V Syntax and Documentation

Arthur Giesel Vedana October 27, 2017

Contents

Int	Introduction											
1	Abstract Syntax and Semantics 1											
	1.1	•										
		1.1.1 Expressions	1									
		1.1.2 Types	5									
		1.1.3 Traits	6									
	1.2	Operational Semantics	8									
		1.2.1 Paths	9									
		1.2.2 Pattern Matching	10									
		1.2.3 Big-Step Rules	11									
	1.3	Type System	21									
		1.3.1 Polymorphism	21									
		1.3.2 Traits	22									
		1.3.3 Type Inference System	22									
2	Exte	extended Language										
	2.1	Abstract Syntax	28									
	2.2	Translation	30									
3	crete Syntax	31										
	3.1	Notation Conventions	31									
	3.2	Basic Structure	31									
	3.3	Identifiers and Operators	32									
	3.4	Terms	33									
	3.5	Functions and Declarations	34									
4	Lang	guage Guide	35									
	4.1	Basic Values	35									
	4.2	Compound Values	36									
		4.2.1 Lists	36									
		4.2.2 Tuples	38									
		4.2.3 Records	38									
	4.3	Identifiers	39									
	4.4	Patterns	39									
	4.5	Constants	40									
	4.6	Type Annotations										
	4.7	Conditionals	41									
	4.8	Match Expressions	41									
	4.9	Operators	42									
		<u> </u>	42									
		4.9.1 Priority	42									
		4.9.1 Priority	42									

		4.9.4	Operators as Functions	43	;
		4.9.5	Functions as Operators	44	Ļ
		4.9.6	Defining new Operators	44	ļ
	4.10	Function	ons	44	ļ
		4.10.1	Named Functions	45	į
		4.10.2	Recursive Named Functions	45	į
		4.10.3	Lambdas	46	į
		4.10.4	Recursive Lambdas	46	į
		4.10.5	Function Type	47	1
	4.11	Application and Currying		1	
			ents		3
	4.13	Librari	es	48	3
5	Ston	dard Li	hrary	50	
3	5.1		ions on Basic Values		
	5.1	5.1.1	Operations on All Values		
		5.1.2	Operations on Numbers		
		5.1.3	Operations on Booleans		
		5.1.4	Operations on Functions		
		5.1.5	Operations on Tuples		
		5.1.6	Operations on Records		
	5.2		ions on Lists		
		5.2.1	Basic Operations		
		5.2.2	Generation Operations		
		5.2.3	Transformation Operations		į
		5.2.4	Reduction Operations		į
		5.2.5	Sublist Operations		į
		5.2.6	Search Operations		5
		5.2.7	Indexing Operations		7
		5.2.8	Sorting Operations		7
		5.2.9	Zipping Operations		7
	5.3	String	Conversion Operations	58	3
6	Cha	ngelog		58	}

Introduction

The *V* programming language is a functional language with eager left-to-right evaluation. It has a simple I/O system supporting only direct string operations. It is a trait based strongly and statically typed language supporting both explicit and implicit typing.

This document both specifies the *V* language and shows its implementation in F#. It is divided into 6 categories:

1. Abstract Syntax and Semantics

This defines the abstract syntax and semantics of the language. It only contains the bare minimum for the language to function, without any syntactic sugar.

2. Extended Language

Defines the extended abstract syntax tree and its translation into the core language.

3. Concrete Syntax

Defines the concrete syntax for the language, describing what are valid expressions and programs.

4. Language Guide

A guide for programming in *V*. This defines all operators, syntactic sugar and other aspects of the language, along with short user-friendly explanations of each language feature (and limitation).

5. Standard Library

Describes all functions provided in the *V* standard library.

6. Changelog

Lists the changes done to the language in each version.

1 Abstract Syntax and Semantics

1.1 Abstract Syntax

1.1.1 Expressions

Programs in V are expressions. Each expression is a member of the abstract syntax tree defined below. The syntax tree will be constructed in parts, with an explanation of what each expression means and their uses. The full syntax tree can be obtained by simply joining all the separate sections.

Functions V, as a functional language, treats functions as first class constructions. This means that functions are regular expressions, and can be passed as arguments, bound to identifiers, etc. Below are both of the function expressions available in V, along with function application.

```
e \qquad ::= \qquad \cdots \\ \mid \qquad func \\ \mid \qquad e_1 \ e_2 \\ \mid \qquad x  func \qquad ::= \qquad \text{fn } x \Rightarrow e \\ \mid \qquad \text{rec } f \ x \Rightarrow e \\ \mid \qquad \text{rec } f : T \ x \Rightarrow e  x \qquad ::= \qquad \{x_0, x_1, \ldots\}
```

All functions in V take exactly one parameter, and so function application evaluates the function by providing a single argument to it.

The first type of function (fn $x \Rightarrow e$) defines a simple unnamed function with a parameter x. x is an identifier from a set of name identifiers.

The body of the function is the expression e, which may or may not contain occurrences of the value passed as the argument of the function.

The other two types of functions are both variants of recursive functions available in V. These functions have a name, f, which is also a member of the set of name identifiers, and this name is used to allow recursive calls from withing their body (e). Like unnamed functions, they take exactly one parameter, x, which may or may not be present in their bodies.

The difference between these variants is in their type declaration: the first variant is implicitly typed, while the second is explicitly typed. In the second variant, the programmer specifies the return type of the function as T (types will be shown later).

There are also expressions to use functions.

The first of them is function application: the first of the expressions is a function, and the second is the argument being passed to the function.

The last expression is simply to allow the use of the parameter defined in a function. An identifier *x* is only considered valid if it has been bound before (either by a function or, as will be seen later, by let declarations of match expressions).

Built-in Functions V has a few built-in functions that provide basic behavior.

These are:

e	::=	 Builtin	
Builtin	::=	+	(add, binary)
		_	(subtract, binary)
		*	(multiply, binary)
	- 1	÷	(divide, binary)
	- 1	_	(negate, unary)
	1	<	(less than, binary)
		\leq	(less than or equal, binary)
	ĺ	>	(more than, binary)
	ĺ	≥	(more than or equal, binary)
	i	=	(equal, binary)
	į	≠	(not equal, binary)
	1	V	(Or, binary)
	i	٨	(And, binary)
		get	(binary)
		set	(ternary)
		stack	(binary)
		distort	(ternary)

Every function has its arity declared. The arity of a function defines how many parameters it needs before it can be fully evaluated.

This behavior does not change the fact that functions receive only one parameter. These particular cases can be thought of as nested functions, each taking a single parameter, until all the necessary parameters have been received.

This means that partially applied built-in functions are also treated as functions, and, therefore, can be passed as arguments, bound to identifiers, etc.

Most of these functions are basic and require no explanation. The last 4, however, will only be explained later, after records and accessors have been introduced.

The boolean functions \vee and \wedge are treated differently than others. Even though they are binary, they have a short-circuit mechanism. This means that, if the result of the application can be known by the first parameter (True in the case of \vee or False in the case of \wedge), the second parameter is not evaluated.

This is in contrast to all other functions in V, which evaluate their arguments before trying to evaluate themselves (This will be explained in more detail in 1.2)

Constructors *V* has another type of special function: Data Constructors. Data Constructors are, as their name suggests, functions that construct values (data).

When fully evaluated, constructors define a form of structured data, storing the values passed to them as arguments. To access these values, it is possible to pattern match (see 1.1.1) on the constructor name when fully evaluated.

Like built-in functions, constructors can be in a partially evaluated state. Partially evaluated constructors are treated as normal functions and, therefore, do not allow matching on their name.

```
e
         ::=
                 con
con
         ::=
                                                    (arity 0)
                 n
                 b
                                                    (arity 0)
                                                    (arity 0)
                 nil
                                                    (arity 0)
                                                    (arity 2)
            Tuple n
                                            (arity n, n \ge 2)
         ::=
                 true | false
                 \mathbb{Z}
         ::=
n
                 'char'
c
         ::=
                 ASCII characters
char
         ::=
```

These functions are *extra* special, however, because they can have arity zero. This means that they "construct" values as soon as they are declared. The basic zero-arity constructors defined in the language are integers, booleans and characters.

The following two constructors (*nil* and ::) are related to lists. The first (with arity 0) is the empty list. The second constructor has arity 2, and extends a list (its second argument) by adding a new value (its first argument) to its head.

The last constructor is actually a family of constructors describing tuples. A constructor Tuplen defines a constructor that has n parameters and evaluates to a tuple with n elements. It is also important to note that $n \ge 2$. This means that only tuple constructors with 2 or more parameters are valid.

Records and Accessors The record system in V is composed of two parts: records and accessors.

Records are a type of structured data composed of associations between labels and values, called fields. Each label is part of an ordered set of labels *l*, and can only appear once in every record. For a record to be valid, the order in which its labels are declared must respect the order in the set of labels.

Accessors are terms that allow access to fields within a record. Accessors view records as trees, where each non-leaf node is a record, and each edge has a name (the label of the field). Accessors, then, define a path on this tree, extracting the node at the end of the path.

```
e ::= \cdots
| \{l_1 : e_1, \dots l_n : e_n\} \quad (n \ge 1)
| \#l 
| \#(e_1, \dots e_n) \quad (n \ge 2)
l ::= \{l_1, l_2, \dots\}
```

The most basic type of path is a simple label #l. An accessor can also be made by combining multiple accessors. The $\#(e_1, \ldots e_n)$ expression creates an accessor for multiple fields of the same record. Another type of composition is vertical, and is obtained by using the built-in function "stack".

Furthermore, paths can be distorted with the "distort" built-in function, specifying a pair of functions to be applied when extracting or inserting values in the field.

More details about how accessors (and paths) work will be provided in a later section.

Insert section

Let and Patterns The let expression is used to bind values to identifiers, allowing them to be reused in a sub-expression. A let expression is divided into 2 parts: the binding and the sub-expression. The binding itself also has 2 parts: the left-hand side, which is a pattern; and the right-hand side, which will be the value to be bound.

Patterns are used to "unpack" values, and can be either explicitly or implicitly typed.

```
e ::= ...

| let p = e_1 in e_2

p ::= patt

| patt ::= x

| _ _ | con p_1, \ldots p_n (constructor pattern, n = arity con)

| {l_1 : p_1, \ldots l_n : p_n} (n \ge 1)

| {l_1 : p_1, \ldots l_n : p_n, \ldots} (partial record, n \ge 1)
```

Much like functions, let expressions allow the use of identifiers in an expression by attaching values to the identifiers. Differently from functions, however, a single let expression can bind multiple identifiers to values by using patterns. Patterns are matched against the values in the right-hand side of the binding, and can, depending on their structure, create any number of identifier bindings.

The pattern x is a simple identifier pattern. It matches any value in the right-hand side, and binds it to the identifier.

The _ also matches any value, but it does not create any new bindings (this is called the ignore pattern).

The *con* pattern matches a completely applied constructor. This pattern is a compound pattern, with the same number of components as the arity of the constructor.

The *con* pattern itself does not create any bindings, but its components might, since they are themselves patterns and, as such, will be matched against the components of the constructor.

The next pattern is a record pattern. This matches a record with *exactly* the same fields as the pattern. Since all labels in a record are ordered, the fields do not need to be reordered for the matching.

For matching any record with *at least* the fields $l_1, \ldots l_n$, one can use the pattern $\{l_1 : p_1, \ldots l_n : p_n, \ldots\}$. This pattern will match any record whose set of labels is a superset of $l_1, \ldots l_n$.

Match Expression A match expression attempts to match a value against a list of patterns. Every pattern is paired with a resulting expression to be evaluated if the pattern matches. Furthermore, it is possible to specify a boolean condition to be tested alongside the pattern matching. This condition will only be tested if the match succeeds, so it can use any identifier bound by the pattern. The matching stops at the first pattern that successfully matches (and any condition is satisfied), and its paired expression is then evaluated.

```
e ::= ...

| match e with match_1, ... match_n (n \ge 1)

match ::= p \to e

| p when e_1 \to e_2
```

Exceptions This expression always evaluates to a runtime error.

Runtime errors usually happen when an expression cannot be correctly evaluated, such as division by zero, accessing an empty list, etc.

Sometimes, however, it can be necessary to directly cause an error. The *raise* expression serves this purpose.

1.1.2 Types

Since V is strongly typed, every (valid) expression has exactly one type associated with it. Some expressions allow the programmer to explicitly declare types, such as patterns and recursive functions. Other expressions, such as $e_1 = e_2$, or even constants, such as 1 or true, have types implicitly associated with them. These types are used by the type system (see 1.3) to check whether an expression is valid or not, avoiding run-time errors that can be detected at compile time.

Types Below are all the types available in V. The first type is a fully applied constructor type. The second type is a function type. The third type is a record type and, finally, the last type is an accessor type.

```
T \quad ::= \quad \cdots \\ \mid \quad conT \ T_1, \ \dots \ T_n \qquad (n = arity \ conT) \\ \mid \quad T_1 \rightarrow T_2 \\ \mid \quad \{l_1 : T_1, \ \dots \ l_n : T_n\} \qquad (n \ge 1) \\ \mid \quad T_1 \# T_2 \qquad \qquad Accessor
```

Most of the types are compound types, and the only scenario in which a type is not compound is for constructor types with arity 0. Function types specify the type of the single parameter (T_1) and the type of output (T_2) .

Record types are also compound types, but they associate every component to its corresponding label. Just like record expressions, the labels must be ordered.

Finally, accessor types define the types of accessor expressions. They have two components: T_1 , which is the type of the record being accessed; and T_2 , the type of the value being accessed. It is read as T_1 accesses T_2 .

Constructor Types These are types associated with constructors. Much like constructors, they can take any amount of arguments to be fully applied, and the number of arguments they take is described by their arity. Instead of taking values as arguments, however, constructor types take types as arguments.

$$\begin{array}{cccc} conT & ::= & \text{Int} & (\text{arity 0}) \\ & | & \text{Bool} & (\text{arity 0}) \\ & | & \text{Char} & (\text{arity 0}) \\ & | & \text{List} & (\text{arity 1}) \\ & | & \text{TupleT } n & (\text{arity } n, \ n \geq 2) \end{array}$$

Variable Types These types represent an unknown constant type. Explicitly typed expressions cannot be given variable types, but they are used by the type system for implicitly typed expressions. In the course of the type inference, the type system can replace variable types for their type.

It is important to realize that variable types already represent a unique type with an unknown identity. This means that a variable type may only be replaced by the specific type which it represents and not any other type. This distinction becomes important when talking about polymorphism, which uses variable types, along with universal quantifiers, to represent a placeholder for any possible type (this is discussed in greater detail in 1.3.1).

$$T ::= \cdots$$
 X^{Traits}
 $X ::= X_1, X_2, ...$

1.1.3 Traits

Types can conform to traits, which define certain behaviors that are expected of said type. Regular types always have their trait information implicitly defined, since this information is included in the language. Variable types, on the other hand, can explicitly

state which traits they possess, restricting the set of possible types they can represent (this is represented by the superscript Traits in a variable type X).

```
Traits ::= \emptyset
| \{Trait\} \cup Traits
Trait ::= Equatable
| Orderable
| \{l: Type\} \quad (Record Label)
```

The information on which types conform to which traits is defined inside the type inference environment. When a type T conforms to a trait T r ait, the notation used is $T \in T$ r ait.

By default, the following rules hold for conformance:

Equatable If a type T is Equatable, expressions of type T can use the equality operators $(=, \neq)$.

To define the set of types that belong to *Equatable*, the following rules are used:

```
\{Int, Bool, Char\} \subset Equatable

T \in Equatable \implies List T \in Equatable

X^{Traits} \in Equatable \implies Equatable \in Traits
```

Orderable If a type T is Orderable, expressions of type T can use the inequality operators $(<, \le, >, \ge)$. Any type that is Orderable is also Equatable.

To define the set of types that belong to *Orderable*, the following rules are used:

```
\{ \text{Int}, \text{Char} \} \subset Orderable 

T \in Orderable \implies List \ T \in Orderable 

X^{Traits} \in Orderable \implies Orderable \in Traits
```

Record Label A type T_1 conforms to a Record Label Trait $\{l: T_2\}$ if it is a record that contains a field with the label l and the type T_2 .

If the type conforms to the trait $\{l: T_2\}$, it can then use the accessor #l. This ensures that accessor for a field can only be used on records that have that field.

To define the set of types that belong to a record label $\{l:T\}$, the following rules are used:

```
 \{l_1: T_1, \dots l_n: T_n, \dots T_k\} \in \{l: T\} \iff l_n = l \wedge T_n = T \qquad (1 \le n \le k)  X^{Traits} \in \{l: T\} \implies \{l: T\} \in Traits
```

1.2 Operational Semantics

The *V* language is evaluated using a big-step evaluation with environments. This evaluation reduces an expression into a value directly, not necessarily having a rule of evaluation for every possible expression. To stop programmers from creating programs that cannot be evaluated, a type inference system will be specified later.

Value A value is the result of the evaluation of an expression in big-step. This set of values is different from the set of expressions of V, even though they share many similarities.

Environment An evaluation environment is a 2-tuple which contains the following information:

1. Arity of constructors

If a constructor has arity n, it requires n arguments to become fully evaluated.

2. Associations between identifiers and values

A new association is created every time a value is bound. This happens in let declarations, function application and match expressions.

Below are the definitions of both values and environments:

```
(arities, vars)
env
arities
                       \{\} \mid \{con \rightarrow n\} \cup arities
                                                                          (n \in \mathbb{N})
             ::=
                       \{\} \mid \{x \rightarrow v\} \cup vars
vars
             ::=
                       con v_1, \ldots v_n
                                                              (n = arity \ con)
                       raise
                      \{l_1: v_1, \ldots l_n: v_n\}
                                                                          (n \ge 1)
                      #path
                       \langle func, env \rangle
                       \ll Builtin . v_1, \ldots v_n \gg (n < arity Builtin)
                       \ll con \cdot v_1, \ldots v_n \gg
                                                        (n < arity \ con)
path
                       path . path
                       (path_1, \dots path_n)
                                                                          (n \ge 2)
                       path[v_1, v_2]
```

The value $\langle func, env \rangle$ defines closures. They represent the result of evaluating functions (and recursive functions) and store the environment at the moment of evaluation. This means that V has static scope, since closures capture the environment at the moment of evaluation and V has eager evaluation.

The values \ll *Builtin* . v_1 , ... $v_n \gg$ and \ll *con* . v_1 , ... $v_n \gg$ are partial applications of built-in functions and constructors, respectively.

Once all the parameters have been defined, they evaluate either to the result of the function (in the case of *Builtin*) or to a fully applied constructor $con v_1, \ldots v_n$.

1.2.1 Paths

Accessors possess structure when treated as values. This structure is built through use of the operations available on accessors, such as the compose expression or the built-in functions. Since accessors view records as trees, this structure is a *path* along a tree.

The most basic structure of a path is a single label. This path describes a field immediately below the root of the tree (the root is viewed as the record itself, and every child is a field of the record). This path is created by using the simple accessor expression #l.

Two paths can be composed vertically, allowing access to subfields of a record. In this scenario, the tree must have at least the same depth as the path (and along the correct field names). Vertical composition is achieved by using the "stack" built-in function, and always combines two paths.

Paths can also be composed horizontally, described as a tuple of paths. When composed horizontally, all paths are used on the root of the record, and the end points of the paths are joined in a tuple for extraction or updating.

Finally, paths can be distorted. This means that the path has two values (functions) associated with it. These functions are then used to transform the values stored in the field of the record (one for extraction, another for updating).

Path Traversal Rules As previously stated, accessors describe paths along a record tree. To use these paths, an auxiliary *traverse* function is used. This functions receives 3 arguments: a path, a record and an update value. The function returns 2 values: the old value associated with the field specified by the path; and an updated record.

This updated record uses the value provided as input to the function to update the field specified by the path. This last argument of the *traverse* function can be omitted and, in such a case, no update is done (that is, the updated record is the same as the input record).

The first rule is for a simple label path. The label must be present in the provided record. A new record is created, associating the provided value with the label specified by the path.

$$\frac{1 \le ||k|| \le ||n||}{r = \{l_1 : v_1, \dots l_k : v, \dots l_n : v_n\}}$$
$$\frac{1 \le ||k|| \le ||n||}{traverse(l_k, \{l_1 : v_1, \dots l_n : v_n\}, v) = v_k, r}$$

For vertically composed paths, three calls to *traverse* are needed.

```
traverse(path_1, \{l_1 : v_1, ...l_n : v_n\}) = rec, r
traverse(path_2, rec, v) = v', rec'
traverse(path_1, \{l_1 : v_1, ...l_n : v_n\}, rec') = rec, r'
traverse(path_1, path_2, \{l_1 : v_1, ...l_n : v_n\}, v) = v', r'
```

The first call omits the update value, and is used to extract a record associated with the first component of the path. This record is then passed, along with the second component of the path and the update value, to the second call of *traverse*. Finally, the third call uses the return of the second call to update the internal record, returning a new updated record.

Joined paths also require multiple calls to *traverse*, but the exact number depends on the amount of paths joined.

$$r_{0} = \{l_{1} : v_{1}, ...l_{n} : v_{n}\}$$

$$\forall i \in [1, n] . traverse(path_{i}, r_{i-1}, v_{i}) = v'_{i}, r_{i}$$

$$traverse((path_{1}, ... path_{n}), \{l_{1} : v_{1}, ...l_{n} : v_{n}\}, (v_{1}, ... v_{n})) = (v'_{1}, ... v'_{n}), r_{n}$$

Pairing the paths with the components of the tuple provided as the update value, each pair is passed as input to a call to *traverse*. This happens from left to right, and each call returns a part of the old value and a partially updated record. Every call uses the partially updated record provided, and the last call to *traverse* returns the fully updated record.

Distorted paths require the evaluation of two applications, one before and one after updating.

$$\begin{cases} \{\} \vdash v_2 \ v \Downarrow v' & traverse(path, \{l_1 : v_1, ...l_n : v_n\}, v') = v'', r \\ & \{\} \vdash v_1 \ v'' \Downarrow v''' \\ \hline & traverse(path[v_1, \ v_2], \{l_1 : v_1, ...l_n : v_n\}, v) = v''', r \end{cases}$$

Initially, the provided update value is applied to the second component (v_2) of the distorted accessor. This value is then provided as the new update value for a call to *traverse*, returning an old value and the updated record. This old value is then provided as argument to the first component (v_1) of the distorted accessor, and the result of this is the output of the whole *traverse* (along with the updated record).

1.2.2 Pattern Matching

For let and match expressions, it is necessary to match a pattern p to a value v. This process, if successful, creates a mapping of identifiers to their corresponding elements of v. If v does not match the pattern p, the process fails.

In the case of a let expression, failing to match means the whole expression evaluates to *raise*. For match expressions, a failed pattern causes the next pattern to be attempted. If there are no more patterns, the expression evaluates to *raise*.

To aid in this matching, a auxiliary "match" function is defined. The function takes a pattern p and a value v, returning a mapping of identifiers to values (the vars of an environment). If the matching fails, the function will return nothing.

The following are the rules for the match function:

$$match(x, v) = \{x \rightarrow v\}$$

 $match(_, v) = \{\}$

$$\frac{con_1 = con_2}{match(con_1 \ v_1, \ \dots v_n, con_2 \ p_1, \ \dots p_n)} = \bigcup_{i=1}^n vars_i$$

$$\frac{k \geq n \qquad \forall \ i \in [1,n] \quad \exists \ j \in [1,k] \quad l_i^1 = l_j^2 \land match(p_i,v_j) = vars_i}{match(\{l_1^1:p_1,\ldots,l_n^1:p_n,\ldots\},\{l_1^2:v_1,\ldots l_k^2:v_k\}) = \bigcup_{i=1}^n vars_i}$$

$$\frac{\forall \ i \in [1,n] \quad l_i^1 = l_i^2 \land match(p_i,v_i) = vars_i}{match(\{l_1^1:p_1,\ldots,l_n^1:p_n\},\{l_1^2:v_1,\ldots l_n^2:v_n\}) = \bigcup_{i=1}^n vars_i}$$

Any other inputs provided to the match function will result in a failed matching. This is represented by:

$$\neg$$
 match(p, v)

1.2.3 Big-Step Rules

Function Expressions Every function evaluates to a closure. This basically stores the function definition and the current environment in a value, allowing the evaluation environment to be restored on function application.

$$env \vdash func \Downarrow \langle func, env \rangle$$
 (BS-F_N)

Built-in functions evaluate to a partially applied built-in without any arguments.

env
$$\vdash$$
 Builtin $\Downarrow \ll$ Builtin . \gg (BS-Builtin)

Similarly, constructors, if they need at least one argument, evaluate to a partially applied constructor. If, however, they do not take any arguments, they immediately evaluate to a fully applied constructor.

$$\frac{\text{env.arities}(con) > 0}{\text{env} \vdash con \parallel \ll con .}$$
 (BS-Con)

$$\frac{\text{env.arities}(con) = 0}{\text{env} + con \parallel con}$$
(BS-Con0)

Application An application expression requires either a closure, a partially applied built-in function or a partially applied constructor for its left-hand operand.

In the case of a closure, there are two different behaviors, depending on whether the function is recursive or not.

When applying non recursive functions, a new association between the parameter identifier (x) and the argument (v_2) is added to the environment stored in the closure (env_1) . The body of the function (e) is then evaluated using this new environment.

$$\frac{\text{env} \vdash e_1 \Downarrow \langle \text{fn } x \Rightarrow e, \text{env}_1 \rangle \qquad \text{env} \vdash e_2 \Downarrow v_2}{\{x \rightarrow v_2\} \cup \text{env}_1 \vdash e \Downarrow v}$$

$$\frac{\{x \rightarrow v_2\} \cup \text{env}_1 \vdash e \Downarrow v}{\text{env} \vdash e_1 e_2 \Downarrow v}$$
(BS-AppFn)

Recursive functions, besides associating the identifier to the argument, also create an association between the function name and its value (i.e the closure itself). This allows the body of the function to call itself, creating a recursive structure.

For operational semantics, there is no difference between the typed and untyped versions of recursive functions, so both have the same evaluation rules.

$$\frac{\operatorname{env} \vdash e_1 \Downarrow \langle \operatorname{rec} f \; x \Rightarrow e, \operatorname{env}_1 \rangle \quad \operatorname{env} \vdash e_2 \Downarrow v_2}{\{x \rightarrow v_2, f \rightarrow \langle \operatorname{rec} f \; x \Rightarrow e, \operatorname{env}_1 \rangle\} \cup \operatorname{env}_1 \vdash e \Downarrow v}{\operatorname{env} \vdash e_1 \; e_2 \Downarrow v}} \quad (BS-AppFnRec)$$

$$\frac{\mathsf{env} \vdash e_1 \Downarrow \langle \mathsf{rec} \ f : T \ x \Rightarrow e, \mathsf{env}_1 \rangle \qquad \mathsf{env} \vdash e_2 \Downarrow v_2}{\{x \to v_2, f \to \langle \mathsf{rec} \ f : T \ x \Rightarrow e, \mathsf{env}_1 \rangle\} \cup \mathsf{env}_1 \vdash e \Downarrow v}} \\ \frac{\{x \to v_2, f \to \langle \mathsf{rec} \ f : T \ x \Rightarrow e, \mathsf{env}_1 \rangle\} \cup \mathsf{env}_1 \vdash e \Downarrow v}}{\mathsf{env} \vdash e_1 \ e_2 \Downarrow v} \ (\mathsf{BS-AppFnRec2})$$

Application on partially applied constructors can behave in two different ways, depending on how many arguments have been already applied.

If the arity of the constructor is larger than the number of arguments already applied (plus the new one being applied), the result of the application is a partially applied constructor with the new value added to the end.

$$\frac{\text{env} \vdash e_1 \Downarrow \ll con \cdot v_1, \dots v_n \gg}{n+1 < \text{env.arities}(con) \quad \text{env} \vdash e_2 \Downarrow v}$$

$$\frac{n+1 < \text{env.arities}(con) \quad \text{env} \vdash e_2 \Downarrow v}{\text{env} \vdash e_1 e_2 \Downarrow \ll con \cdot v_1, \dots v_n, v \gg}$$
(BS-AppCon)

If the arity of the constructor is equal to 1 more than the number of already applied arguments, the application results in a completely applied constructor.

$$\frac{n+1 = \text{env.arities}(con) \quad \text{env} \vdash e_2 \Downarrow v}{\text{env} \vdash e_1 e_2 \Downarrow con \ v_1, \dots v_n, \ v}$$
(BS-AppConTotal)

Application on partially applied built-in functions works similarly, having different rules depending on the number of arguments.

$$\begin{array}{ll} \operatorname{env} \vdash e_1 \Downarrow \ll Builtin \; . \; v_1, \; \ldots v_n \gg \\ \frac{n+1 < \operatorname{arity} Builtin}{\operatorname{env} \vdash e_1 \; e_2 \Downarrow \ll Builtin \; . \; v_1, \; \ldots v_n, \; v \gg \end{array}$$
 (BS-AppBuiltin)

The result of applying the last argument of a built-in function varies depending on what the function does (and what kind of arguments it accepts). These rules will be provided later.

Identifier The evaluation of an identifier depends on the environment in which it is evaluated. If the environment has an association between the identifier and a value, the value is returned. If it does not, the program is malformed and cannot be evaluated (this will be caught in the type system).

$$\frac{\text{env.vars}(x) = v}{\text{env} \vdash x \Downarrow v}$$
 (BS-IDENT)

Records A record construction expression $\{l_1 : e_1, \ldots, l_n : e_n\}$ evaluates each of its sub-expressions individually, resulting in a record value. The order of evaluation is defined by the order of the labels and is done from smallest to largest.

$$\frac{\forall \ k \in [1, n] \quad \text{env} \vdash e_k \Downarrow v_k}{\text{env} \vdash \{l_1 : e_1, \dots l_n : e_n\} \Downarrow \{l_1 : v_1, \dots l_n : v_n\}}$$
(BS-Record)

If any of the sub-expressions evaluate to raise, the whole record also evaluates to raise.

$$\frac{\exists \ k \in [1, n] \ \text{ env} \vdash e_k \Downarrow raise}{\text{env} \vdash \{l_1 : e_1, \dots l_n : e_n\} \Downarrow raise}$$
(BS-RecordRaise)

Accessors There is a different evaluation rule for each type of path available to accessors.

The simplest rule is for a label accessor, which is in itself a value.

env
$$\vdash \#l \Downarrow \#l$$
 (BS-Label)

Joined accessors evaluate each of their sub-expressions, expecting an accessor value as a result.

$$\frac{\forall \ k \in [1, n] \ \text{env} \vdash e_k \Downarrow \#path_k}{\text{env} \vdash \#(e_1, \dots e_n) \Downarrow \#(path_1, \dots path_n)}$$
(BS-Joined)

To create a stacked accessor, the built-in function "stack" must be used. This function has arity 2, and requires both arguments to be accessors. The paths of the accessors are then composed in a stacked accessor, which is the result of the evaluation.

$$\frac{\text{env} \vdash e_1 \Downarrow \ll \text{stack} . \#path_1 \gg \text{env} \vdash e_2 \Downarrow \#path_2}{\text{env} \vdash e_1 e_2 \Downarrow \#path_1 . path_2}$$
 (BS-STACKED)

Similarly, creating distorted accessors requires the built-in function "distort". This function takes 3 arguments, the first being an accessor, and the remaining two being functions. When fully evaluated, the path of the accessor is combined with the function values, creating a distorted accessor.

$$\frac{\text{env} \vdash e_1 \Downarrow \ll \text{distort . } \#path, \ v_1 \gg \qquad \text{env} \vdash e_2 \Downarrow v_2}{\text{env} \vdash e_1 \ e_2 \Downarrow \#path \ [v_1, \ v_2]} \ (\text{BS-Distorted})$$

Using Accessors There are two built-in functions that take accessors as arguments.

Get takes 2 arguments: an accessor and a record. The *traverse* function is then called with the accessor's path and the record (the third argument is omitted), and the first return (i.e. the value associated with the path) is used as the result of the evaluation.

$$\frac{\text{env} \vdash e_1 \Downarrow \ll \text{get. } \#path \gg \qquad \text{env} \vdash e_2 \Downarrow \{l_1 : v_1, \dots l_n : v_n\}}{traverse(path, \{l_1 : v_1, \dots l_n : v_n\}) = v', r'}{\text{env} \vdash e_1 e_2 \Downarrow v'}$$
(BS-Get)

Set takes 3 arguments: an accessor, a generic value and a record. The *traverse* function is then called with the arguments, using the generic value as the update value of the call. The result of the evaluation is the second return of the *traverse* function (i.e. the updated record).

$$\frac{\text{env} \vdash e_1 \Downarrow \ll \text{set .} \#path, \ v \gg \qquad \text{env} \vdash e_2 \Downarrow \{l_1 : v_1, \ \dots \ l_n : v_n\}}{traverse(path, \{l_1 : v_1, \ \dots \ l_n : v_n\}, v) = v', r'}$$

$$\frac{env \vdash e_1 \ e_2 \Downarrow r'}{\text{env} \vdash e_1 \ e_2 \Downarrow r'}$$
 (BS-Set)

Numerical Operations The V language only supports integers, so all operations are done on integer numbers. This means that the division always results in a whole number, truncated towards zero.

$$\frac{\operatorname{env} \vdash e_1 \Downarrow n_1 \qquad \operatorname{env} \vdash e_2 \Downarrow n_2 \qquad ||n|| = ||n_1|| + ||n_2||}{\operatorname{env} \vdash e_1 + e_2 \Downarrow n}$$
 (BS-+)

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \qquad \text{env} \vdash e_2 \Downarrow n_2 \qquad ||n|| = ||n_1|| - ||n_2||}{\text{env} \vdash e_1 - e_2 \Downarrow n}$$
(BS-)

$$\frac{\operatorname{env} \vdash e_1 \Downarrow n_1 \qquad \operatorname{env} \vdash e_2 \Downarrow n_2 \qquad ||n|| = ||n_1|| * ||n_2||}{\operatorname{env} \vdash e_1 * e_2 \Downarrow n}$$
 (BS-*)

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \qquad \text{env} \vdash e_2 \Downarrow 0}{\text{env} \vdash e_1 \div e_2 \Downarrow raise}$$
 (BS-÷Zero)

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2}{\|n_2\| \neq 0 \quad \|n\| = \|n_1\| \div \|n_2\|}$$

$$\frac{\text{env} \vdash e_1 \div e_2 \Downarrow n}{\text{env} \vdash e_1 \div e_2 \Downarrow n}$$
(BS-÷)

Equality Operations The equality operators (= and \neq) allow comparison of certain expressions with other expressions of the same kind. In this way, it is a polymorphic operator, being usable in different contexts. Even so, it is important to realize that it only compares values of the same kind (numbers with numbers, characters with characters, etc).

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad ||n_1|| = ||n_2||}{\text{env} \vdash e_1 = e_2 \Downarrow true} \quad (\text{BS-=NumTrue})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad ||n_1|| \neq ||n_2||}{\text{env} \vdash e_1 = e_2 \Downarrow false} \quad (BS-=NumFalse)$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \qquad \text{env} \vdash e_2 \Downarrow c_2 \qquad ||c_1|| = ||c_2||}{\text{env} \vdash e_1 = e_2 \Downarrow true} \quad \text{(BS-=CharTrue)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad ||c_1|| \neq ||c_2||}{\text{env} \vdash e_1 = e_2 \Downarrow false} \text{ (BS-=CharFalse)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow b_1 \qquad \text{env} \vdash e_2 \Downarrow b_2 \qquad ||b_1|| = ||b_2||}{\text{env} \vdash e_1 = e_2 \Downarrow true} \quad (\text{BS-=BoolTrue})$$

$$\frac{\text{env} \vdash e_1 \Downarrow b_1 \quad \text{env} \vdash e_2 \Downarrow b_2 \quad ||b_1|| \neq ||b_2||}{\text{env} \vdash e_1 = e_2 \Downarrow false} \text{ (BS-=BoolFalse)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 = e_2 \Downarrow true}$$
 (BS-=NilTrue)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 = e_2 \Downarrow false}$$
 (BS-=NilFalse1)

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 = e_2 \Downarrow false}$$
 (BS-=NilFalse2)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_1 = v_3 \Downarrow false}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow false}{\text{env} \vdash e_1 = e_2 \Downarrow false}$$
(BS-=ListFalse)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash v_1 = v_3 \Downarrow true} \quad \frac{\text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_2 = v_4 \Downarrow b}$$
$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow b}{\text{env} \vdash e_1 = e_2 \Downarrow b}$$
(BS-=ListTrue)

$$\begin{array}{cccc} \operatorname{env} \vdash e_1 \Downarrow (v_{11}, \ \ldots \ v_{1n}) & \operatorname{env} \vdash e_2 \Downarrow (v_{21}, \ \ldots \ v_{2k}) \\ \exists \ k \in [1, n] & \operatorname{env} \vdash v_{1k} = v_{2k} \Downarrow false \\ & & \forall \ j \in [1, k) & \operatorname{env} \vdash v_{1j} = v_{2j} \Downarrow true \\ & & & \operatorname{env} \vdash e_1 = e_2 \Downarrow false \end{array} \tag{BS-=TupleFalse}$$

$$\frac{\text{env} \vdash e_1 \Downarrow (v_{11}, \dots v_{1n}) \qquad \text{env} \vdash e_2 \Downarrow (v_{21}, \dots v_{2n})}{\forall k \in [1, n] \quad \text{env} \vdash v_{1k} = v_{2k} \Downarrow true}$$

$$\frac{\forall k \in [1, n] \quad \text{env} \vdash v_{1k} = v_{2k} \Downarrow true}{\text{env} \vdash e_1 = e_2 \Downarrow true}$$
(BS-=TupleTrue)

$$\begin{array}{cccc} \operatorname{env} \vdash e_1 \Downarrow \{l_1^1 : v_1^1, \ \dots \ l_n^1 : v_n^1\} & \operatorname{env} \vdash e_2 \Downarrow \{l_1^2 : v_1^2, \ \dots \ l_n^2 : v_n^2\} \\ \exists \ k \in [1, n] & \ l_k^1 = l_k^2 \land \operatorname{env} \vdash v_k^1 = v_k^2 \Downarrow false \\ & \forall \ j \in [1, k) & \operatorname{env} \vdash v_j^1 = v_j^2 \Downarrow true \\ & & \operatorname{env} \vdash e_1 = e_2 \Downarrow false \end{array} \tag{BS-=RecordFalse}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \{l_1^1 : v_1^1, \ \dots \ l_n^1 : v_n^1\}}{\forall \ k \in [1, n] \quad l_k^1 = l_k^2 \land \text{env} \vdash v_k^1 = v_k^2 \Downarrow \textit{true}} \\ \frac{}{\text{env} \vdash e_1 = e_2 \Downarrow \textit{true}} \text{(BS-=RecordTrue)}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow false}{\text{env} \vdash e_1 \neq e_2 \Downarrow true}$$
 (BS- \neq True)

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow true}{\text{env} \vdash e_1 \neq e_2 \Downarrow false}$$
 (BS- \neq False)

Inequality Operations The inequality operators function much in the same way as the equality operators. The only difference is that they do not allow comparison of certain kinds of expressions (such as booleans) when such expressions do not have a clear ordering to them.

To reduce the number of rules, some rules are condensed for all inequality operators $(<, \le, >, \ge)$. The comparison done on numbers is the ordinary numerical comparison. For characters, the ASCII values are compared numerically.

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad ||n_1|| \, opIneq \, ||n_2||}{\text{env} \vdash e_1 \, opIneq \, e_2 \Downarrow true} \, (BS\text{-IneqNumTrue})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \neg \lVert n_1 \rVert \, opIneq \, \lVert n_2 \rVert}{\text{env} \vdash e_1 \, opIneq \, e_2 \Downarrow true} \, (\text{BS-IneqNumFalse})$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \|c_1\| \, opIneq \, \|c_2\|}{\text{env} \vdash e_1 \, opIneq \, e_2 \Downarrow true} \, (\text{BS-IneqCharTrue})$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \neg \|c_1\| \, opIneq \, \|c_2\|}{\text{env} \vdash e_1 \, opIneq \, e_2 \Downarrow true} \, (\text{BS-IneqCharFalse})$$

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 < e_2 \Downarrow false}$$
 (BS-

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 \leq e_2 \Downarrow true}$$
(BS- \leq Nil)

$$\frac{\text{env} \vdash e_1 \Downarrow nil \quad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 > e_2 \Downarrow false}$$
 (BS->Nil)

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 \ge e_2 \Downarrow true}$$
(BS- \ge Nil)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 < e_2 \Downarrow false}$$
 (BS-

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 \leq e_2 \Downarrow false}$$
 (BS-\leq ListNil)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 > e_2 \Downarrow true}$$
 (BS->ListNil)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow nil}{\text{env} \vdash e_1 \geq e_2 \Downarrow true}$$
 (BS-\ge ListNil)

$$\frac{\text{env} + e_1 \Downarrow nil \qquad \text{env} + e_2 \Downarrow v_1 :: v_2}{\text{env} + e_1 < e_2 \Downarrow true}$$
 (BS-

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 \leq e_2 \Downarrow true}$$
 (BS-\leq NilList)

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 > e_2 \Downarrow false}$$
 (BS->NilList)

$$\frac{\text{env} \vdash e_1 \Downarrow nil \qquad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 \geq e_2 \Downarrow false}$$
 (BS- \geq NILLIST)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash v_1 = v_3 \Downarrow false} \quad \frac{\text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_1 \text{ opIneq } v_3 \Downarrow b}$$

$$\frac{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow b}{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow b}$$
(BS-IneqListHead)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash v_1 = v_3 \Downarrow \textit{true}} \quad \frac{\text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_2 \; opIneq \; v_4 \Downarrow b}}{\text{env} \vdash e_1 \; opIneq \; e_2 \Downarrow b}$$
 (BS-IneqListTail)

Let Expressions These expressions are used to associate an identifier with a specific value, allowing the value to be reused throughout the program. Since V is a functional language, these are not variables, and the values assigned to an identifier will be constant (unless the same identifier is used in a new *let* expression).

After evaluating the expression that is to be associated to the identifier (that is, e_1), resulting in v, the *let* expression evaluates e_2 . For this evaluation, the association of p to v is added to the environment. The result of this evaluation (that is, v_2) is the final result of the evaluation of the entire *let* expression.

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad match(p, v) = true, env_1}{env_1 \cup \text{env} \vdash e_2 \Downarrow v_2}$$
$$\frac{env \vdash \text{let } p = e_1 \text{ in } e_2 \Downarrow v_2}{\text{env} \vdash \text{let } p = e_1 \text{ in } e_2 \Downarrow v_2}$$
(BS-Let)

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad match(p, v) = false, env_1}{\text{env} \vdash \text{let } p = e_1 \text{ in } e_2 \Downarrow raise}$$
 (BS-Let2)

Match Expression The match expression receives a input value and a list of *match*, attempting to pattern match against each one. The first *match* which returns a positive result is considered valid, and its corresponding expression is evaluted as the result of the whole expression.

If no *match* returns a positive result, the whole expression evaluates to *raise*.

$$\exists j \in [1..n] \ multiMatch(v, \text{env}, match_j) = true, v_j$$

$$\forall k \in [1..j] \ multiMatch(v, \text{env}, match_k) = false, v_k$$

$$\text{env} \vdash \text{match} \ e \ \text{with} \ match_1, \dots \ match_n \Downarrow v_j$$

$$\text{env} \vdash e \Downarrow v$$

$$\forall j \in [1..n] \ multiMatch(v, \text{env}, match_j) = false, v_j$$

$$\text{env} \vdash \text{match} \ e \ \text{with} \ match_1, \dots \ match_n \Downarrow raise}$$
 (BS-MATCH2)

In order to properly evaluate a match expression, it is necessary to define an auxiliary function, here called *multiMatch*. This function receives an input value, an environment and a *match*.

If the *match* has a conditional expression, it must evaluate to *true* for the match to be considered valid.

$$\frac{match(p,v) = false, env_1}{multiMatch(v, env, p \to e) = false, v}$$

$$\frac{match(p,v) = true, env_1}{multiMatch(v, env, p \to e) = true, v_2}$$

$$\frac{match(p,v) = true, env_1}{multiMatch(v, env, p \to e) = true, v_2}$$

$$\frac{match(p,v) = false, env_1}{multiMatch(v, env, p \text{ when } e_1 \to e_2) = false, v}$$

$$\frac{match(p,v) = true, env_1}{multiMatch(v, env, p \text{ when } e_1 \to e_2) = false, v}$$

$$\frac{match(p,v) = true, env_1}{multiMatch(v, env, p \text{ when } e_1 \to e_2) = false, v}$$

$$\frac{match(p,v) = true, env_1}{multiMatch(v, env, p \text{ when } e_1 \to e_2) = true, v_2}$$

$$\frac{multiMatch(v, env, p \text{ when } e_1 \to e_2) = true, v_2}{multiMatch(v, env, p \text{ when } e_1 \to e_2) = true, v_2}$$

Exceptions Some programs can be syntactically correct but still violate the semantics of the V language, such as a dividing by zero or trying to access the head of an empty list. In these scenarios, the expression is evaluated as the raise value.

Besides violation of semantic rules, the only other expression that evaluates to the *raise* value is the *raise* expression, using the following rule:

env
$$\vdash raise \Downarrow raise$$
 (BS-Raise)

This value is usually ignored, but it propagates upwards if a "regular" value is expected. This means that expressions that need well-defined sub-expressions, such as numerical and equality operations, evaluate to *raise* if any of these sub-expressions evaluate to *raise*.

The rules for raise propagation are given below.

$$\frac{\text{env} \vdash e_1 \Downarrow raise}{\text{env} \vdash e_1 e_2 \Downarrow raise}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 \qquad \text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash e_1 :: e_2 \Downarrow raise}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 \qquad \text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash e_1 :: e_2 \Downarrow raise}$$
(BS-ListRaise)

$$\frac{\text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash \#l \ e_1 \ e_2 \Downarrow raise}$$
 (BS-RecordAccessRaise)

$$\frac{\text{env} \vdash e_1 \Downarrow raise}{\text{env} \vdash e_1 = e_2 \Downarrow raise}$$
 (BS-=Raise)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 \qquad \text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash e_1 = e_2 \Downarrow raise}$$
 (BS-=Raise2)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \qquad \text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_1 = v_3 \Downarrow raise}$$
$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow raise}{\text{env} \vdash e_1 = e_2 \Downarrow raise}$$
(BS-=ListRaise)

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash v_1 = v_3 \Downarrow true} \quad \frac{\text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_2 = v_4 \Downarrow raise}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow raise}{\text{env} \vdash e_1 = e_2 \Downarrow raise}$$
 (BS-=ListRaise2)

$$\begin{array}{cccc} \operatorname{env} \vdash e_1 \Downarrow \{l_1^1 : v_1^1, \ \dots \ l_n^1 : v_n^1\} & \operatorname{env} \vdash e_2 \Downarrow \{l_1^2 : v_1^2, \ \dots \ l_n^2 : v_n^2\} \\ \exists \ k \in [1, n] & \ l_k^1 = l_k^2 \land \operatorname{env} \vdash v_k^1 = v_k^2 \Downarrow raise \\ & \forall \ j \in [1, k) & \operatorname{env} \vdash v_k^1 = v_k^2 \Downarrow true \\ & & \operatorname{env} \vdash e_1 = e_2 \Downarrow raise \end{array} \tag{BS-=RecordRaise}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow raise}{\text{env} \vdash e_1 \neq e_2 \Downarrow raise}$$
 (BS- \neq Raise)

$$\frac{\text{env} \vdash e_1 \Downarrow raise}{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow raise} \qquad \text{(BS-IneqRaise)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 \quad \text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow raise} \qquad \text{(BS-IneqRaise2)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow v_3 :: v_4}{\text{env} \vdash v_1 = v_3 \Downarrow raise} \qquad \text{(BS-IneqListRaise)}$$

$$\frac{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow raise}{\text{env} \vdash e_1 \text{ opNume}_2 \Downarrow raise} \qquad \text{(BS-NumRaise)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow raise}{\text{env} \vdash e_1 \text{ opNume}_2 \Downarrow raise} \qquad \text{(BS-NumRaise2)}$$

1.3 Type System

V has a strong and static type inference system that checks a program to decide whether or not it is "'well-typed"'. If a program is considered to be well-typed, the type system guarantees that the program will be able to be properly evaluated according to the operational semantics of V. As a side-effect of checking the validity of a program, the type system can also provide the actual type of any implicitly typed expression down to its basic types, be those concrete types or variable types.

1.3.1 Polymorphism

V has support for parametric Damas-Milner polymorphism. This means that functions can have their types be defined with universal quantifiers, allowing their use with any type.

For instance, take the function *count*, which counts the number of elements in a list. This function can be defined as follows:

```
let count = rec count x \Rightarrow if isempty x then 0 else 1 + count (tl x) in count (3::4::nil)
```

In this situation, *count* can be used with a list of any type, not only Int. To allow this, its identifier (*count*) must have a universal association in the environment, defined as so:

```
\forall x. \ x \ list \rightarrow Int
```

The universal quantifier $\forall x$ allows the type variable x to be substituted for any concrete type when the function is called. When creating a polymorphic type, the type system must identify which type variables are free in the function type and which are bound in the environment. This process guarantees that a polymorphic type only universally quantifies those type variables that are not already bound, while still allowing all free variables to be instantiated when the function is called.

1.3.2 Traits

Traits are characteristics that a type can have, defining behaviors expected of that type. Some expressions are polymorphic in a sense that they accept certain types for their operators, but not any type.

1.3.3 Type Inference System

The type inference system is composed of two basic parts:

- Constraint Collection
- Constraint Unification

Constraints are equations between type expressions, which can have both constant types and variable types. To infer the type of a program, the type system recursively collects a set of constraints for every subexpression in that program. This is done in a static way across the expression tree from the nodes to the root, without having to evaluate any of the expressions. To create a valid set of constraints, the system must contain an environment, built from the root to the nodes, to ensure identifiers are properly typed.

Environment Just like the operational semantics, the type system also uses an environment to store information about identifiers. In this case, the environment maps identifiers to type associations. These can be either simple associations or universal associations, which are used for polymorphic functions.

Simple Associations These associate an identifier with a unique type, which can be either constant or a variable type. When the association is called, the type is returned as-is, even if it is a variable type.

Universal Associations This association, also called a type scheme, stores a type which contains at least one variable type bound by a "for all" quantifier (\forall) . When called, this association creates a new variable type for each bound variable and returns a new instance of the type scheme. Universal associations are used exclusively for polymorphic functions.

To create this type of association, the type system must generate a list of "free variables" present in the type that is to be universalized. These are the variable types that are not present in the environment when the identifier is declared. When these free variables are found, they are universally bound. This ensures that only those variable types that are unbound in the environment become universally bound in the resulting association.

Constraint Unification After collecting every type constraint for the program, the type inference system attempts to unify these constraints and find a solution for them. This solution comes in the form of type substitutions, which associate variable types to other types, and type traits, which associate variable types to sets of traits.

If the constraints cannot be unified - that is, if a conflict is found -, the program is deemed not well-typed. Because of how the collection and unification process works, little information is given about where the problem ocurred.

Unification Application After obtaining a valid solution to the set of constraints, the type inference system applies the substitution to the type of the program. This is done recursively until no more substitutions are found, resulting in what is called the principal type. If there are any variable types in the principal type, the traits are applied to them, restricting the set of types that the variable types can represent.

Pattern Matching When a pattern is encountered (such as a let expression or function declaration), it is necessary to match the type of the pattern with the value.

To do this, a "match" function is defined. It takes a pattern *p* and a type *T*, returning a list of constraints and a mapping of identifiers to associations.

The following are the rules for the "match" function:

$$match(x,T) = \{\}, \{x \to T\}$$
 $match(x:T_1,T_2) = \{T_1 = T_2\}, \{x \to T_1\}$
 $match(n,T) = \{T = Int\}, \{\}$
 $match(n:T_1,T_2) = \{T_1 = Int, T_2 = Int\}, \{\}$
 $match(b,T) = \{T = Bool\}, \{\}$
 $match(b:T_1,T_2) = \{T_1 = Bool, T_2 = Bool\}, \{\}$
 $match(c:T_1,T_2) = \{T = Char\}, \{\}$
 $match(c:T_1,T_2) = \{T_1 = Char, T_2 = Char\}, \{\}$
 $match(T) = \{\}, \{\}$
 $match(T_1,T_2) = \{T_1 = T_2\}, \{\}$

$$\frac{X_1 \ is \ new}{match(nil,T) = \{X_1 list = T\}, \{\}}$$
 $\frac{X_1 \ is \ new}{match(nil:T_1,T_2) = \{X_1 list = T_1, T_1 = T_2\}, \{\}}$

$$\frac{X_1 \text{ is new} \quad match(p_1, X_1) = c_1, env_1 \quad match(p_2, X_1 list) = c_2, env_2}{match(p_1 :: p_2, T) = \{X_1 list = T\} \cup c_1 \cup c_2, env_1 \cup env_2\}}$$

$$X_1 \text{ is new} \quad match(p_1, X_1) = c_1, env_1 \quad match(p_2, X_1 list) = c_2, env_2 \\ match(p_1 :: p_2 : T_1, T_2) = \{X_1 list = T_1, T_1 = T_2\} \cup c_1 \cup c_2, env_1 \cup env_2 \}$$

$$\frac{\forall i \in [1, n] \ X_i \ is \ new \land match(p_i, X_i) = c_i, env_i}{match((p_1, ...p_n), T) = \{(X_i, ...X_n) = T\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

$$\forall i \in [1, n] \ X_i \text{ is new } \land match(p_i, X_i) = c_i, env_i$$

$$\overline{match((p_1,...p_n): T_1, T_2) = \{(X_i,...X_n) = T_1, T_1 = T_2\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

$$\frac{\forall i \in [1, n] \ X_i \ is \ new \land match(p_i, X_i) = c_i, env_i \qquad X_0^{\{\{l_i: X_i\}\} \forall i \in [1, n]}}{match(\{l_1: p_1, \dots, l_n: p_n, \dots\}, T) = \{X_0 = T\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

$$\frac{\forall i \in [1,n] \ X_i \ is \ new \land match(p_i,X_i) = c_i, env_i \ X_0^{\{\{l_i:X_i\}\}\forall i \in [1,n]}}{match(\{l_1:p_1,\ldots,l_n:p_n,\ldots\}:T_1,T_2) = \{X_0 = T_1,T_1 = T_2\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

$$\forall i \in [1, n] \ X_i \ is \ new \land match(p_i, X_i) = c_i, env_i$$

$$\frac{\forall i \in [1, n] \ X_i \ is \ new \land match(p_i, X_i) = c_i, env_i}{match(\{l_1 : p_1, \dots, l_n : p_n\}, T) = \{\{l_1 : X_1, \dots, l_n : X_n\} = T\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

$$\forall i \in [1, n] \ X_i \text{ is new } \land match(p_i, X_i) = c_i, env_i$$

$$\frac{\forall i \in [1, n] \ X_i \ is \ new \land match(p_i, X_i) = c_i, env_i}{match(\{l_1 : p_1, \dots, l_n : p_n\} : T_1, T_2) = \{\{l_1 : X_1, \dots, l_n : X_n\} = T_1, T_1 = T_2\} \cup \bigcup_{i=1}^n c_i, \bigcup_{i=1}^n env_i}$$

Constraint Collection Rules Every expression in V has a rule for constraint collection, similar to how every expression has a rule for its semantic evaluation.

$$\Gamma \vdash n : \text{Int} \mid \{\}$$
 (T-Num)

$$\Gamma \vdash b : Bool \mid \{\}$$
 (T-Bool)

$$\Gamma \vdash c : \text{Char} \mid \{\}$$
 (T-Char)

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T \mid \{\}}$$
 (T-IDENT)

$$\frac{\forall \ k \in [1, n] \quad \Gamma \vdash e_k : T_k \mid C_k}{\Gamma \vdash (e_1, \dots e_n) : (T_1, \dots T_n) \mid C_1 \cup \dots C_n}$$
 (T-Tuple)

$$\frac{\forall k \in [1, n] \quad \Gamma \vdash e_k : T_k \mid C_k}{\Gamma \vdash \{l_1 : e_1, \dots l_n : e_n\} : \{l_1 : T_1, \dots l_n : T_n\} \mid C_1 \cup \dots C_n}$$
 (T-Record)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{\{l:T_1\}\}} \text{ is new}}{\Gamma \vdash \#l \ e_1 \ e_2 : (T_1, T_2) \mid C_1 \cup C_2 \cup \{X_1 = T_2\}} \text{ (T-RecordAccess)}$$

$$\frac{X_1 \text{ is new}}{\Gamma \vdash nil : X_1 \text{ list } | \{\}}$$
 (T-N_{IL})

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 :: e_2 : T_1 \ list \mid C_1 \cup C_2 \cup \{T_1 \ list = T_2\}}$$
 (T-List)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 + e_2 : \operatorname{Int} \mid C_1 \cup C_2 \cup \{T_1 = \operatorname{Int}; T_2 = \operatorname{Int}\}}$$
 (T-+)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 - e_2 : \operatorname{Int} \mid C_1 \cup C_2 \cup \{T_1 = \operatorname{Int}; T_2 = \operatorname{Int}\}}$$
 (T-)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 * e_2 : \operatorname{Int} \mid C_1 \cup C_2 \cup \{T_1 = \operatorname{Int}; T_2 = \operatorname{Int}\}}$$
 (T-*)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 \div e_2 : \operatorname{Int} \mid C_1 \cup C_2 \cup \{T_1 = \operatorname{Int}; T_2 = \operatorname{Int}\}}$$
 (T-÷)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Equatable\}} \text{ is new}}{\Gamma \vdash e_1 = e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Equatable\}} = T_2\}}$$
 (T-=)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Equatable\}} \text{ is new}}{\Gamma \vdash e_1 \neq e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Equatable\}} = T_2\}}$$
 (T-\neq)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Orderable\}} \text{ is new}}{\Gamma \vdash e_1 < e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Orderable\}} = T_2\}}$$
 (T-<)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Orderable\}} \text{ is new}}{\Gamma \vdash e_1 \leq e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Orderable\}} = T_2\}}$$
 (T-\leq)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Orderable\}} \text{ is new}}{\Gamma \vdash e_1 > e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Orderable\}} = T_2\}}$$
 (T->)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1^{\{Orderable\}} \text{ is new}}{\Gamma \vdash e_1 \geq e_2 : \text{Bool} \mid C_1 \cup C_2 \cup \{T_1 = T_2; X_1^{\{Orderable\}} = T_2\}}$$
 (T-\geq)

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X_1 \text{ is new}}{\Gamma \vdash e_1 \mid e_2 : X \mid C_1 \cup C_2 \cup \{T1 = T_2 \to X_1\}}$$
 (T-App)

$$\frac{X \text{ is new} \quad match(p, X) = C, env \quad env \cup \Gamma \vdash e : T_1 \mid C_1}{\Gamma \vdash \text{fn } p \Rightarrow e : X \rightarrow T_1 \mid C \cup C_1} \tag{T-Fn}$$

$$\begin{array}{ll} X \ is \ new & match(p,X) = C, env \\ \{x \rightarrow (X \rightarrow T)\} \cup env \cup \Gamma \vdash e : T_1 \mid C_1 \\ \hline \Gamma \vdash \texttt{rec} \ x : T \ p \Rightarrow e : X \rightarrow T_1 \mid C \cup C_1 \cup \{T_1 = T\} \end{array} \tag{T-Rec}$$

$$\begin{array}{ccc} X_1 \ is \ new & X_2 \ is \ new & match(p,X_1) = C,env \\ \hline \{x \rightarrow X_2\} \cup env \cup \Gamma \vdash e : T_1 | C_1 \\ \hline \Gamma \vdash \mathbf{rec} \ x \ p \Rightarrow e : X_1 \rightarrow T_1 \mid C \cup C_1 \cup \{X_2 = X_1 \rightarrow T_1\} \end{array} \tag{T-Rec2}$$

$$\begin{split} \Gamma \vdash e_1 : T_1 \mid C_1 & match(p, T_1) = C, env \\ & env \cup \Gamma \vdash e_2 : T_2 \mid C_2 \\ \hline \Gamma \vdash \mathsf{let} \ p = e_1 \ \mathsf{in} \ e_2 : T_2 \mid C \cup C_1 \cup C_2 \end{split} \tag{T-Let}$$

$$\begin{array}{c|c} \Gamma \vdash e : T \mid C & X_1 \text{ is new} \\ \hline \\ \forall j \in [1..n] \text{ } \textit{multiMatch}(T, X_1, \Gamma, \textit{match}_j) = C_j \\ \hline \\ \Gamma \vdash \mathsf{match} \text{ } e \text{ with } \textit{match}_1, ... \text{ } \textit{match}_n : X_1 \mid C \cup \bigcup_{i=1}^n C_i \end{array}$$

$$\frac{match(p, T_1) = C, \Gamma_1 \qquad \Gamma_1 \cup \Gamma \vdash e : T_3 \mid C_3}{multiMatch(T_1, T_2, \Gamma, p \rightarrow e) = C \cup C_3 \cup \{T_3 = T_2\}}$$

$$\frac{\mathit{match}(p, T_1) = C, \Gamma_1 \qquad \Gamma_1 \cup \Gamma \vdash e_1 : T_3 \mid C_3 \qquad \Gamma_1 \cup \Gamma \vdash e_2 : T_4 \mid C_4}{\mathit{multiMatch}(T_1, T_2, \Gamma, p \text{ when } e_1 \rightarrow e_2) = C \cup C_3 \cup C_4 \cup \{T_3 = \mathit{Bool}, T_4 = T_2\}}$$

$$\frac{X_1 \ is \ new}{\Gamma \vdash raise : X_1 \mid \{\}} \tag{T-Raise}$$

2 Extended Language

In order to facilitate programming, it is useful to define an extended language. A program is first parsed into this language, and the resulting tree is translated into the regular abstract syntax.

This allows the core language to be concise, reducing the complexity of type inference and evaluation. Complex constructs (such as comprehensions and multi-parameter functions) can be included only in the extended language, and it suffices to provide a translation into the core language.

This translation does have the drawback of reducing the formality of evaluation. Since there are no evaluation rules for the additional constructs, it is impossible to prove the correctness of the translation rules. This does not in any way affect the correctness of the core language type inference and evaluation, and the advantages gained by this method far outweigh the drawbacks, so it is still a net positive to the language.

The following two sections will describe the abstract syntax tree for the extended language and how it translates to a syntax tree in the core language.

2.1 Abstract Syntax

The extended language has terms which are similar to (if not exactly the same as) ones existing in the core language. These terms are presented in their entirety here, and, since they are directly extrated from the core language, no explanation will be given for them.

All patterns are extracted from the core language without any structural differences, and are defined below.

Most types are, like patterns, extracted from the language.

$$\begin{array}{lll} T' & ::= & \text{Int} \\ & | & \text{Bool} \\ & | & \text{Char} \\ & | & T_1' \to T_2' \\ & | & T' \ list \\ & | & (T_1', \ \dots \ T_n') & (n \geq 2) \\ & | & \{l_1: T_1', \ \dots \ l_n: T_n'\} & (n \geq 1) \end{array}$$

The first addition to the extended language is the concept of *type aliases*. These types are simple renames of existing types, and can be used in programs as a way to simplify type declarations.

$$T'$$
 ::= \cdots τ τ ::= $\{\tau_0, \tau_1, \ldots\}$

A conditional expression, which translates to a match expression on the patterns *true* and *false*, has been added.

$$e'$$
 ::= \cdots
| if e'_1 then e'_2 else e'_3

Functions have been extended to allow multiple parameters, removing the necessity of declaring nested functions. These functions still require at least one parameter.

$$\begin{array}{lll} e' & ::= & \cdots \\ & \mid & \operatorname{fn} p_1', \, \dots \, p_n' \Rightarrow e' & (n \geq 1) \\ & \mid & \operatorname{rec} x : T \ p_1', \, \dots \, p_n' \Rightarrow e' & (n \geq 1) \\ & \mid & \operatorname{rec} x \, p_1', \, \dots \, p_n' \Rightarrow e' & (n \geq 1) \end{array}$$

The let expression is also extended, and a new construction (*decl'*) is needed. Besides the basic value binding, 4 new function bindings are allowed. These correspond

to all combinations of typed, untyped, recursive and non-recursive functions, with at least one parameter.

Along with value and function bindings, a new type alias binding was added. This binding creates a new type alias that can be used further down in the syntax tree.

```
\begin{array}{lll} e' & ::= & \cdots \\ & | & \text{let } decl' \text{ in } e \\ \\ decl' & ::= & p' = e' \\ & | & x \; p_1', \; \dots \; p_n' = e' & (n \geq 1) \\ & | & \text{rec } x \; p_1', \; \dots \; p_n' = e' & (n \geq 1) \\ & | & x : T' \; p_1', \; \dots \; p_n' = e' & (n \geq 1) \\ & | & \text{rec } x : T' \; p_1', \; \dots \; p_n' = e' & (n \geq 1) \\ & | & \text{type alias } \tau = T' \end{array}
```

2.2 Translation

3 Concrete Syntax

3.1 Notation Conventions

The following conventions are used for presenting the syntax of programs in V:

The syntax is described using a BNF grammar, with each production having the form:

```
\langle sentence \rangle ::= \langle pat_1 \rangle | \langle pat_2 \rangle | \dots | \langle pat_n \rangle
```

Whitespace is always explicitly expressed in productions with the _ character. It is used as a shorthand for the \(\lambda \text{whitespace} \rangle \) production. Literal characters will always be written in terminal font, so | and [] mean the literal characters, while | and [] are the choice and option pattern, respectively.

3.2 Basic Structure

```
::= \langle whitespace \rangle \langle expression \rangle \langle eof \rangle
⟨program⟩
\langle library \rangle
                                ::= \langle whitespace \rangle \{ \langle declaration \rangle \} \langle eof \rangle
                                ::= \{ \langle whitechars \rangle \mid \langle comment \rangle \}
(whitespace)
(whitechars)
                                ::= \langle whitechar \rangle \{ \langle whitechar \rangle \}
(whitechar)
                                ::= \langle space \rangle | \langle tab \rangle | \langle newline \rangle
⟨newline⟩
                                ::= \langle return \rangle \langle linefeed \rangle | \langle return \rangle | \langle linefeed \rangle
\langle space \rangle
                                ::= space (' ')
                                ::= horizontal tab ('\t')
\langle tab \rangle
                                ::= carriage return ('\r')
⟨return⟩
                                ::= line feed ('\n')
\langle linefeed \rangle
⟨comment⟩
                                ::= // \{ \langle any \rangle_{\langle newline \rangle} \} \langle newline \rangle
                                ::= any ASCII character
\langle any \rangle
```

3.3 Identifiers and Operators

```
::= (\langle idstart \rangle \{\langle idcontinue \rangle \}) \langle reservedid \rangle
⟨identifier⟩
\langle idstart \rangle
                         ::= \langle small \rangle |_{-}
                         ::= \langle small \rangle | \langle large \rangle | \langle digit \rangle | ' |_{-}| ?
⟨idcontinue⟩
\langle reservedid \rangle
                         ::= let|true|false|if|then|else
                           | rec|nil|raise|when|match|with
                           | try|except|for|in|import|infix
                           | infixl|infixr|type|alias
                         ::= (\langle typeidentstart \rangle \{ \langle idcontinue \rangle \}) \langle reserved type \rangle
⟨typeident⟩
⟨typeidentstart⟩
                        ::= \(\large\) | _
⟨reservedtype⟩
                         ::= Int | Bool | Char
\langle small \rangle
                         ::= a | b | ... | z
                         ::= A \mid B \mid \ldots \mid Z
\langle large \rangle
                         ::= 0 | 1 | ... | 9
\langle digit \rangle
\langle operator \rangle
                         ::= \langle symbol \rangle \{ \langle symbol \rangle \}
                         ::= \langle operator \rangle_{\langle reservedop \rangle}
⟨customop⟩
\langle reservedop \rangle
                         ::= + | - | * | / | <= | < | =
                          | !=|>=|>|::
\langle symbol \rangle
                         ::= : | ? | ! | % | $ | & | * | + | -
                          | . | / | < | = | > | @ | ^ | | | ~
```

3.4 Terms

```
\langle term \rangle
                              ::= \langle identifier \rangle
                                    true | false
                                                                                                                           (booleans)
                                    \langle number \rangle
                                    nil
                                                                                                                          (empty list)
                                    raise
                                                                                                                          (exception)
                                    \langle char \rangle
                                    ⟨string⟩
                                    ⟨parentheses⟩
                                    \langle record \rangle
                                    # \(\dentifier\)
                                                                                                                    (record access)
                                    ⟨squarebrackets⟩
                                    if \( \langle expression \rangle \) then \( \langle expression \rangle \) else \( \langle expression \rangle \)
                                    \langle match \rangle
                                    \langle lambda \rangle
                                    \langle reclambda \rangle
                                    \langle let \rangle
\langle number \rangle
                             ::= \langle decimal \rangle \mid \langle binary \rangle \mid \langle octal \rangle \mid \langle hexadecimal \rangle
\langle decimal \rangle
                             ::= \langle digit \rangle \{ \langle digit \rangle \}
\langle binary \rangle
                             ::= 0 (b | B) \langle bindigit \rangle \{ \langle bindigit \rangle \}
\langle octal \rangle
                             ::= \emptyset (o|0) \langle octdigit \rangle \{ \langle octdigit \rangle \}
⟨hexadecimal⟩
                             ::= 0 (x | X) \langle hexdigit \rangle \{ \langle hexdigit \rangle \}
⟨bindigit⟩
                             ::= 0 | 1
⟨octdigit⟩
                             ::= 0 | 1 | ... | 7
                             ::= \langle digit \rangle | a | \dots | f | A | \dots | F
⟨hexdigit⟩
\langle char \rangle
                             ::= ' (\langle escape \rangle | \langle any \rangle_{\langle 1 \rangle}) '
                             ::= " \{ \langle escape \rangle | \langle any \rangle_{\langle "| \setminus \rangle} \} "
⟨string⟩
                             ::= \langle (b|n|r|t \rangle | "|')
\langle escape \rangle
⟨parentheses⟩
                           (parenthesised expression)
                               (tuple, n \ge 2)
                                    ( ¬⟨operator⟩ ¬)¬
                                                                                                                  (prefix operator)
                             ::= \{ \ \Box \langle recordcomp \rangle_1 \ , \ \Box ... \ , \ \Box \langle recordcomp \rangle_n \ \} \ \Box
\langle record \rangle
                                                                                                                                 (n \ge 1)
⟨recordcomp⟩
                             ::= \langle identifier \rangle \bot : \bot \langle expression \rangle
```

```
(list, n \ge 0)
                | [ _⟨expression⟩ for _⟨pattern⟩ in ⟨expression⟩ ] _
                   (comprehension)
                  ⟨match⟩
               ::= match \( \( \lambda \) (expression \) with \( \lambda \) (matchcomp \( \rangle \) }
⟨matchcomp⟩
               ⟨expression⟩
               ::= \langle operand \rangle \{ \langle operator \rangle \, \bot \langle operand \rangle \}
               ::= \langle application \rangle
⟨operand⟩
               | - ¬⟨application⟩
                                                           (unary negation)
⟨application⟩
               ::= \langle term \rangle \, \bot \{ \langle term \rangle \, \bot \}
3.5 Functions and Declarations
               \langle lambda \rangle
```

```
\langle reclambda \rangle
                      \langle let \rangle
                       ::= \langle declaration \rangle; \Box \langle expression \rangle
                       ::= let \_\langle pattern \rangle = \_\langle expression \rangle
⟨declaration⟩
                        | \text{let} \_[\text{rec} \_] \langle \text{funcname} \rangle \{ \langle \text{parameter} \rangle \} [ : \_ \langle \text{type} \rangle ] = \_ \langle \text{expression} \rangle
                            import ¬⟨string⟩ ¬
                        | type _alias _\(\text{typeident}\) _= _\(\text{type}\)
⟨funcname⟩
                       ::= \langle identifier \rangle \, \bot
                        \langle fixity \rangle
                       ::= infixl|infixr|infix
                      ::= \langle pattuple \rangle \, \bot | \, (\, \bot \langle pattern \rangle \, ) \, \bot | \, \langle patvalue \rangle \, \bot
⟨parameter⟩
⟨patvalue⟩
                       ::= \langle identifier \rangle
                                                                                       (wildcard pattern)
                            true | false
                                                                                                 (booleans)
                            \langle number \rangle
                        | nil
                                                                                               (empty list)
                        |\langle char \rangle|
                            (string)
                        | \langle pattuple \rangle
                        | \langle patrecord \rangle
                        (list, n \ge 0)
```

```
⟨pattuple⟩
                                                                                                  (n \ge 2)
                      ::= { \Box \langle patrecordcomp \rangle_1 , \Box... , \Box \langle patrecordcomp \rangle_n } \Box (n \ge 1)
⟨patrecord⟩
                       | \{ \Box \langle patrecordcomp \rangle_1, \Box \dots, \Box \langle patrecordcomp \rangle_n, \dots \Box \} \Box
                           (partial record)
\langle patrecordcomp \rangle ::= \langle identifier \rangle \Box : \Box \langle pattern \rangle
⟨pattern⟩
                      ::= \langle patvalue \rangle \bot : \bot \langle type \rangle
                       | \langle patvalue \rangle \_ \{ :: \_ \langle patvalue \rangle \_ \}
⟨typevalue⟩
                      ::= Int
                       | Bool
                       Char
                       | \(\lambda\typeident\rangle\)
                       (tuple, n \ge 2)
                       (list)
                       | \langle typerecord \rangle
⟨typerecord⟩
                      \langle typerecordcomp \rangle ::= \langle identifier \rangle \bot : \bot \langle type \rangle
                      ::= \langle typevalue \rangle \bot \{ -> \langle typeValue \rangle \bot \}
\langle type \rangle
```

4 Language Guide

4.1 Basic Values

There are 4 types of basic values available in the *V* language:

- 1. Integers
- 2. Booleans
- 3. Character
- 4. Strings

Integers Only positive integers (plus zero) are recognized. They are always specified in decimal format, using only the digits from 0 to 9.

Booleans Two values are available: true for true, and false for false.

Characters and Strings A character literal is a single Unicode character surround by single quotes ('). A string literal is a sequence of zero or more Unicode characters surrounded by double quotes ("). Technically, strings are not basic values, since they are just syntactic sugar for a list of characters.

```
"abc" \rightarrow' a' ::' b' ::' c' :: nil
```

Some characters must be escaped in order to insert them into either character or string literals. "Escaping" a character means preceding it by the backslash character (\). For character literals, the single quote must be escaped (\'), while string literals require the escaping of the double quote (\").

There is also support for ASCII escape codes to insert special characters in literals. These are the allowed escape codes and their resulting characters:

Escape code	Character
\b	backspace
\n	newline (line feed
\r	carriage return
\t	horizontal tab
//	backslash
\'	single quote
\"	double quote

Any escape code can be used in either character or string literals. Furthermore, the special characters can be inserted directly into the literal. This means that multi line strings are supported by V. This also means that a single quote followed by a new line and a single quote is interpreted as a valid character literal (i.e. '\n').

4.2 Compound Values

4.2.1 Lists

Lists are ordered collections of values of the same type. There are no limits on the size of a list, even accepting lists with 0 values (the empty list).

Creating Lists An empty list can be created using either the nil keyword or the empty list literal, which is written as [] (empy square brackets).

To create a list with values, simply enclose the sequence of values, each separated by a comma, between square brackets.

```
[] // Empty list
[1, 2, 3] // List containing 3 values
```

Expanding Lists It is possible to add a value to the start of a list by using the list construction operator (::). The append function allowing the addition of a value to the end of a list. It is also possible to create a new list by using the concatenation operator (@), which adds two lists together.

```
let x = 0 :: [1, 2, 3];
// x is equal to [0, 1, 2, 3]
let y = append 4 [1, 2, 3];
// y is equal to [1, 2, 3, 4]
let z = [1, 2] @ [3, 4];
// z is equal to [1, 2, 3, 4]
```

Accessing Lists Any element of a list can be accessed by using the index (!!) operator. Lists are 0-indexed, which means the first value of a list is at index 0.

```
["a", "b", "c"] !! 0 // Returns "a"
```

An attempt to access an index outside the range of a list (that is, indexes equal to or greater than the size of the list) will result in a runtime error.

```
["a", "b", "c"] !! 5 // Runtime error
```

There are many other operations available for accessing elements of a list, including head (returns the first value of a list), last, filter, maximum, etc.

Complex Operations Although the V language does not directly support complex operations on lists, the standard library (see 5) provides a number of functions to manipulate lists. Among these are functions like map, filter, sort, fold, sublist, which provide basic functionality for performing computations with lists.

Ranges Ranges allow the easy creation of lists of integers in a arithmetic progression. There are two variants of ranges, one for simple integer counting and one for more complex progressions.

The first variant specifies the first and last value for the list. The list is then composed with every integer number between these values. Because of this, the first value must be smaller then the last

```
[1..5] // [1,2,3,4,5]
[3..7] // [3,4,5,6,7]
[5..3] // Invalid
```

The second variants specifies the first, second and last value for the list. The increment is the difference between the second and first value of the list, which can even be negative.

The increment is then added to each element until the largest possible value which is smaller than or equal to the last value. If the increment is negative, the list stops at the smallest possible value which is larger than or equal to the last value.

```
[1,3..10] // [1,3,5,7,9]
[5,4..1] // [5,4,3,2,1]
[5,3..0] // [5,3,1]
```

Comprehensions List comprehensions are a simple way to transform every value in a list, creating a new list.

In the example below, 1s is a list containing every number from 1 to 10, inclusive. Using a list comprehension, new is a list containing every number from 2 to 11, since the code x+1 is executed for every value in 1s.

```
let ls = [1..10];
let new = [x+1 for x in ls];
```

4.2.2 Tuples

Tuples group multiple values, possibly of different types, into a single compound value. The minimum size of a tuple is 2, but there is no limit on its maximum size.

Tuples are specified inside parenthesis, with each of its values separated by com-

```
(1, `hello'')
(true, `c', 43)
```

Tuples are immutable, which means they cannot be changed once they are created. There is no way to add or remove elements from a tuple.

To access a specific field of a tuple, it must be deconstructed using a pattern. This can be done either by using a let expression or by pattern matching in a function definition.

```
let (x, y) = (1, ``hello'')
(\&\&(x, y, z) \rightarrow x) (true, `c', 43)
```

Tuples are extremely useful as return values for functions that must convey more than one piece of information. Since every function can only return one value, tuples can be used to group the different values that the function must return.

4.2.3 Records

Records are, like tuples, groupings of multiple values of possibly different types. Unlike tuples, however, each value has its unique label. The smallest size for a record is 1, but there is no limit on its maximum size.

To construct a record, each value must be preceded by a label and a colon. Each label-value pair is separated by a comma, and the whole record is enclosed in curly brackets ({ }).

```
{name: ``Martha'', age: 32}
{day: 1, month: 1, year: 2000}
```

To get the value of a single field in a record, one can use the get function, passing the name of the field and the record. As can be seen below, the name of the field must be prefixed by the # character.

```
get #age {name: ``Martha'', age: 32} // Returns 32
get #month {day: 1, month: 1, year: 2000} // Returns 1
```

Unlike tuples, it is possible to alter a single field in a record. This is done by using the set function, which takes the name of the field (with the # character), the value to be set and the record.

It is also possible to modify a value of a field by using the modify function, which applies a function given as its parameter to the existing value.

It is important to note that these function do not change the original record, but instead returns a new, modified record.

```
set 3 #age {name: ``Martha'', age: 32}
// Returns {name: ``Martha'', age: 3}
set 8 #month {day: 1, month: 1, year: 2000}
// Returns {day: 1, month: 8, year: 2000}
modify #age (\x -> x * 2) {name: ``Martha'', age: 32}
// Returns {name: ``Martha'', age: 64}
```

4.3 Identifiers

Identifiers are used to name constants (in let declarations), functions and function arguments. When an identifier is expected, the identifier is defined as the longest possible sequence of valid characters. Any Unicode character is considered valid, with the exception of the following:

	,	;	:	!	@	&
+	-	/	*	<	=	>
()	{	}	[]	
%	\	,	"	\n	\r	\t
_	•					

Numerical digits are not allowed at the start of an identifier, but they can be used in any other position.

Furthermore, V has some reserved names that cannot be used by any identifier. They are the following:

	let	true	false	if	then	else
	rec	nil	raise	when	match	with
Ì	for	in	import	infix	infixl	infixr

4.4 Patterns

Patterns are rules for deconstructing values and binding their parts to identifiers. They can be used in constant declarations and function parameters, simplifying the extraction of data from compound values.

Pattern	Examples	Comments
Identifier	x, y, z	Matches any value and binds to the identifier
Number	1, 3	Matches and ignores the number
Boolean	true, false	Matches and ignores the boolean
Character	'c', 'f'	Matches and ignores the character
Identifier	x, y, z	Matches any value and binds to the identifier
Wildcard	_	Matches and ignores any value
Tuple	(x, _, y)	Matches tuples with corresponding size
Record	{a: _, b: (x,y), c: x}	Matches records with corresponding labels
Partial Record	{a: _, b: (x,y), c: x,}	Matches records with at least the specified labels
Nil	nil, []	Matches the empty list
Cons	x :: y	Matches the head and tail of a non-empty list
List	[x, y, z]	Matches lists of corresponding size

Compound patterns, such as Tuple, Record and List, are composed of other patterns separated by commas. All patterns can have optional type annotations added to explicitly declare their types.

One thing to notice is that all patterns related to lists (List, Cons and Nil) and values (numbers, characters) can fail. If an attempt to match a pattern fails (e.g matching a non-empty list with Nil), the expression will raise an exception.

4.5 Constants

Constants are associations of identifiers to values. The value associated to a particular identifier cannot be changed after it is declared.

The keyword let is used to start a constant declaration, and a semicolon ends it. After the let keyword, any pattern can be used.

Below are examples of constant declarations:

```
let name = ``Steve'';
let age: Int = 32;
let (x: Int, y) = (4, true);
```

4.6 Type Annotations

Type annotations are used to explictly state the type of a constant, function argument or function return value. They are not necessary for most programs, since the interpreter can infer the type of any expression.

Sometimes, the programmer may want to create artificial constraints on a function argument, and type annotations allow this.

The table below shows every type that can be specified in type annotations. These types align with the types available in the V language, since every type can be used in a type annotation.

Type	Example Values	Comments
Int	1, 0, -3	
Bool	true, false	
Char	'c', ' '	
String	"abc", ""	This is syntactic sugar for [Char]
[Type]	[1, 2, 3], nil	List Type
(<i>Type</i> , <i>Type</i>)	(1, true, 'a')	Tuple Type
{id: <i>Type</i> , id: <i>Type</i> }	{a: 3, b: false}	Record Type
Type -> Type		Function Type (see 4.10.5)

4.7 Conditionals

V provides a conditional expression (if ... then ... else) to control the flow of a program. This expression tests a condition and, if its value is true, executes the first branch (known as the then clause). If the condition is false, the expression executes the second branch (the else clause).

```
if b then
  1+3 // Will execute if b is true
else
  2 // Will execute if b is false
```

The only accepted type for the condition of a conditional is Boolean. All types are accepted in the then and else branch, but they must be of the same type.

```
// This conditional is invalid code, since 4 and "hello" are of different type
if true then
   4
else
   "hello"
```

Unlike imperative languages, every conditional in V must specify both branches. This ensures that the conditional will always return a value.

It is possible to chain multiple conditionals together.

```
if grade > 10 then
  "The grade cannot be higher than 10"
else if grade < 0
  "The grade cannot be lower than 0"
else
  "The grade is valid"</pre>
```

4.8 Match Expressions

Match expressions are another way to control the flow of a program based on comparison with a list of patterns. Any number (greater or equal to 1) of patterns can be specified, and each one has a corresponding result expression.

```
match value with
| pattern1 -> result-expression1
| pattern2 when condition -> result-expression2
```

Each pattern is tested from top to bottom, stopping the comparison as soon as a valid match is found. When this happens, the corresponding result expression is evaluated and returned.

It is also possible to specify an aditional condition that must be satisfied for a pattern to be accepted. This condition can use any identifiers declared in its corresponding pattern, and it is not evaluated unless the pattern pattern returns a correct match.

4.9 Operators

V contains a number of infix binary operators to manipulate data.

Along with them, there is only one prefix unary operator, the negation operator. This operator is handled differently from a function application, both in its priority and its associativity.

4.9.1 Priority

Every operator is ordered within a priority system, in which operators at a higher priority level are evaluated first. The levels are ordered in numerical order (i.e. priority 9 is the highest level). For different operators at the same priority level, the evaluation is always done from left to right.

4.9.2 Associativity

Some operators can be composed several times in a row, such as addition or function application. For these operators, it is necessary to define how they are interpreted to return the desired value. There are 2 possible associativities that an operator can have:

```
• Left-associative
((a + b) + c) + d
```

• Right-associative a + (b + (c + d))

4.9.3 Table of Operators

Below is a summary of every operator available in the language, along with a small description and their associativities (if any). The table is ordered by decreasing priority level (the first operator has the highest priority).

Priority	Operator	Meaning	Associativity
10	f x	Function Application	Left
9	f.g	Function Composition	Right
	x !! y	List Indexing	Left
8	х * у	Multiplication	Left
	х / у	Division	Left
	х % у	Remainder	Left
7	x + y	Addition	Left
	х - у	Subtraction	Left
	- x	Unary Negation	None
6	х :: у	List Construction	Right
5	х@у	List Concatenation	Right
4	x == y	Equals	None
	x != y	Not Equals	None
	x > y	Greater Than	None
	x >= y	Greater Than Or Equal	None
	x < y	Less Than	None
	x <= y	Less Than Or Equal	None
3	х && у	Logical AND	Right
2	x y	Logical OR	Right
1	х \$ у	Function Application	Right

4.9.4 Operators as Functions

It is possible to use any operator as a function by wrapping it in parenthesis. The left-hand operand becomes the first argument of the function, and the right-hand operand becomes the second argument.

This is useful mainly when passing operators as arguments to functions.

```
// Both expressions are equivalent zipWith (x y \rightarrow x + y) [1,2,3] [3,2,1] zipWith (+) [1,2,3] [3,2,1] // Adds 2 to every element in the list map ((+) 2) [1,2,3]
```

As is shown in the last example, it is possible to provide the left-hand operand to obtain a partially applied function. If one wishes to provide the right-hand operand instead, it is possible to use the flip function, which changes the order of a function that takes two parameters.

```
// Divides 2 by every value in the list map ((/) 2) [1,2,3]

// Divides every value in the list by 2 map (flip (/) 2) [1,2,3]
```

4.9.5 Functions as Operators

Wrapping a function name in backticks (') will turn it into an infix binary operator. The first parameter of the function will become the left-hand operand, while the second parameter will become the right-hand operand.

```
4 `add` 5 add 4 5
```

It is possible to use this with functions that take more than 2 parameters, but then it becomes necessary to use parenthesis to pass the remaning parameters. This greatly reduces the readability of the code, and is therefore not encouraged.

4.9.6 Defining new Operators

Is is possible to define new operators to be used like regular operators. The syntax for this is the same as creating a new function, but the operator must be enclosed in parenthesis.

```
let (\%+) x y = x \% y + 1;
5 \%+ 4 // 2
```

When declaring an operator, it is possible to also define its associativity and priority. This is done by using the keywords infix1 (left associative), infixr (right associative) and infix (non-associative), followed by a number from 1 to 9.

```
let infix1 1 ($) f x = f x;
let f x = x + 2;
```

If the associativity and priority information is not provided, the operator will have priority 1 and be left associative.

The following are the list of characters allowed for operators.

?	!	%	&	*	+
-		/	<	=	/
@	٨		~		

4.10 Functions

There are 4 types of functions that a programmer can declare:

- 1. Named functions
- 2. Recursive Named functions
- 3. Lambdas (unnamed functions)
- 4. Recursive Lambdas

4.10.1 Named Functions

These are functions that have a name by which they can be called after their definition. After the name, the programmer must specify one or more parameters, which can be any pattern. If an explicitly typed pattern is used, it must be enclosed in parenthesis. After every argument, the programmer can specify the return type of the function.

The body of a function can use any parameter declared in its definition to compute a return value. Since every expression in the language returns a value, any valid expression is accepted as the body of a function. The only constraint is that, if the definition specifies a return type, the value must be of that type.

Below are three examples of named functions:

```
let add x y =
    x + y
;

let duplicate (x: Int): Int =
    x * 2
;

let addTuple (x, y) =
    x + y
:
```

4.10.2 Recursive Named Functions

These functions differ from regular named functions by the fact that they can be called from within their own body. This means that the function can be called recursively, iterating over a certain value (or values). To indicate that a function is recursive, the keyword rec is added before its name. Below are two examples of recursive named functions:

```
let rec count ls =
   if empty? ls then
    0
   else
    1 + count (tail ls);
;
let rec factorial (x: Int): Int =
   if x == 0 then
    1
   else
    x * factorial (x - 1);
```

Here, both functions perform a test that determines whether the end condition is met. If the end condition is met, the function returns a simple value. If the end condition

is not met, the function recursively calls itself with a modified value, continuing the iteration.

In the case of the count function, the recursion terminates when the input is an empty list. For the factorial function, an input equal to 0 terminates the recursion.

4.10.3 Lambdas

These are simple unnamed functions with a compact syntax that allows them to be written in a single line most of the time. This is useful mostly when passing lambdas as arguments to other functions, since they do not require creating a full named declaration.

The general syntax of a lambda is as follows:

```
\param1 param2 ... -> body
```

A backslash (\) indicates the start of a lambda, followed immediately by its parameters. Like in named functions, these can be any valid pattern. Unlike named functions, however, the return type of a lambda is never specified.

After the parameters, an arrow (->) indicates the start of the function body, which extends as far to the right as possible. Because of this, lambdas are usually enclosed in parenthesis to limit their scope.

Below are the same examples shown in the named functions section, but defined using lambda expressions. Notice that, without the use of parenthesis to enclose each lambda, the first function would try to include everything inside its body, resulting in a parsing error.

```
\\ add
(\x y -> x + y)
\\ duplicate
(\(x: Int) -> x * 2)
\\ AddTuple
(\(x, y) -> x + y)
```

4.10.4 Recursive Lambdas

Just like there is a recursive variant of named functions, there is a recursive variant of lambdas. These are compact expressions to define recursive functions. Like regular lambdas, they are used mostly to be passed as arguments to other functions, and are usually enclosed in parenthesis.

Unlike for regular lambdas, it is necessary to specify a name for a recursive lambda. Without a name, it would be impossible to call itself within its body. It is important to realize that, unlike with recursive named functions, this name is limited in scope to the inside of the lambda definition.

```
(rec fac x \rightarrow if x == 0 then 1 else x * fac (x - 1))
```

```
fac 4 // This is invalid code
```

With the example above, we see that the programmer tried to call a recursive lambda outside its definition. The offending code is outside the scope in which fac is available, resulting in invalid code.

4.10.5 Function Type

Every function has a type consisting of its parameter types and return type. Every parameter type is separated by an arrow (->), and the return type is also separated by a single arrow from the parameter types.

The syntax for a function type is as follows:

```
param1 -> param2 -> ... -> return
```

If one of the parameters is itself a function, it is possible to use parenthesis to indicate this.

The following function type defines a function that takes two parameters. The first is a function of type Int -> Int. The second parameter is an Int, and the return type is also Int.

```
(Int \rightarrow Int) \rightarrow Int \rightarrow Int
```

If the parenthesis were ommited, the type would describe a function that takes 3 parameters of type Int.

4.11 Partial Application and Currying

Technically, every function in *V* takes only one parameter. When a function is defined as having multiple parameters, it is actually a curried function.

As an example, take the following function, which returns the largest of two numbers:

```
let max x y =
   if x > y then
    x
   else
   y
;
```

This appears to be a function that takes two integers and returns an integer. In reality, max is a function that takes one integer and returns another function. This returned function takes one integer as a parameter and returns another integer.

This allows what is called *partial application*, which is when a function is called with too few arguments. This creates a function that "fixes" the applied arguments and returns a function that takes the remaining arguments.

Using the example above, we can write max 5 to create a new function that takes only one argument. This function will then return the largest between its argument and the number 5.

It can then be bound to a name, just like any other function, and used elsewhere. This is also useful for quickly creating new functions with fixed data or to be passed as arguments.

```
let max5 = max 5;
max5 3 // Returns 5
max5 10 // Returns 10
```

String Conversions There are available functions to convert integers and booleans into and from strings. There are no included functions to convert compound types, but it is possible to create custom ones for each use case.

To convert strings to integers, the function is parseInt.

To convert integers to strings, the function is printInt.

To convert strings to booleans, the function is parseBool.

To convert booleans to strings, the function is printBool.

4.12 Comments

Comments are text that is ignored by the interpreter. They can be used to add notes or reminders for yourself or anyone that reads the source code.

Currently, only single line comments are avaiable. They begin with two forward-slashes (//) and continue until the end of the current line

```
// This is a comment on its own line.

3 + 4 // This line has code and a comment.
```

4.13 Libraries

Libraries are collections of constant and function declarations designed to be reused in multiple programs. These files can then be compiled to increase loading times when interpreting programs, or be loaded as parseable text files on their own.

To import a library in a program, the following syntax is used:

```
import "library"
```

The name of the library must be a string indicating the path of the library file. This path can either be relative to the program that is being executed or absolute. If a file extension is not provided for the file, it is assumed to be vl, which is the default extension used for compiled V libraries, or v, which is the extension for source code files in V.

Libraries can be imported anywhere in a program, and their functions will have their scope limited to wherever they were imported.

In the example below, we have a library with a single function double. This library is then imported inside a function in a program. Because the library was imported inside the scope of the function body, none of its functions can be called outside of it.

```
// math.vl
let double x = x * 2;

//-----
// Program.v
let quadruple x =
  import "math"
  double (double x) // Valid;

double 4 // Invalid
```

5 Standard Library

The Standard Library, called stdlib, is always imported in every V program. It provides basic functions for a number of use cases, ranging from numerical operations to function manipulation.

Some basic language features, such as list comprehensions and ranges, depend on the existence of the stdlib. This means that, while it is possible to create programs without importing the stdlib, doing so will most likely break any existing program.

5.1 Operations on Basic Values

5.1.1 Operations on All Values

The operations below are helper functions, designed to allow cleaner code.

```
id :: a \rightarrow a
```

Identity function.

```
const :: a \rightarrow b \rightarrow a
```

Always returns the first parameter it is passed.

5.1.2 Operations on Numbers

The 4 basic operations (addition, subtraction, multiplication and division) are built into the language. Other operations must be defined in terms of these.

One important thing to note is that the unary negation operator (-) is tightly coupled with the negate function defined in the stdlib. While the operator is defined inside the language, it depends on the presence of the stdlib to function.

```
remainder :: Int \rightarrow Int \rightarrow Int
```

Integer remainder, satisfying:

$$(x / y) * y + (remainder x y) = x$$

(%) ::
$$Int \rightarrow Int \rightarrow Int$$
 | left-associative, priority 8 |

Infix version of remainder

```
negate :: Int \rightarrow Int
```

Unary negation, satisfying:

```
x + (negate x) = 0
```

```
abs :: Int \rightarrow Int
```

Absolute value

5.1.3 Operations on Booleans

Below are all the operations on booleans defined in the Standard Library.

```
and :: Bool \rightarrow Bool \rightarrow Bool
```

Boolean "AND"

```
(&&) :: Bool \rightarrow Bool \rightarrow Bool | right-associative, priority 3 |
```

Infix version of and

```
or :: Bool \rightarrow Bool \rightarrow Bool
```

Boolean "OR"

```
(||) :: Bool \rightarrow Bool \rightarrow Bool | right-associative, priority 2 |
```

Infix version of or

```
not :: Bool \rightarrow Bool
```

```
Boolean "not"
not True = False
not False = True
```

```
xor :: Bool \rightarrow Bool \rightarrow Bool
```

```
Boolean "xor"

xor True True = False

xor True False = True

xor True False = True

xor False False = False
```

5.1.4 Operations on Functions

Basic manipulation of functions and application. Most of the usefulness of these functions come from their infix versions. They allow more compact and easier to read code

to be written, mainly reducing the need for parentheses.

flip ::
$$(a \rightarrow b \rightarrow c) \rightarrow b \rightarrow a \rightarrow c$$

flip f takes its first two arguments in reverse order of f.

flip
$$f x y = f y x$$

apply ::
$$(a \rightarrow b) \rightarrow a \rightarrow b$$

This function simply applies its second argument to its first. While this seems redundant (after all, apply f x is the same as f x), it can be used higher order situations.

(\$) ::
$$(a \rightarrow b) \rightarrow a \rightarrow b$$
 | right-associative, priority 1 |

Infix version of apply. While it has the same functionality as normal function application, it is right-associative with the lowest possible priority.

In some situations, this allows parentheses to be omitted.

$$f$$
\$ g \$ h $x = f$ (g (h x))

compose ::
$$(b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$$

Function composition. Applies the third argument to the second one, applying the resulting value to the first argument.

compose
$$f g x = f (g x)$$

(.) ::
$$(b \to c) \to (a \to b) \to a \to c$$
 | right-associative, priority 9 |

Infix version of compose.

Can be used with \$ to reduce the number of parentheses needed.

$$f . g . h \$ x = f (g (h x))$$

5.1.5 Operations on Tuples

The stdlib also provides basic functions for manipulating tuples with 2 components. For larger tuples, it is necessary to create custom functions.

fst ::
$$(a,b) \rightarrow a$$

Returns the first component of a pair.

snd :: $(a,b) \rightarrow b$

Returns the second component of a pair.

swap :: $(a,b) \rightarrow (b,a)$

Swap the components of a pair.

5.1.6 Operations on Records

The functions below are used with record accessors (#label) to get, set and change individual fields in a record. While the functions themselves have a more generic type and can, therefore, be used in more circumstances, they were created with records in mind.

get ::
$$(a \rightarrow b \rightarrow (c,d)) \rightarrow b \rightarrow c$$

Returns the value of a field in a record.

get #label record

set ::
$$(a \rightarrow b \rightarrow (c,d)) \rightarrow a \rightarrow b \rightarrow d$$

Returns the inputed record, modifying a single field value.

set #label value record

modify ::
$$(a \rightarrow b \rightarrow (c, d)) \rightarrow (c \rightarrow a) \rightarrow b \rightarrow d$$

Returns the inputed record, modifying a single field value by applying the old value to the specified function.

modify #label function record

5.2 Operations on Lists

5.2.1 Basic Operations

Basic functions to aid in using lists.

head :: $[a] \rightarrow a$

Returns the first element of a list, which must have at least one element.

last :: $[a] \rightarrow a$

Returns the last element of a list, which must have at least one element.

tail ::
$$[a] \rightarrow [a]$$

Removes the first element of a list, which must have at least one element.

init ::
$$[a] \rightarrow [a]$$

Removes the last element of a list, which must have at least one element.

tail ::
$$[a] \rightarrow [a]$$

Removes the first element of a list, which must have at least one element.

empty? ::
$$[a] \rightarrow Bool$$

Returns True if the list is empty, and False otherwise.

length ::
$$[a] \rightarrow Int$$

Returns the number of elements in the list.

append ::
$$a \rightarrow [a] \rightarrow [a]$$

Adds an element to the end of a list.

concat ::
$$[a] \rightarrow [a] \rightarrow [a]$$

Appends two lists, maintaining order.

(@) ::
$$[a] \rightarrow [a] \rightarrow [a]$$
 | right-associative, priority 5 |

Infix version of concat.

5.2.2 Generation Operations

These operations create lists based on input values.

range ::
$$Int \rightarrow Int \rightarrow Int \rightarrow [Int]$$

range start finish increment generates a list of the form [start, start + increment, start + 2 * increment, ..., n], where

```
\begin{array}{ll} \text{increment} > 0 \implies \text{n} \leq \text{finish} \\ \text{increment} < 0 \implies \text{n} \geq \text{finish} \end{array}
```

5.2.3 Transformation Operations

These operations transform a list, altering its elements, their order, or both.

reverse ::
$$[a] \rightarrow [a]$$

Returns the elements of the input in reverse order.

map ::
$$(a \rightarrow b) \rightarrow [a] \rightarrow [b]$$

map f 1s returns a list by applying the function f to each element of the list 1s.

5.2.4 Reduction Operations

These operations take a list and reduce it to a simple value.

fold ::
$$(b \rightarrow a \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$$

fold f acc ls reduces the list using the function f, applying it to an accumulator (acc) and each element of the list, from left to right.

reduce ::
$$(a \rightarrow a \rightarrow a) \rightarrow [a] \rightarrow a$$

The same as fold, but using the first element of the list as the acc

$$\mathbf{all} \ :: \ (a \to Bool) \to [a] \to Bool$$

Checks whether all elements of a list satisfy a predicate. An empty list returns true.

any ::
$$(a \rightarrow Bool) \rightarrow [a] \rightarrow Bool$$

Checks whether any elements of a list satisfy a predicate. An empty list returns false.

$$\mathbf{maximum} \ :: \ \mathit{Orderable} \ a \Longrightarrow [a] \to a$$

Returns the largest element of the list.

minimum :: Orderable $a \Rightarrow [a] \rightarrow a$

Returns the smallest element of the list.

5.2.5 Sublist Operations

These operations return smaller segments of an existing list.

take ::
$$Int \rightarrow [a] \rightarrow [a]$$

take n 1s returns the first n elements of 1s.

drop ::
$$Int \rightarrow [a] \rightarrow [a]$$

drop n 1s returns the list resulting from removing the first n elements of 1s.

takeWhile ::
$$(a \rightarrow Bool) \rightarrow [a] \rightarrow [a]$$

takeWhile p 1s returns the longest prefix of 1s such that every element satisfies p.

dropWhile ::
$$(a \rightarrow Bool) \rightarrow [a] \rightarrow [a]$$

dropWhile p ls returns the suffix that remains after takeWhile p ls.

sublist ::
$$Int \rightarrow Int \rightarrow [a] \rightarrow [a]$$

sublist start length 1s drops the first start elements of 1s, and then takes the first length elements of the resulting list.

5.2.6 Search Operations

These operations search for specific elements in a list.

exists :: Equatable
$$a \Rightarrow a \Rightarrow [a] \Rightarrow Bool$$

Tests whether the given element exists in the list.

filter ::
$$(a \rightarrow Bool) \rightarrow [a] \rightarrow [a]$$

filter p 1s returns a sublist of 1s such that every element satisfies p.

5.2.7 Indexing Operations

Manipulate a list through the index of its elements

indexOf :: Equatable
$$a \Rightarrow a \rightarrow [a] \rightarrow Int$$

indexOf t 1s returns the index of the first occurrence of t in 1s. If the element does not occur, returns -1.

nth ::
$$Int \rightarrow [a] \rightarrow a$$

nth n 1s returns the element of 1s at position n. If n is negative or larger than length 1s, an exception is raised.

(!!) ::
$$[a] \rightarrow Int \rightarrow a$$
 | left-associative, priority 9 |

The infix version of nth. It receives its operands in reverse order, allowing for expressions in the form 1s !! n.

5.2.8 Sorting Operations

Sort lists.

sort :: Orderable
$$a \Rightarrow [a] \rightarrow [a]$$

Sorts a list in ascending order.

5.2.9 Zipping Operations

Operations that deal with tuples and lists.

zip ::
$$[a] \to [b] \to [(a,b)]$$

Takes two lists and returns a list composed of corresponding pairs. It the lists have different lengths, elements of the larger one are discarded.

zipWith ::
$$(a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$$

Takes two lists and a function, returning a list composed of the result of applying the function to corresponding elements in each list. It the lists have different lengths, elements of the larger one are discarded.

unzip ::
$$[(a,b)] \to ([a],[b])$$

Takes a list of pairs, returning a pair of lists, each containing the corresponding components of the original list.

5.3 String Conversion Operations

Converts values from and to strings.

parseInt :: $String \rightarrow Int$

Converts a string into an integer. The only representation accepted is decimal (without a leading +), and the function raises an exception if parsing fails.

printInt :: $Int \rightarrow String$

Converts an integer value into a string.

parseBool :: $String \rightarrow Bool$

Converts a string into a boolean. Valid strings are "true" and "false".

printBool :: $Bool \rightarrow String$

Converts a boolean value into a string.

6 Changelog

v0.2

May 1, 2017

Additions

- Match Expressions Abstract Syntax Operational Semantics Type System Concrete Syntax

 This is a structure to control the flow of a program by attempting to match a value against a list of patterns.
- Prefix Notation for Operators Concrete Syntax
 Wrapping any infix operator in parenthesis turns it into a function that takes two parameters.
- Infix Notation for Functions Concrete Syntax

 Wrapping any function name in backticks (') turns it into a binary operator.

• Creating new Operators Concrete Syntax

When declaring a function, wrapping a name in parenthesis makes it a operator. Only a small number of characters are allowed for operator names, and it is possible to define the associativity and priority of the newly created operator (if this information is ommitted, the default values are left associative and priority 9).

• Partial Record Pattern Abstract Syntax Operational Semantics Type System

Concrete Syntax

This pattern does not create an equality constraint on records. Instead, it creates trait constraints, allowing for ad-hoc subtyping.

• Extended Language Abstract Syntax

Created an extended language (syntax tree). This languaged is used for parsing, and then is translated into the core language.

• Type Aliases Concrete Syntax

Allows the programmer to specify type aliases, simplifying type declarations

Changes

• List Operations Abstract Syntax Operational Semantics Type System

The list operations hd, t1 and isempty have been removed from the language defition. They have been added to the stdlib, and use pattern matching to recreate their functionality.

- Non-Strict Semantics Operational Semantics
 - Functions can now be non-strict, and compound values (lists, tuples, records) can contain undefined (raise) components.
- Boolean Operations Abstract Syntax Operational Semantics Type System

 The boolean operators ∧ (AND) and ∨ (OR) have been removed from the definition and added to the stdlib.
- Record Projection (Accessor) Abstract Syntax Operational Semantics Type System Replaced record projection with record accessor, allowing for editing of individual fields in a record.
- Conditionals Abstract Syntax Operational Semantics Type System
 Removed conditionals from core language, adding them to the extended language.

Removals

• Input and Output Abstract Syntax Operational Semantics Type System Concrete Syntax Removed the input and output expressions. This was done because they are not compatible with the functional nature of V. IO will (probably) be added again in the future, but for now it must be removed.

• Sequence Operator, Skip and Unit

Type System | Concrete Syntax | Abstract Syntax | Operational Semantics

Since these are not used by any expressions, they have been removed.

- Try Abstract Syntax Operational Semantics Type System Concrete Syntax

 Exceptions can no longer be handled. This was done because of the change to non-strict semantics.
- Project Index (TuplePosition Trait) Abstract Syntax Operational Semantics

 Type System Concrete Syntax

Removed the ability to project a specific component of a tuple, making patterns the only way to decompose tuples.

As a result of this, the TuplePosition Trait has been removed.