NCERT Discrete - 11.9.5.8

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Question : 11.9.5.8

In a potato race, a bucket is placed at the starting point, which is 5m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line. There are ten potatoes in the line.
A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Hint: To pick up the first potato and the second potato, the total distance (in meters) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$.

Solution: The distances covered for each term are:

$$\text{1st term: 10m} \tag{1}$$
$$\text{2nd term: 16m} \tag{2}$$
$$\text{3rd term: 22m} \tag{3}$$
$$\tag{4}$$

It forms an AP sequence with:

Parameter	Value	Description
$x(0)$	10	First term
d	6	Common difference

Applying Z transform:

$$x(z) = \frac{10}{1-z^{-1}} + \frac{6z^{-1}}{(1-z^{-1})^2} \tag{5}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{6}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = x(n) * u(n) \tag{7}$$

Applying Z transform on both sides

$$Y(z) = X(z)U(z) \tag{8}$$
$$= \frac{10}{(1-z^{-1})^2} + \frac{6z^{-1}}{(1-z^{-1})^3} \tag{9}$$

Using contour integration to find inverse Z transform:

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \tag{10}$$
$$= \frac{1}{2\pi j} \oint_C \left[\frac{10}{(1-z^{-1})^2} + \frac{6z^{-1}}{(1-z^{-1})^3} \right] z^8 \, dz \tag{11}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{12}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$R_1 = \frac{1}{(2-1)!} \left. \frac{d(10z^{10})}{dz} \right|_{z=1} \tag{13}$$
$$= 10 * 10 = 100 \tag{14}$$

$$R_2 = \frac{1}{(3-1)!} \left. \frac{d^2(6z^{10})}{dz^2} \right|_{z=1} \tag{15}$$
$$= \frac{6}{2}(10)(9) = 270 \tag{16}$$

The sum of terms is given by R_i:

$$100 + 270 = 370 \tag{17}$$