Basic principles - Geometry

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Basic principles - Geometry

- Projective geometry
  - Projective, Affine, Homography
  - Pinhole camera model and triangulation
- Epipolar geometry
  - Essential and Fundamental Matrix (8-point algorithm)
- Absolute pose problem
  - Linear pose, P3P
- Robust estimation
  - RANSAC
- Model selection
  - H vs. E vs. F (GRIC, QDEGSAC, etc.)

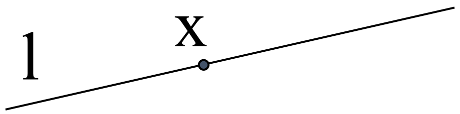
2D points and lines

- Homogeneous representations

$$ax + by + c = 0 \qquad (a,b,c)^\mathsf{T} \sim \lambda (a,b,c)^\mathsf{T}, \forall \lambda \neq 0$$

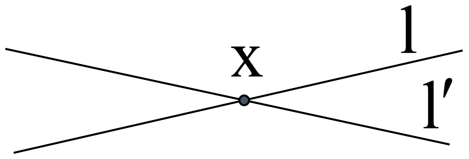
$$(a,b,c)(x,y,1)^\mathsf{T} = 0 \qquad (x,y,1)^\mathsf{T} \sim \mu (x,y,1)^\mathsf{T}, \forall \mu \neq 0$$

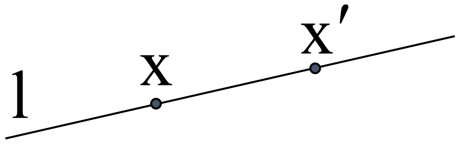
$$\mathbf{l}^\mathsf{T}\mathbf{x} = 0$$



- Join of points and lines

$$\mathbf{x} = \mathbf{l} \times \mathbf{l}' \qquad \mathbf{l} = \mathbf{x} \times \mathbf{x}'$$



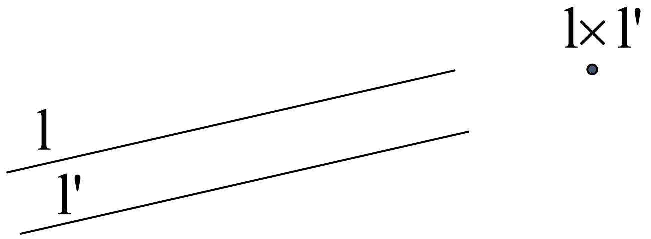


Ideal points and the line at infinity

- Intersection of parallel lines

$$\mathbf{l} = (a, b, c)^\mathsf{T} \text{ and } \mathbf{l}' = (a, b, c')^\mathsf{T}$$

$$\mathbf{l} \times \mathbf{l}' = (b, -a, 0)^\mathsf{T}$$



Ideal points $(x_1, x_2, 0)^\mathsf{T}$

Line at infinity $\mathbf{l}_\infty = (0,0,1)^\mathsf{T}$

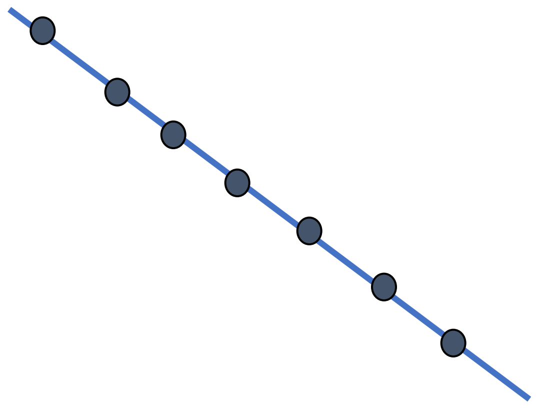
$$\mathbf{P}^2 = \mathbf{R}^2 \cup \mathbf{l}_\infty$$

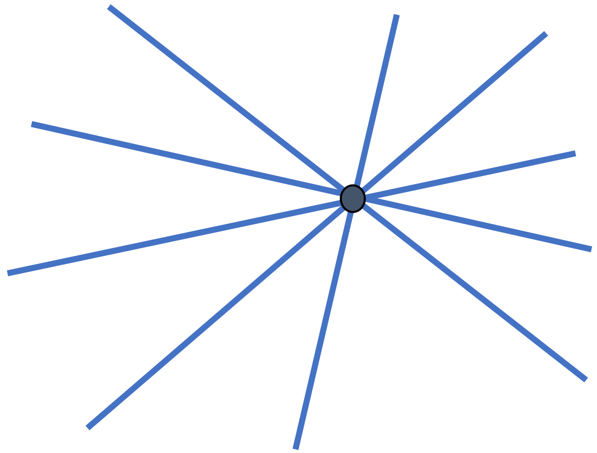
Note that in $\mathbf{P}^2$ there is no distinction between ideal points and others

Duality of points and lines in P^2

- Homogeneous representation identical
- Equations symmetric

$$l^T x = 0 \qquad x = l \times l' \qquad l = x \times x'$$





2D projective transformations

<u>Definition:</u>

A *projectivity* is an invertible mapping h from P^2 to itself such that three points x_1,x_2,x_3 lie on the same line if and only if $h(\mathrm{x}_1)$,$h(\mathrm{x}_2)$,$h(\mathrm{x}_3)$ do.

<u>Theorem:</u>

A mapping $h{:}\mathrm{P}^2\rightarrow\mathrm{P}^2$ is a projectivity if and only if there exist a non-singular 3x3 matrix $\mathbf{H}$ such that for any point in P^2 reprented by a vector x it is true that $h(\mathrm{x})=\mathbf{H}\mathrm{x}$

<u>Definition:</u> Projective transformation

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \text{or} \quad \mathrm{x}' = \mathbf{H}\,\mathrm{x}$$

8DOF

projectivity=collineation=projective transformation=homography

Transformation of 2D points and lines

For a point transformation

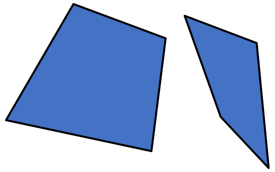
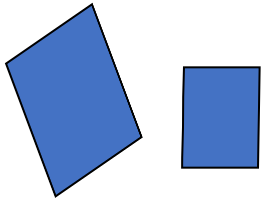
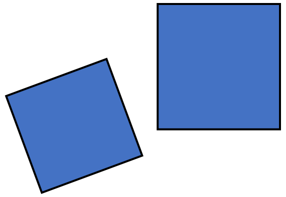
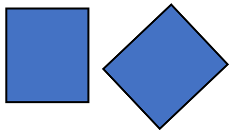
$$\mathrm{x}' = \mathbf{H}\,\mathrm{x}$$

Transformation for lines

$$\mathrm{l}' = \mathbf{H}^{-T}\,\mathrm{l}$$

Notice: $\mathrm{l}'^{\mathrm{T}}\mathrm{x}' = \mathrm{l}^{\mathrm{T}}\mathbf{H}^{-1}\mathbf{H}\mathrm{x} = \mathrm{l}^{\mathrm{T}}\mathrm{x}$

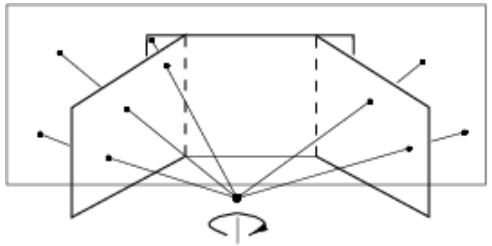
Hierarchy of 2D transformations

		transformed squares	invariants
Projective 8dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, order of contact (intersection, tangency, inflection, etc.), cross ratio
Affine 6dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallellism, ratio of areas, ratio of lengths on parallel lines (e.g midpoints), linear combinations of vectors (centroids). **The line at infinity $\mathbf{l}_\infty$**
Similarity 4dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratios of lengths, angles. **The circular points I,J**
Euclidean 3dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		lengths, areas.

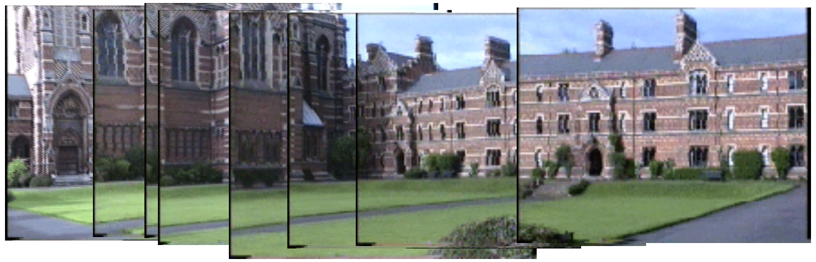
Homography application 1: Planar rectification

(notice uncertainty for transformed point depends on which points used for computation of homography)

Homography application 2: Panoramic images



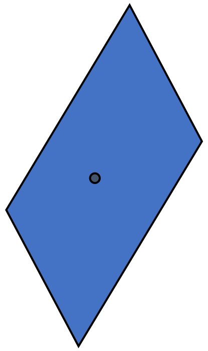
pure rotation



3D points, lines and planes

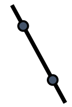
- Representation of points and planes

$$AX + BY + CZ + D = 0 \qquad \Pi^{\mathsf{T}} X = 0$$

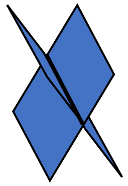


- Representation of lines
  - Join of two points
  - Intersection of two planes

$$\lambda X + \mu X' = 0$$



$$\lambda \Pi + \mu \Pi' = 0$$

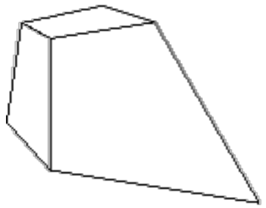


- Duality: points and planes, lines and lines

Projective transformations

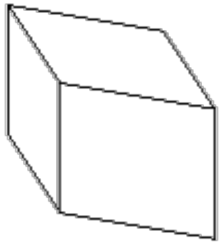
- Projective transformations

$$X' = HX$$



- Affine transformations

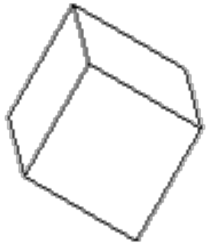
$$X' = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix} X$$



plane at infinity fixed

- Similarity transformations

$$X' = \begin{bmatrix} \sigma R & t \\ 0 & 1 \end{bmatrix} X$$



absolute conic fixed

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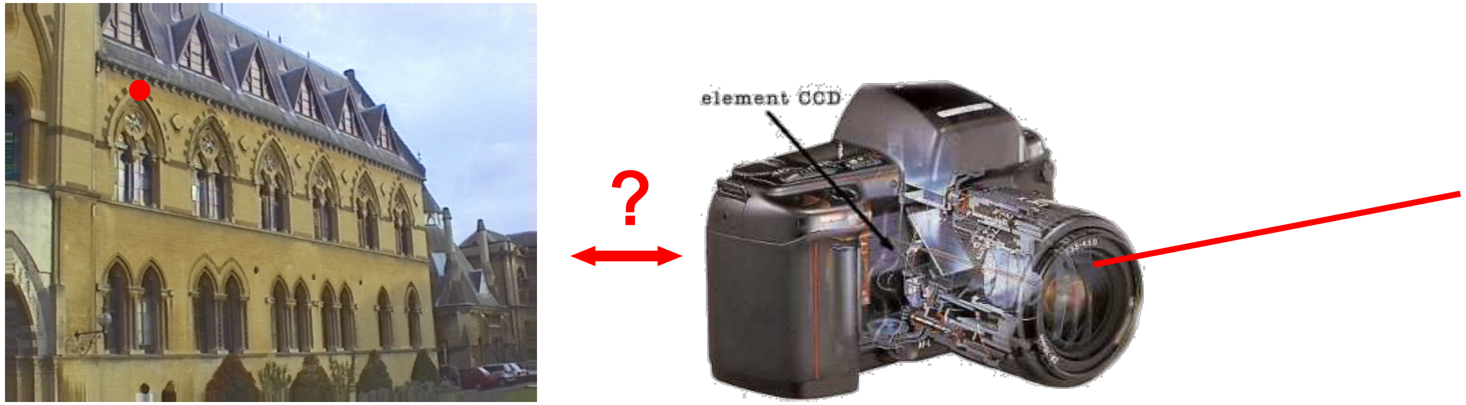
Camera model

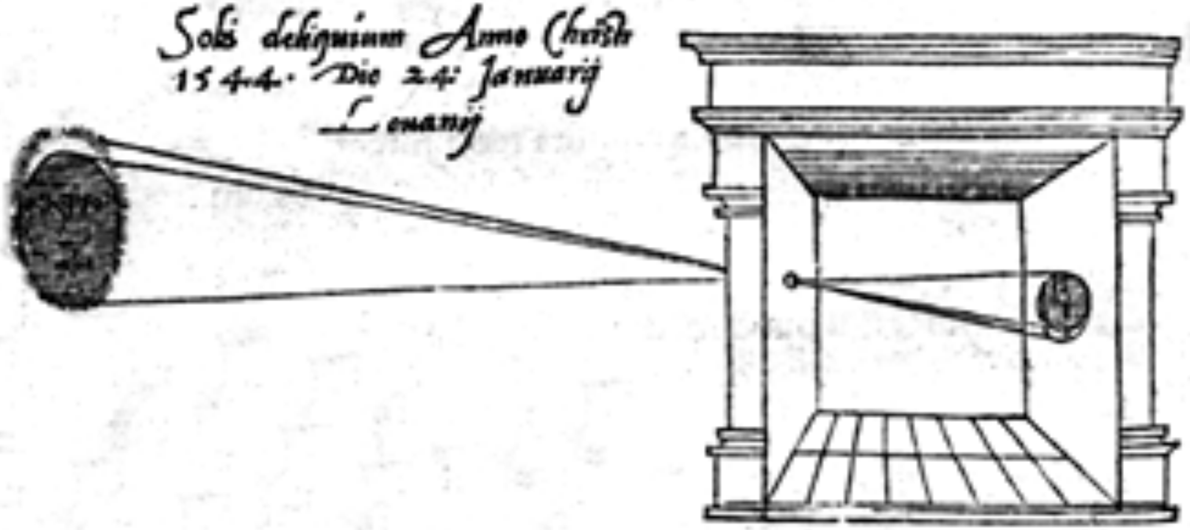
Relation between pixels and rays in space

(i.e. between 2D image and 3D world)



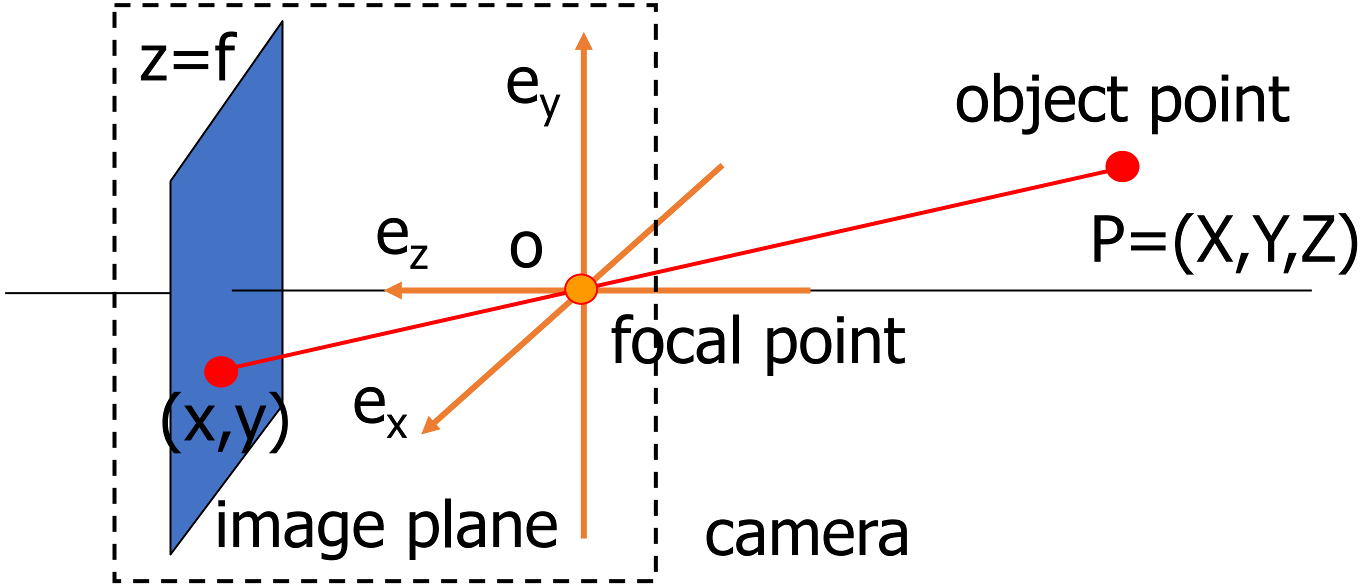
Pinhole camera

illum in tabula per radios Solis, quàm in cœlo contingit: hoc est, si in cœlo superior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.



Sic nos exactè Anno .1544. Louanii eclipsim Solis obseruauimus, inuenimusq; deficere paulò plus q̃ dex-

The pinhole camera



Uniform triangles gives:

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Projection matrix

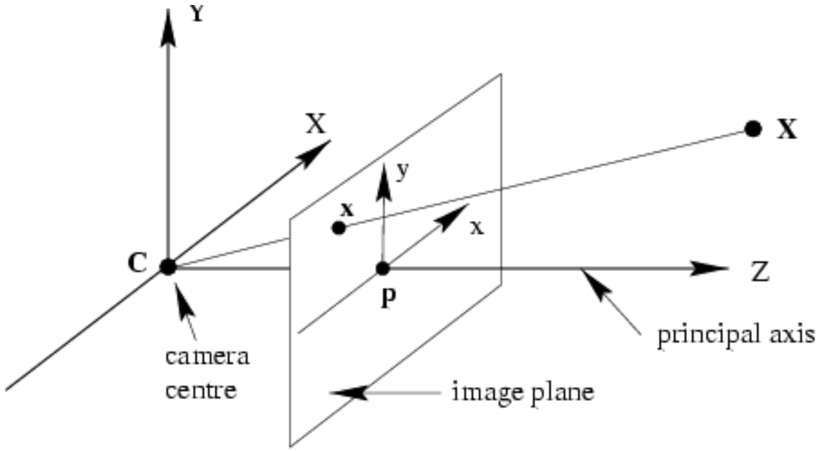
- Include coordinate transformation and camera intrinsic parameters

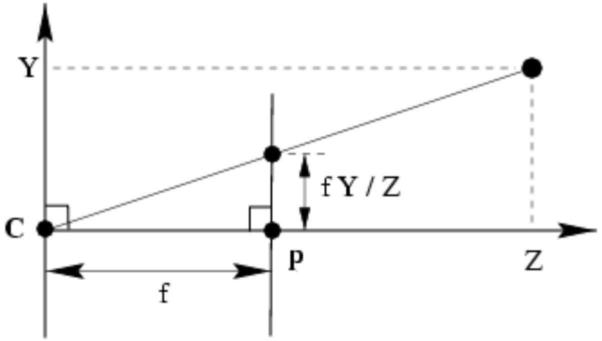
$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \gamma f & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \left[R^\top \mid -R^\top t \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda x = K\left[R^\top \mid -R^\top t \right] X$$

$$\lambda x = PX$$

Pinhole camera model



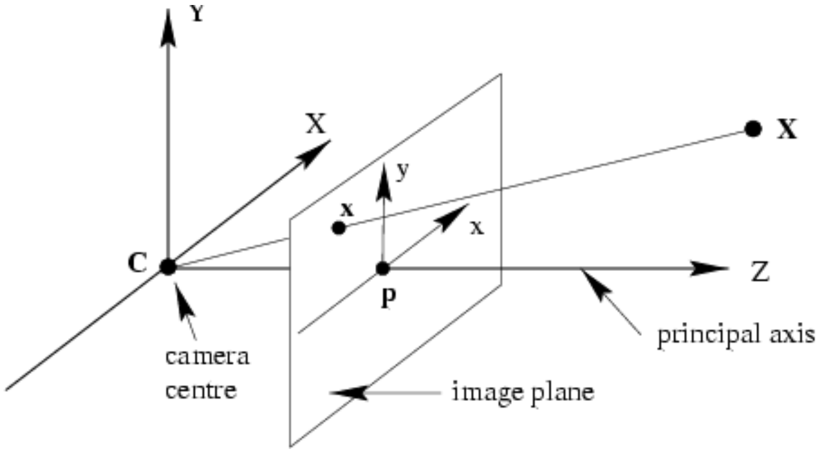


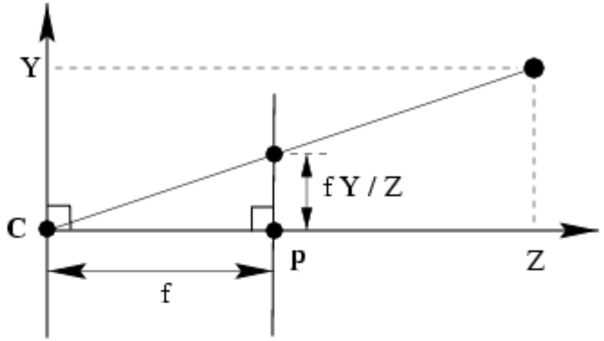
$$(X, Y, Z)^T \mapsto (fX/Z,\, fY/Z)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

linear projection in homogeneous coordinates!

Pinhole camera model



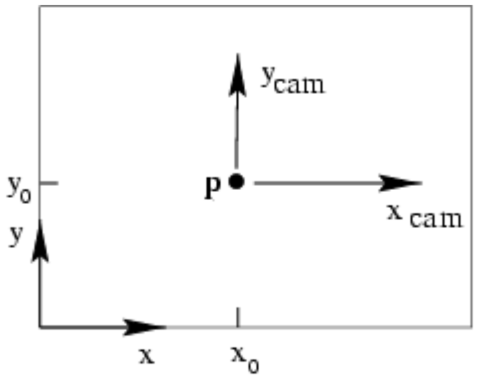


$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{PX}$$

$$\mathrm{P} = \mathrm{diag}(f, f, 1)\left[\mathrm{I} \mid 0\right]$$

Principal point offset

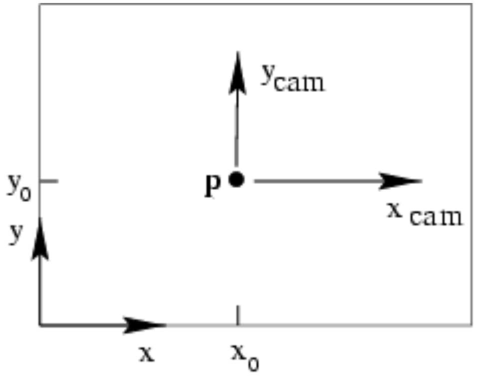


$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

$(p_x, p_y)^T$ principal point

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal point offset

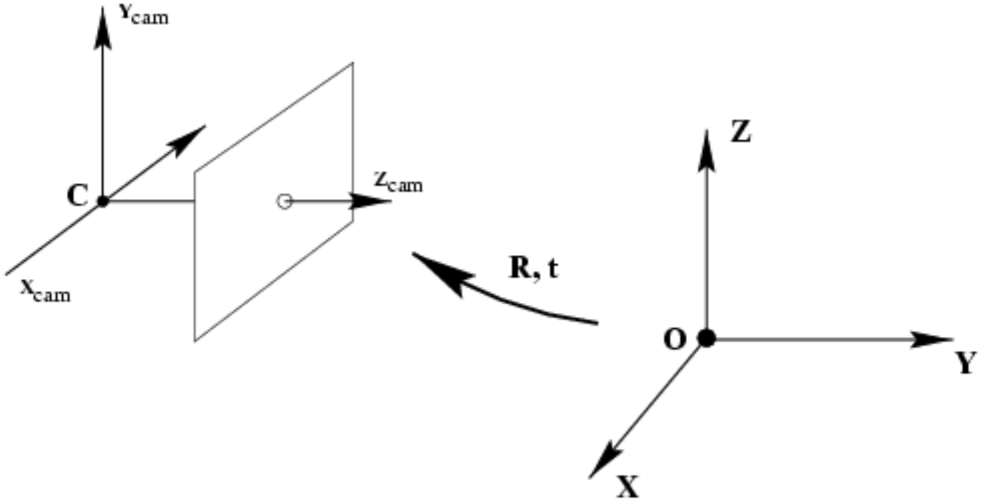


$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{K}\left[\mathrm{I} \mid 0\right]\mathrm{X}_{cam}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$ calibration matrix

Camera rotation and translation



$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

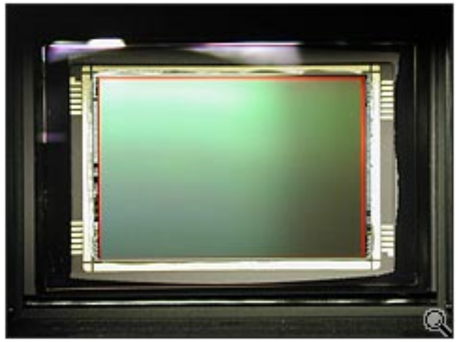
$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$\mathbf{x} = \mathbf{K}\left[I \mid 0\right] X_{cam}$$

$$\mathbf{x} = \mathbf{K}R\left[I \mid -\tilde{C}\right] X$$

$$x = PX \qquad P = K\left[R \mid t\right] \qquad t = -R\tilde{C}$$

CCD camera



$$K = \begin{bmatrix} \alpha_x & & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

General projective camera

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_x & p_y \\ & & 1 \end{bmatrix}$$

$$\mathrm{P} = \underbrace{\mathrm{KR}}_{\text{non-singular}}\left[\mathrm{I} \mid \quad \tilde{\mathrm{C}}\right]$$

11 dof (5+3+3)

$$\mathrm{P} = \mathrm{K}\left[\mathrm{R} \mid \mathrm{t}\right]$$

intrinsic camera parameters
extrinsic camera parameters

Action of projective camera on points and lines

projection of point

$$x = PX$$

forward projection of line

$$X(\mu) = P(A + \mu B) = PA + \mu PB = a + \mu b$$

back-projection of line

$$\Pi = P^T l$$

$$\Pi^T X = l^T PX \qquad \left(l^T x = 0;\ x = PX\right)$$

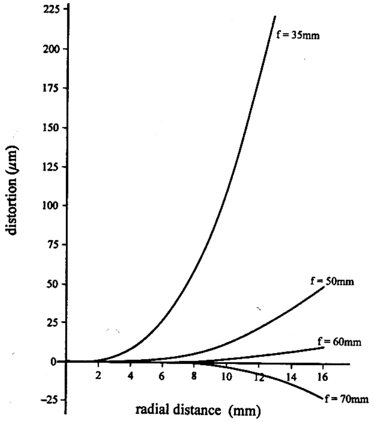
Radial distortion

- Due to spherical lenses (cheap)
- Model:

$$\begin{bmatrix}x\\y\\1\end{bmatrix} \sim \begin{bmatrix} f_x & s & c_x\\0&f_y&c_y\\0&0&1\end{bmatrix} \mathrm{R}\left(\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}\mathbf{R}^\top & -\mathbf{R}^\top \mathbf{t}\\0_3^\top & 1\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}\right)$$

$$\mathrm{R} \qquad (x,y) = (1+K_1(x^2+y^2)+K_2(x^2+y^2)^2+\ldots)\begin{bmatrix}x\\y\end{bmatrix}$$

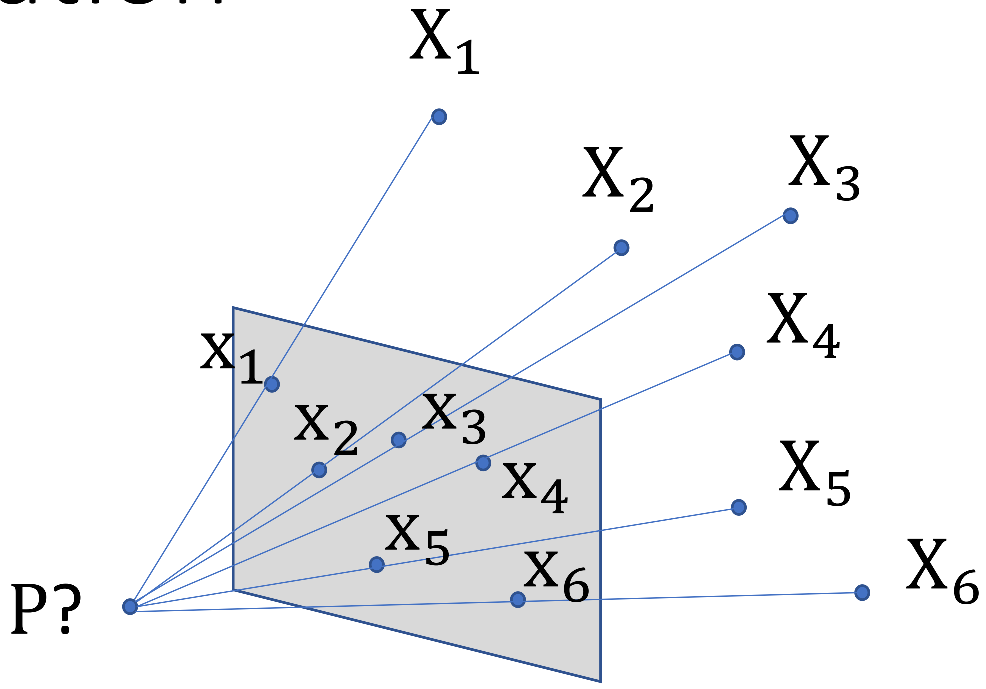
straight lines are not straight anymore



http://foto.hut.fi/opetus/260/luennot/11/atkinson_6-11_radial_distortion_zoom_lenses.jpg

Linear camera pose estimation

Given 2D and 3D points, compute the camera pose



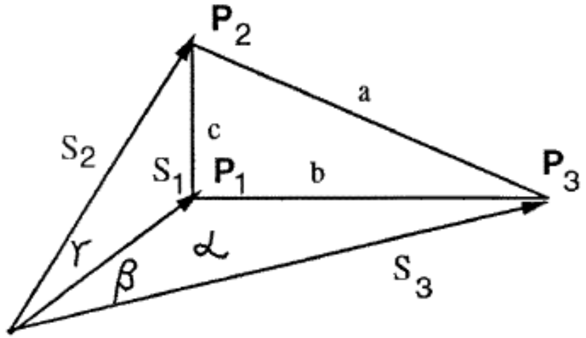
$$\lambda \mathrm{x} = \mathrm{PX}$$

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \mathrm{X}$$

$$\begin{cases} P_3 \mathrm{X} x = P_1 \mathrm{X} \\ P_3 \mathrm{X} y = P_2 \mathrm{X} \end{cases} \qquad \begin{bmatrix} \mathrm{X}^T & 0 & \mathrm{X}^T x \\ 0 & \mathrm{X}^T & \mathrm{X}^T y \end{bmatrix} \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix}$$

Three points perspective pose – p3p

(Haralick et al., IJCV94)



$$s_2^2 + s_3^2 - 2s_2s_3\cos\alpha = a^2$$
$$s_1^2 + s_3^2 - 2s_1s_3\cos\beta = b^2$$
$$s_1^2 + s_2^2 - 2s_1s_2\cos\gamma = c^2$$

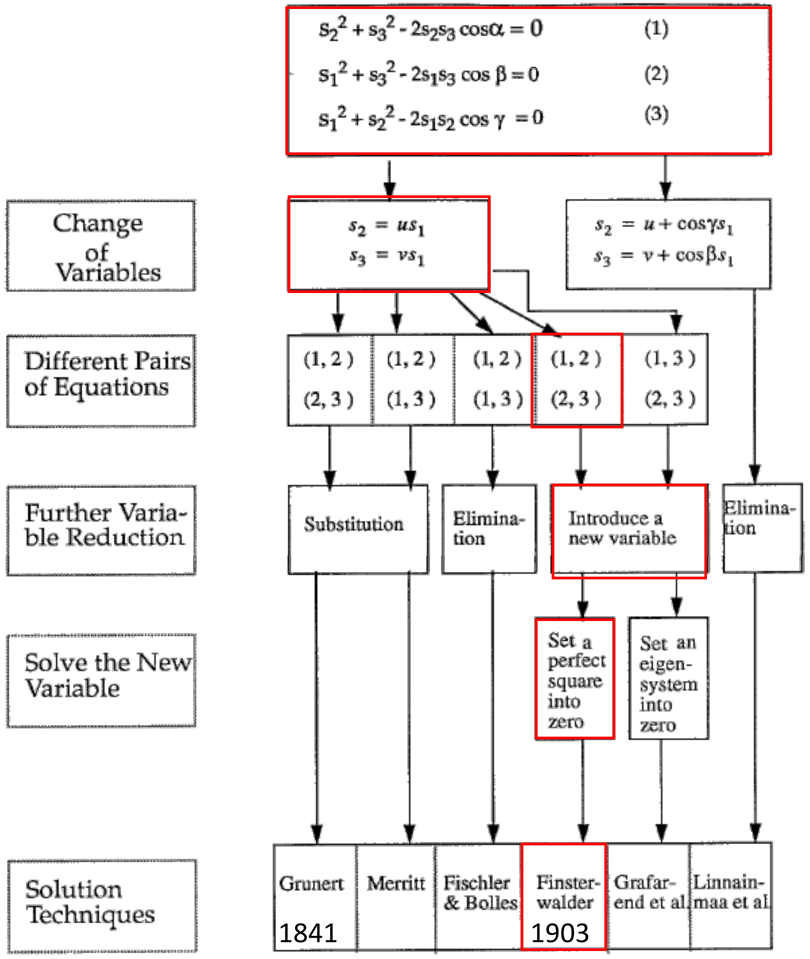
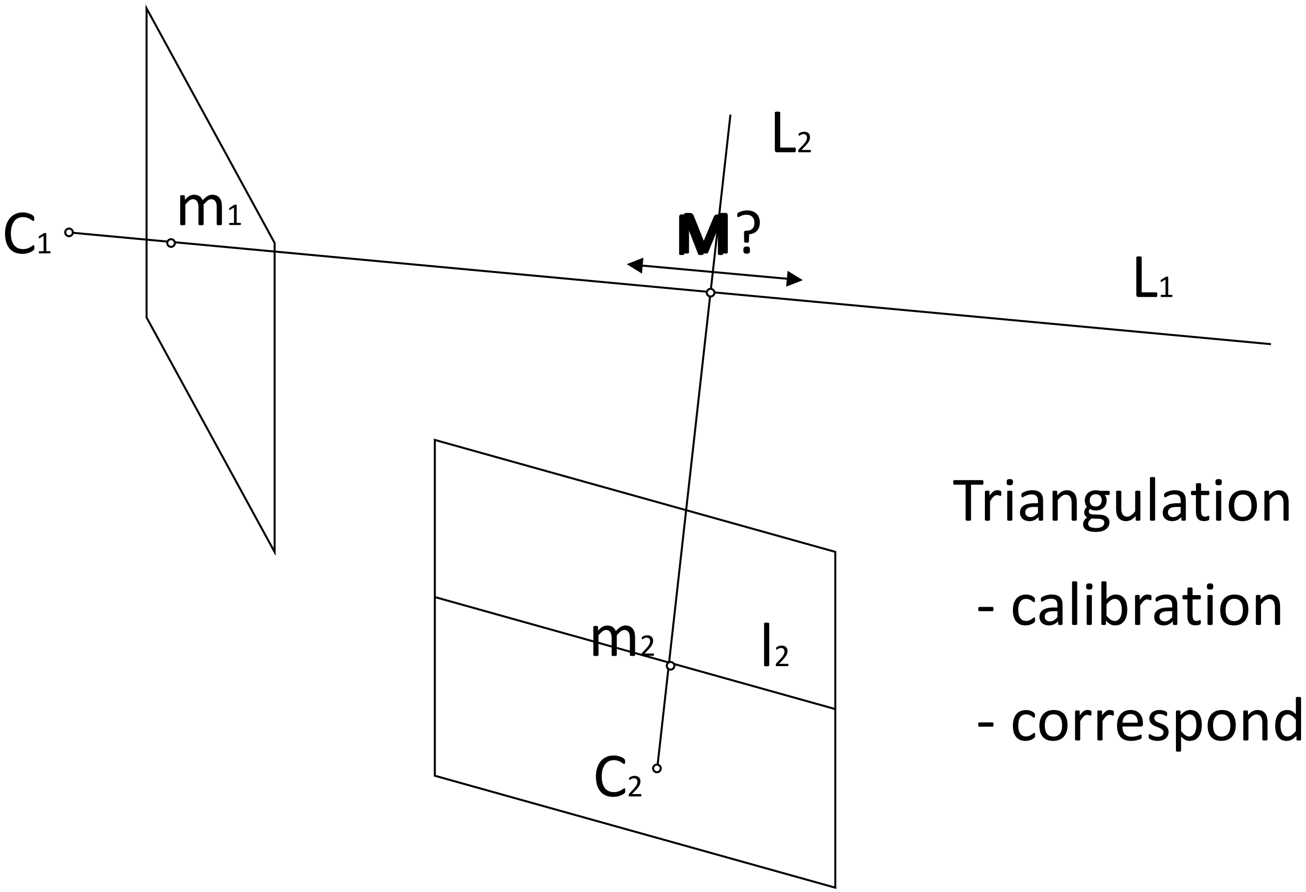


Fig. 2. Shows the differences of a algebraic derivations among six solution techniques.

All techniques yield 4th order polynomial

Haralick et al. recommends using Finsterwalder's technique as it yields the best results numerically

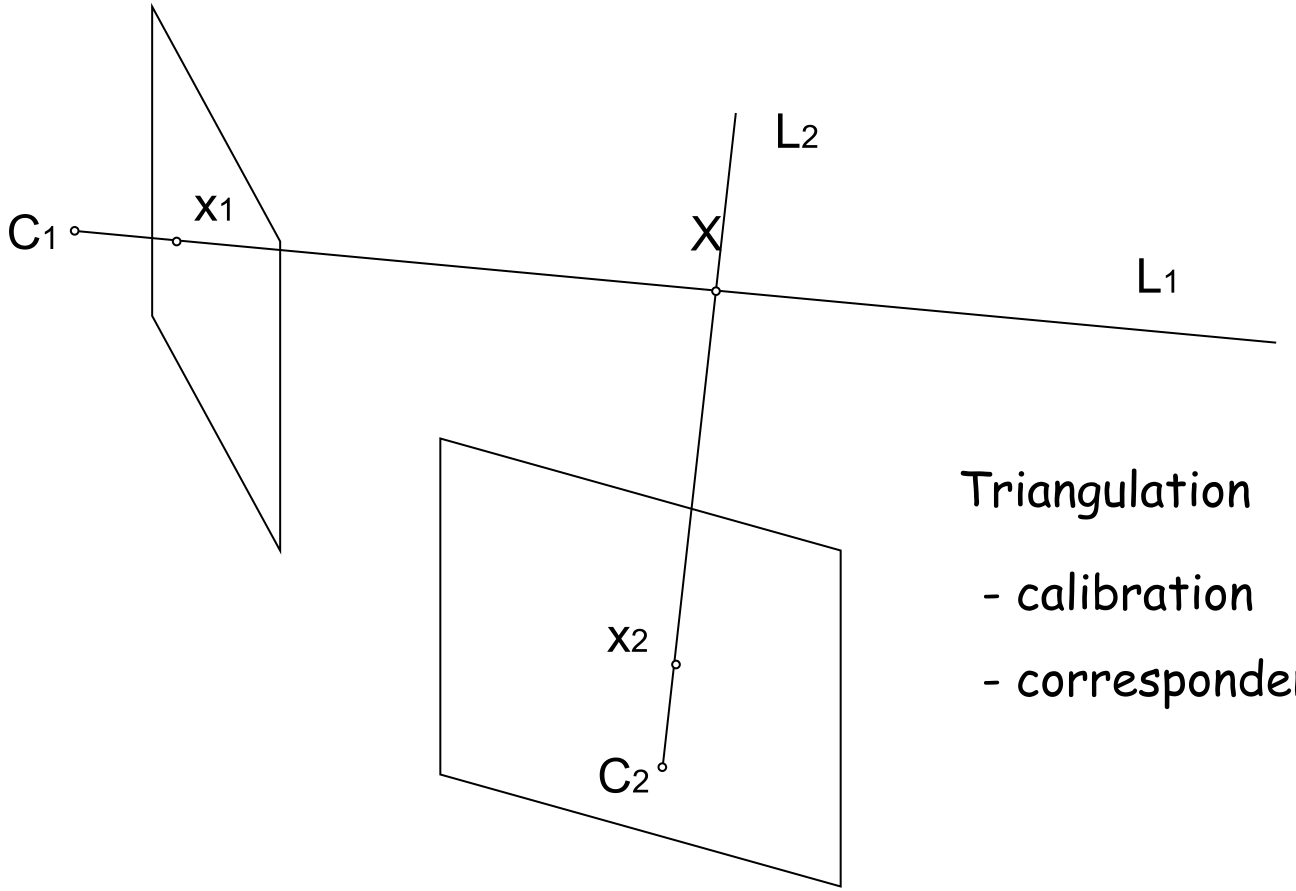
3D from images



Triangulation

- calibration
- correspondences

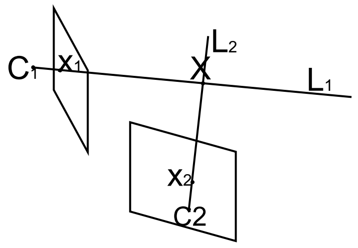
Triangulation



Triangulation

- calibration
- correspondences

Triangulation



- Backprojection

$$\lambda \mathtt{x} = \mathbf{P}\mathtt{X}$$

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathtt{P}_1 \\ \mathtt{P}_2 \\ \mathtt{P}_3 \end{bmatrix} \mathtt{X}$$

$$\begin{aligned} \mathtt{P}_3\mathtt{X}x &= \mathtt{P}_1\mathtt{X} \\ \mathtt{P}_3\mathtt{X}y &= \mathtt{P}_2\mathtt{X} \end{aligned}$$

$$\begin{bmatrix} \mathtt{P}_3 x - \mathtt{P}_1 \\ \mathtt{P}_3 y - \mathtt{P}_2 \end{bmatrix} \mathtt{X} = 0$$

- Triangulation

$$\begin{bmatrix} \mathtt{P}_3 x - \mathtt{P}_1 \\ \mathtt{P}_3 y - \mathtt{P}_2 \\ \mathtt{P}'_3 x' - \mathtt{P}'_1 \\ \mathtt{P}'_3 y' - \mathtt{P}'_2 \end{bmatrix} \mathtt{X} = 0$$

$$\begin{bmatrix} \frac{1}{\mathtt{P}_3\tilde{\mathtt{X}}} \begin{pmatrix} \mathtt{P}_3 x - \mathtt{P}_1 \\ \mathtt{P}_3 y - \mathtt{P}_2 \end{pmatrix} \\ \frac{1}{\mathtt{P}'_3\tilde{\mathtt{X}}} \begin{pmatrix} \mathtt{P}'_3 x - \mathtt{P}'_1 \\ \mathtt{P}'_3 y - \mathtt{P}'_2 \end{pmatrix} \end{bmatrix} \mathtt{X} = 0$$

Iterative least-squares

- Maximum Likelihood Triangulation

$$\arg\min_{\mathtt{X}} \sum_i \left(\mathtt{x}_i - \lambda^{-1}\mathbf{P}_i\mathtt{X}\right)^2$$

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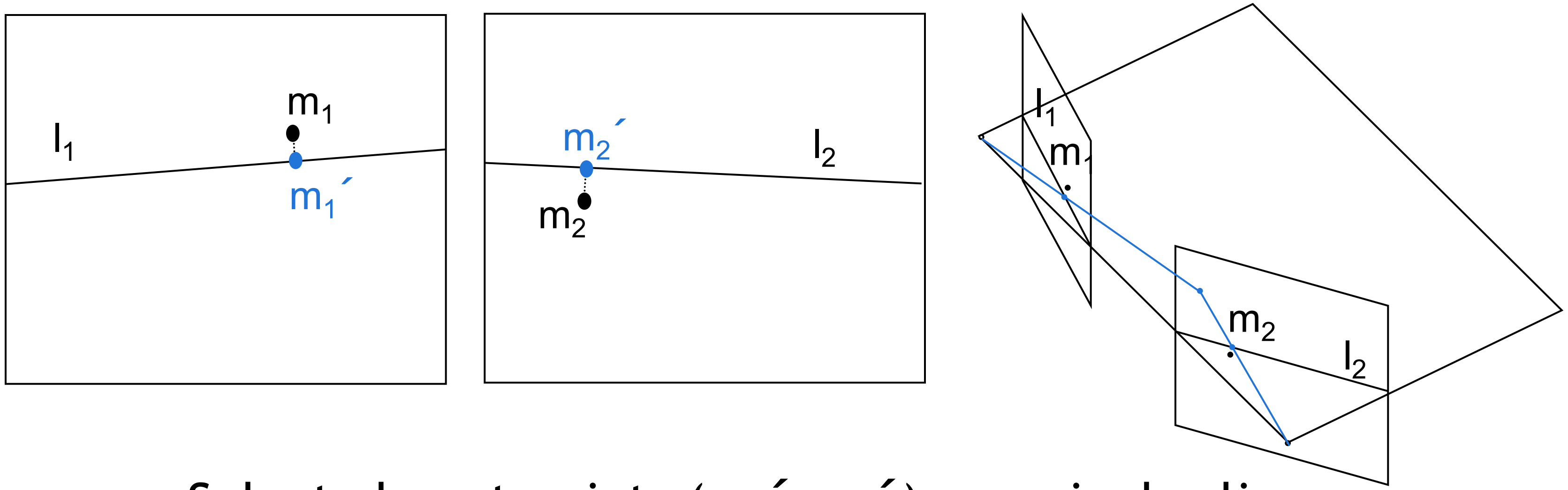
Optimal 3D point in epipolar plane

- Given an epipolar plane, find best 3D point for (m_1,m_2)



Select closest points $(m_1´,m_2´)$ on epipolar lines

Obtain 3D point through exact triangulation

Guarantees minimal reprojection error (given this epipolar plane)

Non-iterative optimal solution

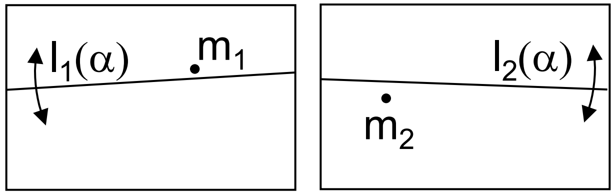
- Reconstruct matches in projective frame by minimizing the reprojection error

$$D(\mathbf{m}_1, \mathbf{P}_1\mathbf{M})^2 + D(\mathbf{m}_2, \mathbf{P}_2\mathbf{M})^2 \quad \textbf{3DOF}$$

- Non-iterative method (Hartley and Sturm, CVIU´97)

  Determine the epipolar plane for reconstruction

$$D(\mathbf{m}_1, \mathbf{l}_1(\alpha))^2 + D(\mathbf{m}_2, \mathbf{l}_2(\alpha))^2 \text{ (polynomial of degree 6)}$$



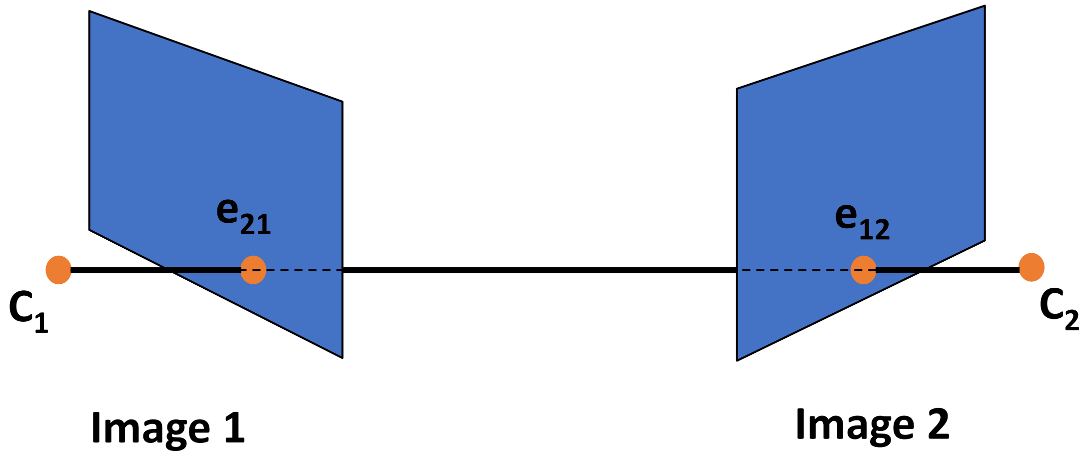
1DOF

Reconstruct optimal point from selected epipolar plane

Note: only works for two views

The epipoles

The *epipole* is the projection of the focal point of one camera in another image.

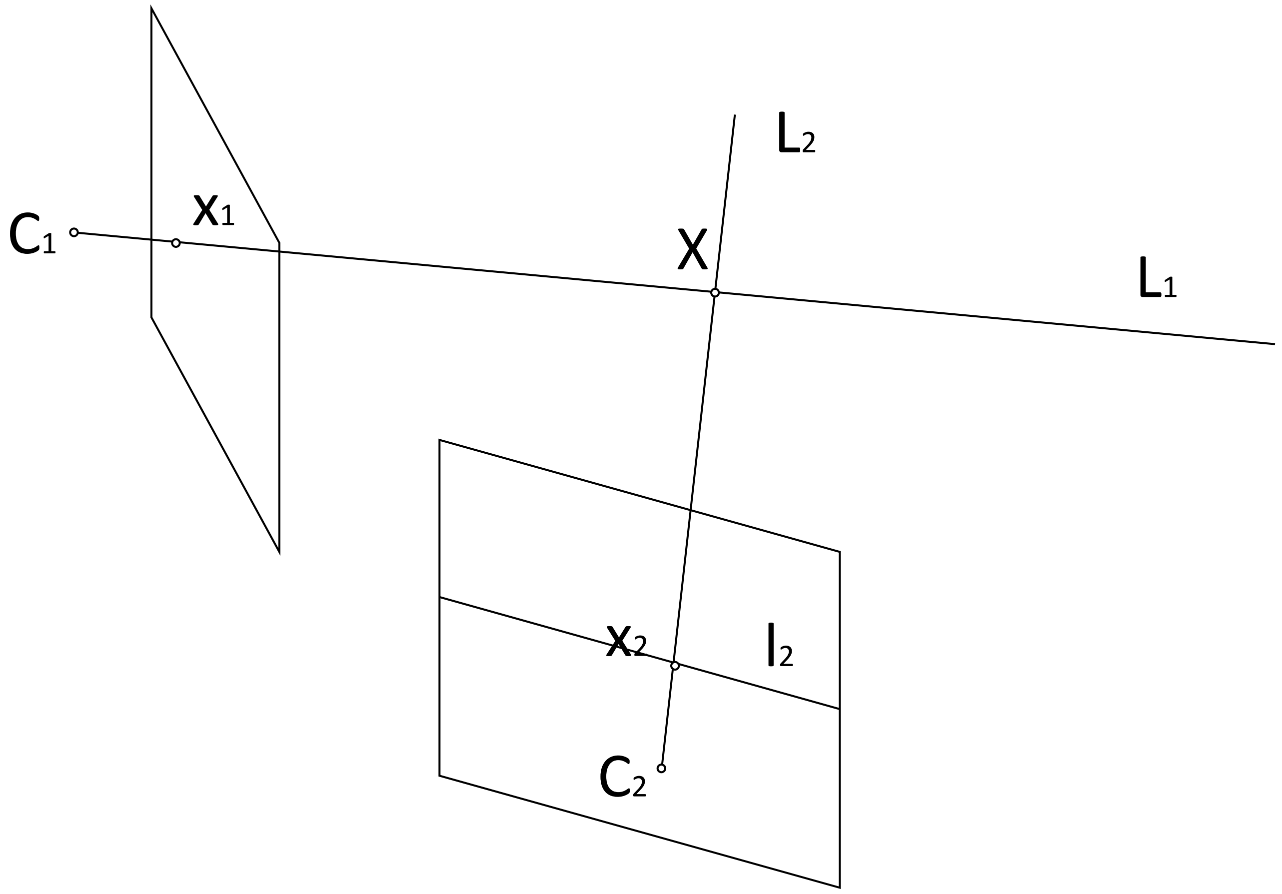


$$P_1 = [A_1 \mid b_1]$$
$$P_2 = [A_2 \mid b_2]$$

$$P_1 C_1 = [A_1 \mid b_1]\begin{bmatrix} -A_1^{-1} b_1 \\ 1 \end{bmatrix} = 0$$

$$P_2 C_1 = [A_2 \mid b_2]\begin{bmatrix} -A_1^{-1} b_1 \\ 1 \end{bmatrix} = b_2 - A_2 A_1^{-1} b_1$$

3D from images



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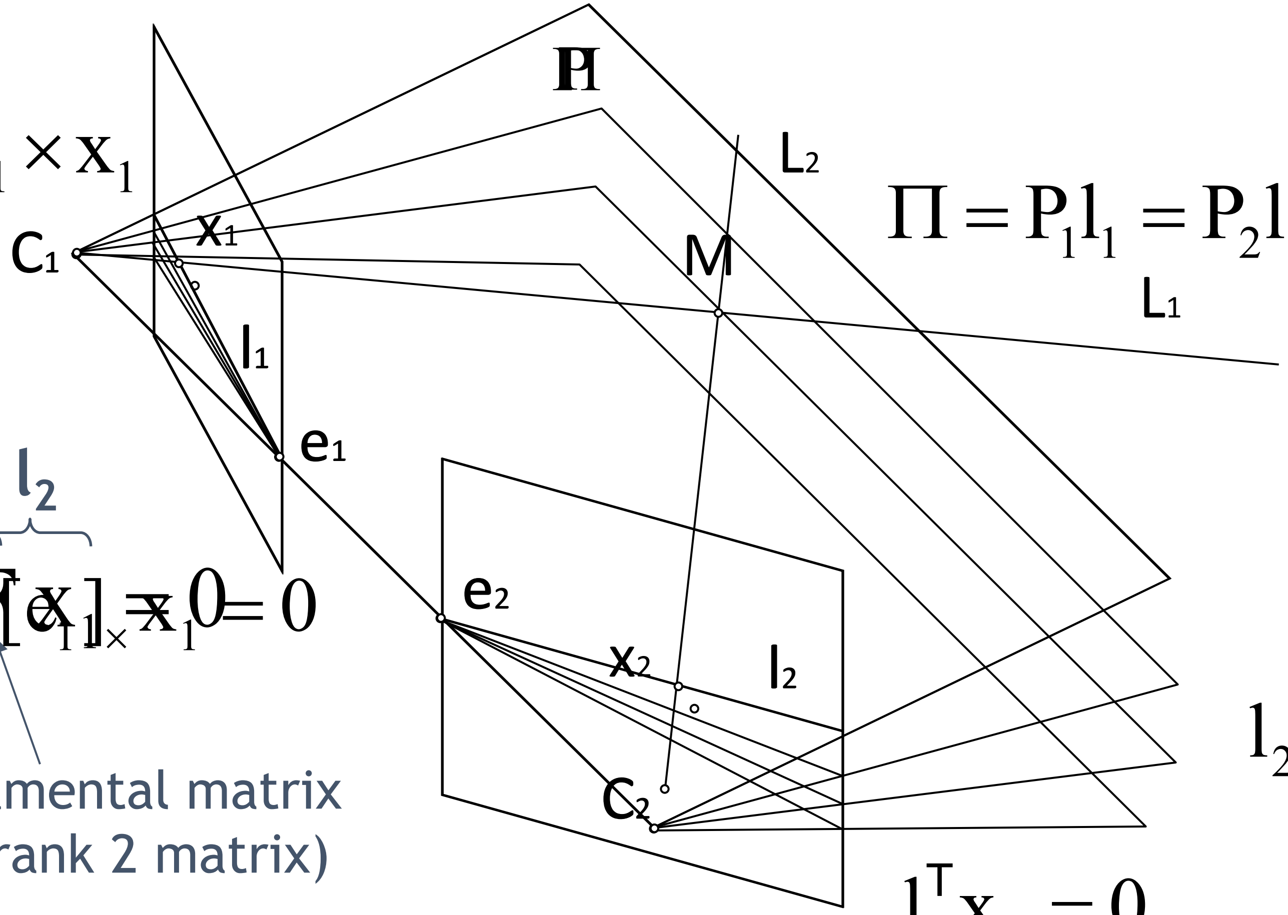
Epipolar geometry

$$l_1 = e_1 \times x_1$$



$$\Pi = P_1 l_1 = P_2 l_2$$

$$l_2 = P_2^{+} P_1 l_1$$

$$l_2^{T} x_2 = 0$$

$$x_2^{T} \underbrace{P_2^{+T}}_{} \underbrace{P_1^{T}}_{} [e_1]_\times x_1 = 0$$

$$\overbrace{x_2^{T} P_2^{+T}}^{l_2^{T}} \; \overbrace{P_1^{T} [e_1]_\times x_1}^{}$$

$$x_2^{T} F x_1 = 0$$

Fundamental matrix
(3x3 rank 2 matrix)

F-matrix computation – 8 point

- **For every match (m,m´):**

$$\mathbf{m}'^{\mathsf{T}} \mathbf{F} \mathbf{m} = 0$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} xx' & yx' & x' & xy' & yy' & y' & x & y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

F-matrix computation – 8 point

- Stack data rows

  $\Rightarrow$ Linear system of equations

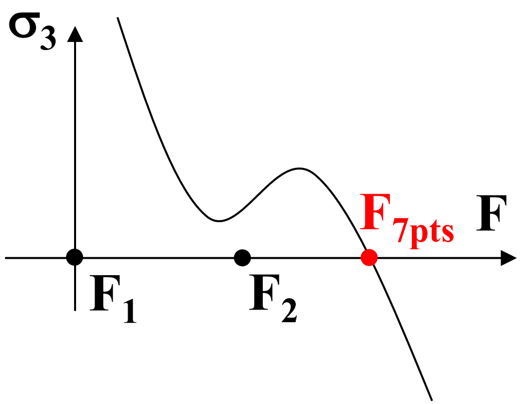
$$\begin{array}{c}\begin{bmatrix} x_1x_1' & y_1x_1' & x_1' & x_1y_1' & y_1y_1' & y_1' & x_1 & y_1 & 1 \\ x_2x_2' & y_2x_2' & x_2' & x_2y_2' & y_2y_2' & y_2' & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx_n' & y_nx_n' & x_n' & x_ny_n' & y_ny_n' & y_n' & x_n & y_n & 1 \end{bmatrix} \\ \begin{array}{ccccccccc} \sim 10000 & \sim 10000 & \sim 100 & \sim 10000 & \sim 10000 & \sim 100 & \sim 100 & \sim 100 & 1 \end{array}\end{array} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

Orders of magnitude difference
Between column of data matrix
$\rightarrow$least-squares yields poor results
$\rightarrow$Need to normalized (Hartley PAMI'97)

F-matrix computation – 7 point

Linear equations yield 1 parameter family of solutions

$F_1+\lambda F_2$



obtain 1 or 3 solutions

$$\det(\mathbf{F}_1 + \lambda \mathbf{F}_2) = a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

(cubic equation)

Essential matrix

For calibrated camera and normalized image coordinates the fundamental matrix becomes the essential matrix

$$E = R[t]_x$$

The essential matrix allows to directly derive R and t (up to scale)

E-matrix computation – 5 point

(Nister CVPR'03)

Linear equations yield 3 parameter family of solutions

$E_1+\lambda_2 E_2+\lambda_3 E_3+\lambda_4 E_4$

9 cubic constraint on E

$$EE^\top E - \frac{1}{2}trace(EE^\top)E = 0$$

Eventually yields 10 degree polynomial to solve

Can deal with planar data

Requires calibration

E-matrix computation – 5 point

(Nister, CVPR03)

- Linear equations for 5 points

$$\begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ x'_2x_2 & x'_2y_2 & x'_2 & y'_2x_2 & y'_2y_2 & y'_2 & x_2 & y_2 & 1 \\ x'_3x_3 & x'_3y_3 & x'_3 & y'_3x_3 & y'_3y_3 & y'_3 & x_3 & y_3 & 1 \\ x'_4x_4 & x'_4y_4 & x'_4 & y'_4x_4 & y'_4y_4 & y'_4 & x_4 & y_4 & 1 \\ x'_5x_5 & x'_5y_5 & x'_5 & y'_5x_5 & y'_5y_5 & y'_5 & x_5 & y_5 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{E}_{11} \\ \mathbf{E}_{12} \\ \mathbf{E}_{13} \\ \mathbf{E}_{21} \\ \mathbf{E}_{22} \\ \mathbf{E}_{23} \\ \mathbf{E}_{31} \\ \mathbf{E}_{32} \\ \mathbf{E}_{33} \end{bmatrix} = 0$$

- Linear solution space

$$\mathrm{E} = \mathrm{xX} + \mathrm{yY} + \mathrm{zZ} + \mathrm{wW}$$

scale does not matter, choose $\mathrm{w} = 1$

- Non-linear constraints

$$\left.\begin{aligned} &\det(\mathbf{E}) = 0 \\ &2\mathbf{E}\mathbf{E}^T\mathbf{E} - tr\left(\mathbf{E}\mathbf{E}^T\right)\mathbf{E} = 0 \end{aligned}\right\}$$

10 cubic polynomials

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E-matrix computation – 5 point

(Nister, CVPR03)

- Perform Gauss-Jordan elimination on polynomials

[n] represents polynomial of degree n in z

	A	x^3	y^3	x^2y	xy^2	x^2z	x^2	y^2z	y^2	xyz	xy	x	y	1
	$\langle a \rangle$	1	.	.	.	.	.	.	.	.	.	[2]	[2]	[3]
	$\langle b \rangle$		1	.	.	.	.	.	.	.	.	[2]	[2]	[3]
	$\langle c \rangle$			1	.	.	.	.	.	.	.	[2]	[2]	[3]
	$\langle d \rangle$				1	.	.	.	.	.	.	[2]	[2]	[3]
$\langle k \rangle$	$\langle e \rangle$					1						[2]	[2]	[3]
-z	$\langle f \rangle$						1					[2]	[2]	[3]
$\langle l \rangle$	$\langle g \rangle$							1				[2]	[2]	[3]
-z	$\langle h \rangle$								1			[2]	[2]	[3]
$\langle m \rangle$	$\langle i \rangle$									1		[2]	[2]	[3]
-z	$\langle j \rangle$										1	[2]	[2]	[3]

$$\langle k \rangle \equiv \langle e \rangle - z \langle f \rangle$$
$$\langle l \rangle \equiv \langle g \rangle - z \langle h \rangle$$
$$\langle m \rangle \equiv \langle i \rangle - z \langle j \rangle$$

B	x	y	1
$\langle k \rangle$	[3]	[3]	[4]
$\langle l \rangle$	[3]	[3]	[4]
$\langle m \rangle$	[3]	[3]	[4]

$$\boxed{\langle n \rangle \equiv det(B)}$$

Minimal relative pose with know vertical

Fraundorfer, Tanskanen and Pollefeys, ECCV2010

Vertical direction can often be estimated

- inertial sensor
- vanishing point

$$E = \begin{bmatrix} t_z \sin(y) & -t_z \cos(y) & t_y \\ t_z \cos(y) & t_z \sin(y) & -t_x \\ -t_y \cos(y) - t_x \sin(y) & t_x \cos(y) - t_y \sin(y) & 0 \end{bmatrix}$$

5 linear unknowns $\rightarrow$ linear 5 point algorithm

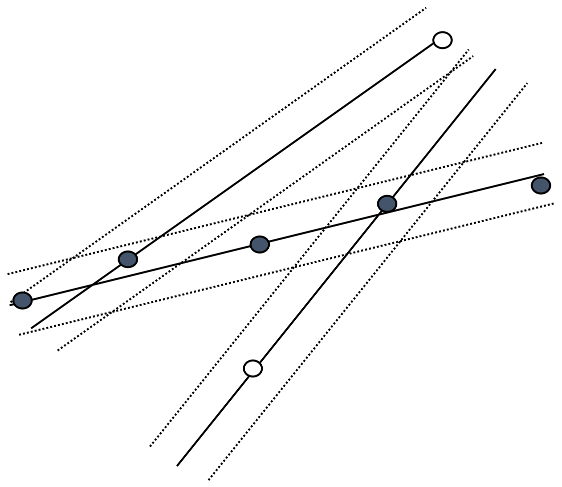
3 unknowns $\rightarrow$ quartic 3 point algorithm

robust estimation (RANSAC)

To avoid outliers, generate hypothesis with as few data as possible

Keep doing until succesful

e.g. line fitting



Robust F-matrix computation (RANSAC)

Step 1. Extract features

Step 2. Compute a set of potential matches

Step 3. do

Step 3.1 select minimal sample (i.e. 7 matches)

Step 3.2 compute solution(s) for F

(generate hypothesis)

Step 3.3 determine inliers (verify hypothesis)

until Γ(*#inliers*,*#samples*)<95%

Step 4. Compute F based on all inliers

Step 5. Look for additional matches

Step 6. Refine F based on all correct matches

$$\Gamma = 1 - \left(1 - \left(\frac{\#inliers}{\#matches}\right)^7\right)^{\#samples}$$

#inliers	90%	80%	70%	60%	50%
#samples	5	13	35	106	382

Some recent work: QDEGSAC (Frahm and Pollefeys CVPR06) , ARRSAC (Raguram et al.ECCV08) , RANSAC with uncertainty (Raguram et al ICCV'09), USAC (Raguram et al TPAMI)

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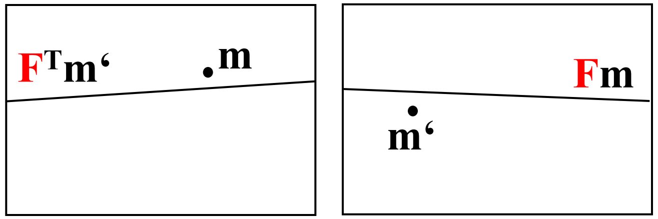
F-matrix non-linear refinement

Minimize geometric instead of algebraic error

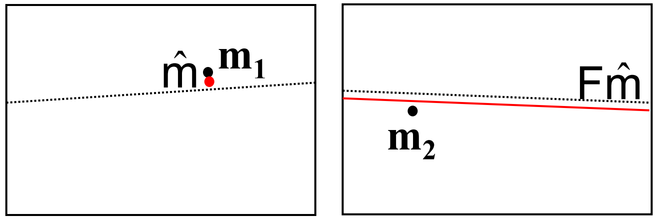
$$\min_{\mathbf{F}} \sum \left(D(\mathbf{m}', \mathbf{F}\mathbf{m})^2 + D(\mathbf{m}, \mathbf{F}^{\mathrm{T}}\mathbf{m}')^2 \right) \quad \leftrightarrow \quad \min_{\mathbf{F}} \sum \mathbf{m}'^{\mathrm{T}} \mathbf{F}\mathbf{m}$$



Meaningfull error in image space

$$\min_{\mathbf{F}, \hat{\mathbf{m}}} \sum \left(D(\mathbf{m}', \mathbf{F}\hat{\mathbf{m}})^2 + D(\mathbf{m}, \hat{\mathbf{m}})^2 \right)$$

Maximum Likelihood Estimastion (given Gaussian noise error model)

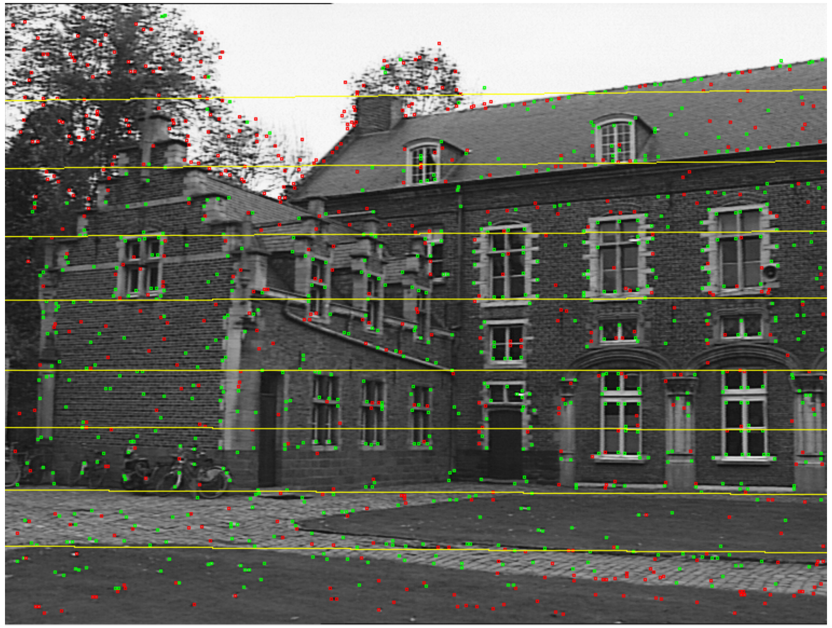


(Initialize with linear algorithm; Use e.g. Levenberg-Marquardt)

Finding more matches for F-matrix

restrict search range to neighborhood of epipolar line (±1.5 pixels)
relax disparity restriction (along epipolar line)

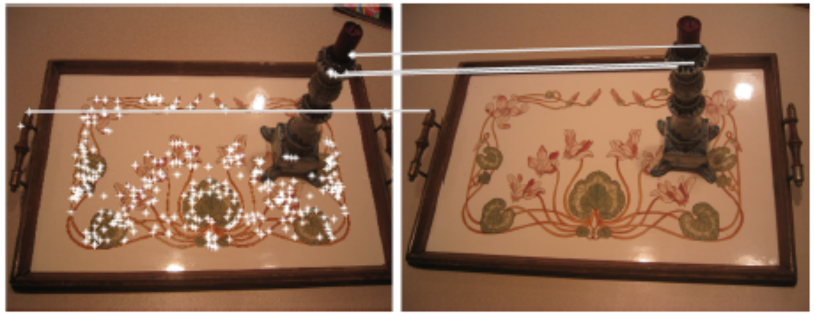
Computed F-matrix & matches



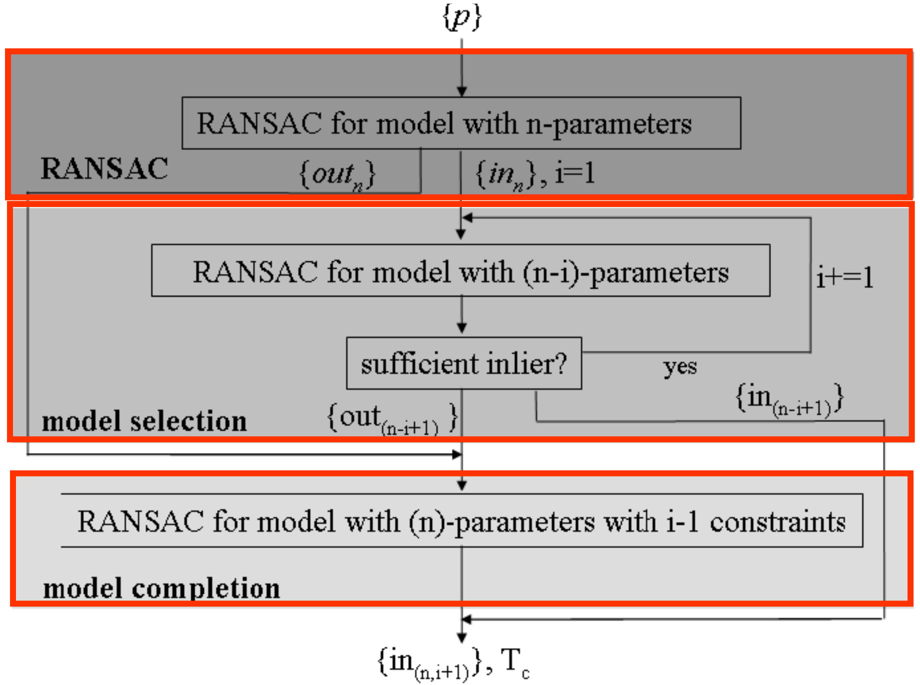
geometric relations between two views is fully described by recovered 3x3 matrix F

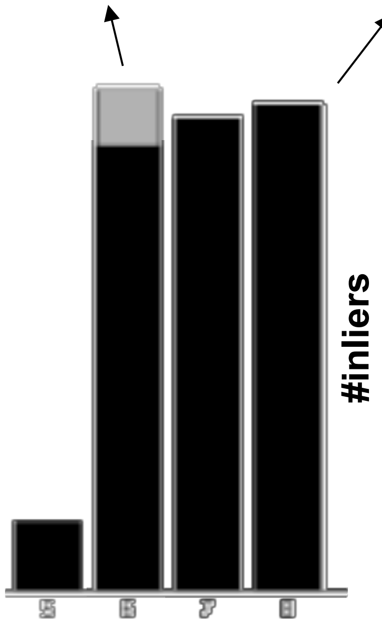
Computing F for quasi-planar scenes QDEGSAC

(Frahm & Pollefeys CVPR06)

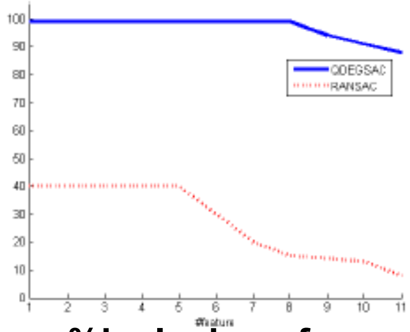


337 matches on plane, 11 off plane





data rank



%inclusion of out-of-plane inliers

17% success for RANSAC

100% for QDEGSAC

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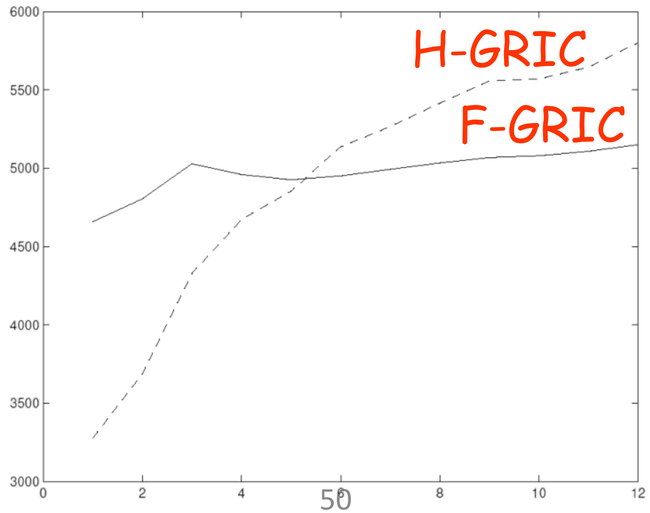
Key-frame selection

Select key-frame when F yields a better model than H

- Use Robust Geometric Information Criterion (Torr '98)

$$\text{GRIC} = \underbrace{\sum \rho(e_i^2)}_{\text{bad fit penalty}} + \underbrace{(nd\ln(r) + k\ln(rn))}_{\text{model complexity}}$$

- Given view i as a key-frame, pick view j as next key-frame for first view where $\text{GRIC}(F_{ij})>\text{GRIC}(H_{ij})$ (or a few views later)

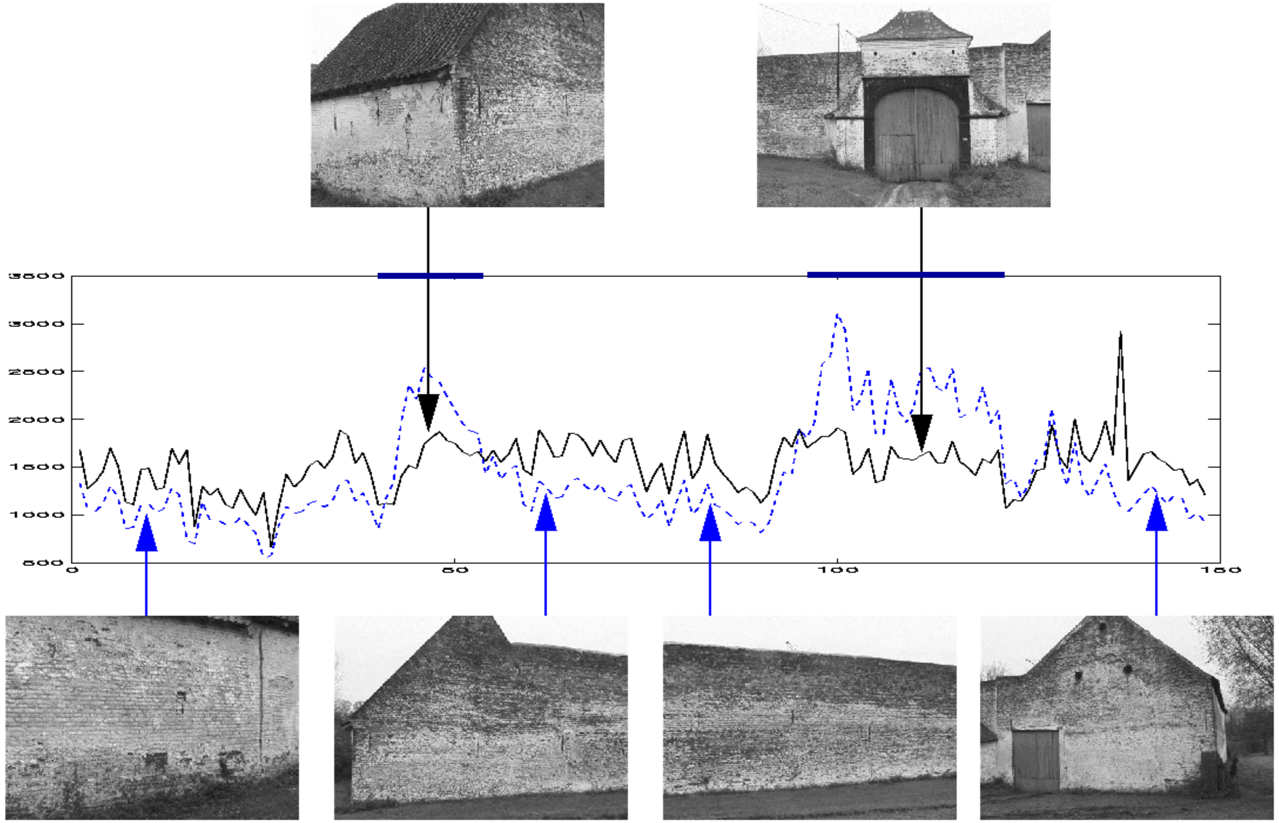


(Pollefeys et al.'02)

Dealing with dominant planar scenes

(Pollefeys et al., ECCV'02)

- USaM fails when common features are all in a plane
- Solution: <u>part 1</u> Model selection to detect problem



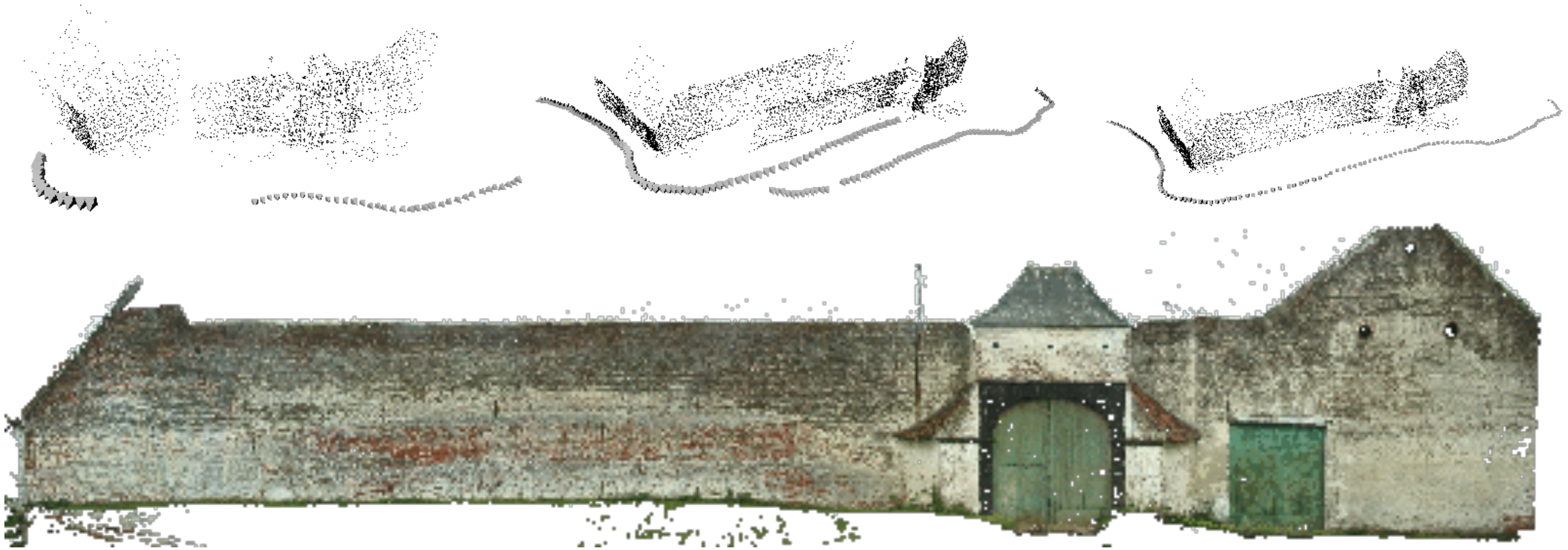
Dealing with dominant planar scenes

(Pollefeys et al., ECCV'02)

- USaM fails when common features are all in a plane
- Solution: <u>part 2</u> Delay ambiguous computations

  until after self-calibration

  (couple self-calibration over all 3D parts)



Questions?