

Question 5

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1 Computing 84 coordinates

For each of the digits from 0-9, compute mean μ and covariance matrix C .

$$\mu = \frac{\sum_{i=1}^n X_i}{n} \text{ and } C = \frac{\sum_{i=1}^n (X_i - \mu)(X_i - \mu)^T}{n}$$

Now to do the principal component analysis we will diagonalize the covariance matrix C and get the eigenvalues and eigenvectors of C .

$$D = V^T C V$$

where, $V = [v_1 \ v_2 \ \dots \ v_{84}]$ and v_i are eigenvectors of C . Diagonal of D will contain the corresponding eigenvalues.

Now we have to choose a 84-dimensional basis for some 84-dimensional hyperplane within the original Euclidean space, such that the chosen 84-dimensional hyperplane maximizes the total dispersion of the original data. Such a 84-dimensional basis will be formed by the 84 eigenvectors corresponding to the 84 largest eigenvalues of C .

WLOG, consider v_1, v_2, \dots, v_{84} to be the eigenvectors corresponding to 84 largest eigenvalues.

Let, $X = [x_1 \ x_2 \ \dots \ x_{784}]^T$ be the vector representating the original image and $c = [c_1 \ c_2 \ \dots \ c_{84}]^T$ be the new coordinates after dimensionality reduction. We will get those 84 coordinates by taking projection of original image on those 84 eigenvectors.

$$\begin{aligned} c_1 &= X \cdot v_1 = v_1^T X \\ c_2 &= X \cdot v_2 = v_2^T X \\ &\vdots \\ c_{84} &= X \cdot v_{84} = v_{84}^T X \end{aligned}$$

2 Reconstructing image from 84 coordinates

Now that we have got 84 coordinates, we need an algorithm to reconstruct the image from those 84 coordinates. Let the vector representating the reconstructed image be I .

$$I = c_1 v_1 + c_2 v_2 + \dots + c_{84} v_{84}$$

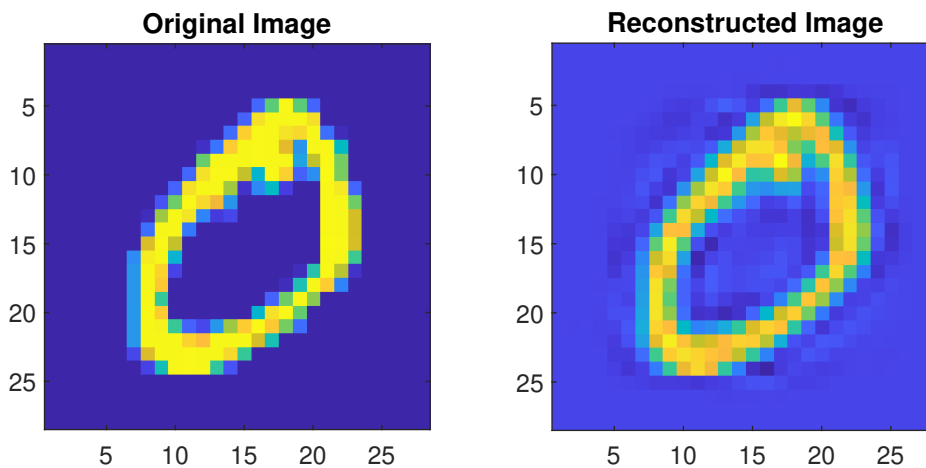
This equation in matrix form is given by,

$$I = [v_1 \ v_2 \ \dots \ v_{84}] c$$

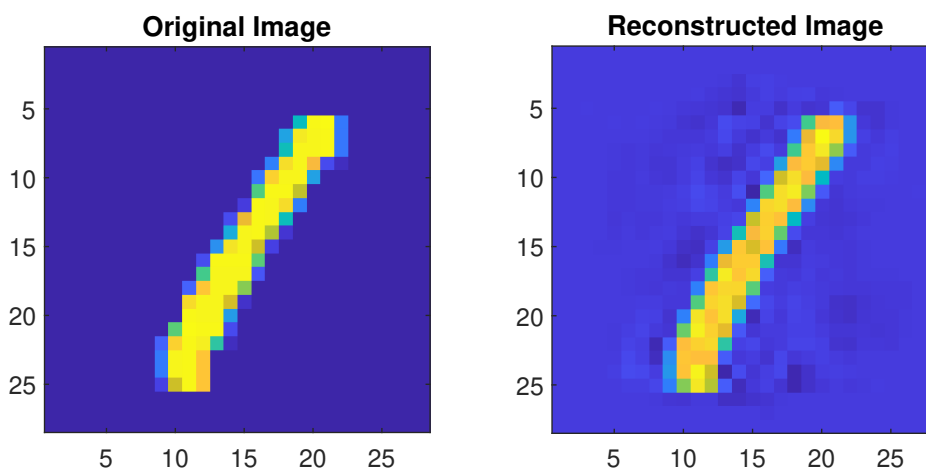
3 Original Image vs Reconstructed Image

- (5 points) Give an algorithm for regenerating / reconstructing the image using those 84 coordinates (and the knowledge of the designed 84-dimensional basis). For each of the ten digits (0–9), pick an image, and show the original and the reconstructed images side by side.

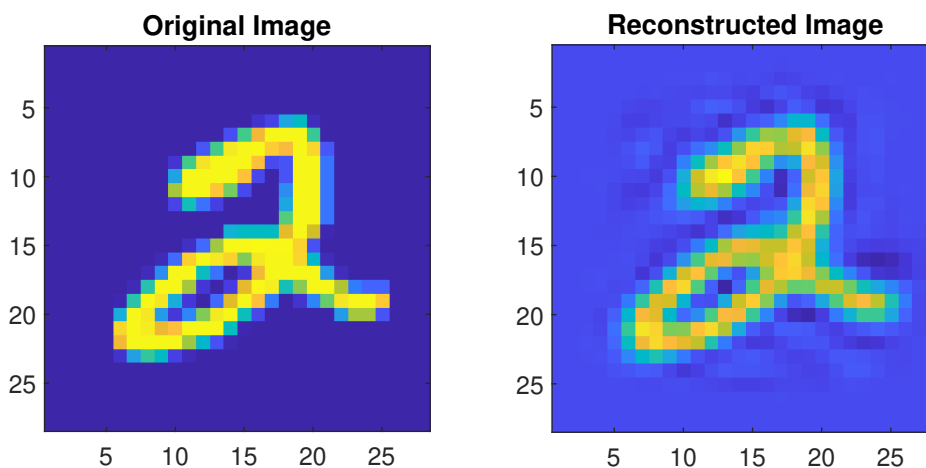
Images For the digit 0:



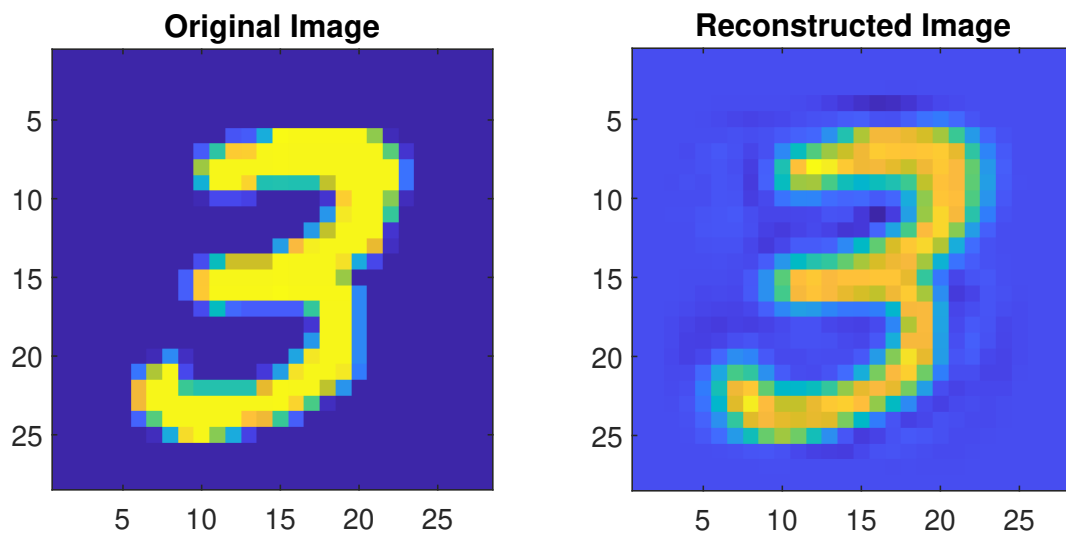
Images For the digit 1:



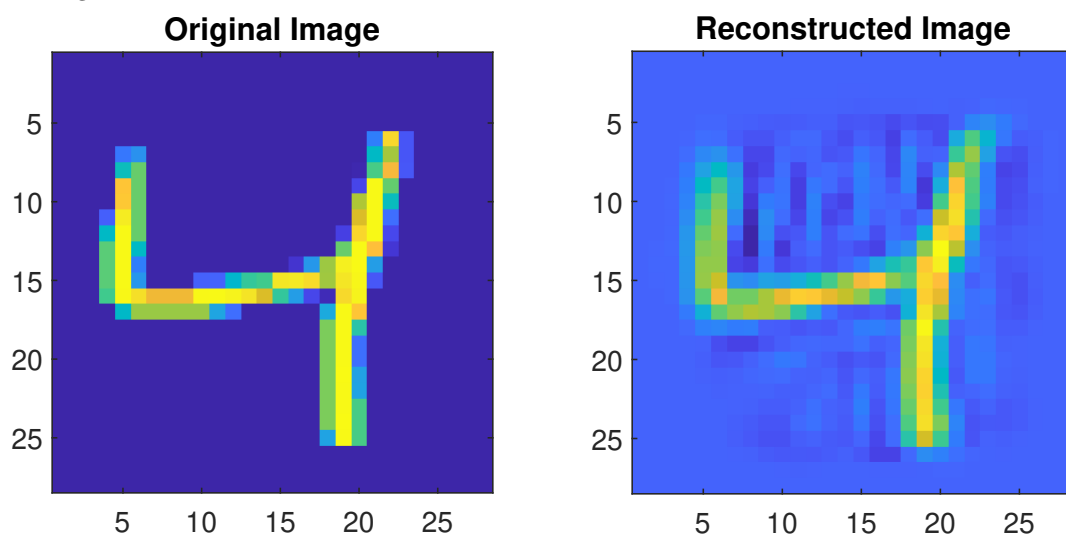
Images For the digit 2:



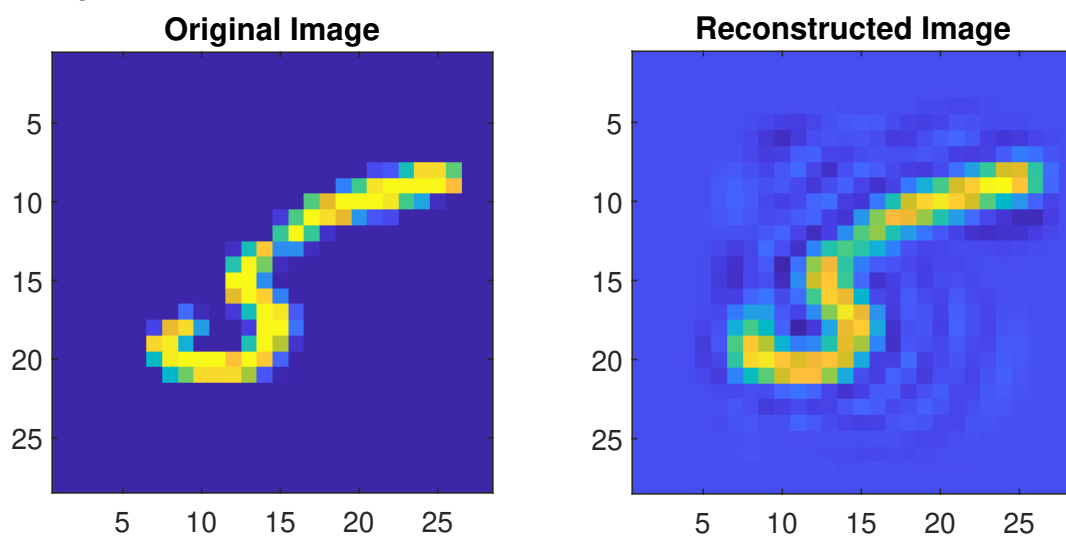
Images For the digit 3:



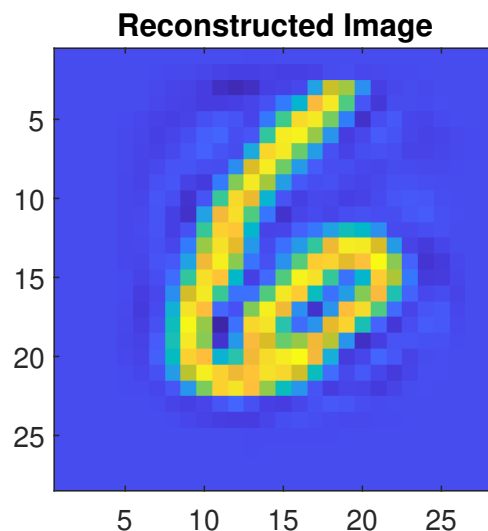
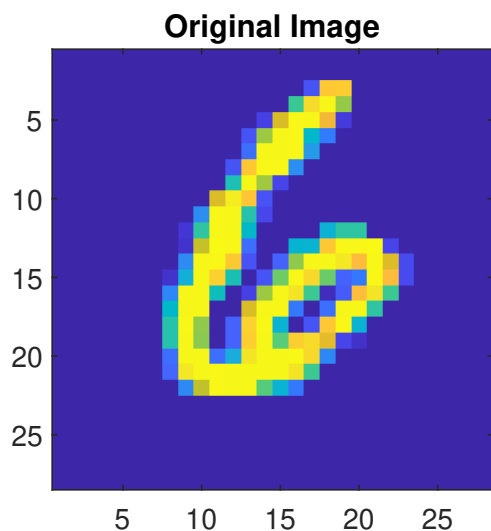
Images For the digit 4:



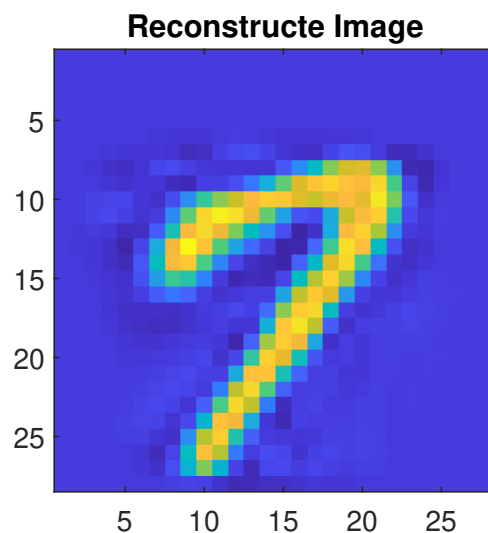
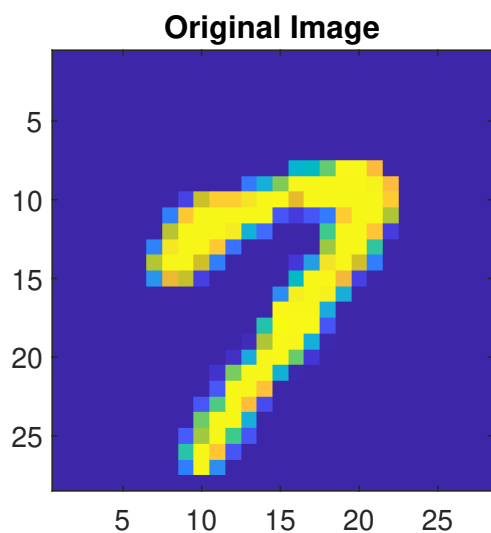
Images For the digit 5:



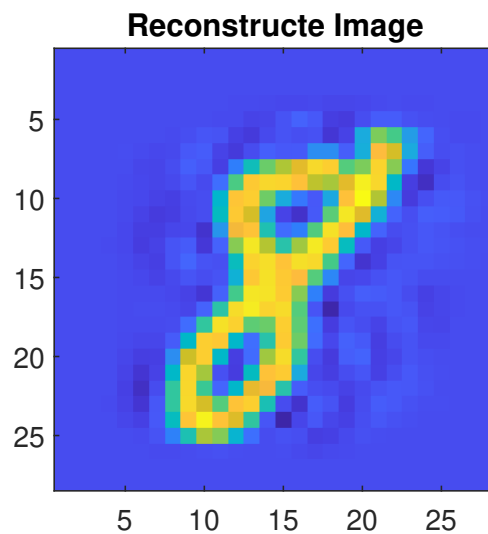
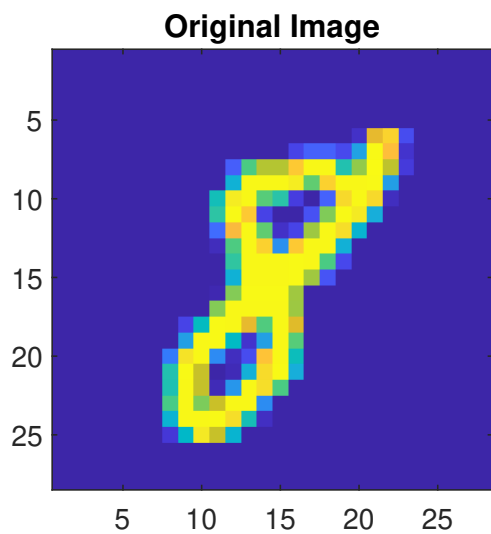
Images For the digit 6:



Images For the digit 7:



Images For the digit 8:



Images For the digit 9:

