

Discrete Assignment (10.5.3.20)

Avani Chouhan
EE23BTECH11205

January 25, 2024

Question :

The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

Representing the series using Z-transform:

$$X(z) = 5 \sum_{n=0}^{\infty} (2^n) z^{-n} \quad (8)$$

Solution :

Given:

$$a = 5 \quad (\text{first term}) \quad (1)$$

$$r = 2 \quad (\text{common ratio}) \quad (2)$$

$$S_n = 315 \quad (\text{sum of the GP}) \quad (3)$$

Number of terms (n):

$$315 = \frac{5(2^n - 1)}{2 - 1} \quad (4)$$

Solving for n :

$$\begin{aligned} 315 &= 5(2^n - 1) \\ 63 &= 2^n - 1 \\ 64 &= 2^n \\ n &= 6 \end{aligned} \quad (5)$$

Last term (T_n):

$$\begin{aligned} T_n &= a \cdot r^{(n-1)} \\ T_n &= 5 \cdot 2^{(6-1)} \end{aligned} \quad (6)$$

Calculating:

$$\begin{aligned} T_n &= 5 \cdot 32 \\ T_n &= 160 \end{aligned} \quad (7)$$

Therefore, the number of terms is $n = 6$ (5) and the last term is $T_n = 160$ (7).