

# L1 Syntax and Documentation

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## Introduction

The *L1* programming language is a functional language with eager left-to-right evaluation. It has a simple I/O system supporting only direct string operations. It is a trait based strongly and statically typed language supporting both explicit and implicit typing.

This document both specifies the *L1* language and shows its implementation in F#. It is divided into 4 categories:

1. Abstract Syntax and Semantics

This defines the abstract syntax and semantics for the functional language. It only contains the bare minimum for the language to function, without any syntactic sugar.

2. Concrete Syntax

This is the actual syntax when programming for *L1*. This defines all operators, syntactic sugar and other aspects of the language.

3. Implementation

Technical aspects on how *L1* is implemented in F#, showing the interpreter, evaluator and type inference.

4. Change log

A chronological list of changes made both to the language definition and its implementation.

# 1 Abstract Syntax and Semantics

## 1.1 Abstract Syntax

### 1.1.1 Expressions

Programs in  $L1$  are expressions. Each expression is a member of the abstract syntax tree defined below. The syntax tree will be constructed in parts, with an explanation of what each expression means and their uses. The full syntax tree can be obtained by simply joining all the separate sections.

**Constants and Variables**  $L1$  has support for a few basic constants and variables.

$e$	$::=$	$n$
		$ $
		$b$
		$ $
		$c$
		$ $
		$x$
$x$	$::=$	$\{x_0, x_1, \dots\}$
$b$	$::=$	$true \mid false$
$n$	$::=$	$\mathbb{Z}$
$c$	$::=$	$'char'$
$char$	$::=$	ASCII characters

The constants available for the language are:

- Booleans
- Integers
- Characters

For variables, one can define any number of identifiers to be used throughout the program. These variables must be associated to other terms in some way (functions, let declarations, etc) as they cannot be evaluated on their own.

**Conditional** Like most functional languages,  $L1$  provides a conditional expression. This expression, like all others, always returns a value.

$e$	$::=$	$\dots$
		$ $
		$\text{if } e_1 \text{ then } e_2 \text{ else } e_3$

**Binary Operations**  $L1$  provides a series of binary operations built-in into the language.

$$\begin{aligned}
e & ::= \dots \\
& \quad | \quad e_1 \text{ op } e_2 \\
\\
\text{op} & ::= \text{opNum} \mid \text{opEq} \mid \text{opIneq} \mid \text{opBool} \\
\text{opNum} & ::= + \mid - \mid * \mid \div \\
\text{opEq} & ::= = \mid \neq \\
\text{opIneq} & ::= < \mid \leq \mid > \mid \geq \\
\text{boolOp} & ::= \wedge \mid \vee
\end{aligned}$$

They are divided into separate categories, each one requiring terms of specific types. The following list describes the requirements and meaning of each category of built-in operators:

- Numerical  
These operators require numbers and also return numbers.
- Equality  
These operators ( $=$ ,  $\neq$ ) compare two different values for equality, returning a boolean. The values type must conform to the Equatable trait.
- Inequality  
These operators compare two different values for order, returning a boolean. The values type must conform to the Equatable and Orderable trait.
- Boolean Operators  
These operators perform logical operations on boolean values.

**Functions** The expressions below all relate to function and function application in the *L1* language.

$$\begin{aligned}
e & ::= \dots \\
& \quad | \quad \text{fn } x : T \Rightarrow e \\
& \quad | \quad \text{fn } x \Rightarrow e \\
& \quad | \quad \text{rec } x_1 : T_1 \rightarrow T_2 \ x_2 : T_1 \Rightarrow e \\
& \quad | \quad \text{rec } x_1 \ x_2 \Rightarrow e \\
& \quad | \quad e_1 \ e_2
\end{aligned}$$

The first two expressions define simple unnamed functions that take exactly one parameter,  $x$ . The first of these explicitly says what the type of this parameter is, while the second one leaves the job of inferring the type to the compiler. When a value  $v$  is applied to a function, all occurrences of  $x$  in  $e$  are replaced by  $v$  and then the expression is evaluated. (In reality, this replacement only occurs as needed to be more efficient (see 1.2), but the result is the same).

The following two expressions define recursive functions that also take one parameter. In these expressions,  $x_1$  is the name of the function that can be recursively called inside  $e$ .  $x_2$  is the identifier of the single parameter for the function. As with unnamed

functions, recursive functions have two variations: explicitly and implicitly typed. In the explicitly typed version,  $x_1$  has to be typed as a function that takes the type of  $x_2$  (that is,  $T_1$ ) and returns another type  $T_2$ .

The last expression is the application of  $e_2$  to a function  $e_1$ .

**Let declarations** The expressions below are used to declare identifiers for sub-expressions. This both helps increase readability and reduce repetition when writing programs.

```

e ::= ...
    | let x : T = e1 in e2
    | let x = e1 in e2

```

There are two versions of the `let` expression: one that is explicitly typed and one that is implicitly typed, just as with function expressions. Again, similar to functions, this expression works by replacing all occurrences of  $x$  by the value of  $e_1$  in  $e_2$  and then evaluating the resulting expression.

**Exceptions** Exceptions are used when an expression cannot be evaluated correctly because of reasons outside of the normal. These reasons include division by zero, accessing an empty list, etc. Since these situations cannot be known before evaluating a program, there is no way to defend against them in a type system. To deal with this, we must have an expression to deal with exceptions. It is also a good idea to be able to create exceptions inside a program, so we have an expression to do just that.

```

e ::= ...
    | raise
    | try e1 with e2

```

The first expression simply evaluates to an exception that will be propagated to its parent expression.

The second expression evaluates  $e_1$  and, if an exception is encountered, evaluates  $e_2$ . If the evaluation of  $e_1$  does not encounter an exception, the resulting value is used, and  $e_2$  is discarded.

**Lists** The  $L1$  language has built-in lists. Each list is a homogeneous finite-length collection of values, meaning that a list contains only elements of the same type. There are also basic operations on lists, such as appending, obtaining the first element of a list, etc.

```

e ::= ...
    | nil
    | e1 :: e2
    | isempty e
    | hd e
    | tl e

```

The first expression is the empty list. It is the only zero-length list possible, and all other lists are constructed on top of it.

The second expression is the append operation, adding  $e_1$  to the front of the list  $e_2$ .

The last three expressions are operations on lists.

The first one tests whether  $e$  is the empty list (i.e. *nil*), returning true if positive and false if negative.

The second one returns the first element of a non-empty list. If the list is empty, an exception is raised.

The third one returns the list obtained by removing the first element of a non-empty list. If the list is empty, an exception is raised.

**Input and Output** These expressions allow a program to interact with a user. Because of this, they are inherently nondeterministic.

$$\begin{array}{lcl} e & ::= & \dots \\ & | & \text{input} \\ & | & \text{output } e \end{array}$$

The **input** expression receives a line of text from the user. It always evaluates to a list of characters, but the length of this list is only known at run-time.

The **output** expression prints a list of characters to the user. It is only evaluated for its side-effect of printing the list, resulting in a useless value. This is dealt with by the next set of expressions in the language.

**Sequence** Up until now, no expressions in *L1* had side effects, so all expressions were written to obtain their resulting value. With the introduction of input and (especially) output expressions, now there exists expressions that are evaluated only for their side effect. Because of this, it is important to enable a programmer to ignore the resulting value and continue evaluating other expressions.

$$\begin{array}{lcl} e & ::= & \dots \\ & | & \text{skip} \\ & | & e_1 ; e_2 \end{array}$$

The first new expression (*skip*) is an empty value, always meant to be ignored. It is the result of evaluating an **output** expression and any other expression that is only evaluated for its side-effects.

The second expression evaluates  $e_1$ , ignores its results and evaluates  $e_2$ . It requires that  $e_1$  evaluates to no useful value (i.e. evaluates to *skip*).

### 1.1.2 Types

Since *L1* is strongly typed, every (valid) expression has exactly one type associated with it. Some expressions require the programmer to explicitly declare types of identifiers, such as **let** declarations and functions. Other expressions, such as  $e_1 = e_2$ , or even constants, such as 1 or *true*, have types implicitly associated with them. These

types are used by the type system (see 1.3) to check whether an expression is valid or not, avoiding runtime errors that can be detected in compile time.

**Traits** *L1* has a trait system to define certain behaviors for types. Every type can have multiple traits.

$T$	$::=$	$X$
		Int
		Bool
		Char
		Unit
		$T_1 \rightarrow T_2$
		$T \text{ list}$
$X$	$::=$	$X_1, X_2, \dots$



## 1.2 Operational Semantics

The *L1* language is evaluated using a big-step evaluation with environments. This evaluation reduces an expression into a value directly, not necessarily having a rule of evaluation for every possible expression. To stop programmers from creating programs that cannot be evaluated, a type inference system will be specified later.

**Value** A value is the result of the evaluation of an expression in big-step. This set of values is different from the set of expressions of *L1*, even though they share many similarities.

**Environment** An environment is a mapping of identifiers to values that is extended each time a *let* declaration is encountered. Because the environment stores only values, this means that *L1* has eager evaluation.

Below are the definitions of both values and environments:

$$env ::= \{\} \mid \{x \rightarrow v\} \cup env$$
$$v ::= \begin{array}{l} n \\ b \\ c \\ nil \\ v_1 :: v_2 \\ raise \\ skip \\ \langle x, e, env \rangle \\ \langle x_1, x_2, e, env \rangle \end{array}$$

The values  $\langle x, e, env \rangle$  and  $\langle x_1, x_2, e, env \rangle$  are closures and recursive closures, respectively. They represent the result of evaluating functions and recursive functions, both and store the environment at the moment of evaluation. This means that *L1* has static scope, since closures capture the environment at the moment of evaluation and *L1* has eager evaluation.

Closures also store the identifier for the parameter of the respective function (as  $x$ ), along with the function body (as  $e$ ). Recursive closures, besides storing the identifier for the parameter (as  $x_2$ ) and the function body, also store the name of the function (as  $x_1$ ). This allows the function to be called inside its own body, something that the simple closure does not allow.

### 1.2.1 Big-Step Rules

$$env \vdash n \Downarrow n \quad (\text{BS-Num})$$
$$env \vdash b \Downarrow b \quad (\text{BS-Bool})$$

$$\text{env} \vdash c \Downarrow c \quad (\text{BS-CHAR})$$

$$\frac{\text{env}(x) = v}{\text{env} \vdash x \Downarrow v} \quad (\text{BS-IDENT})$$

**Numerical Operations** The *L1* language only supports integers, so all operations are done on integer numbers. This means that the division always results in a whole number, truncated towards zero.

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n\| = \|n_1\| + \|n_2\|}{\text{env} \vdash e_1 + e_2 \Downarrow n} \quad (\text{BS-+})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n\| = \|n_1\| - \|n_2\|}{\text{env} \vdash e_1 - e_2 \Downarrow n} \quad (\text{BS-})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n\| = \|n_1\| * \|n_2\|}{\text{env} \vdash e_1 * e_2 \Downarrow n} \quad (\text{BS-*})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow 0}{\text{env} \vdash e_1 \div e_2 \Downarrow \text{raise}} \quad (\text{BS-}\div\text{ZERO})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n_2\| \neq 0 \quad \|n\| = \|n_1\| \div \|n_2\|}{\text{env} \vdash e_1 \div e_2 \Downarrow n} \quad (\text{BS-}\div)$$

**Equality Operations** The equality operators (= and  $\neq$ ) allow comparison of certain expressions with other expressions of the same kind. In this way, it is a polymorphic operator, being usable in different contexts. Even so, it is important to realize that it only compares values of the same kind (numbers with numbers, characters with characters, etc).

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n_1\| = \|n_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{true}} \quad (\text{BS-}=\text{NUMTRUE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n_1\| \neq \|n_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \quad (\text{BS-}=\text{NUMFALSE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \|c_1\| = \|c_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{true}} \quad (\text{BS-}=\text{CHARTRUE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \|c_1\| \neq \|c_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \text{ (BS-CHARFALSE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow b_1 \quad \text{env} \vdash e_2 \Downarrow b_2 \quad \|b_1\| = \|b_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{true}} \text{ (BS-BOOLTRUE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow b_1 \quad \text{env} \vdash e_2 \Downarrow b_2 \quad \|b_1\| \neq \|b_2\|}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \text{ (BS-BOOLFALSE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 = e_2 \Downarrow \text{true}} \text{ (BS-NILTRUE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \text{ (BS-NILFALSE1)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \text{ (BS-NILFALSE2)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow v_3 :: v_4 \quad \text{env} \vdash v_1 = v_3 \Downarrow \text{false}}{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}} \text{ (BS-LISTFALSE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow v_3 :: v_4 \quad \text{env} \vdash v_1 = v_3 \Downarrow \text{true} \quad \text{env} \vdash v_2 = v_4 \Downarrow b}{\text{env} \vdash e_1 = e_2 \Downarrow b} \text{ (BS-LISTTRUE)}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow \text{false}}{\text{env} \vdash e_1 \neq e_2 \Downarrow \text{true}} \text{ (BS-NOTTRUE)}$$

$$\frac{\text{env} \vdash e_1 = e_2 \Downarrow \text{true}}{\text{env} \vdash e_1 \neq e_2 \Downarrow \text{false}} \text{ (BS-NOTFALSE)}$$

**Inequality Operations** The inequality operators function much in the same way as the equality operators. The only difference is that they do not allow comparison of certain kinds of expressions (such as booleans) when such expressions do not have a clear ordering to them.

To reduce the number of rules, some rules are condensed for all inequality operators ( $<$ ,  $\leq$ ,  $>$ ,  $\geq$ ). The comparison done on numbers is the ordinary numerical comparison. For characters, the ASCII values are compared numerically.

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \|n_1\| \text{opIneq} \|n_2\|}{\text{env} \vdash e_1 \text{opIneq} e_2 \Downarrow \text{true}} \text{ (BS-INEQNUMTRUE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow n_2 \quad \neg \|n_1\| \text{opIneq} \|n_2\|}{\text{env} \vdash e_1 \text{opIneq} e_2 \Downarrow \text{true}} \text{ (BS-INEQNUMFALSE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \|c_1\| \text{opIneq} \|c_2\|}{\text{env} \vdash e_1 \text{opIneq} e_2 \Downarrow \text{true}} \text{ (BS-INEQCHARTRUE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow c_1 \quad \text{env} \vdash e_2 \Downarrow c_2 \quad \neg \|c_1\| \text{opIneq} \|c_2\|}{\text{env} \vdash e_1 \text{opIneq} e_2 \Downarrow \text{true}} \text{ (BS-INEQCHARFALSE)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 < e_2 \Downarrow \text{false}} \text{ (BS-<NIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 \leq e_2 \Downarrow \text{true}} \text{ (BS-≤NIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 > e_2 \Downarrow \text{false}} \text{ (BS->NIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 \geq e_2 \Downarrow \text{true}} \text{ (BS-≥NIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 < e_2 \Downarrow \text{false}} \text{ (BS-<LISTNIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 \leq e_2 \Downarrow \text{false}} \text{ (BS-≤LISTNIL)}$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 > e_2 \Downarrow \text{true}} \quad (\text{BS-}>\text{LISTNIL})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 \geq e_2 \Downarrow \text{true}} \quad (\text{BS-}\geq\text{LISTNIL})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 < e_2 \Downarrow \text{true}} \quad (\text{BS-}<\text{NILLIST})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 \leq e_2 \Downarrow \text{true}} \quad (\text{BS-}\leq\text{NILLIST})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 > e_2 \Downarrow \text{false}} \quad (\text{BS-}>\text{NILLIST})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil} \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 \geq e_2 \Downarrow \text{false}} \quad (\text{BS-}\geq\text{NILLIST})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow v_3 :: v_4 \quad \text{env} \vdash v_1 = v_3 \Downarrow \text{false} \quad \text{env} \vdash v_1 \text{ opIneq } v_3 \Downarrow b}{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow b} \quad (\text{BS-INEQLISTHEAD})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2 \quad \text{env} \vdash e_2 \Downarrow v_3 :: v_4 \quad \text{env} \vdash v_1 = v_3 \Downarrow \text{true} \quad \text{env} \vdash v_2 \text{ opIneq } v_4 \Downarrow b}{\text{env} \vdash e_1 \text{ opIneq } e_2 \Downarrow b} \quad (\text{BS-INEQLISTTAIL})$$

**Logical Operations** The logical operators  $\wedge$  (AND) and  $\vee$  (OR) both have a short-circuit evaluation. This means that, if the result of the operation can be determined from the first operand, the second one is not evaluated.

$$\frac{\text{env} \vdash e_1 \Downarrow \text{true}}{\text{env} \vdash e_1 \vee e_2 \Downarrow \text{true}} \quad (\text{BS-}\vee\text{SHORT})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{false} \quad \text{env} \vdash e_2 \Downarrow b}{\text{env} \vdash e_1 \vee e_2 \Downarrow b} \quad (\text{BS-}\vee)$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{false}}{\text{env} \vdash e_1 \wedge e_2 \Downarrow \text{false}} \quad (\text{BS-}\wedge\text{SHORT})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{true} \quad \text{env} \vdash e_2 \Downarrow b}{\text{env} \vdash e_1 \wedge e_2 \Downarrow b} \quad (\text{BS-}\wedge)$$

**Conditional Expression**  $L_1$  supports conditional expressions, which always return a value, but not conditional statements. Because of this, all conditional expressions must have both a *then* and an *else* branch. Evaluation is done only on the condition and the proper branch, avoiding the evaluation of the unused branch.

$$\frac{\text{env} \vdash e_1 \Downarrow \text{true} \quad \text{env} \vdash e_2 \Downarrow v}{\text{env} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \quad (\text{BS-IFTRUE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{false} \quad \text{env} \vdash e_3 \Downarrow v}{\text{env} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \quad (\text{BS-IFFALSE})$$

**Function Expressions** There are two types of function expressions, each having two variants, with implicit or explicit typing.

The first is for a simple unnamed function that takes exactly one parameter. This parameter can occur anywhere inside the function body ( $e$ ), and will be replaced by the argument when the function is called.

The typed variant specifies only the type of the input, not of the output.

$$\text{env} \vdash \text{fn } x : T \Rightarrow e \Downarrow \langle x, e, \text{env} \rangle \quad (\text{BS-FN})$$

$$\text{env} \vdash \text{fn } x \Rightarrow e \Downarrow \langle x, e, \text{env} \rangle \quad (\text{BS-FN2})$$

The second type of function is a recursive function that also takes exactly one parameter ( $x_2$ ). Unlike the unnamed function, a recursive function also specifies its own name ( $x_1$ ), such that it can be called within the function body.

The typed variant must specify the types of both the input and output. The type  $T_1$  is the type of the input, and  $T_2$  is the type of the output. In  $L_1$ , the programmer specifies the type of the function (that is,  $T_1 \rightarrow T_2$ ), instead of specifying the output type directly.

$$\text{env} \vdash \text{rec } x_1 : T_1 \rightarrow T_2 \ x_2 : T_1 \Rightarrow e \Downarrow \langle x_1, x_2, e, \text{env} \rangle \quad (\text{BS-REC})$$

$$\text{env} \vdash \text{rec } x_1 \ x_2 \Rightarrow e \Downarrow \langle x_1, x_2, e, \text{env} \rangle \quad (\text{BS-REC2})$$

**Application** An application expression requires either a closure or a recursive closure for its left-hand operand. The right-hand operand (argument) is always evaluated using the current environment, resulting in a value  $v_2$ .

In the case of a simple closure, the body of the function ( $e$ ) is evaluated using the stored closure, adding an association between the parameter identifier ( $x$ ) and the argument ( $v_2$ ).

$$\frac{\text{env} \vdash e_1 \Downarrow \langle x, e, \text{env} \rangle \quad \text{env} \vdash e_2 \Downarrow v_2 \quad \{x \rightarrow v_2\} \cup \text{env} \vdash e \Downarrow v}{\text{env} \vdash e_1 \ e_2 \Downarrow v} \quad (\text{BS-APPFN})$$

In the case of a recursive closure, there are two new associations added to the stored closure. The first is, as with a simple closure, the parameter identifier ( $x_2$ ) and the argument ( $v_2$ ). The second is the function identifier ( $x_1$ ) and the closure itself. This ensures that the function body can call the recursive function again, since its closure is included in the environment.

$$\frac{\text{env} \vdash e_1 \Downarrow \langle x_1, x_2, e, \text{env} \rangle \quad \text{env} \vdash e_2 \Downarrow v_2}{\text{env} \vdash e_1 e_2 \Downarrow v} \quad (\text{BS-APPREC})$$

**Let Expressions** These expressions are used to associate an identifier to a specific value, allowing the value to be reused throughout the program. Since  $L1$  is a functional languages, these are not variables, and the values assigned to an identifier will be constant (unless the same identifier is used in a new *let* expression).

After evaluating the expression that is to be associated to the identifier (that is,  $e_1$ ), resulting in  $v$ , the *let* expression evaluates  $e_2$ . For this evaluation, the association of  $x$  to  $v$  is added to the environment. The result of this evaluation (that is,  $v_2$ ) is the final result of the evaluation of the entire *let* expression.

There are two variants of the *let* expression, one with explicit typing and one with implicit typing. For the purposes of evaluation, both have the same behavior.

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad \{x \rightarrow v\} \cup \text{env} \vdash e_2 \Downarrow v_2}{\text{env} \vdash \text{let } x : T = e_1 \text{ in } e_2 \Downarrow v_2} \quad (\text{BS-LET})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad \{x \rightarrow v\} \cup \text{env} \vdash e_2 \Downarrow v_2}{\text{env} \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v_2} \quad (\text{BS-LET2})$$

**Lists** The expression *nil* always evaluates to the value *nil*, which represents an empty list. The append operation ( $::$ ) accepts any value as its first operand ( $e_1$ ), but the second operand ( $e_2$ ) must evaluate to either the empty list (*nil*) or a non-empty list (represented by the value  $v_1 :: v_2$ ).

$$\text{env} \vdash \text{nil} \Downarrow \text{nil} \quad (\text{BS-NIL})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad \text{env} \vdash e_2 \Downarrow \text{nil}}{\text{env} \vdash e_1 :: e_2 \Downarrow v :: \text{nil}} \quad (\text{BS-LIST})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v \quad \text{env} \vdash e_2 \Downarrow v_1 :: v_2}{\text{env} \vdash e_1 :: e_2 \Downarrow v :: (v_1 :: v_2)} \quad (\text{BS-LIST2})$$

The *empty* expression returns *true* if and only if its operand ( $e_1$ ) evaluates to the empty list (*nil*).

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil}}{\text{env} \vdash \text{isempty } e_1 \Downarrow \text{true}} \quad (\text{BS-EMPTYTRUE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash \text{isempty } e_1 \Downarrow \text{false}} \quad (\text{BS-EMPTYFALSE})$$

The *head* expression attempts to obtain the first element of a non-empty list. The *tail* expression is the mirror of the *head* expression, removing the first element of the list and returning the remaining list. If the list evaluates to *nil*, a run-time exception is thrown for both of these expressions.

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash \text{hd } e_1 \Downarrow v_1} \quad (\text{BS-HEAD})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil}}{\text{env} \vdash \text{hd } e_1 \Downarrow \text{raise}} \quad (\text{BS-HEADEMPTY})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v_1 :: v_2}{\text{env} \vdash \text{tl } e_1 \Downarrow v_2} \quad (\text{BS-TAIL})$$

$$\frac{\text{env} \vdash e_1 \Downarrow \text{nil}}{\text{env} \vdash \text{tl } e_1 \Downarrow \text{raise}} \quad (\text{BS-TAILEMPTY})$$

**Exceptions** Some programs can be syntactically correct but still violate the semantics of the *L1* language, such as a dividing by zero or trying to access the head of an empty list. In these scenarios, the expression is evaluated as the *raise* value.

Besides violation of semantic rules, the only other expression that evaluates to the *raise* value is the *raise* expression, using the following rule:

$$\text{env} \vdash \text{raise} \Downarrow \text{raise} \quad (\text{BS-RAISE})$$

This value is propagated by (almost) all expressions, climbing up the evaluation tree. This means that, for every evaluation rule, there is an alternative rule that, when a sub-expression evaluates to *raise*, evaluates the whole expression to *raise*. To avoid cluttering this document with the repetition of rules, these are not shown in their entirety. Below are a few examples of these propagation rules:

$$\frac{\text{env} \vdash e_1 \Downarrow \text{raise}}{\text{env} \vdash \text{let } x : T = e_1 \text{ in } e_2 \Downarrow \text{raise}} \quad (\text{BS-LETRAISE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow n_1 \quad \text{env} \vdash e_2 \Downarrow \text{raise}}{\text{env} \vdash e_1 + e_2 \Downarrow \text{raise}} \quad (\text{BS-+RAISE})$$



The only expression that does not propagate the *raise* value is the *try* exception. Its evaluation rules are the following:

$$\frac{\text{env} \vdash e_1 \Downarrow \text{raise} \quad \text{env} \vdash e_2 \Downarrow v}{\text{env} \vdash \text{try } e_1 \text{ with } e_2 \Downarrow v} \quad (\text{BS-TRYRAISE})$$

$$\frac{\text{env} \vdash e_1 \Downarrow v}{\text{env} \vdash \text{try } e_1 \text{ with } e_2 \Downarrow v} \quad (\text{BS-TRY})$$

**Sequential Evaluation** The expression  $e_1 ; e_2$  is used when the expression  $e_1$  is only evaluated for its side effects, without any regard for its resulting value.

$$\frac{\text{env} \vdash e_1 \Downarrow \text{skip} \quad \text{env} \vdash e_2 \Downarrow v}{\text{env} \vdash e_1 ; e_2 \Downarrow v} \quad (\text{BS-SEQUENTIAL})$$

The value *skip* can be obtained either by the expression *skip*

$$\text{env} \vdash \text{skip} \Downarrow \text{skip} \quad (\text{BS-SKIP})$$

or by evaluating other expressions (such as *output e*, as shown below).

**Input and Output** Since both input and output deal with side effects, specifying their evaluation rules is tricky.

The *output* expression does not evaluate to any significant value, so we represent with the value *skip*.

$$\frac{\text{env} \vdash e \Downarrow \text{nil}}{\text{env} \vdash \text{output } e \Downarrow \text{skip}} \quad (\text{BS-OUTPUTEMPTY})$$

$$\frac{\text{env} \vdash e \Downarrow c :: v_1 \quad \text{env} \vdash \text{output } v_1 \Downarrow \text{skip}}{\text{env} \vdash \text{output } e \Downarrow \text{skip}} \quad (\text{BS-OUTPUT})$$

This cannot be represented in the semantic rules, but the value  $c$  that results from evaluating  $e$  in rule **BS-Output** is printed on the output stream (typically the console). The rule **BS-OutputEmpty** prints a new line character on the output stream, effectively making *output* similar to "printLine" found in other languages.

The *input* expression does evaluate to a significant value, but the value is non-deterministic, since it depends on the input of a user. The only guarantees that exist are that it will be either *nil* or  $c :: v_2$ , making this similar to "readLine" found in other languages.

$$\text{env} \vdash \text{input} \Downarrow v \quad (\text{BS-INPUT})$$

## 1.3 Type System

### 1.3.1 Type Inference Rules

$$\Gamma \vdash n : \text{Int} \mid [] \quad (\text{T-Num})$$

$$\Gamma \vdash b : \text{Bool} \mid [] \quad (\text{T-Bool})$$

$$\Gamma \vdash c : \text{Char} \mid [] \quad (\text{T-Char})$$

$$\Gamma \vdash \text{skip} : \text{Unit} \mid [] \quad (\text{T-Skip})$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T \mid []} \quad (\text{T-IDENT})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 + e_2 : \text{Int} \mid [T_1 = \text{Int}; T_2 = \text{Int}]} \quad (\text{T-+})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 - e_2 : \text{Int} \mid [T_1 = \text{Int}; T_2 = \text{Int}]} \quad (\text{T-})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 * e_2 : \text{Int} \mid [T_1 = \text{Int}; T_2 = \text{Int}]} \quad (\text{T-*})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 \div e_2 : \text{Int} \mid [T_1 = \text{Int}; T_2 = \text{Int}]} \quad (\text{T-}\div)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Equatable}}{\Gamma \vdash e_1 = e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-=})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Equatable}}{\Gamma \vdash e_1 \neq e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-}\neq)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Orderable}}{\Gamma \vdash e_1 < e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-<})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Orderable}}{\Gamma \vdash e_1 \leq e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-}\leq)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Orderable}}{\Gamma \vdash e_1 > e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-}>)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \in \text{Orderable}}{\Gamma \vdash e_1 \geq e_2 : \text{Bool} \mid [T_1 = T_2; X = T_2]} \quad (\text{T-}\geq)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 \vee e_2 : \text{Bool} \mid [T_1 = \text{Bool}; T_2 = \text{Bool}]} \quad (\text{T-}\vee)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 \wedge e_2 : \text{Bool} \mid [T_1 = \text{Bool}; T_2 = \text{Bool}]} \quad (\text{T-}\wedge)$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad \Gamma \vdash e_3 : T_3}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \text{Bool} \mid [T_1 = \text{Bool}; T_2 = T_3]} \quad (\text{T-If})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2 \quad X \text{ is new}}{\Gamma \vdash e_1 \ e_2 : X \mid [T_1 = T_2 \rightarrow X]} \quad (\text{T-App})$$

$$\frac{\{x \rightarrow T\} \cup \Gamma \vdash e : T_1}{\Gamma \vdash \text{fn } x : T \Rightarrow e : T \rightarrow T_1 \mid []} \quad (\text{T-Fn})$$

$$\frac{X \text{ is new} \quad \{x \rightarrow X\} \cup \Gamma \vdash e : T_1}{\Gamma \vdash \text{fn } x \Rightarrow e : X \rightarrow T_1 \mid []} \quad (\text{T-Fn2})$$

$$\frac{\{x_1 \rightarrow (T_1 \rightarrow T_2), x_2 \rightarrow T_1\} \cup \Gamma \vdash e : T_3}{\Gamma \vdash \text{rec } x_1 : T_1 \rightarrow T_2 \ x_2 : T_1 \Rightarrow e : T_1 \rightarrow T_2 \mid [T_3 = T_2]} \quad (\text{T-Rec})$$

$$\frac{X \text{ is new} \quad Y \text{ is new} \quad \{x_1 \rightarrow X, x_2 \rightarrow Y\} \cup \Gamma \vdash e : T}{\Gamma \vdash \text{rec } x_1 \ x_2 \Rightarrow e : Y \rightarrow T \mid [X = Y \rightarrow T]} \quad (\text{T-Rec2})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \{x \rightarrow T\} \cup \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{let } x : T = e_1 \text{ in } e_2 : T_2 \mid [T = T_1]} \quad (\text{T-Let})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \{x \rightarrow T_1\} \cup \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : T_2 \mid []} \quad (\text{T-LET2})$$

$$\frac{X \text{ is new}}{\Gamma \vdash \text{nil} : X \text{ list} \mid []} \quad (\text{T-NIL})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 :: e_2 : T_1 \text{ list} \mid [T_1 \text{ list} = T_2]} \quad (\text{T-LIST})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad X \text{ is new}}{\Gamma \vdash \text{isempty } e_1 : \text{Bool} \mid [T_1 = X \text{ list}]} \quad (\text{T-EMPTY})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad X \text{ is new}}{\Gamma \vdash \text{head } e_1 : X \mid [T_1 = X \text{ list}]} \quad (\text{T-HEAD})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad X \text{ is new}}{\Gamma \vdash \text{tail } e_1 : X \mid [T_1 = X \text{ list}]} \quad (\text{T-TAIL})$$

$$\frac{X \text{ is new}}{\Gamma \vdash \text{raise} : X \mid []} \quad (\text{T-RAISE})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{try } e_1 \text{ with } e_2 : T_2 \mid [T_1 = T_2]} \quad (\text{T-TRY})$$

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 ; e_2 : T_2 \mid [T_1 = \text{Unit}]} \quad (\text{T-SEQUENTIAL})$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \text{output } e : \text{Unit} \mid [T = \text{Char list}]} \quad (\text{T-OUTPUT})$$

$$\Gamma \vdash \text{input} : \text{Char list} \mid [] \quad (\text{T-INPUT})$$