TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61)

Monday November 5, 2012, 9:00–12:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(2) 1. Show that the following abstract proposition is a contingency (i.e., not a tautology and not a contradiction):

$$(a \Rightarrow (b \wedge c)) \wedge (\neg d \Leftrightarrow (b \vee c)) \wedge (a \wedge \neg e) \ .$$

2. Determine for each of the following propositions whether it is true or false. Give arguments for your answers.

(1) (a) $\forall_x[x \in \mathbb{Z} : \exists_y[y \in \mathbb{Z} : 2x - y = 3]]$;

(1) (b) $\exists_y[y \in \mathbb{Z} : \forall_x[x \in \mathbb{Z} : 2x - y = 3]]$.

(3) 3. Prove that the abstract propositions

$$(P \Leftrightarrow \neg Q) \quad \text{and} \quad (P \vee \neg(Q \Rightarrow P))$$

are *comparable* (i.e., the left-hand side formula is stronger than the right-hand side formula, or the right-hand side formula is stronger than the left-hand side formula).

(2) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$(\forall_x[P(x)] \wedge \neg\exists_y[P(y) \wedge Q(y)]) \Rightarrow \neg\exists_x[Q(x)]$$

is a tautology.

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5. Check for each of the following formulas whether it holds for all sets A, B and C. If so, then give a proof; if not, then give a counterexample.

(1) (a) $((A \cap (B \backslash C)) = \emptyset) \;\Rightarrow\; ((A \cap B) = (A \cap C))$;

(1) (b) $((A \cap B) = (A \cap C)) \;\Rightarrow\; ((A \cap (B \backslash C)) = \emptyset)$.

6. Consider the mapping $F : \mathbb{Z} \to \mathbb{Z}$ defined for all $x \in \mathbb{Z}$ by

$$F(x) = 2x^2 + 3 \ .$$

(1) (a) Determine $F(\{-1, 0, 1\})$.

(1) (b) Is F a bijection? (Motivate your answer!)

(3) 7. Let R be an equivalence relation on V, and let $a, b, c \in V$. Prove that

$$(c \in K(a) \wedge \neg(c \in K(b))) \Rightarrow \neg(b \in K(a)) \ .$$

(As usual, $K(x)$ denotes the equivalence class of x.)

(4) 8. Prove that $4n^3 - 4n$ is divisible by 3 for every natural number n.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.