Notes on Integration of The Planck Function

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The Planck function is given by

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}\,. \tag{1}$$

In case of using multigroup approximation for the radiative transfer equation, one needs to integrate the Planck function over $\nu_p \le \nu \le \nu_{p+1}$ and compute

$$\int_{\nu_p}^{\nu_{p+1}} B_\nu d\nu\,. \tag{2}$$

Let us cast Eq. (2) in the following form:

$$\int_{\nu_p}^{\nu_{p+1}} B_\nu d\nu = \frac{2h}{c^2}\int_{\nu_p}^{\nu_{p+1}}\frac{\nu^3 d\nu}{e^{\frac{h\nu}{kT}}-1} = \left\{x=\frac{h\nu}{kT}\right\} = \frac{2k^4T^4}{h^3c^2}\int_{\frac{h\nu_p}{kT}}^{\frac{h\nu_{p+1}}{kT}}\frac{x^3dx}{e^x-1} = \frac{1}{\pi}\sigma_p(T,\nu_p,\nu_{p+1})T^4\,, \tag{3}$$

$$\sigma_p(T,\nu_p,\nu_{p+1}) = \frac{2\pi k^4}{h^3c^2}\int_{\frac{h\nu_p}{kT}}^{\frac{h\nu_{p+1}}{kT}}\frac{x^3dx}{e^x-1} = \frac{2\pi k^4}{h^3c^2}\left(\sigma\left(\frac{h\nu_{p+1}}{kT}\right)-\sigma\left(\frac{h\nu_p}{kT}\right)\right), \tag{4}$$

where

$$\sigma\left(\frac{h\nu_p}{kT}\right) = \int_0^{\frac{h\nu_p}{kT}}\frac{x^3dx}{e^x-1}\,. \tag{5}$$

The approximate formula for (5) (V. Gol'din & B. Chetverushkin) is the following:

$$\sigma(z) = \begin{cases} z^3\left(\frac{1}{3}-\frac{z}{8}+\frac{z^2}{62.4}\right), & \text{for } z \le 2\,,\\ 6.4939 - e^{-z}\left(z^3+3z^2+6z+7.28\right), & \text{for } z > 2\,. \end{cases} \tag{6}$$