Question 2

Avadhoot Jadhav - 210050027
Hrishikesh Jedhe Deshmukh - 210050073

October 2022

1 Method for generating sample points from 2D Gaussian

Let,

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$C_X = E[XX^T] - E[X]E[X]^T$$

Let us define a new random vector Y,

$$Y = PX$$

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = P \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\begin{aligned}
C_Y &= E[YY^T] - E[Y]E[Y]^T \\
C_Y &= E[PX(PX)^T] - E[PX]E[PX]^T \\
C_Y &= E[PXX^TP^T] - PE[X](PE[X])^T \\
C_Y &= PE[XX^T]P^T - PE[X]E[X]^TP^T \\
C_Y &= PC_XP^T
\end{aligned}$$

We know every real symmetric matrix can be diagonalized by an orthogonal matrix.
As C_X is a real symmetric matrix, we can choose an orthogonal matirx P such that C_Y is a diagonal matrix.
Further, we also know that the diagonal entries of C_Y will be the eigenvalues of C_X.

Now, we will use `eig(.)` function in MATLAB to do `[V, D]=eig(`C_X`)`
Here, $V = P^T = P^{-1}$ as P is an orthogonal matrix and $D = C_Y$.

As C_Y, which is the covariance matrix of Y is a diagonal matrix, $Cov(Y_1, Y_2) = 0$.
As Y is a 2-D gaussian, Covariance=0 implies independence.
Note that the above statement is not true for general random variables.

So now, we can generate random draws from Y_1 and Y_2 using `randn(.)` as both are independent.
We will get X by taking the inverse transformation as follows,

$$X = P^{-1}Y$$

$$X = VY$$

To get draws of Y_1 and Y_2, we will first draw from `randn(.)` and then scale it appropriately by standard deviation of Y_1 and Y_2 respectively and add respective means to them.

2 Calculating errors in mean and covariance

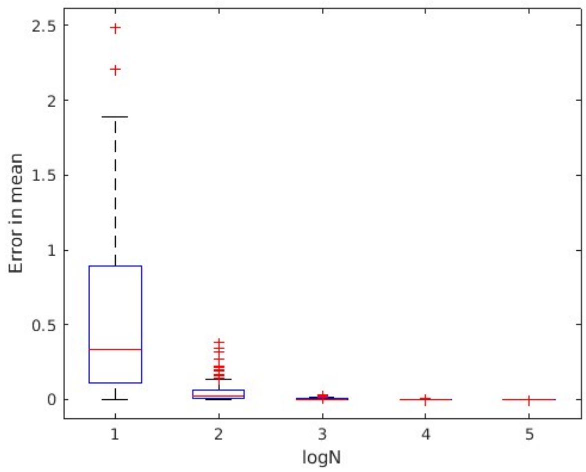
Error in mean is defined as,

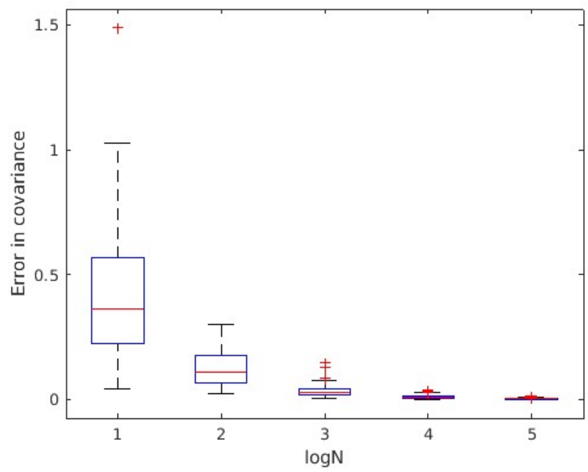
$$\frac{||\mu - \hat{\mu_N}||_2}{||\mu||_2} = \frac{(\mu - \hat{\mu_N})^T(\mu - \hat{\mu_N})}{\mu^T\mu}$$

Error in covariance is defined as,

$$\frac{||C - \hat{C_N}||_{Fro}}{||C||_{Fro}} = \frac{\sqrt{\text{sum of squares of entries of } C - \hat{C_N}}}{\sqrt{\text{sum of squars of entries of } C}}$$

We will calculate these errors and plot a box-plot of these values.





3 Scatter Plots