Question 4

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1 Idea

For a particular number to calculate mean we'll add all the matrices having label equal to that digit and divide it by number of such matrices.

$$\mu = \frac{\sum X_i}{N}$$

Covariance can be calculated by the formula:

$$C = \frac{\sum (X_i - \mu)(X_i - \mu)^T}{N}$$

Now since C (covariance matrix) is symmetric then according to spectral theorem there exist two matrices V and D such that $C = V^{-1}DV$, Where D is a diagonal matrix with diagonal entries as eigenvalues of C and columns of V are eigenvector of C.The eigenvector corresponding to maximum eigenvalue will be the first principal component.

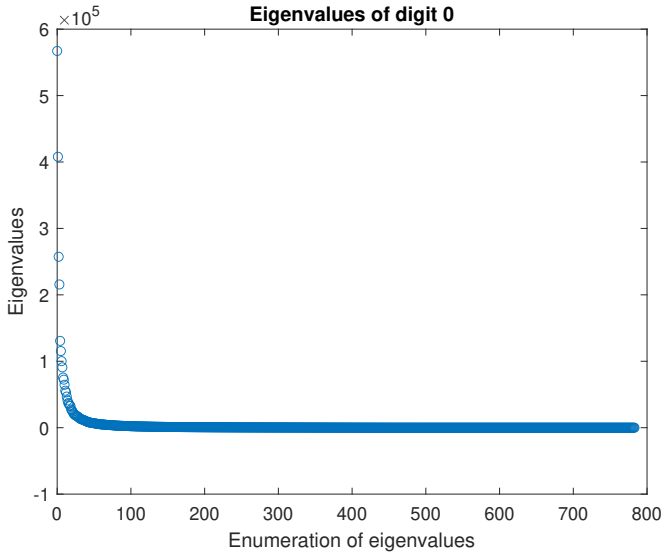
2 Graphs of sorted eigenvalues:

Comment and justify what you observe. How many "principal" / significant modes of variation (i.e., number of "large" eigenvalues) do you find, for each digit ? Are the significant modes of variation equal to 28^2 or far less ? Why ?

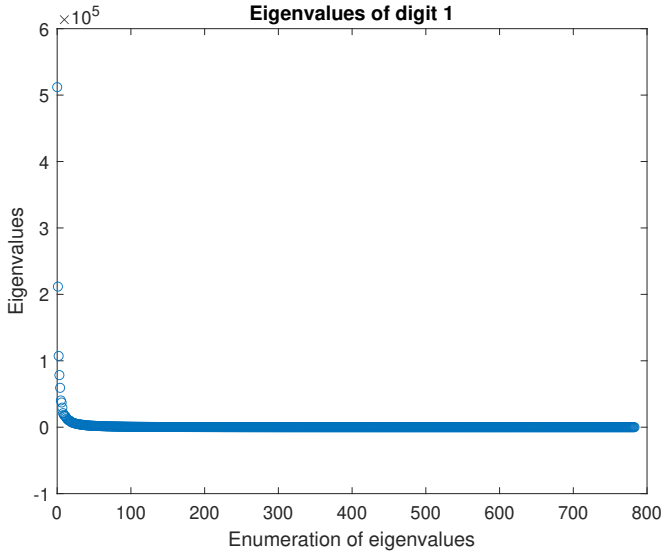
The number of *large* eigenvalues for each digit are very less compared to total eigenvalues. As we can see from the graphs below, there are only 5 to 10 large eigenvalues for each digit compared to total of 784 eigenvalues. For *easy* to draw digits like **0** and **1**, significant modes of variations are less than that of say **4** which is a bit complicated to write. This is due to the fact that there cannot be much variability in writing **0** and **1** from person-to-person, but there can be a large variation in writing **4** from person-to-person.

Now as to why the significant modes of variation are far less than 784, there are only a handful of pixels in an image where the difference in writing digit can show up. Almost all the digits are written similarly by everyone thus reducing variability in large number of pixels. So the variation only shows up in small number of pixels from overall pixels due to minor differences in writing the digits.

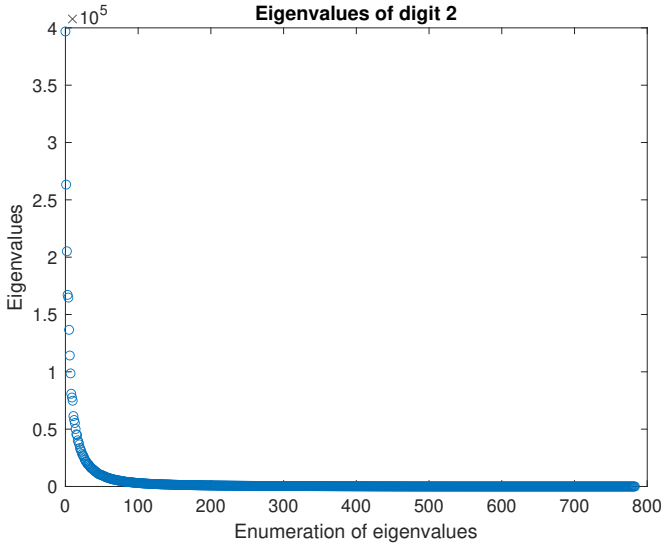
Graph of eigenvalues For the digit 0:



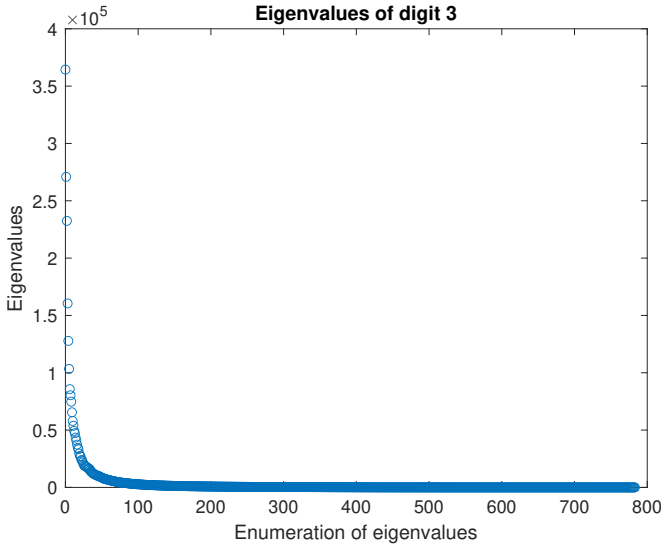
Graph of eigenvalues For the digit 1:



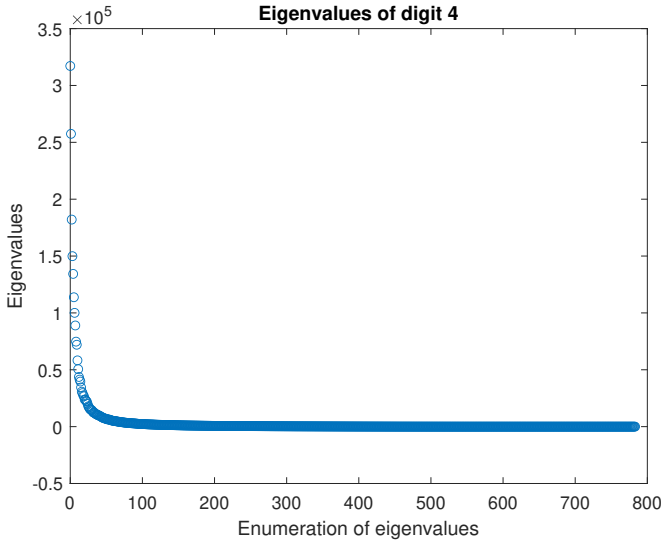
Graph of eigenvalues For the digit 2:



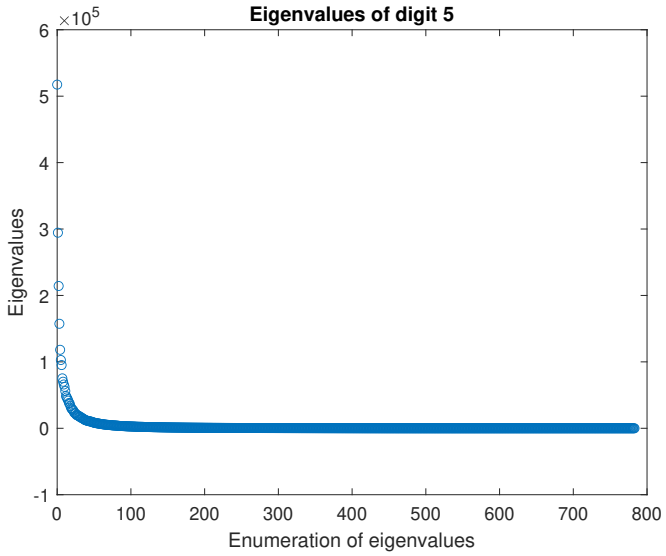
Graph of eigenvalues For the digit 3:



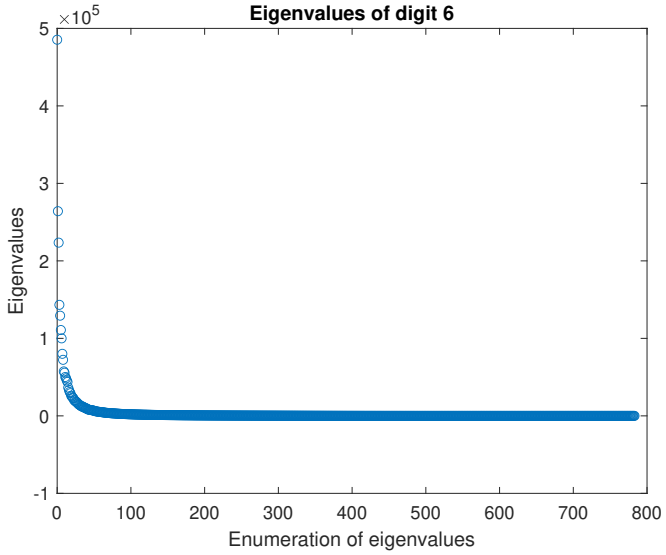
Graph of eigenvalues For the digit 4:



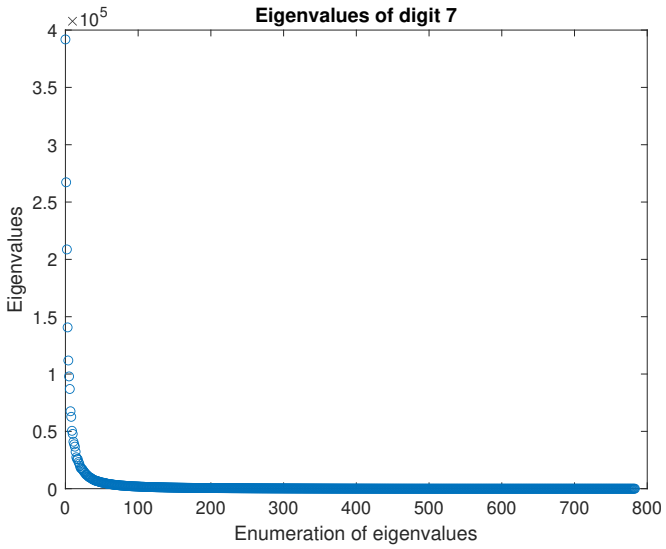
Graph of eigenvalues For the digit 5:



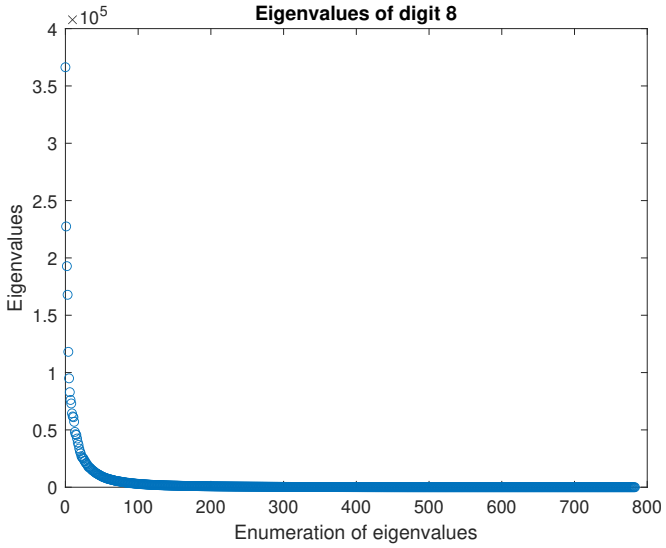
Graph of eigenvalues For the digit 6:



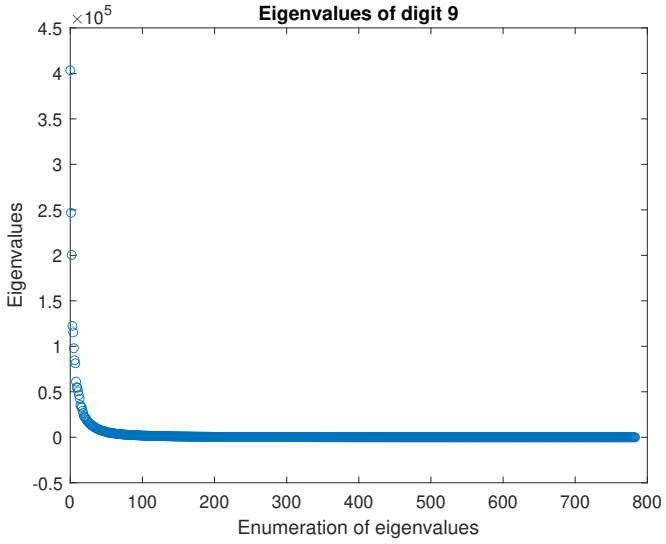
Graph of eigenvalues For the digit 7:



Graph of eigenvalues For the digit 8:



Graph of eigenvalues For the digit 9:



3 Principal mode of variation of digits around mean:

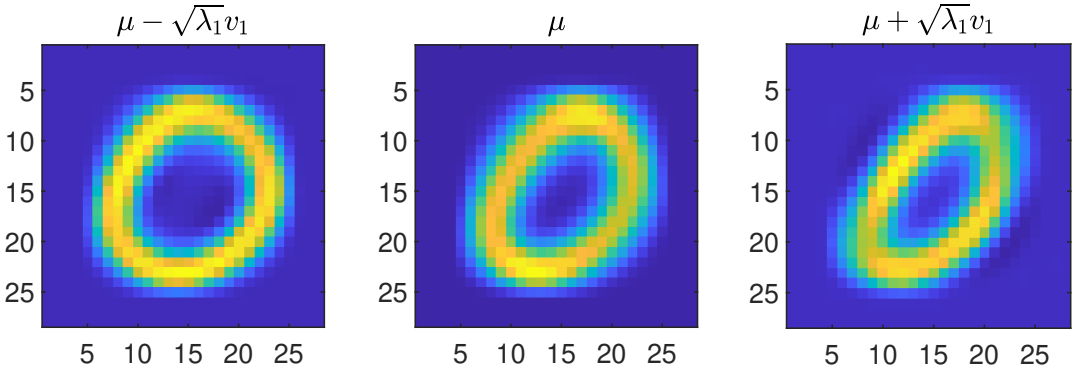
For each of the images, $\mu - \sqrt{\lambda_1}v_1$, μ and $\mu + \sqrt{\lambda_1}v_2$ comment and justify what you observe.

Principal mode of variation contains those pixels which show the most variability. That is the areas of image which change a lot from person to person while writing that digit.

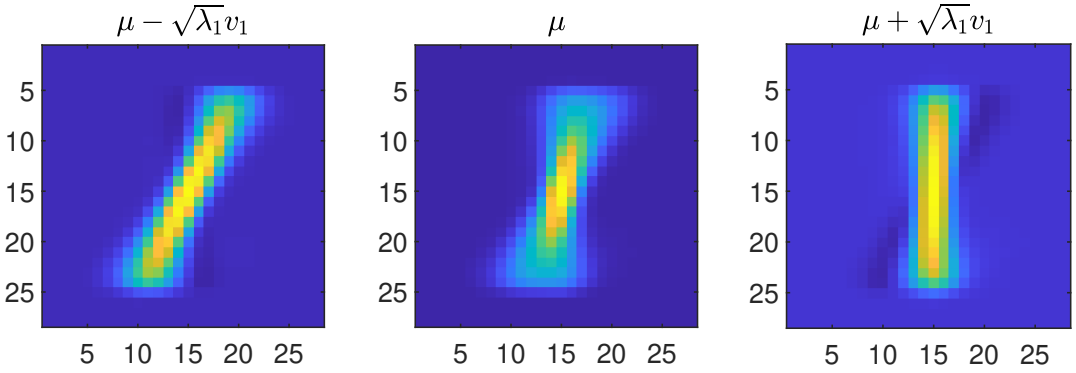
Let's take example of digit '2'. While writing '2' many people make a loop, while many don't. So this feature has a lot of variability which shows up in Principal Mode of Variation. So one of the images will have a clear loop in it while other will try to erase the loop. That's exactly what we observe in those three images of '2'. Left image is the one in which the features of loop are being erased, while right is the one in which the loop is being shown prominently.

Also an interesting trend can be seen in almost all digits. All of the digits are tilted a bit to right. So Principal Mode of Variation captures this. In one of the images, digit becomes a bit straight while in other digit becomes even more tilted towards right. We think, this tilt towards right is caused by having a lot more right-handed people than left-handed people.

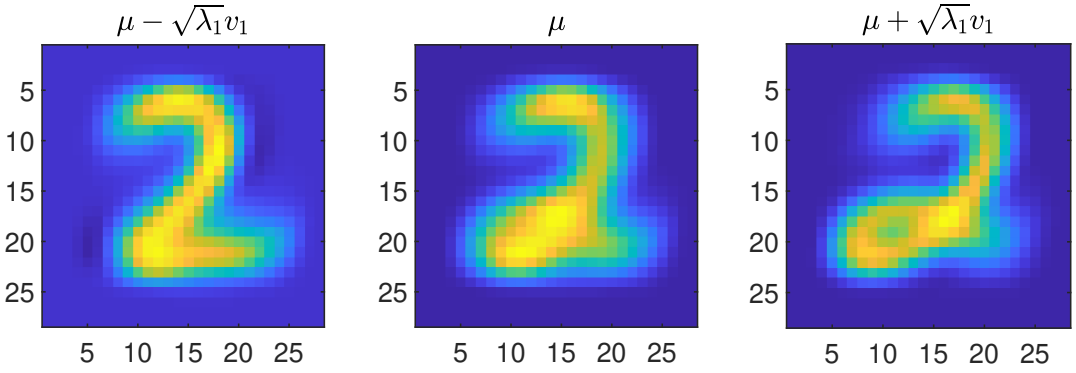
Images For the digit 0:



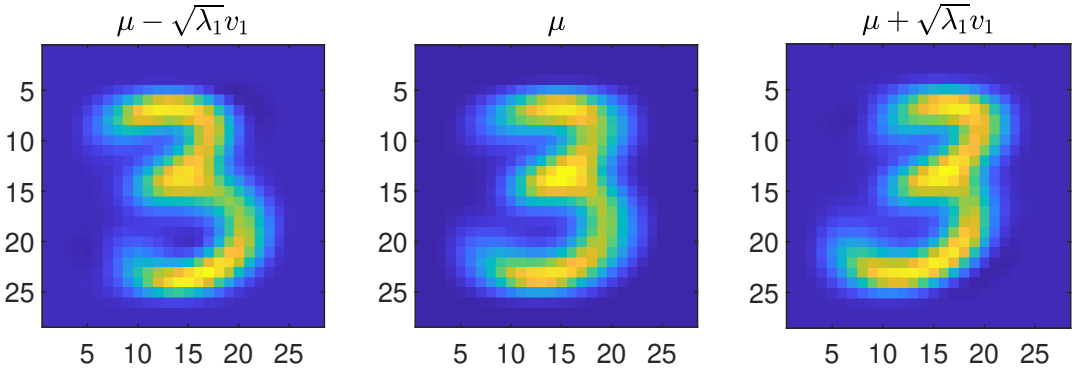
Images For the digit 1:



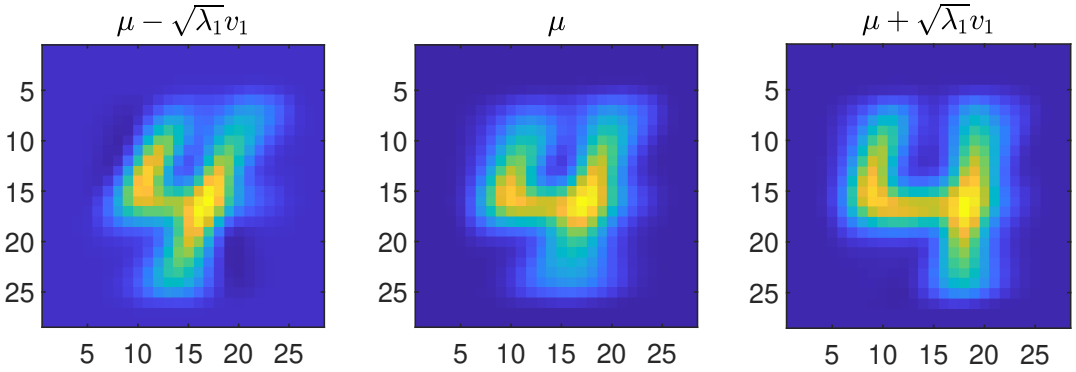
Images For the digit 2:



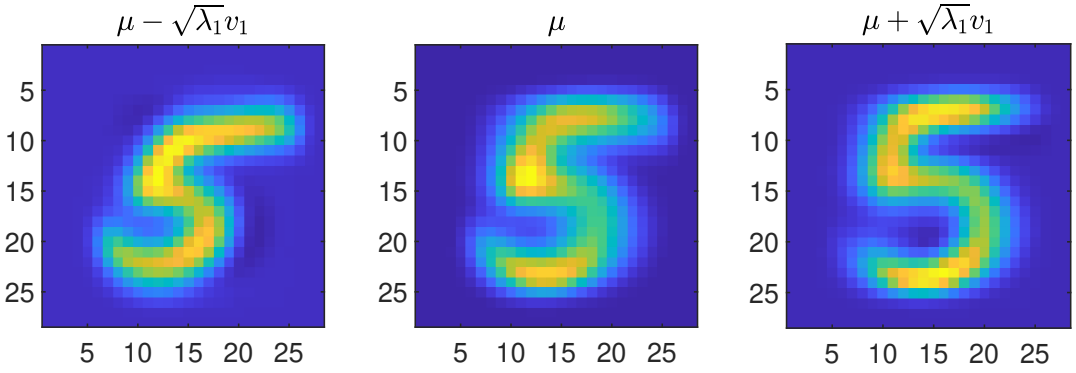
Images For the digit 3:



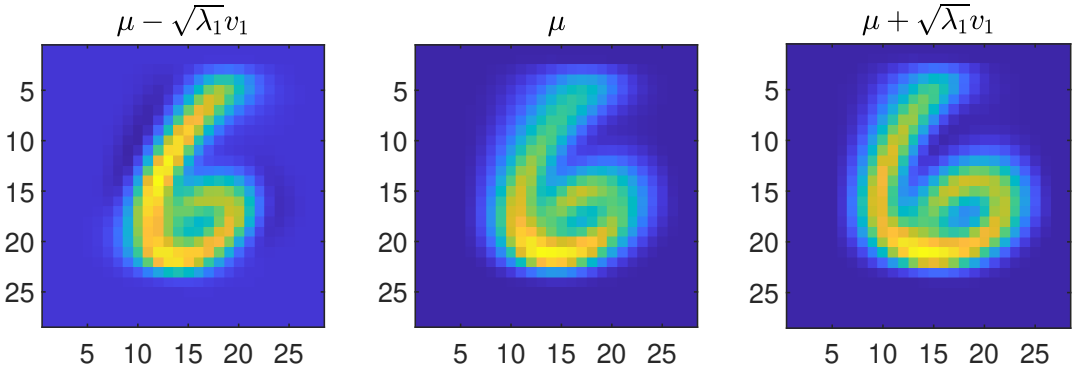
Images For the digit 4:



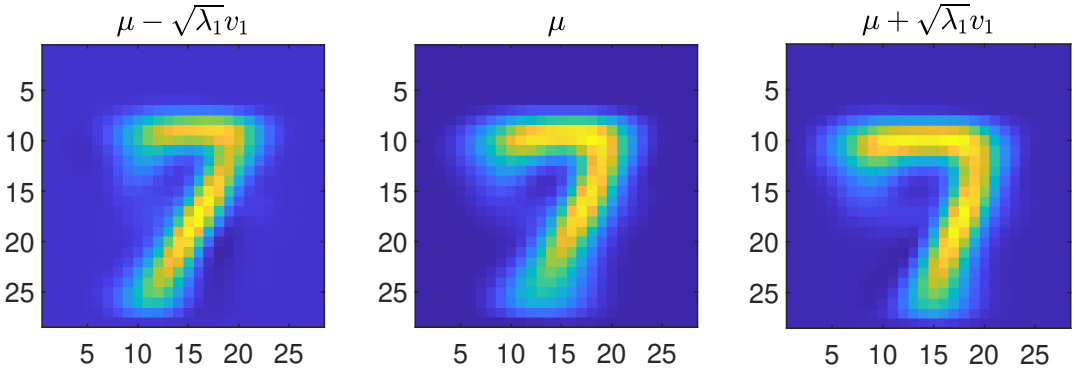
Images For the digit 5:



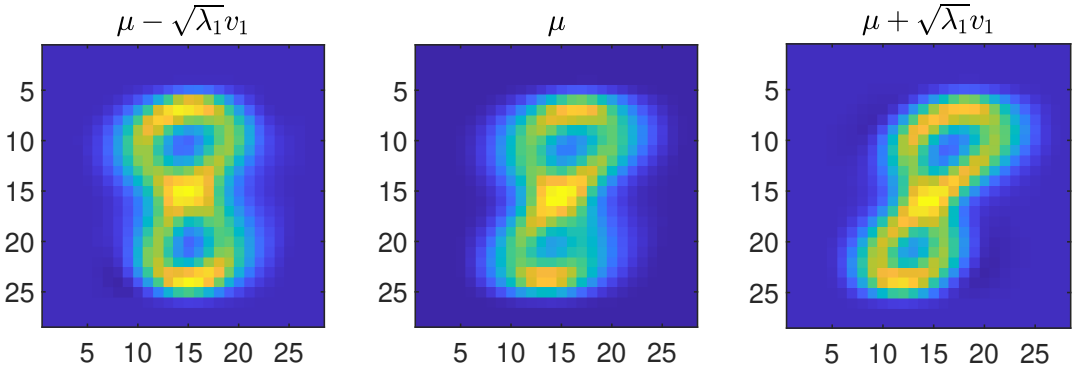
Images For the digit 6:



Images For the digit 7:



Images For the digit 8:



Images For the digit 9: