Sr. No. of Question Paper : 772   G   Your Roll No................

Unique Paper Code   : 234103

Name of the Paper   : Discrete Structures (CSHT-102)

Name of the Course   : **B.Sc. (H) Computer Science**
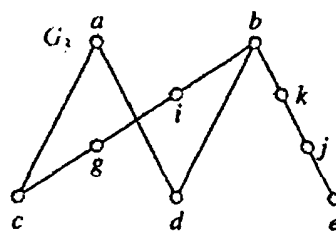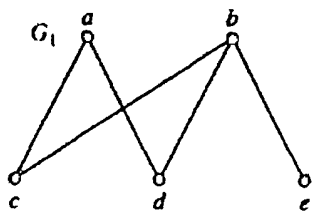
Semester   : I

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **Section A** is compulsory.

3. Attempt any **4** questions from **Section B**.

4. Please attempt parts of a question together.

SECTION A

1. (a) Show that the graphs G1 and G3 (as given) are homeomorphic.   (3)

   (b) Can there be a complete graph with 20 vertices and 180 edges ? Justify your answer.   (3)

   (c) Draw the graph G whose adjacency matrix $A = [a_{ij}]$ is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(3)

P.T.O.

(d) Determine the discrete numeric function for the given generating function

$$A(x) = \frac{2 + 3z - 6z^2}{1 - 2z}$$

(3)

(e) Given that the value of $p \to q$ is true, can you determine the value of $\bar{p} \lor (p \leftrightarrow q)$? (3)

(f) Let A = {a,b,c} be a set and the relation R on A is defined as :

R = {(a,a), (a,b), (b,c), (c,c)}.

Is R (i) reflexive, (ii) symmetric, and (iii) transitive ? (3)

(g) Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. (4)

(h) Show that $\lnot p \to (q \to r)$ and $q \to (p \lor r)$ are logically equivalent using the laws of logical equivalence. (4)

(i) Find f + g and fg for the functions f and g where $f(x) = x^2 + 1$ and $g(x) = x + 2$. (4)

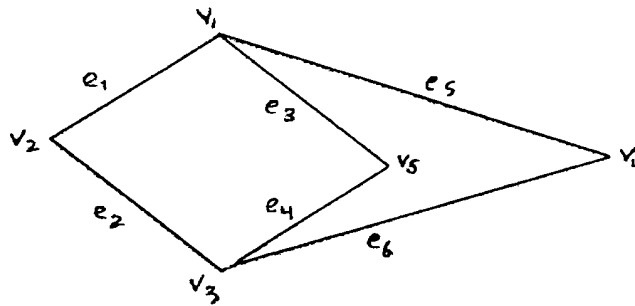(j) Use master method to find the running time for the given $T(n) = 124\, T(n/5) + n^3$. (5)

SECTION B

2. (a) A candidate is required to answer 6 out of 10 questions which are divided into two groups containing 5 questions each and he is permitted to attempt not more than four questions from any group. In how many ways can he select the questions ? (5)
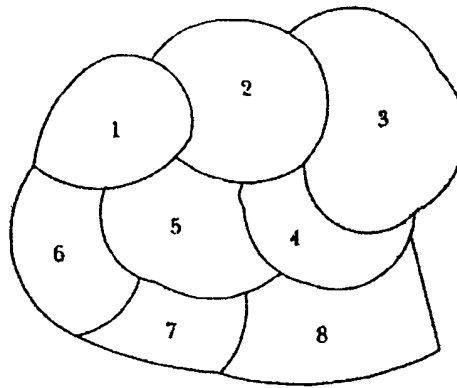
(b) Prove by the principle of mathematical induction that

$$P(n) : 1 \cdot 2 + 2 \cdot 2^2 + \dots + n \cdot 2^n = (n-1)\, 2^{n+1} + 2$$

(5)

3. (a) Determine whether the given graph is bipartite or not. Justify your answer. (3)

(b) Show that if 11 numbers are chosen from the set $\{1,2,...20\}$, then one of them will be multiple of another. (2)

(c) Define the chromatic number of a graph G. Find $\chi(G)$. (5)

4. (a) Let D_n denote the set of all positive divisors of n. Draw the Digraph and Hasse Diagram for the poset $(D_{30}, \leq)$. (5)

(b) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$. Satisfy the recurrence relation

$$a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0.$$

Determine a_r. (5)

5. (a) Show that $\bar{p}$ is tautologically implied by $\overline{p \wedge \bar{q}}$, $\bar{q} \vee r$, $\bar{r}$. (4)

(b) Suppose that the number of bacteria in a colony triples every hour.

(i) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

      (ii) If 100 bacteria are used to begin a new colony, how many bacteria
           will be in the colony in 10 hours ?   (4)

  (c) Give the converse and the inverse of the statement "If it rains today, then I
      will drive to work".   (2)

6. (a) List all the steps used to binary search for 9 in the given sequence of
      numbers 1, 3, 4, 5, 6, 8, 9, 11, 13, 15, 17.   (5)

  (b) Define big O notation. Prove that $f(n) = n^2 + 4n$ and $g(n) = n^2$, $n \geq 0$.

                                                                                           (5)

7. (a) Show that the following premises are inconsistent :—   (5)

      (i) If jack misses many classes through illness, then he fails high school.

      (ii) If jack fails high school, then he is uneducated.

      (iii) If jack reads a lot of books, then he is not educated.

      (iv) Jack misses many classes through illness and reads a lot of books.

  (b) Suppose that f1(x) is O(g1(x)) and f2(x) is O(g2(x)). Then (f1 + f2) (x) is
      O(max (|g1(x)|, |g2(x)|).   (5)