Question 1

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1 Ellipse

1.1 Algorithm

Main idea is:
Calculate marginal pdf of θ
Find inverse of the CDF of obtained density function.
The obtained inverse is mapping function for θ
given θ generate random value of r by following the same process.

Equation of ellipse in polar coordinates is given by:

$$r(\theta) = \frac{ab}{\sqrt{(b\cos\theta)^2 + (a\sin\theta)^2}}$$

So PDF of marginal of θ will be:

$$\int \frac{dA}{A}$$

$$f(\theta) = \frac{ab}{2\pi((b\cos\theta)^2 + (a\sin\theta)^2)}$$

now the random θ can be generated by taking inverse of CDF of above density function The CDF of marginal will be:

$$\int_0^\theta \frac{ab}{2\pi((b\cos\theta)^2 + (a\sin\theta)^2)}\, d\theta$$

$$F(\theta) = \frac{1}{2\pi}\tan^{-1}\frac{a\tan\theta}{b}$$

Taking inverse of this:

$$\theta = \tan^{-1}\frac{b\tan 2\pi x}{a}$$

where x is random number between 0 and 1

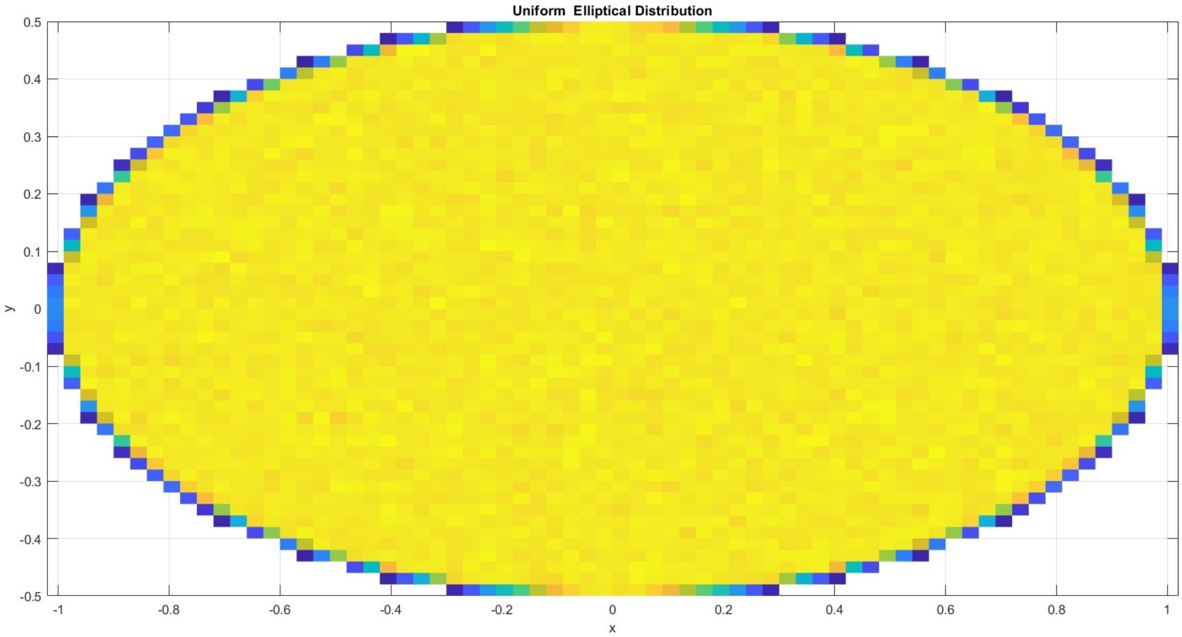
But range of arctan is $-\pi/2$ to $\pi/2$, So to generate random θ from $-\pi/2$ to $3\pi/2$ we'll take random number from 0 to 1 and map the obtained θ uniformly as: If number is less than 0.25 we'll take negative of θ, else for number less than 0.5 we'll add π to negated value of θ, else if for number less than 0.75 we'll add π to θ. In this way we can get random values of θ.

Now given a 'θ'; probability of $R = r$ is proportional to r; so CDF is proportional to r^2 and hence CDF inverse will be $\sqrt{rand()}$
For a particular θ, range of r is 0 to $\frac{ab}{\sqrt{(b\cos\theta)^2+(a\sin\theta)^2}}$

so random r will be given as:

$$r = \frac{ab}{\sqrt{(b\cos\theta)^2 + (a\sin\theta)^2}}\sqrt{rand()}$$

1.2 Plot



Above plot is obtained by generating random numbers by given algorithm. It can be seen that the distribution indeed is uniform.

2 Triangle

2.1 Algorithm

Here also the idea is same, that is
Calculate marginal PDF of x (for simplicity marginal PDF is scaled by area, and hence random number generated will be from 0 to Area instead of 0 to 1)
Find inverse of the CDF of obtained density function.
The obtained inverse is mapping function for x
given x generate random value of y taking a number between 0 andY(x) uniformly.

marginal PDF of X = x: (scaled by Area)

$$\begin{cases} \frac{3ex}{\pi} & 0 \le x \le \pi/3 \\ \frac{-3e(x-\pi)}{2\pi} & \pi/3 \le x \le \pi \end{cases}$$

marginal CDF of X = x:

$a = \frac{-3e}{4\pi}$, $b = \frac{3e}{2}$, $c = \frac{-e\pi}{4}$

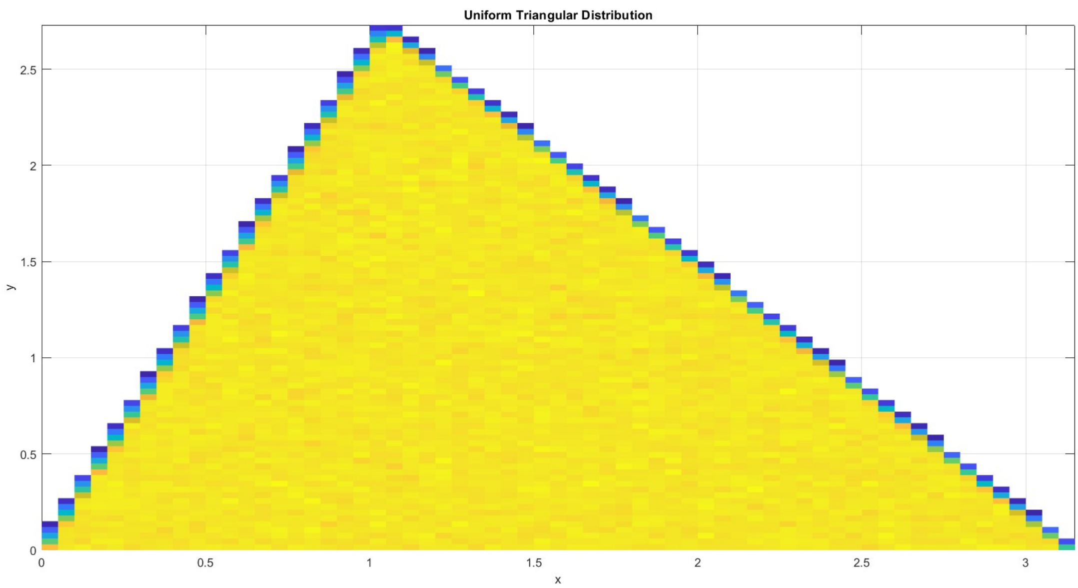
$$\begin{cases} \frac{3ex^2}{2\pi} & 0 \le x \le \pi/3 \\ ax^2 + bx + c & \pi/3 \le x \le \pi \end{cases}$$

inverse = **mapping function for x :**

$$\begin{cases} \sqrt{\frac{2x\pi}{3e}} & 0 \le x \le \pi e/6 \\ \frac{-b+\sqrt{b^2-4a(c-x)}}{2a} & \pi e/6 \le x \le \pi e/2 \end{cases}$$

So we will take numbers from 0 to πe/2 (area of triangle) and map it to 0 to π according to above mapping function.
Now given x,y will be chosen uniformly from o to y(x) that is **mapping function for y uniform(0,Y(x))**

2.2 Plot



Above plot is obtained on generating (x,y) using above algorithm. The obtained distribution is uniform.