# Notes on Calculation of Group Opacities in F&C Test

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$$\varkappa_{B,g} = \frac{\int_{\nu_g}^{\nu_{g+1}} \varkappa_\nu B_\nu d\nu}{\int_{\nu_g}^{\nu_{g+1}} B_\nu d\nu}\,, \tag{1}$$

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}\,, \tag{2}$$

$$\varkappa_\nu(T) = \frac{\varkappa^*}{(h\nu)^3}\Big(1-e^{-\frac{h\nu}{kT}}\Big)\,, \quad \varkappa^* = 27\,. \tag{3}$$

We define $\hat{\nu} = h\nu$ and $\hat{T} = kT$ in eV.

$$B_\nu = \frac{2(h\nu)^3}{h^2c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} = \frac{2}{h^2c^2}\frac{\hat{\nu}^3}{e^{\frac{\hat{\nu}}{\hat{T}}}-1}\,, \tag{4}$$

$$\varkappa_\nu(T) = \frac{\varkappa^*}{\hat{\nu}^3}\Big(1-e^{-\frac{\hat{\nu}}{\hat{T}}}\Big)\,, \tag{5}$$

$$\varkappa_{B,g} = \frac{\int_{\hat{\nu}_g}^{\hat{\nu}_{g+1}} \varkappa_\nu(\hat{\nu},\hat{T}) B_\nu(\hat{\nu},\hat{T}) d\hat{\nu}}{\int_{\nu_g}^{\nu_{g+1}} B_\nu(\hat{\nu},\hat{T}) d\hat{\nu}}\,. \tag{6}$$

$$\varkappa_{B,g} = \varkappa^* \frac{\int_{\hat{\nu}_g}^{\hat{\nu}_{g+1}} e^{-\frac{\hat{\nu}}{\hat{T}}} d\hat{\nu}}{\int_{\nu_g}^{\nu_{g+1}} \frac{\hat{\nu}^3 e^{-\frac{\hat{\nu}}{\hat{T}}}}{1-e^{-\frac{\hat{\nu}}{\hat{T}}}} d\hat{\nu}}\,, \tag{7}$$

In case $\frac{\hat{\nu}}{\hat{T}} \gg 1$, we multiply both nominator and denominator by $e^{\frac{\hat{\nu}_g}{\hat{T}}}$.

$$\varkappa_{B,g} = \varkappa^* \frac{\int_{\hat{\nu}_g}^{\hat{\nu}_{g+1}} e^{-\frac{\hat{\nu}-\hat{\nu}_g}{\hat{T}}} d\hat{\nu}}{\int_{\nu_g}^{\nu_{g+1}} \frac{\hat{\nu}^3 e^{-\frac{\hat{\nu}-\hat{\nu}_g}{\hat{T}}}}{1-e^{-\frac{\hat{\nu}}{\hat{T}}}} d\hat{\nu}} = \varkappa^* \frac{\hat{T}\left(1-e^{-\frac{\hat{\nu}_{g+1}-\hat{\nu}_g}{\hat{T}}}\right)}{\int_{\nu_g}^{\nu_{g+1}} \frac{\hat{\nu}^3 e^{-\frac{\hat{\nu}-\hat{\nu}_g}{\hat{T}}}}{1-e^{-\frac{\hat{\nu}}{\hat{T}}}} d\hat{\nu}} \tag{8}$$

$$\varkappa_{B,g} = \varkappa^* \frac{\hat{T}\left(1-e^{-\frac{\hat{\nu}_{g+1}-\hat{\nu}_g}{\hat{T}}}\right)}{\hat{T}^4 e^{\frac{\hat{\nu}_g}{\hat{T}}} \int_{\frac{\hat{\nu}_g}{\hat{T}}}^{\frac{\hat{\nu}_{g+1}}{\hat{T}}} \frac{x^3}{e^x-1} dx} = \varkappa^* \frac{\left(1-e^{-\frac{\hat{\nu}_{g+1}-\hat{\nu}_g}{\hat{T}}}\right)}{\hat{T}^3 e^{\frac{\hat{\nu}_g}{\hat{T}}} \left(\sigma\Big(\frac{\hat{\nu}_{g+1}}{\hat{T}}\Big) - \sigma\Big(\frac{\hat{\nu}_g}{\hat{T}}\Big)\right)}\,, \tag{9}$$

$$e^{\frac{\hat{\nu}_g}{\hat{T}}}\left[\sigma\Big(\frac{\hat{\nu}_{g+1}}{\hat{T}}\Big) - \sigma\Big(\frac{\hat{\nu}_g}{\hat{T}}\Big)\right] = \tilde{\sigma}\Big(\frac{\hat{\nu}_{g+1}}{\hat{T}}, \frac{\hat{\nu}_g}{\hat{T}}\Big)\,, \tag{10}$$

$$\tilde{\sigma}\Big(z_1, z_2\Big) = -e^{-(z_1-z_2)}(z_1^3+3z_1^2+6z_1+7.28) + (z_2^3+3z_2^2+6z_2+7.28)\,, \tag{11}$$

where $z_1 > 2$ and $z_2 > 2$.

$$\varkappa_{B,g} = \frac{\varkappa^*\left(1-e^{-\frac{\hat{\nu}_{g+1}-\hat{\nu}_g}{\hat{T}}}\right)}{\hat{T}^3\tilde{\sigma}\Big(\frac{\hat{\nu}_{g+1}}{\hat{T}}, \frac{\hat{\nu}_g}{\hat{T}}\Big)} \quad \text{for} \quad \frac{\hat{\nu}_g}{\hat{T}} \gg 1 \quad \text{and} \quad \frac{\hat{\nu}_{g+1}}{\hat{T}} \gg 1\,. \tag{12}$$