Question 2 - Report

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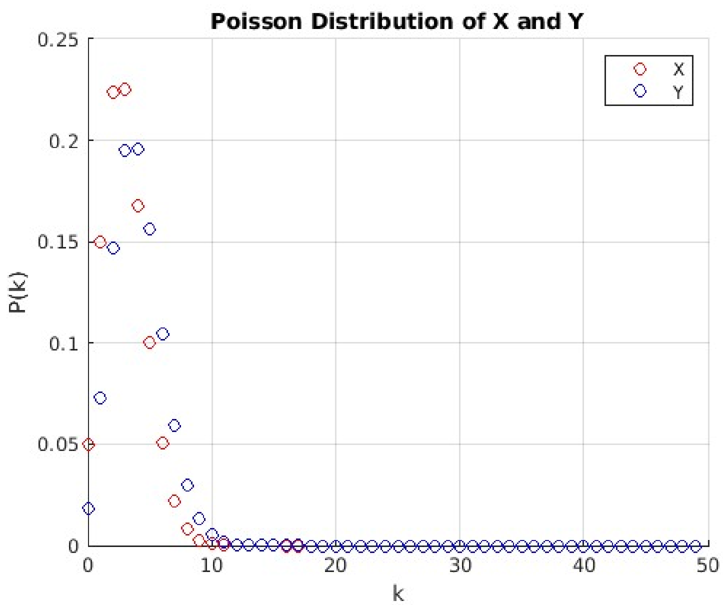
August 22, 2022

1 Addition of Poisson random variables



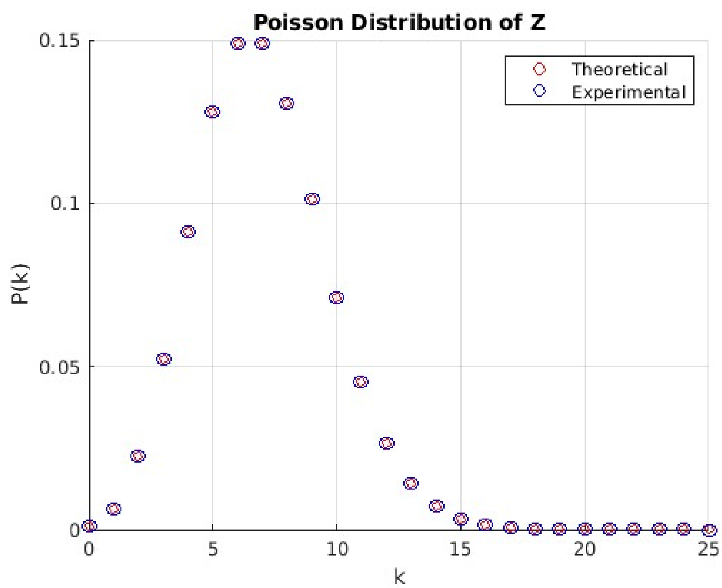
This is plot of poisson random variables X and Y, with $\lambda_x = 3$ and $\lambda_y = 4$. Now we define a new random variable Z as $Z = X + Y$. To calculate Z empirically, we add up the values of X and Y and then plot Z.

Now, to calculate $P(Z)$,

$$\begin{aligned} P(Z = k) &= \sum_{j=0}^{k} P(X = j, Y = k - j) \\ &= \sum_{j=0}^{k} P(X = j)P(Y = k - j) \end{aligned} \tag{1}$$

$$
\begin{aligned}
P(Z=k) &= \sum_{j=0}^{k} \frac{e^{-\lambda_X}\lambda_X^j}{j!}\frac{e^{-\lambda_Y}\lambda_Y^{k-j}}{(k-j)!} \\
&= \frac{e^{-(\lambda_X+\lambda_Y)}}{k!}\sum_{j=0}^{k}\frac{k!}{j!(k-j)!}\lambda_X^j\lambda_Y^{k-j} \\
&= \frac{e^{-(\lambda_X+\lambda_Y)}}{k!}\sum_{j=0}^{k}\binom{k}{j}\lambda_X^j\lambda_Y^{k-j} \\
P(Z=k) &= e^{-(\lambda_X+\lambda_Y)}\frac{(\lambda_x+\lambda_Y)^k}{k!}
\end{aligned}
\tag{2}
$$

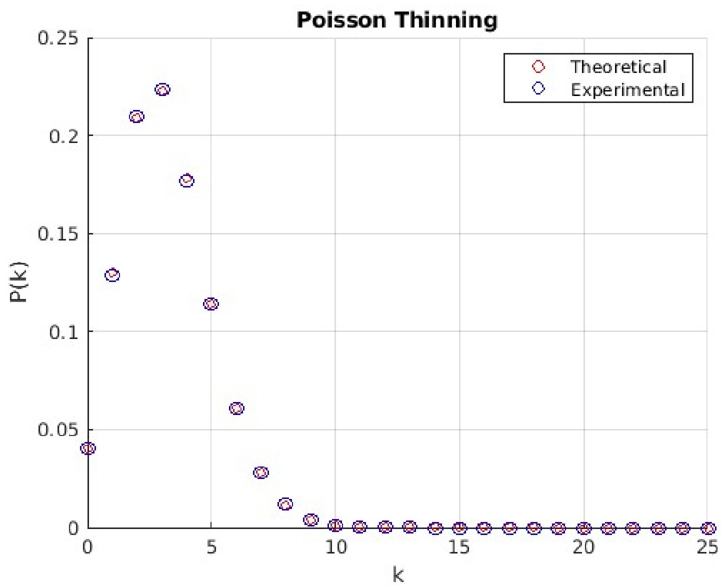
Now plotting random variable Z both theoretically and empirically,



Idea::
We have used `poissrnd(.)` function to generate poisson random numbers with given rate. Then we plotted their probability on a scatter plot by adding individual probabilities. To plot a new random variable $Z = X + Y$, we will sum over multiplication of probabilities of of X and Y.

Conclusion::
When we add two independent poisson random variables, new random variable that we obtain has a rate equal to sum of the former two.

2 Poisson Thinning



Above plot displays both Theoretical and Empirical distribution of thinned variable Z.

Theoretically, $P(Z)$ is given by,

$$\begin{aligned} P(Z=k) &= \sum_{j=k}^{\infty} P(Y=j, Z=k) \\ &= \sum_{j=k}^{\infty} P(Z=k|Y=j)P(Y=j) \\ &= \sum_{j=k}^{\infty} \frac{e^{-\lambda}\lambda^j}{j!}\binom{j}{k}p^k(1-p)^{j-k} \\ &= \frac{e^{-\lambda}(\lambda p)^k}{k!}\sum_{j=k}^{\infty}\frac{(\lambda(1-p))^{j-k}}{(j-k)!} \\ P(Z=k) &= e^{-\lambda p}\frac{(\lambda p)^k}{k!} \end{aligned} \tag{3}$$

Idea::

Random variable Y was plotted in the last part. In this part we had to select the subset of arrivals with probability 0.8. To do this we used `binornd(.)` function. Now we plot the new random variable Z using these subsets of arrivals.

Conclusion:: New randoom variable has rate equal to λp, where λ is rate of original poisson random variable.