Question-3 Report

Avadhoot Jadhav - 210050027
Hrishikesh Jedhe Deshmukh - 210050073

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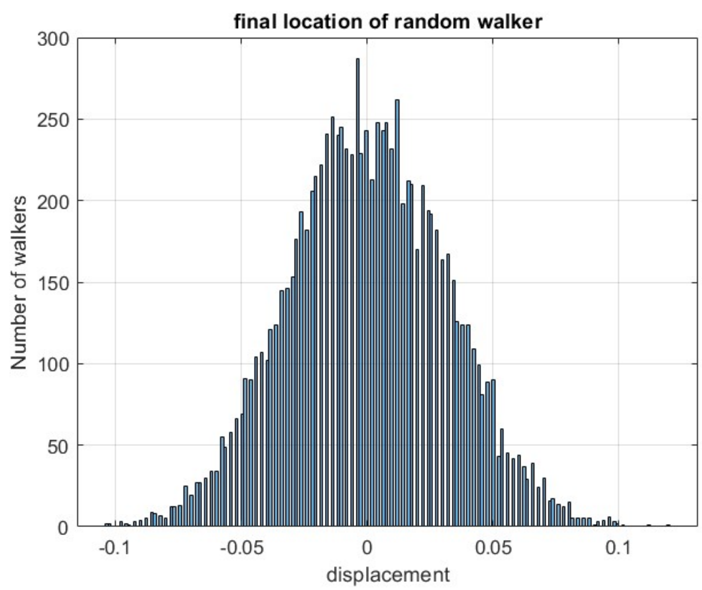
1 Random Walkers : Final position



Above plot is histogram of final displacement from origin of walker and number of walkers.

Idea ::
I generated random numbers from 0 to 1. This distribution will be uniform. if the number is greater than 0.5, increase the position of walker by 10^{-3} that is walker moves towards right ; else towards left.Then added all the displacements to get the final position of walker. Here I used $sum(.)$ function to sum values.

Conclusion::
It is clear from the graph that final position of random walkers follows Gaussian distribution.and probability of walker moving a distance more than 0.1 is almost

zero.

Mean of final positions of random walkers calculated experimentally:

$$E[X] = \sum_{i=1}^{10000} x_i P(x_i) = 0.0001386$$

Variance of final positions of random walkers calculated experimentally :

$$Var(X) = \sum_{i=1}^{10000} (x_i - E[X])^2 P(x_i) = 0.00099093$$

Let,
n = number of steps;
l = length of steps;
r = the number of steps toward right (+ve direction);
then:
final position = $x = (2r - n)l$
So mean =

$$\sum_{r=1}^{n} x_i P(x_i) =$$

$$= \sum_{r=1}^{n} (2r - n)l \binom{n}{r} (\frac{1}{2})^n$$

$$= (\frac{1}{2})^n l \quad \left(\sum_{r=1}^{n} 2r \binom{n}{r} \quad - \quad \sum_{r=1}^{n} n \binom{n}{r} \right)$$

$$= (\frac{1}{2})^n l \left(2n(2^{n-1}) - n2^n \right) = 0$$

True mean = 0

And variance =

$$\sum_{r=1}^{n} x_i^2 P(x_i) =$$

$$= \sum_{r=1}^{n} (2r-n)^2 l^2 \binom{n}{r} (\frac{1}{2})^n$$

$$= (\frac{1}{2})^n l^2 \quad \left(\sum_{r=1}^{n} 4r^2 \binom{n}{r} \quad - \quad \sum_{r=1}^{n} 4rn \binom{n}{r} \quad + \quad \sum_{r=1}^{n} n^2 \binom{n}{r} \right)$$

$$= (\frac{1}{2})^n l^2 \left(4n(n+1)2^{n-2} - 4n^2(2^{n-1}) - n22^n \right) = nl^2$$

True variance $= nl^2$;

Here, $n = 1000$ and $l = 10^{-3}$
so,
Theoretical mean = 0 and Theoretical variance = 10^{-3}
Error in mean = 0.0001386
Error in variance = 0.00000907

2 Law of Large Numbers

Law of Large Numbers states that if $X_1,\ X_2,\ ...\ \ X_n$ are independent and identically distributed random variables, with finite expected value $EX_i = \mu < \infty$, then for any $\epsilon > 0$,

$$\lim_{n\to\infty} P(|\overline{X} - \mu| \geq \epsilon) = 0$$

Now for the question, we define a new random variable, to model N independent draws from X.

$$\hat{M} = \frac{\sum_{i=1}^{N} X_i}{N}$$

All the draws have same expected value, so we can use the Law of Large Numbers,

$$\lim_{N\to\infty} P(|\hat{M} - Mean(X)| \geq \epsilon) = 0$$

$$\lim_{N\to\infty} \hat{M} \to Mean(X)$$

Now we define a new random variable,

$$\hat{V} = \frac{\sum_{i=1}^{N} (X_i - \hat{M})^2}{N}$$

And we have to prove the following,

$$\lim_{N\to\infty} \hat{V}(X) = Var(X)$$

$$E(\hat{V}) = \frac{\sum_{i=1}^{N} E[(X_i - \hat{M})^2]}{N}$$

Also we know that,

$$Var(X) + [E(X)]^2 = E(X^2)$$

$$E(\hat{V}) = \frac{1}{N}\sum_{i=1}^{N}[Var(X_i - \hat{M}) + [E(X_i - \hat{M})]^2]$$

$$E(X_i - \hat{M}) = 0$$

$$E(\hat{V}) = \frac{1}{N}\sum_{i=1}^{N} Var(X_i - \hat{M})$$

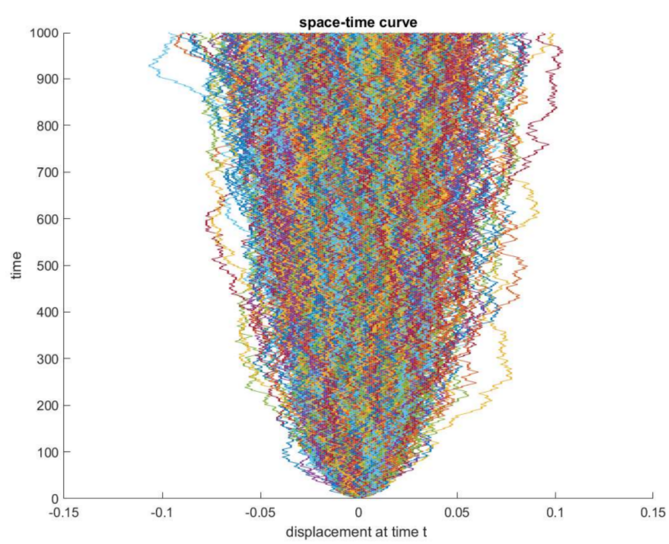
$$E(\hat{V}) = Var(X_i - \hat{M})$$

$$E(\hat{V}) = Var(X_i) - Var(\hat{M})$$

Now by Law of Large Numbers, $Var(\hat{M}) \to 0$ as $N \to \infty$

$$\lim_{N\to\infty} \hat{V} = Var(X)$$

3 Space - Time curve



Above plot is the space-time curve for 1st 1000 walkers.

Idea::
For this graph; I first generated a matrix with random numbers and as above mp them to 1 or -1 based on if they are more than 0.5 or not.Now to find the position of walker at time t that is after $n = t$ steps , we need to find the cumulative sum of all steps till that time. the $cumsum(.)$ function in matlab is useful to do so. In this way I calculated the position of walker at time t for 1^{st}1000 walkers and plot those against time to get the above plot.

Conclusion::
from the graph we can see that the position of walker at any time is between -0.1 to 0.1. that is the probability of walker going to the same direction (left or right) continuously is very less. Also, we can see that the distribution is symmetric with respect to origin this reflects the fact that probability of walker going to right or left is same.