Homography

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Contents

1 Vanishing point and distances

How is the 3D distance related to the distance on the image taken by a pinhole camera? Consider the following figure.

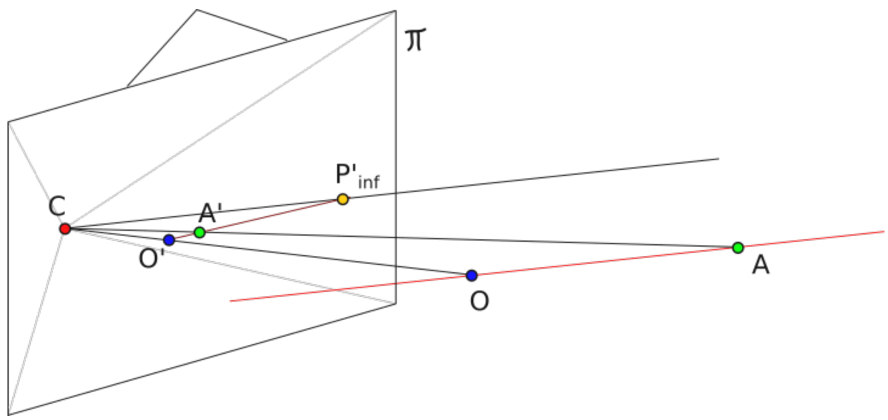

Figure 1: Camera geometry.

C is the camera center. The red line crossing through O and A, is an arbitrary line in 3D, that is not parallel to the image plane π. Consider O as the origin on the red line, and as we slide A on the line, A' slides as well on the image plane. As A tends to infinity, A' tends to P'_{inf}. All the points marked on the figure are on one plane that can be seen on the next figure.

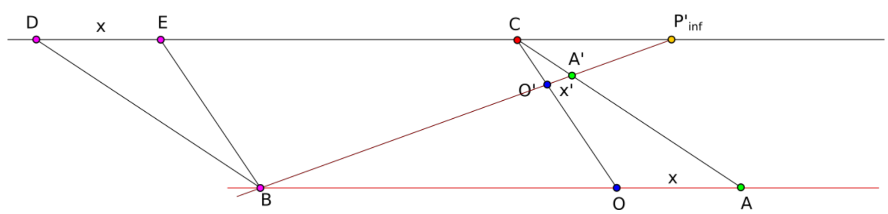

Figure 2: Plane geometry.

Point B is the intersection of the image plane and the red line. Drawing a parallel line with AC from B, intersects the line CP'_{inf} at point D. Similarly, but parallel with OC we get the point E. Thus $|DE| = |OA| \equiv x$.

Triangle $A'CP'_{\text{inf}}$ is similar to the triangle BDP'_{inf}:

$$\frac{|A'P'_{\text{inf}}|}{|CP'_{\text{inf}}|} = \frac{|BP'_{\text{inf}}|}{|DP'_{\text{inf}}|} \tag{1}$$

Triangle $O'CP'_{\text{inf}}$ is similar to the triangle BEP'_{inf}:

$$\frac{|O'P'_{\text{inf}}|}{|CP'_{\text{inf}}|} = \frac{|BP'_{\text{inf}}|}{|EP'_{\text{inf}}|} \tag{2}$$

Dividing (1) by (2) we get

$$\frac{|A'P'_{\text{inf}}|}{|O'P'_{\text{inf}}|} = \frac{|EP'_{\text{inf}}|}{|DP'_{\text{inf}}|} \tag{3}$$

Denote $|O'P'_{\text{inf}}| \equiv d'_{\text{inf}}$, $O'A' \equiv x'$ and $EP'_{\text{inf}} \equiv \delta$.

With these we can write:

$$\frac{d'_{\text{inf}} - x'}{d'_{\text{inf}}} = \frac{\delta}{\delta + x} \tag{4}$$

From this we get:

$$\boxed{\frac{x'}{d'_{\text{inf}}} = \frac{x}{\delta + x}} \tag{5}$$

Let's plot x'/d'_{inf} against x/δ:

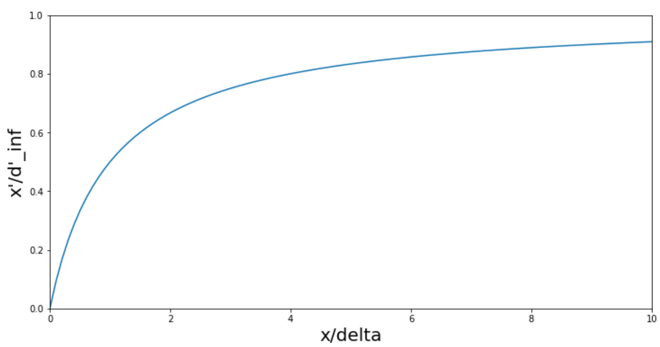

Figure 3: Plot.

1.1 getting the vanishing point.

Now assume that instead of having one point A on the red line, we have A_1 and A_2 such that $|OA_1| = |A_1A_2| \equiv \Delta x$. (See figure)

Using (5) we can write these equations:

$$\frac{\Delta d_1}{d'_{\text{inf}}} = \frac{\Delta x}{\delta + \Delta x} \tag{6}$$

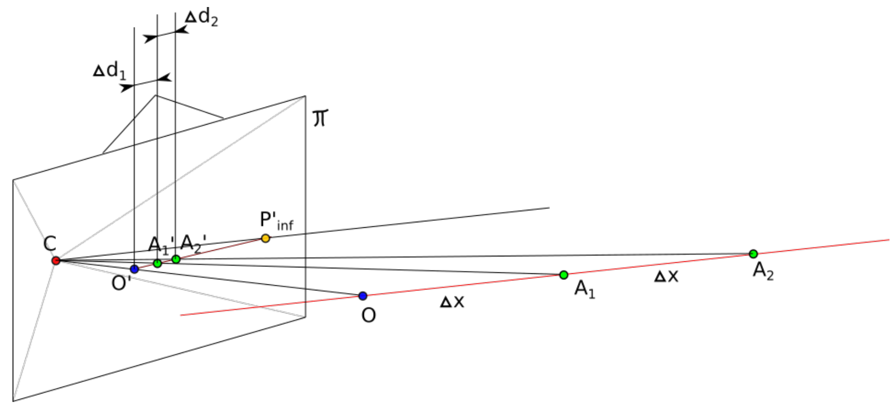

Figure 4: 3D geometry for calculating the vanishing point.

$$\frac{\Delta d_1 + \Delta d_2}{d'_{\text{inf}}} = \frac{2\Delta x}{\delta + 2\Delta x} \tag{7}$$

From these we can derive a formula for the vanishing distance:

$$\boxed{d'_{\text{inf}} = \frac{\Delta d_1(\Delta d_1 + \Delta d_2)}{\Delta d_1 - \Delta d_2}} \tag{8}$$

Note that we do not need to know Δx, the distances in 3D space, only the distances on the image (Δd_1, Δd_2).

1.2 application example

Let's look at an example (look at the image below), where we have $\Delta d_1 = 57.55$, $\Delta d_2 = 32.25$. With these we can substitute into (8)

$$d'_{\text{inf}} = \frac{\Delta d_1(\Delta d_1 + \Delta d_2)}{\Delta d_1 - \Delta d_2} = \frac{57.55 \cdot (57.55 + 32.25)}{57.55 - 32.25} = 204.27 \tag{9}$$

Figure 5: Example of calculating the vanishing point position.