TU/e Technische Universiteit Eindhoven University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Logic and Set Theory (final examination) | Course code: 2IT61/2IT07

Date: April 14, 2015

Start time: 18:00 | End time : 21:00

Number of pages: 2

Number of questions: 7

Maximum number of points/distribution of points over questions:20 (the number between parentheses in front of a problem indicates how many points you score with a correct answer to it)

Method of determining final grade: the grade for this examination will be determined by dividing the total number of scored points by 2

Answering style: open questions

Exam inspection: a review session will be organised

Other remarks: A partially correct answer is sometimes awarded with a fraction of the points.

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☐ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☐ Other:

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

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(2) 1. Show that the following abstract proposition is a contingency (*i.e.*, neither a tautology, nor a contradiction):

$$((\neg a \vee c) \Rightarrow (a \Leftrightarrow b)) \wedge ((\neg c \wedge d) \Rightarrow e) \ .$$

2. Let $\mathbf{L}$ be the set of all traffic lights at a crossing. Furthermore, let O be a binary predicate on $\mathbf{L}$, and let G, Y, and R be unary predicates on $\mathbf{L}$ with the following interpretations for all $\ell, \ell' \in \mathbf{L}$:

$O(\ell, \ell')$ means 'ℓ and ℓ' are opposite to each other' ,
$G(\ell)$ means 'ℓ shows green' ,
$Y(\ell)$ means 'ℓ shows yellow' , and
$R(\ell)$ means 'ℓ shows red' .

Give formulas of predicate logic that express the following statements:

(1) (a) Every traffic light shows green or red or yellow.

(1) (b) Distinct traffic lights do not both show green, unless they are opposite to each other.

(3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$\forall_x[P(x) \vee \neg Q(x)] \Rightarrow (\exists_y[\neg P(y)] \Rightarrow \exists_z[Q(z) \Rightarrow R(z)])$$

is a tautology.

(2) 4. Check whether the following formula holds for all sets A, B and C:

$$A \in \mathcal{P}(B \cup C) \Rightarrow A \in \mathcal{P}(B) \ .$$

If so, then give a proof; if not, then provide a counterexample.

(1) 5. (a) Give the formula for '$H : C \to B$ is a surjection'.

(3) (b) Let $H : C \to B$ be a surjection, and let $G_1 : B \to A$ and $G_2 : B \to A$ be mappings. Prove that

$$\forall_z[z \in C : G_1(H(z)) = G_2(H(z))] \Rightarrow \forall_y[y \in B : G_1(y) = G_2(y)] \ .$$

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(3) 6. Prove with induction on $n \geq 1$ that 7 is a divisor of $8^n - 1$.

7. Define the relation R on $\mathbb{N}^+ \times \mathbb{N}^+$ by

$$(k, \ell)\ R\ (m, n) \text{ if, and only if, } k \mid m \wedge (k = m \Rightarrow \ell \leq n)\ .$$

(2) (a) Prove that R is a (reflexive) ordering.

(2) (b) Make a Hasse diagram of $\langle \{1, 2, 3, 4\} \times \{1, 2\}, R \rangle$.