Notes

On Scaled Equations of Nonlinear Radiative Transfer Problems and Computational Units

Dmitriy Anistratov
anistratov@ncsu.edu
March 17, 2016

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1 Computational Units

Let us introduce the following scaling by means of special computational units:

- $t = \rho_t \hat{t}$,
- $I_g = \rho_I \hat{I}_g$,
- $B_g = \rho_B \hat{B}_g$,
- $\varepsilon = \rho_\varepsilon \hat{\varepsilon}$.

2 Scaled Equations

The multigroup radiative transfer equations with scaling are given by

$$\frac{1}{\rho_t c}\frac{\partial \hat{I}_g}{\partial \hat{t}} + \mathbf{\Omega} \cdot \nabla \hat{I}_g + \varkappa_{E,g}(T)\hat{I}_g = \frac{\rho_B}{\rho_I}\varkappa_{B,g}(T)\hat{B}_g(T)\,, \tag{1a}$$

$$\hat{I}_g\Big|_{\mathbf{r}=\mathbf{r}_\gamma} = \frac{1}{\rho_I} I_g^{in}\,, \quad \text{for } \mathbf{r}_\gamma \in \partial G \text{ and } \mathbf{\Omega}\cdot\mathbf{e}_n < 0\,, \quad \hat{t} \geq \frac{1}{\rho_t} t_0\,, \tag{1b}$$

$$\hat{I}_g\Big|_{\hat{t}=\rho_t^{-1}t_0} = \frac{1}{\rho_I} I_g^0\,, \quad \text{for } \mathbf{r} \in G \text{ and all } \mathbf{\Omega}\,, \tag{1c}$$

where

$$\hat{B}_g(T) = \frac{\hat{\sigma}_{R,g}(T)}{\pi} T^4 \tag{2}$$

and

$$\hat{\sigma}_{R,g}(T) = \frac{1}{\rho_B}\sigma_{R,g}(T)\,. \tag{3}$$

The moments of the scaled intensity are

$$\hat{E}_g = \frac{1}{c}\int_{4\pi} \hat{I}_g d\mathbf{\Omega}\,, \tag{4}$$

$$\hat{\mathbf{F}}_g = \int_{4\pi} \mathbf{\Omega}\hat{I}_g d\mathbf{\Omega}\,. \tag{5}$$

The scaled energy balance equation has the following form:

$$\frac{\rho_\varepsilon}{\rho_t}\frac{\partial \hat{\varepsilon}(T)}{\partial \hat{t}} = c\sum_{g=1}^{N_g}\Big(\varkappa_{E,g}(T)\rho_I \hat{E}_g - \varkappa_{B,g}(T)\rho_B \hat{E}_g^{pl}\Big)\,, \tag{6}$$

$$\hat{\varepsilon}(T) = \hat{c}_v T\,, \tag{7}$$

$$T|_{\hat{t}=\rho_t^{-1}t_0} = T^0\,, \tag{8}$$

where

$$T^0 = T(t_0)\,, \tag{9}$$

$$\hat{c}_v = \frac{1}{\rho_\varepsilon}c_v\,, \tag{10}$$

$$\hat{E}_g^{pl} = \frac{4\pi}{c}\hat{B}_g(T) = \frac{4\hat{\sigma}_{R,g}(T)}{c}T^4\,. \tag{11}$$

We note that

$$\hat{E} = \sum_{g=1}^{N_g}\hat{E}_g\,, \tag{12}$$

$$\hat{\mathbf{F}} = \sum_{g=1}^{N_g}\hat{\mathbf{F}}_g\,, \tag{13}$$

$$\hat{E}^{pl}(T) = \frac{4\hat{\sigma}_R}{c}T^4\,, \tag{14}$$

$$\hat{\sigma}_R = \frac{1}{\rho_B}\sigma_R\,. \tag{15}$$

3 A Particular Case of Computational Units

Consider the following case:

- $t = [shakes]$,
- $\rho_t = 1$,
- $\rho_B = \rho_I$,
- $\rho_\varepsilon = \rho_I$,

where, for instance, $\rho_I = 10^{13}$. Using *shakes* we obtain

- c=299.792458 $\left[\frac{cm}{sh}\right]$.
- $h = 6.62613\times 10^{-19}$ erg $\cdot$ sh

- $\sigma_R = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67\times 10^{-13} \frac{erg}{cm^2 \ sh \ (^\circ K)^4} = 1.0266 \times 10^4 \frac{erg}{cm^2 \ sh \ eV^4}$
- specific intensity $I(\mathbf{x}, \mathbf{\Omega}, \nu, t) = ch\nu n(\mathbf{x}, \mathbf{\Omega}, \nu, t)$: $\left[\frac{erg}{cm^2 \ sh \ Hz \ ster}\right]$,
- monochromatic radiation energy $E_\nu = E(\mathbf{x}, \nu, t) = \int_{4\pi} h\nu n d\mathbf{\Omega} = \frac{1}{c}\int_{4\pi} I d\mathbf{\Omega}$: $\left[\frac{erg}{cm^3 \ Hz}\right]$.

We now define the scaled Stefan-Boltzmann constant

$$\hat{\sigma}_R = \frac{1}{\rho_I}\sigma_R \,, \tag{16}$$

the scaled function

$$\hat{\sigma}_{R,g}(T) = \frac{1}{\rho_I}\sigma_{R,g}(T) \,, \tag{17}$$

scaled heat capacity constant

$$\hat{c}_v = \frac{1}{\rho_I} c_v \,, \tag{18}$$

and scaled intensities at boundaries and at the initial moment of time

$$\hat{I}_g^{in} = \frac{1}{\rho_I} I_g^{in} \,, \quad \hat{I}_g^0 = \frac{1}{\rho_I} I_g^0 \,. \tag{19}$$

The set of scaled equations (1) and (6) get the form of the original NRT equations, namely,

$$\frac{1}{c}\frac{\partial \hat{I}_g}{\partial \hat{t}} + \mathbf{\Omega}\cdot\nabla \hat{I}_g + \varkappa_{E,g}(T)\hat{I}_g = \varkappa_{B,g}(T)\hat{B}_g(T) \,, \tag{20a}$$

$$\hat{I}_g\Big|_{\mathbf{r}=\mathbf{r}_\gamma} = \hat{I}_g^{in} \,, \quad \text{for } \mathbf{r}_\gamma \in \partial G \text{ and } \mathbf{\Omega}\cdot\mathbf{e}_n < 0 \,, \quad \hat{t} \geq \rho_t^{-1} t_0 \,, \tag{20b}$$

$$\hat{I}_g\Big|_{\hat{t}=\rho_t^{-1} t_0} = \hat{I}_g^0 \,, \quad \text{for } \mathbf{r} \in G \text{ and all } \mathbf{\Omega} \,. \tag{20c}$$

$$\frac{\partial \hat{\varepsilon}(T)}{\partial \hat{t}} = c\sum_{g=1}^{N_g}\Big(\varkappa_{E,g}(T)\hat{E}_g - \varkappa_{B,g}(T)\hat{E}_g^{pl}\Big) \,, \tag{20d}$$