NCERT Discrete - 10.5.3.20

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Question : 10.5.3.20 The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

Solution:

Given:

$$x(0) = 5 \tag{1}$$
$$r = 2 \tag{2}$$
$$x(n) = x(0)r^n \tag{3}$$
$$x(z) = \frac{x(0)}{1 - rz^{-1}} \tag{4}$$
$$S(z) = X(z)U(z) \tag{5}$$
$$S(z) = \frac{x(0)(\frac{r}{1-rz^{-1}} - \frac{1}{1-z^{-1}})}{r-1} \tag{6}$$

By contour integration:

$$s(n) = x(0)\left(\frac{r^{n+1} - 1}{r - 1}\right)u(n) \tag{7}$$

From (7):

$$315 = 5(2^{n+1} - 1) \tag{8}$$
$$63 = 2^{n+1} - 1 \tag{9}$$
$$64 = 2^{n+1} \tag{10}$$
$$n = 5 \tag{11}$$

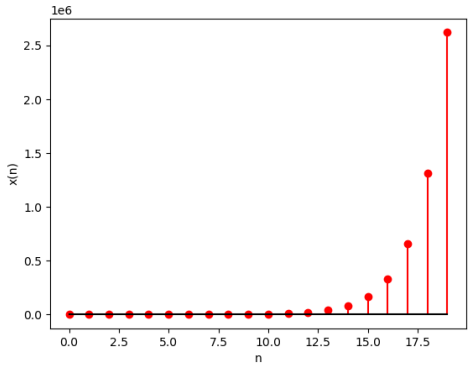

Fig. 0. Stem plot of GP

$$x(n) = x(0)r^n \tag{12}$$
$$x(5) = 5\left(2^5\right) \tag{13}$$
$$= 160 \tag{14}$$

Therefore, the number of terms is 6, and the last term is 160.