TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07)

Thursday October 30, 2014, 9:00–12:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(2) 1. Prove that the formulas

$$a \Rightarrow (b \vee c) \quad \text{and} \quad (b \Rightarrow a) \wedge \neg c$$

are incomparable.

(1) 2. Prove with a *calculation* (i.e., using the formal system based on standard equivalences described in *Part I* of the book) that

$$P \Rightarrow \neg Q \stackrel{val}{==} \neg(P \wedge Q) \ .$$

3. Let $\mathbb{P}$ be the set of all people, let Anna denote a particular person in $\mathbb{P}$, and let $\mathbb{B}$ be the set of all books. Furthermore, let R and L be predicates on $\mathbb{P} \times \mathbb{B}$ with the following interpretations for all $p \in \mathbb{P}$ and $b \in \mathbb{B}$:

$R(p, b)$ means 'p has read b', and
$L(p, b)$ means 'p liked b'.

Give formulas of predicate logic that express the following statements:

(1) (a) Everybody has read a book.
(1) (b) Anna has only read books she liked.

(3) 4. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$(\forall_x[P(x) \Rightarrow \neg Q(x)] \wedge \exists_y[P(y) : Q(y) \vee R(y)]) \Rightarrow \exists_z[P(z) \wedge R(z)]$$

is a tautology.

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(2) 5. Prove that the following formula holds for all sets A, B and C:

$$A \cap B = A \cap C \;\Rightarrow\; A \cap (B \setminus C) = \emptyset \ .$$

(3) 6. Let the sequence $a_0, a_1, a_2, \ldots$ be inductively defined by

$$\begin{aligned} a_0 &:= 0 \\ a_1 &:= 1 \\ a_{i+2} &:= 3a_{i+1} - 2a_i \qquad (i \in \mathbb{N}). \end{aligned}$$

Prove that $a_n = 2^n - 1$ for all $n \in \mathbb{N}$.

7. Let A and B be sets, and let $F : A \to B$ and $G : B \to A$ be mappings such that $\forall_x[x \in A : G(F(x)) = x]$.

(2) (a) Prove that F is an injection.

(1) (b) Show, with a counterexample, that F is not necessarily a surjection.

8. We define

$$V := \mathcal{P}(\{0,1,2\}) \backslash \{\{0,1\}, \{1,2\}, \{0,1,2\}\} \ .$$

(1) (a) Determine V.

(2) (b) Make a Hasse diagram of $\langle V, \subseteq \rangle$.

(1) (c) What are the minimal elements of V in $\langle V, \subseteq \rangle$?
What are the maximal elements of V in $\langle V, \subseteq \rangle$?

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.