

Denote the following base quantities:

$c_i^{q,P}$	—	\$-value of collateral asset i owned by user q on platform P
$l_j^{q,P}$	—	\$-value of loan asset j owed by user q on platform P
λ_i^P	—	Liquidation loan-to-value ratio (i.e. collateral factor) of asset i on platform P
inc_i^P	—	Liquidation incentive for asset i on platform P

From the base quantities described above,
let us define the following derived quantities:

$L^{q,P} = \sum_j l_j^{q,P}$	—	total \$-value of assets owed by user q on platform P
$r_i^{q,P} = \frac{\lambda_i^P \cdot c_i^{q,P}}{\sum_i \lambda_i^P \cdot c_i^{q,P}}$	—	share of loans collateralized by asset i owned by user q on platform P
$L_i^P = \sum_q r_i^{q,P} \cdot L^{q,P}$	—	Total \$-value collateralized by asset i on platform P
$\langle inc \rangle_i = \frac{\sum_P inc_i^P \cdot L_i^P}{\sum_P L_i^P}$	—	Loan weighted average of liquidation incentives for asset i across all platforms
$\langle \lambda \rangle_i = \frac{\sum_P \lambda_i^P \cdot L_i^P}{\sum_P L_i^P}$	—	Loan weighted average of liquidation loan-to-value ratio for asset i across all platforms
$\tilde{c}_{i,j}^P = \sum_q r_i^{q,P} \cdot l_j^{q,P}$	—	Total \$-value of asset i collateralizing asset j loans on platform P

Lastly, let us define the following two quantities pertaining
to swap liquidity for the i - j pair:

$s_j(s_i)$	—	\$-value of asset j quantity s_j obtained by swapping asset i quantity s_i
$\mathcal{L}_{i \rightarrow j} = \operatorname{argmax}_{s_i} s_j(s_i) > (\langle \lambda \rangle_i + \langle inc \rangle_i) \cdot s_i$	—	Available liquidity for healthy liquidations across lending platforms offering asset i as collateral

We are now in a position to define the 'Toxicity Number' $\tau_{i,j}$ for the pair i - j :

$$\tau_{i,j} = \frac{\sum_P \tilde{c}_{i,j}^P}{\mathcal{L}_{i \rightarrow j}}$$