formula

$$(a \Rightarrow \neg b) \vee (\neg c \wedge d) \vee ((d \wedge e) \Leftrightarrow \texttt{False}) \ .$$

Show that φ is contingent (i.e., neither a tautology, nor a contradiction).

2. Let $\mathbf{H}$ be the set of all *humans*, let $\mathbf{W}$ be the set of all *weekly tests* and let $\mathbf{F}$ be the set of all *final examinations*. Furthermore, let Adam denote a particular human, i.e., Adam $\in \mathbf{H}$, and let $S \subseteq \mathbf{H}$ and $P \subseteq \mathbf{H} \times (\mathbf{W} \cup \mathbf{F})$ be the predicates with the following interpretation

$S(x)$ means '*x is a student*' ,
$P(x, y)$ means '*x has passed y*' .

Give formulas of predicate logic that express the following statements:

(1) (a) Adam has passed all weekly tests but no final examination.

(1) (b) If a student has passed all weekly tests, then this student has also passed a final examination.

(3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\exists_x \forall_y [P(x, y) \Rightarrow Q(y)] \Rightarrow \forall_z [\neg Q(z) : \exists_u [\neg P(u, z)]]$$

is a tautology.

(2) 4. Does the equivalence

$$\forall_x [x \in \mathbb{N} : \exists_y [y \in \mathbb{N} : P(x, y)]] \stackrel{val}{==} \exists_y [y \in \mathbb{N} : \forall_x [x \in \mathbb{N} : P(x, y)]]$$

hold for all binary predicates P on natural numbers? (Give a proof or a counterexample.)

(2) 5. Check whether the formula

$$A \times A \subseteq B \times C \Rightarrow (A \backslash B) \cap C = \emptyset$$

holds for all sets A, B and C. If so, then give a proof; if not, then give a counterexample.

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6. Consider the mapping (function) $f : \mathbb{R} \to (\mathbb{R}^+ \cup \{0\})$ defined for all $x \in \mathbb{R}$ by

$$f(x) = x^2 - 6x + 9 \ .$$

(1) (a) Show that f is not an injection.

(1) (b) Let $X \subseteq \mathbb{R}$. Give the formula for '$f : X \to (\mathbb{R}^+ \cup \{0\})$ is a bi-jection'.

(1) (c) Give an $X \subseteq \mathbb{R}$ such that $f : X \to (\mathbb{R}^+ \cup \{0\})$ is a bi-jection.

(3) 7. Prove that $7^n - 1$ is divisible by 6 for all $n \geq 1$.

(3) 8. We define a binary relation R on $\mathbb{N}^+$, the set of all positive natural numbers, for all $x, y \in \mathbb{N}^+$ by

$$x \ R \ y \text{ if, and only if, there exist } m, n \in \mathbb{N}^+ \text{ such that } x^m = y^n.$$

Prove that R is an equivalence relation.