Question-4 Report

Avadhoot Jadhav - 210050027
Hrishikesh Jedhe Deshmukh - 210050073

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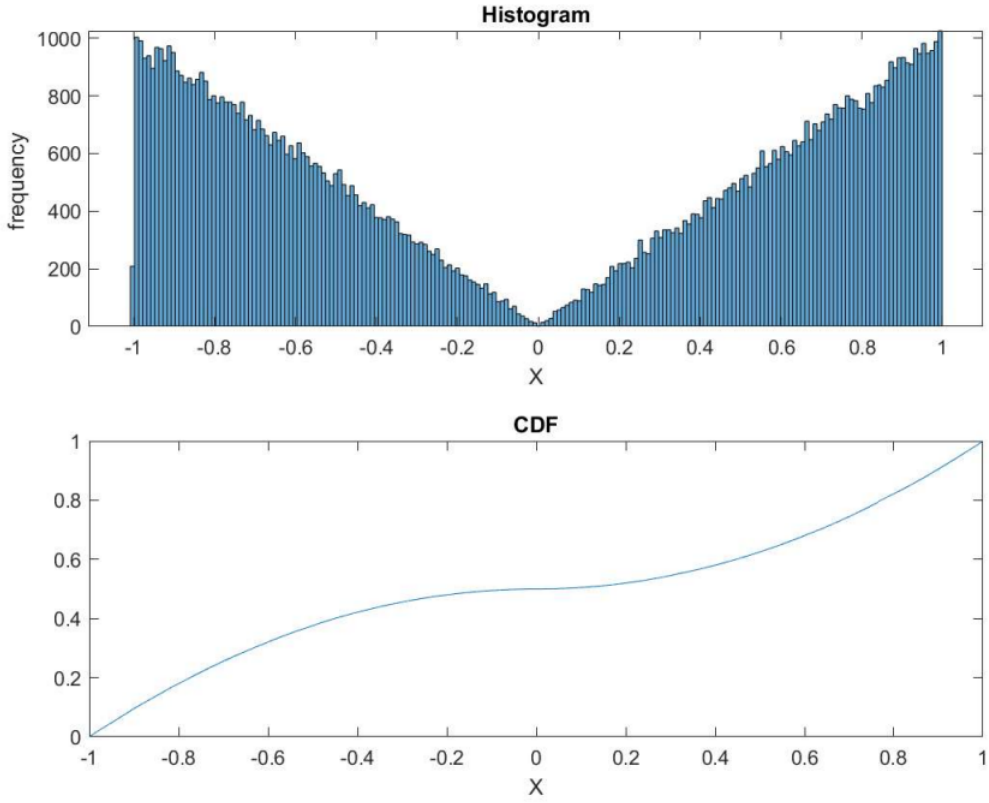
1 Generating draws from $P_X(.)$



Above plot is histogram and CDF of the number generated(10^5). these plots follows the same distribution function as $P_X(.)$

Idea::
To generate independent draws similar to $P_X(.)$ idea was to generate random numbers from 0 to 1 and then using a function f map these values to -1 to 1 such that it's distribution will be similar to $P_X(.)$. This function can be calculated by:

1. Find cumulative distribution function of $P_X(.)$
2. Inverse of the CDF
3. map the function accordingly

the CDF of given function is:

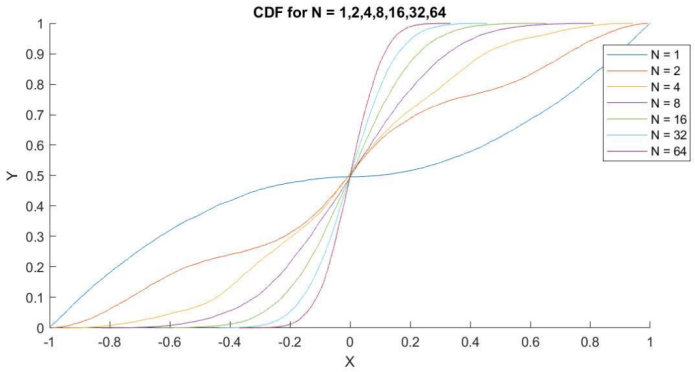
$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1-x^2}{2} & -1 < x \leq 0 \\ \frac{1+x^2}{2} & 0 < x \leq 1 \\ 1 & x > 1 \end{cases}$$

since the range of CDF is $[0, 1]$ the domain of mapping function should be $[0, 1]$. so we will take values from 0 to 1 and map the numbers $[0, 0.5]$ to $[-1, 0]$ and $[0.5, 1]$ to $[0, 1]$. Thus, our mapping function is inverse of CDF as :-

$$f(x) = \begin{cases} -\sqrt{1-2x} & 0 < x \leq 0.5 \\ \sqrt{1+2x} & 0.5 < x \leq 1 \end{cases}$$

Conclusion::
We can clearly see that the histogram generated after taking random numbers and mapping them according to above mentioned function is M shaped as mentioned in the question and have same probability distribution function as that of $P_X(.)$

2 CDFs and Histograms for $P_{Y_N}(.)$



The above graph contains cumulative distribution function for varying N and 10^4 draws.

Idea::
To generate independent draws from $P_{Y_N}(.)$ I made a function $aver(.)$ which takes N and number of draws as input and then uses mapping same as above part to generate N random draws from $P_X(.)$. then the average of those N values is calculated and in this way all the 10^4 draws are generated.

To calculate CDF of data, first we need to find how many times a particular number appeared in the data; for that I used $Unique(.)$ function from matlab. Then by using for loop I calculated frequency of all these numbers, and then using $cumsum(.)$ function calculated the cumulative frequency. This cumulative frequency on dividing with total number of draws (10^4) gave the required CDF.

Observation:: It can be observed that as the value of N is increasing; distribution is getting more and more similar to Gaussian distribution.
following Histograms also follows this pattern

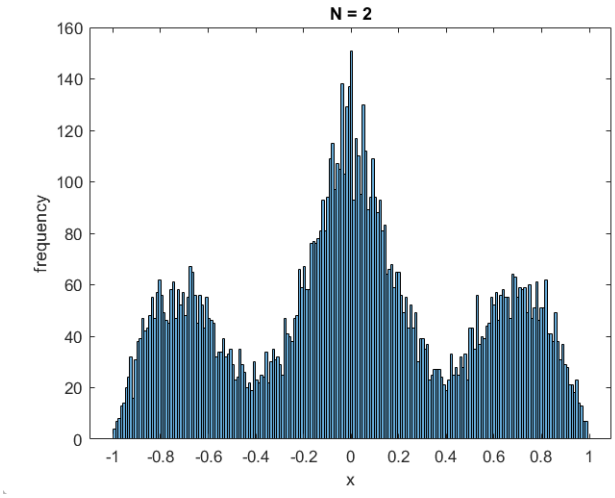


Figure 1: Histogram for N = 2

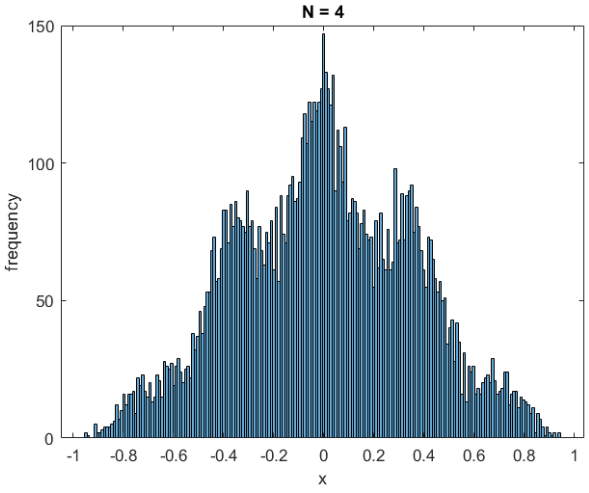


Figure 2: Histogram for N = 4

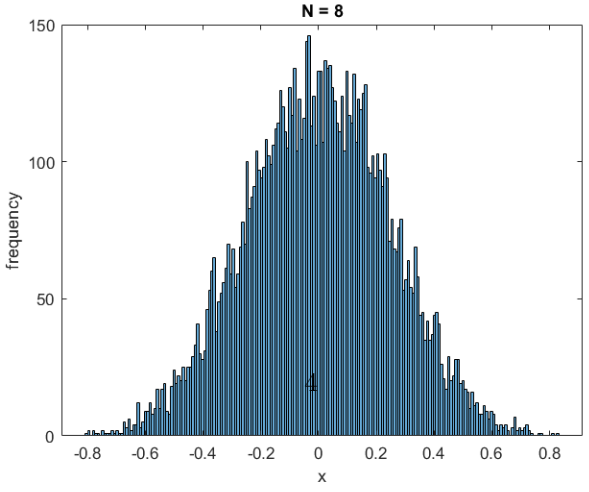


Figure 3: Histogram for N = 8

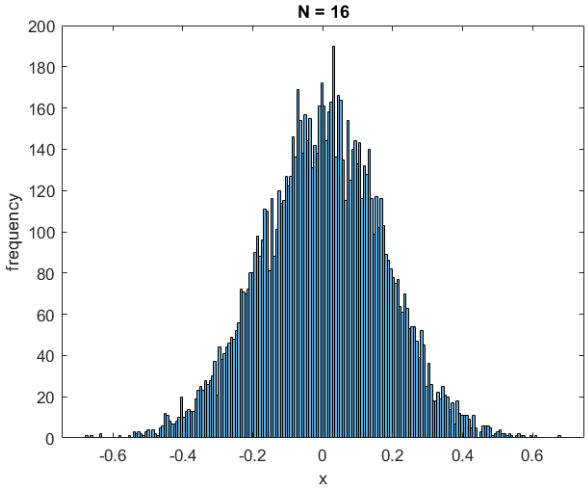


Figure 4: Histogram for N = 16

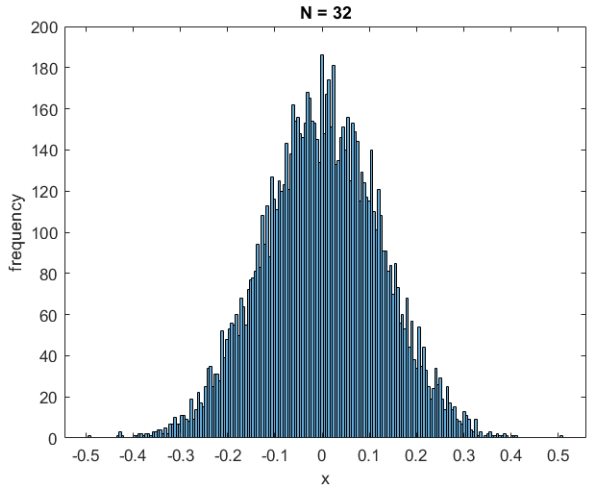


Figure 5: Histogram for N = 32

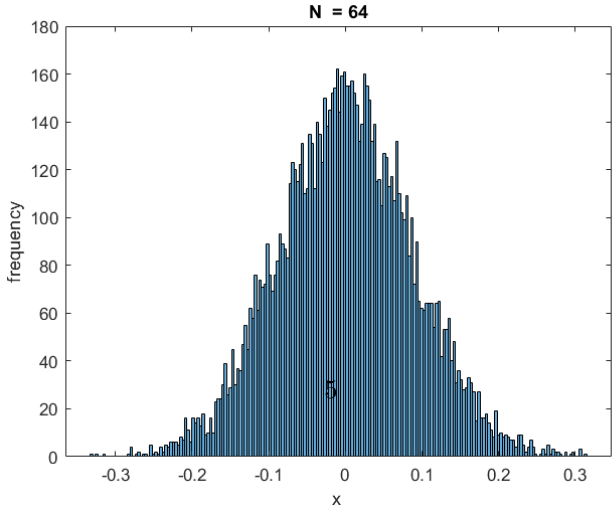


Figure 6: Histogram for N = 64