TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Final examination Logic & Set Theory (2IT61/2IT07)

Thursday January 23, 2014, 14:00–17:00 hrs.

You are **not** allowed to use any books, notes, or other course material. Your solutions to the problems have to be formulated and written down in a clear and precise manner.

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(2) 1. Prove that the formulas

$$(P \Rightarrow Q) \vee R \quad \text{and} \quad P \vee \neg(Q \vee R)$$

are incomparable.

(2) 2. Prove with a *calculation* (i.e., using the methods described in Part 1 of the book) that

$$P \Rightarrow (Q \Rightarrow R) \stackrel{val}{==} (Q \vee R) \Rightarrow (P \Rightarrow R).$$

3. Write the following sentences as a formula of predicate logic (for (b), you may use $\mathbb{P}$ to denote the set of all primes):

(1) (a) There is no natural between 3 and 7 that is a square.

(1) (b) Every odd natural can be written as the sum of the squares of at most three primes.

(2) 4. Give three different elements of the set $\mathcal{P}(\{3,7\} \times \mathbb{N} \times \{\emptyset\})$.

(3) 5. Let $F : A \to B$ be an injective mapping. Prove with a *derivation* (i.e., using the methods described in *Part II* and *III* of the book) that

$$\forall_{X,Y}[X, Y \subseteq A : X \cap Y = \emptyset \Rightarrow F(X) \cap F(Y) = \emptyset].$$

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(4) 6. The sequence $a_0, a_1, a_2, \ldots$ is inductively defined by

$$\begin{aligned} a_0 &:= 4 \\ a_{n+1} &:= 3 \cdot a_n - 2 \cdot n - 5 \end{aligned}$$

Prove that $a_n = 3^n + n + 3$ for all $n \in \mathbb{N}$.

7. Define the relation R on $\mathbb{N} \times \mathbb{N}$ by

$$(a, b)\ R\ (x, y) \text{ if, and only if, } a < x \wedge b \geq y \quad (\text{for } a, b, x, y \in \mathbb{N}) \ .$$

(2) (a) Prove that $\langle \mathbb{N} \times \mathbb{N}, R \rangle$ is an *irreflexive ordering*.

(2) (b) Draw a Hasse diagram of $\langle \{2, 3\} \times \{5, 6, 7\}, R \rangle$.

(1) (c) Give the minimal elements of $\langle \mathbb{N} \times \mathbb{N}, R \rangle$.

The number between parentheses in front of a problem indicates how many points you score with a correct answer to it. A partially correct answer is sometimes awarded with a fraction of those points. The grade for this examination will be determined by dividing the total number of scored points by 2.