

# 1. data

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## data 1

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In[1]:= S1 = {11 → 17, 17 → 19, 19 → 13, 13 → 11, 14 → 18, 18 → 16, 16 → 12, 12 → 14, 31 → 61, 61 → 43,
43 → 53, 53 → 31, 33 → 63, 63 → 41, 41 → 51, 51 → 33, 32 → 62, 62 → 42, 42 → 52, 52 → 32};
S2 = {34 → 64, 64 → 46, 46 → 56, 56 → 34, 35 → 65, 65 → 45, 45 → 55,
55 → 35, 36 → 66, 66 → 44, 44 → 54, 54 → 36};
S3 = {21 → 27, 27 → 29, 29 → 23, 23 → 21, 24 → 28, 28 → 26, 26 → 22, 22 → 24, 37 → 67, 67 → 49,
49 → 59, 59 → 37, 39 → 69, 69 → 47, 47 → 57, 57 → 39, 38 → 68, 68 → 48, 48 → 58, 58 → 38};
S4 = {51 → 57, 57 → 59, 59 → 53, 53 → 51, 54 → 58, 58 → 56, 56 → 52, 52 → 54, 11 → 31, 31 → 27,
27 → 47, 47 → 11, 17 → 37, 37 → 21, 21 → 41, 41 → 17, 14 → 34, 34 → 24, 24 → 44, 44 → 14};
S5 = {12 → 32, 32 → 28, 28 → 48, 48 → 12, 18 → 38, 38 → 22, 22 → 42,
42 → 18, 15 → 35, 35 → 25, 25 → 45, 45 → 15};
S6 = {61 → 67, 67 → 69, 69 → 63, 63 → 61, 64 → 68, 68 → 66, 66 → 62, 62 → 64, 13 → 33, 33 → 29,
29 → 49, 49 → 13, 19 → 39, 39 → 23, 23 → 43, 43 → 19, 16 → 36, 36 → 26, 26 → 46, 46 → 16};
S7 = {31 → 33, 33 → 39, 39 → 37, 37 → 31, 32 → 36, 36 → 38, 38 → 34, 34 → 32, 17 → 61, 61 → 29,
29 → 57, 57 → 17, 19 → 67, 67 → 27, 27 → 51, 51 → 19, 18 → 64, 64 → 28, 28 → 54, 54 → 18};
S8 = {14 → 62, 62 → 26, 26 → 58, 58 → 14, 16 → 68, 68 → 24, 24 → 52,
52 → 16, 15 → 65, 65 → 25, 25 → 55, 55 → 15};
S9 = {41 → 43, 43 → 49, 49 → 47, 47 → 41, 42 → 46, 46 → 48, 48 → 44, 44 → 42, 11 → 63, 63 → 23,
23 → 59, 59 → 11, 13 → 69, 69 → 21, 21 → 53, 53 → 13, 12 → 66, 66 → 22, 22 → 56, 56 → 12};
StartSet = {31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48,
49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69,
11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29};
Exch[list_] := Module[{len = Length[list[[1]]], per = {}},
  For[Exchi = 1, Exchi ≤ Length[list[[1]]], Exchi++, If[list[[1, Exchi]] ≠ list[[2, Exchi]],
    per = per ∪ {list[[1, Exchi]] → list[[2, Exchi]]}];
  per
]
IS1 = Exch[{StartSet, StartSet /. S1 /. S1 /. S1}];
IS2 = Exch[{StartSet, StartSet /. S2 /. S2 /. S2}];
IS3 = Exch[{StartSet, StartSet /. S3 /. S3 /. S3}];
IS4 = Exch[{StartSet, StartSet /. S4 /. S4 /. S4}];
IS5 = Exch[{StartSet, StartSet /. S5 /. S5 /. S5}];
IS6 = Exch[{StartSet, StartSet /. S6 /. S6 /. S6}];
IS7 = Exch[{StartSet, StartSet /. S7 /. S7 /. S7}];
IS8 = Exch[{StartSet, StartSet /. S8 /. S8 /. S8}];
IS9 = Exch[{StartSet, StartSet /. S9 /. S9 /. S9}];
prmt(*mypermutation*) =
  {S1, S2, S3, S4, S5, S6, S7, S8, S9, IS1, IS2, IS3, IS4, IS5, IS6, IS7, IS8, IS9};

T1 = Exch[{StartSet, StartSet /. S8 /. S3 /. IS8}](S8-1S3 S8*);
IT1 = Exch[{StartSet, StartSet /. T1 /. T1 /. T1}];
T2 = Exch[{StartSet, StartSet /. S7 /. S3 /. IS7}](S7-1S3 S7*);
IT2 = Exch[{StartSet, StartSet /. T2 /. T2 /. T2}];
T3 = Exch[{StartSet, StartSet /. S5 /. S3 /. IS5}](S5-1S3 S5*);
IT3 = Exch[{StartSet, StartSet /. T3 /. T3 /. T3}];
T4 = Exch[{StartSet, StartSet /. S5 /. S3 /. IS5 /. S3 /. S3 /. S5 /. S3 /. IS5}]
  (S5-1S3 S5 S32 S5-1S3 S5*);
IT4 = Exch[{StartSet, StartSet /. T4 /. T4}];
T5 = Exch[{StartSet, StartSet /. S6 /. S3 /. S3 /. IS6 /. S3 /. S6 /. S3 /. IS6}]
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(*S6-1S3 S6 S3 S6-1S32S6*);
IT5 = Exch[{StartSet, StartSet /. T5 /. T5 /. T5 /. T5 /. T5}];
T6 = Exch[{StartSet, StartSet /. S4 /. S5 /. S6 /. T4 /. IS4 /. IS5 /. IS6}]
(*S4-1S5-1S6-1S5-1S3 S5 S32 S5-1S3 S5 S4 S5 S6*);
IT6 = Exch[{StartSet, StartSet /. T6 /. T6}];
T7 = Exch[{StartSet,
  StartSet /. IS1 /. IS2 /. IS3 /. S4 /. S5 /. S6 /. T4 /. IS4 /. IS5 /. IS6 /. S1 /. S2 /. S3}]
(*S1 S2 S3 S4-1S5-1S6-1S5-1S3 S5 S32 S5-1S3 S5 S4 S5 S6 S1-1S2-1S3-1*);
IT7 = Exch[{StartSet, StartSet /. T7 /. T7}];
T8 =
  Exch[{StartSet, StartSet /. S1 /. S1 /. S2 /. S2 /. S3 /. S3 /. S4 /. S5 /. S6 /. T4 /. IS4 /.
    IS5 /. IS6 /. S1 /. S1 /. S2 /. S2 /. S3 /. S3}]
  (*S12 S22 S32 S4-1S5-1S6-1S5-1S3 S5 S32 S5-1S3 S5 S4 S5 S6 S12S22S32*);
IT8 = Exch[{StartSet, StartSet /. T8 /. T8}];
T9 = Exch[{StartSet, StartSet /. T8 /. T8 /. S3 /. T8}]
(*T8 S3 T8-1=S12 S22 S32 S4-1S5-1S6-1S5-1S3 S5 S32 S5-1S3 S5 S4 S5 S6 S12S22S32
  S3 S12S22S32S4-1S5-1S6-1S5-1S3-1S5 S32S5-1S3-1S5 S4 S5 S6 S12 S22 S32 *);
IT9 = Exch[{StartSet, StartSet /. T9 /. T9 /. T9}];
T10 = Exch[{StartSet, StartSet /. S6 /. S9 /. IS3 /. IS9 /. S3 /. IS6}]
(*S6-1S3 S9-1S3-1S9 S6*);
IT10 = Exch[{StartSet, StartSet /. T10 /. T10 /. T10 /. T10 /. T10}];
T11 = Exch[{StartSet,
  StartSet /. S6 /. S3 /. S3 /. IS6 /. S3 /. S6 /. IS3 /. IS6 /. S3 /. S6 /. S3 /. IS6}]
  (*S6-1S3 S6 S3 S6-1S3-1S6 S3 S6-1S32S6*);
IT11 = Exch[{StartSet, StartSet /. T11 /. T11}];
T12 = Exch[{StartSet, StartSet /. S6 /. S3 /. T4 /. T10 /. IS6}]
(*S6-1T10 T4 S3 S6=S6-1S6-1S3 S9-1S3-1S9 S6 S5-1S3 S5 S32 S5-1S3 S5 S3 S6*);
IT12 = Exch[{StartSet,
  StartSet /. T12 /. T12 /. T12 /. T12 /. T12 /. T12 /. T12 /. T12 /. T12 /. T12 /. T12}];
T13 = Exch[{StartSet, StartSet /. T12 /. T11}] (*T11 T12=S6-1S3 S6 S3 S6-1S3-1
  S6 S3 S6-1S32S6 S6-1S6-1S3 S9-1S3-1S9 S6 S5-1S3 S5 S32 S5-1S3 S5 S3 S6*);
IT13 = Exch[{StartSet, StartSet /. T13 /. T13 /. T13 /. T13 /. T13 /. T13 /. T13 /. T13 /. T13 /.
  T13 /. T13}];
T14 = Exch[{StartSet, StartSet /. T10 /. IS3}] (*S3-1T10=S3-1S6-1S3 S9-1S3-1S9 S6*);
IT14 = Exch[{StartSet,
  StartSet /. T14 /. T14 /. T14 /. T14 /. T14 /. T14 /. T14 /. T14 /. T14 /. T14}];
T15 = Exch[{StartSet, StartSet /. S3 /. S3 /. T14 /. T5}]
(*T5 T14 S32=S6-1S3 S6 S3 S6-1S32S6 S3-1S6-1S3 S9-1S3-1S9 S6 S32*);
IT15 = Exch[{StartSet,
  StartSet /. T15 /. T15 /. T15 /. T15 /. T15 /. T15 /. T15 /. T15 /. T15 /. T15}];
T16 = Exch[{StartSet, StartSet /. T5 /. T5 /. T12}] (*T12 T52=S6-1S6-1S3 S9-1S3-1S9
  S6 S5-1S3 S5 S32 S5-1S3 S5 S3 S6 S6-1S3 S6 S3 S6-1S32S6 S6-1S3 S6 S3 S6-1S32S6=
  S6-1S6-1S3 S9-1S3-1S9 S6 S5-1S3 S5 S32 S5-1S3 S5 S3 S3 S6 S3 S6-1S3-1 S6 S3 S6-1S32S6*);
IT16 = Exch[{StartSet, StartSet /. T16 /. T16 /. T16 /. T16 /. T16 /. T16 /. T16 /. T16 /. T16 /.
  T16 /. T16}];
(*IT13=Exch[{StartSet,StartSet/.IT11/.IT12}];
IT14=Exch[{StartSet,StartSet/.S3/.IT10}];
IT15=Exch[{StartSet,StartSet/.IT5/.IT14/.S3/.S3}];
IT16=Exch[{StartSet,StartSet/.IT12/.IT5/.IT5}];*)
T17 = Exch[{StartSet, StartSet /. T16 /. IT13}] (*T13-1T16=T12-1T11-1T12 T52*);
IT17 = Exch[{StartSet, StartSet /. T17 /. T17 /. T17 /. T17 /. T17}];
T18 = Exch[{StartSet, StartSet /. IT15 /. T14}] (*T14 T15-1=S3-1T10 S32 T14-1T5-1*);
IT18 = Exch[{StartSet, StartSet /. T18 /. T18}];
T19 = Exch[{StartSet, StartSet /. T14 /. T14 /. IT13}]
(*T13-1T142=T12-1T11-1S3-1T10 S3-1T10*);
IT19 = Exch[{StartSet, StartSet /. T19 /. T19 /. T19}];
T20 = Exch[{StartSet, StartSet /. T19 /. T19}]

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(*T192=T12-1T11-1S3-1T10 S3-1T10 T12-1T11-1S3-1T10 S3-1T10*);
IT20 = Exch[{StartSet, StartSet /. T20}];
T21 = Exch[{StartSet, StartSet /. T17 /. T17 /. T17}];
(*T173=T12-1T11-1T12 T52T12-1T11-1T12 T52T12-1T11-1T12 T52*);
IT21 = Exch[{StartSet, StartSet /. T21}];
T22 = Exch[
  {StartSet, StartSet /. S1 /. S1 /. S2 /. S2 /. S3 /. S3 /. T4 /. S1 /. S1 /. S2 /. S2 /. S3 /.
    S3}]( *S12S22S32T4 S12S22S32*);
IT22 = Exch[{StartSet, StartSet /. T22 /. T22}];
T23 = Exch[{StartSet, StartSet /. T4 /. T20 /. T4 /. T22}];
(*S12S22S32T4 S12S22S32 T4 T20 T4*);
IT23 = Exch[{StartSet, StartSet /. T23}];
T24 = Exch[{StartSet, StartSet /. IS5 /. T20 /. S5 /. S5 /. T3 /. S3 /. T3 /. T3 /. T21}];
(*T21 T32S3 T3 S52T20 S5-1*);
IT24 = Exch[{StartSet, StartSet /. T24 /. T24 /. T24}];
T25 = Exch[{StartSet, StartSet /. S5 /. S5 /. S2 /. S5 /. S5 /. IS2}]( *S2-1S52S2 S52*);
IT25 = Exch[{StartSet, StartSet /. T25}];
T26 = Exch[{StartSet, StartSet /. S2 /. S2 /. S5 /. S2 /. S2 /. IS5}]( *S5-1S22S5 S22*);
IT26 = Exch[{StartSet, StartSet /. T26}];
T27 = Exch[{StartSet, StartSet /. IS1 /. IS2 /. S1 /. S5 /. S2 /. IS5}];
(*S5-1S2 S5 S1 S2-1S1-1*);
IT27 = Exch[{StartSet, StartSet /. T27 /. T27}];

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## data 2

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In[76]:= llstrttSt = {11, 12, 13, 14, 16, 17, 18, 19, 34, 36, 44, 46, 21, 23, 27, 22, 24, 26, 15, 35};
TrnsfrmStfrGrph = {S1 ∪ S2 ∪ S3 ∪ S4 ∪ S5 ∪ S6 ∪ S7 ∪ S8 ∪ S9, S2 ∪ S3 ∪ S5 ∪ S6 ∪ S7 ∪ S8,
  S2 ∪ S3 ∪ S6 ∪ S7 ∪ S8, S2 ∪ S3 ∪ S7 ∪ S8, S2 ∪ S3 ∪ S7 ∪ T1, S2 ∪ S3 ∪ S7, S2 ∪ S3 ∪ T2 ∪ T3,
  S2 ∪ S3 ∪ T2 ∪ T4, S2 ∪ S3 ∪ T4 ∪ T5 ∪ T6, S3 ∪ T4 ∪ T5 ∪ T7 ∪ T8, S3 ∪ T4 ∪ T5 ∪ T8,
  S3 ∪ T4 ∪ T5 ∪ T9, S3 ∪ T4 ∪ T5 ∪ T10 ∪ T11 ∪ T12, T4 ∪ T13 ∪ T14 ∪ T15 ∪ T16,
  T4 ∪ T17 ∪ T18 ∪ T19, T4 ∪ T20 ∪ T21, T22 ∪ T23, T23 ∪ T24, T25 ∪ T26 ∪ T27, T25};
TrnsfrmStfrPrmttn = {{S1, S2, S3, S4, S5, S6, S7, S8, S9}, {S2, S3, S5, S6, S7, S8},
  {S2, S3, S6, S7, S8}, {S2, S3, S7, S8}, {S2, S3, S7, T1}, {S2, S3, S7}, {S2, S3, T2, T3},
  {S2, S3, T2, T4}, {S2, S3, T4, T5, T6}, {S3, T4, T5, T7, T8}, {S3, T4, T5, T8},
  {S3, T4, T5, T9}, {S3, T4, T5, T10, T11, T12}, {T4, T13, T14, T15, T16},
  {T4, T17, T18, T19}, {T4, T20, T21}, {T22, T23}, {T23, T24}, {T25, T26, T27}, {T25}};
TrnsfrmStfrshrtPrnt = {"S1", "S2", "S3", "S4", "S5", "S6", "S7", "S8", "S9"},
  {"S2", "S3", "S5", "S6", "S7", "S8"}, {"S2", "S3", "S6", "S7", "S8"},
  {"S2", "S3", "S7", "S8"}, {"S2", "S3", "S7", "T1"}, {"S2", "S3", "S7"},
  {"S2", "S3", "T2", "T3"}, {"S2", "S3", "T2", "T4"}, {"S2", "S3", "T4", "T5", "T6"},
  {"S3", "T4", "T5", "T7", "T8"}, {"S3", "T4", "T5", "T8"},
  {"S3", "T4", "T5", "T9"}, {"S3", "T4", "T5", "T10", "T11", "T12"},
  {"T4", "T13", "T14", "T15", "T16"}, {"T4", "T17", "T18", "T19"}, {"T4", "T20", "T21"},
  {"T22", "T23"}, {"T23", "T24"}, {"T25", "T26", "T27"}, {"T25}};
TrnsfrmStfrlngPrnt = {"S1", "S2", "S3", "S4", "S5", "S6", "S7", "S8", "S9"},
  {"S2", "S3", "S5", "S6", "S7", "S8"}, {"S2", "S3", "S6", "S7", "S8"},
  {"S2", "S3", "S7", "S8"}, {"S2", "S3", "S7", "S8 S3 S8-1"},
  {"S2", "S3", "S7"}, {"S2", "S3", "S7 S3 S7-1", "S5 S3 S5-1"},
  {"S2", "S3", "S7 S3 S7-1", "S5 S3 S5-1S32S5 S3 S5-1"},
  {"S2", "S3", "S5 S3 S5-1S32S5 S3 S5-1", "S6 S32S6-1S3 S6 S3 S6-1",
    "S4 S5 S6 S5 S3 S5-1S32S5 S3 S5-1S6-1S5-1S4-1"},
  {"S3", "S5 S3 S5-1S32S5 S3 S5-1", "S6 S32S6-1S3 S6 S3 S6-1",
    "S1-1S2-1S3-1S4 S5 S6 S5 S3 S5-1S32S5 S3 S5-1S6-1S5-1S4-1S1 S2 S3",
    "S12S22S32S4 S5 S6 S5 S3 S5-1S32S5 S3 S5-1S6-1S5-1S4-1S12S22S32"},

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{"S3", "S5 S3 S5-1S32S5 S3 S5-1", "S6 S32S6-1S3 S6 S3 S6-1",
 "S12S22S32S4 S5 S6 S5 S3 S5-1S32S5 S3 S5-1S6-1S5-1S4-1S12S22S32"},
{"S3", "S5 S3 S5-1S32S5 S3 S5-1", "S6 S32S6-1S3 S6 S3 S6-1",
 "S12 S22 S32S4 S5 S6 S5 S3-1S5-1S32S5 S3-1S5-1S4-1S5-1S6-1S12S22S32S3
 S12S22S32 S4 S5 S6 S5 S3 S5-1S32 S5 S3 S5-1 S4-1S5-1S6-1S12 S22 S32"},
{"S3", "S5 S3 S5-1S32S5 S3 S5-1", "S6 S32S6-1S3 S6 S3 S6-1", "S6 S9 S3-1S9-1S3 S6-1",
 "S6 S32S6-1S3 S6 S3-1S6-1S3 S6 S3 S6-1",
 "S6 S3 S5 S3 S5-1S32 S5 S3 S5-1S6 S9 S3-1S9-1S3 S6-1S6-1"},
{"S5 S3 S5-1S32S5 S3 S5-1", "T12 T11", "T10 S3-1", "S32T14 T5", "T52T12"},
{"S5 S3 S5-1S32S5 S3 S5-1", "T52T12 T11-1T12-1",
 "T5-1T14-1S32T10 S3-1", "T10 S3-1T10 S3-1T11-1T12-1"},
{"S5 S3 S5-1S32S5 S3 S5-1", "T10 S3-1T10 S3-1T11-1T12-1T10 S3-1T10 S3-1T11-1T12-1",
 "T52T12 T11-1T12-1T52T12 T11-1T12-1T52T12 T11-1T12-1"}, {"S12S22S32T4 S12S22S32",
 "T4 T10 S3-1T10 S3-1T11-1T12-1T10 S3-1T10 S3-1T11-1T12-1 T4 S12S22S32T4 S12S22S32"},
{"T4 T10 S3-1T10 S3-1T11-1T12-1T10 S3-1T10 S3-1T11-1T12-1 T4 S12S22S32T4 S12S22S32",
 "T52T12 T11-1T12-1T52T12 T11-1T12-1T52T12 T11-1T12-1 T32S3 T3
 S52T10 S3-1T10 S3-1T11-1T12-1T10 S3-1T10 S3-1T11-1T12-1 S5-1"},
{"S2-1S52S2 S52", "S5-1S22S5 S22", "S1-1S2-1S1 S5 S2 S5-1"}, {"S2-1S52S2 S52"}};
TrnsfrmStfrrlst = {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}},
{{2}, {3}, {5}, {6}, {7}, {8}}, {{2}, {3}, {6}, {7}, {8}}, {{2}, {3}, {7}, {8}},
{{2}, {3}, {7}, {8, 3, 17}}, {{2}, {3}, {7}}, {{2}, {3}, {7, 3, 16}}, {5, 3, 14}},
{{2}, {3}, {7, 3, 16}}, {5, 3, 14, 3, 3, 5, 3, 14}}, {{2}, {3}, {5, 3, 14, 3, 3, 5, 3, 14}},
{6, 3, 3, 15, 3, 6, 3, 15}}, {4, 5, 6, 5, 3, 14, 3, 3, 5, 3, 14, 15, 14, 13}},
{{3}, {5, 3, 14, 3, 3, 5, 3, 14}}, {6, 3, 3, 15, 3, 6, 3, 15}},
{10, 11, 12, 4, 5, 6, 5, 3, 14, 3, 3, 5, 3, 14, 15, 14, 13, 1, 2, 3},
{1, 1, 2, 2, 3, 3, 4, 5, 6, 5, 3, 14, 3, 3, 5, 3, 14, 15, 14, 13, 1, 1, 2, 2, 3, 3}},
{{3}, {5, 3, 14, 3, 3, 5, 3, 14}}, {6, 3, 3, 15, 3, 6, 3, 15}},
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{{3}, {5, 3, 14, 3, 3, 5, 3, 14}}, {6, 3, 3, 15, 3, 6, 3, 15}}, {6, 9, 12, 18, 3, 15}}, {6, 3, 3,
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 15, 15, 6, 3, 3, 15, 3, 6, 12, 15, 3, 6, 3, 15}}, {6, 9, 12, 18, 3, 15, 12}},
{3, 3, 6, 9, 12, 18, 3, 15, 12, 6, 3, 3, 15, 3, 6, 3, 15}}, {6, 3, 3, 15, 3, 6, 3, 15,
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{{5, 3, 14, 3, 3, 5, 3, 14}}, {6, 3, 3, 15, 3, 6, 3, 15, 6, 3, 3, 15, 3, 6, 3, 15,
 6, 3, 5, 3, 14, 3, 3, 5, 3, 14, 6, 9, 12, 18, 3, 15, 15, 6, 12, 15, 12, 6, 3, 15,
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{6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12, 15, 12, 6, 3, 15, 12,
 6, 12, 12, 15, 6, 6, 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12, 14, 12, 15}},
{{5, 3, 14, 3, 3, 5, 3, 14}}, {6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6,
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 12, 14, 12, 15, 6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12, 15, 12, 6, 3,
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 6, 3, 15, 12, 6, 12, 12, 15, 6, 6, 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12, 14, 12, 15}},
{{1, 1, 2, 2, 3, 3, 5, 3, 14, 3, 3, 5, 3, 14, 1, 1, 2, 2, 3, 3}},

```

```

{5, 3, 14, 3, 3, 5, 3, 14, 6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12, 15,
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 3, 14, 3, 3, 5, 3, 14, 1, 1, 2, 2, 3, 3, 5, 3, 14, 3, 3, 5, 3, 14, 1, 1, 2, 2, 3, 3}},
{{5, 3, 14, 3, 3, 5, 3, 14, 6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12,
 15, 12, 6, 3, 15, 12, 6, 12, 12, 15, 6, 6, 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12,
 14, 12, 15, 6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12, 15, 12, 6, 3, 15,
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 3, 14, 3, 3, 5, 3, 14, 1, 1, 2, 2, 3, 3, 5, 3, 14, 3, 3, 5, 3, 14, 1, 1, 2, 2, 3, 3}},
{14, 6, 9, 12, 18, 3, 15, 12, 6, 9, 12, 18, 3, 15, 12, 6, 12, 15, 12, 6, 3, 15, 12, 6,
 12, 12, 15, 6, 6, 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12, 14, 12, 15, 6, 9, 12, 18, 3,
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 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12, 14, 12, 15, 6, 3, 3, 15, 3, 6, 3, 15, 6, 3, 3,
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 3, 15, 12, 6, 12, 12, 15, 6, 6, 12, 9, 3, 18, 15, 5, 12, 14, 12, 12, 5, 12, 14, 12, 15}},
{{11, 5, 5, 2, 5, 5}, {14, 2, 2, 5, 2, 2}, {10, 11, 1, 5, 2, 14}}, {{11, 5, 5, 2, 5, 5}}};

```

---

## data 3

```

In[82]:= square = {{{Glow[Blue], Black, Cuboid[{0, 2, 3}, {1, 3, 3.001}]},
  {Glow[Blue], Black, Cuboid[{1, 2, 3}, {2, 3, 3.001}]},
  {Glow[Blue], Black, Cuboid[{2, 2, 3}, {3, 3, 3.001}]},
  {Glow[Blue], Black, Cuboid[{0, 1, 3}, {1, 2, 3.001}]},
  {Glow[Blue], Black, Cuboid[{1, 1, 3}, {2, 2, 3.001}]},
  {Glow[Blue], Black, Cuboid[{2, 1, 3}, {3, 2, 3.001}]},
  {Glow[Blue], Black, Cuboid[{0, 0, 3}, {1, 1, 3.001}]},
  {Glow[Blue], Black, Cuboid[{1, 0, 3}, {2, 1, 3.001}]},
  {Glow[Blue], Black, Cuboid[{2, 0, 3}, {3, 1, 3.001}]},
  {{Glow[Yellow], Black, Cuboid[{0, 2, 0}, {1, 3, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{1, 2, 0}, {2, 3, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{2, 2, 0}, {3, 3, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{0, 1, 0}, {1, 2, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{1, 1, 0}, {2, 2, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{2, 1, 0}, {3, 2, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{0, 0, 0}, {1, 1, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{1, 0, 0}, {2, 1, 0.001}]},
  {Glow[Yellow], Black, Cuboid[{2, 0, 0}, {3, 1, 0.001}]},
  {{Glow[Green], Black, Cuboid[{0, 0, 2}, {1, 0.001, 3}]},
  {Glow[Green], Black, Cuboid[{1, 0, 2}, {2, 0.001, 3}]},
  {Glow[Green], Black, Cuboid[{2, 0, 2}, {3, 0.001, 3}]},
  {Glow[Green], Black, Cuboid[{0, 0, 1}, {1, 0.001, 2}]},
  {Glow[Green], Black, Cuboid[{1, 0, 1}, {2, 0.001, 2}]},
  {Glow[Green], Black, Cuboid[{2, 0, 1}, {3, 0.001, 2}]},
  {Glow[Green], Black, Cuboid[{0, 0, 0}, {1, 0.001, 1}]},
  {Glow[Green], Black, Cuboid[{1, 0, 0}, {2, 0.001, 1}]},
  {Glow[Green], Black, Cuboid[{2, 0, 0}, {3, 0.001, 1}]},
  {{Glow[Purple], Black, Cuboid[{0, 3, 2}, {1, 3.001, 3}]},
  {Glow[Purple], Black, Cuboid[{1, 3, 2}, {2, 3.001, 3}]},
  {Glow[Purple], Black, Cuboid[{2, 3, 2}, {3, 3.001, 3}]},
  {Glow[Purple], Black, Cuboid[{0, 3, 1}, {1, 3.001, 2}]},
  {Glow[Purple], Black, Cuboid[{1, 3, 1}, {2, 3.001, 2}]},
  {Glow[Purple], Black, Cuboid[{2, 3, 1}, {3, 3.001, 2}]},
  {Glow[Purple], Black, Cuboid[{0, 3, 0}, {1, 3.001, 1}]},
  {Glow[Purple], Black, Cuboid[{1, 3, 0}, {2, 3.001, 1}]},
  {Glow[Purple], Black, Cuboid[{2, 3, 0}, {3, 3.001, 1}]},
  {{Glow[Red], Black, Cuboid[{0, 0, 2}, {0.001, 1, 3}]},
  {Glow[Red], Black, Cuboid[{0, 1, 2}, {0.001, 2, 3}]},
  {Glow[Red], Black, Cuboid[{0, 2, 2}, {0.001, 3, 3}]},
  {Glow[Red], Black, Cuboid[{0, 0, 1}, {0.001, 1, 2}]},
  {Glow[Red], Black, Cuboid[{0, 1, 1}, {0.001, 2, 2}]},
  {Glow[Red], Black, Cuboid[{0, 2, 1}, {0.001, 3, 2}]},
  {Glow[Red], Black, Cuboid[{0, 0, 0}, {0.001, 1, 1}]},
  {Glow[Red], Black, Cuboid[{0, 1, 0}, {0.001, 2, 1}]},
  {Glow[Red], Black, Cuboid[{0, 2, 0}, {0.001, 3, 1}]},
  {{Glow[Orange], Black, Cuboid[{3, 0, 2}, {3.001, 1, 3}]},
  {Glow[Orange], Black, Cuboid[{3, 1, 2}, {3.001, 2, 3}]},
  {Glow[Orange], Black, Cuboid[{3, 2, 2}, {3.001, 3, 3}]},
  {Glow[Orange], Black, Cuboid[{3, 0, 1}, {3.001, 1, 2}]},
  {Glow[Orange], Black, Cuboid[{3, 1, 1}, {3.001, 2, 2}]},
  {Glow[Orange], Black, Cuboid[{3, 2, 1}, {3.001, 3, 2}]},
  {Glow[Orange], Black, Cuboid[{3, 0, 0}, {3.001, 1, 1}]},
  {Glow[Orange], Black, Cuboid[{3, 1, 0}, {3.001, 2, 1}]},
  {Glow[Orange], Black, Cuboid[{3, 2, 0}, {3.001, 3, 1}]}}};

```

## 2. function

In[83]:= (\*提取变换前后的集合\*)

```

ExtractSet[StartSet_, Tans_] :=
Module[{StrtSt = StartSet, ndSt = {}, myi = 1}, ndSt = StrtSt /. Tans;
While[myi ≤ Length[StrtSt], If[StrtSt[[myi]] == ndSt[[myi]],
StrtSt = Delete[StrtSt, myi]; ndSt = Delete[ndSt, myi]; myi--; myi++];
{StrtSt, ndSt}
];

Vsl[StartP_] := Module[{nP = {}, pthfrshrtPrnt = {},
pthfrlngPrnt = {}, pthfrlst = {}, g = {}, pth = {}, lst = {}},
nP = StartP;
For[i = 1, i ≤ Length[l1strtSt], i++, g = Graph[TrnsfrmStfrGrph[[i]],
VertexLabels → "Name", GraphStyle → "SmallNetwork", VertexSize → Small,
DirectedEdges → True, GraphLayout → "CircularEmbedding", ImageSize → 450];
pth = FindShortestPath[g, l1strtSt[[i]] /. nP, l1strtSt[[i]]]; (*用于消元的路径*)
lst = If[pth ≠ {},
Flatten[Table[Select[Table[ifrntrgr, {ifrntrgr, Length[TrnsfrmStfrPrmttn[[i]]}],
(pth[[ifrpth]] /. TrnsfrmStfrPrmttn[[i, #]] == pth[[ifrpth + 1]] &,
1], {ifrpth, Length[pth] - 1}], {}];
(*lst: nP /. TrnsfrmStfrPrnt[[i, lst1]] /. TrnsfrmStfrPrnt[[i, lst2]] ... *)
For[ifrnP = 1, ifrnP ≤ Length[lst], ifrnP++,
nP = Exch[{StartSet, StartSet /. nP /. TrnsfrmStfrPrmttn[[i, lst[[ifrnP]]]}];
AppendTo[pthfrshrtPrnt, TrnsfrmStfrshrtPrnt[[i, lst[[ifrnP]]]];
AppendTo[pthfrlngPrnt, TrnsfrmStfrlngPrnt[[i, lst[[ifrnP]]]];
AppendTo[pthfrlst, TrnsfrmStfrlst[[i, lst[[ifrnP]]]]];
];
Flatten[pthfrlst]
];

pdtVsl[StartP_] := Module[{lst = Vsl[StartP], ifrlst = 1, nwlst = {}, bl = True},
While[bl,
While[ifrlst ≤ Length[lst],
nwlst = Vsl[PermutateMultiply[StartSet /. StartP, lst[[1 ;; ifrlst]]];
If[Length[nwlst] < Length[lst] - ifrlst,
lst = Flatten[{lst[[1 ;; ifrlst]], nwlst}], ifrlst++];
];
If[lst == Dlt[lst], bl = False];
If[bl, While[lst != Dlt[lst], lst = Dlt[lst]]];
];
lst
];

TS[ifrs_] [sqr_, nm_] := Module[{sq = sqr, lst = {}},
If[ifrs == 1 || ifrs == 10,
lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪ {{1, 5}}];
If[ifrs == 3 || ifrs == 12, lst =
IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪ {{2, 5}}];
If[ifrs == 4 || ifrs == 13, lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪
{{5, 5}}];
If[ifrs == 6 || ifrs == 15, lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪
{{6, 5}}];
If[ifrs == 7 || ifrs == 16, lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪
{{3, 5}}];
If[ifrs == 9 || ifrs == 18, lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]] ∪
{{4, 5}}];
If[ifrs == 2 || ifrs == 5 || ifrs == 8 || ifrs == 11 || ifrs == 14 || ifrs == 17,
lst = IntegerDigits[ExtractSet[StartSet, prmt[[ifrs]]][[1]]];
Do[sq[[lst[[ifrd, 1]], lst[[ifrd, 2]], 3]] =

```

```

Rotate[sqr[[lst[[ifrd, 1]], lst[[ifrd, 2]], 3]], Sign[9.5 - ifrs] 90 nm Degree,
  {Boole[4 ≤ ifrs ≤ 6 || 13 ≤ ifrs ≤ 15], Boole[7 ≤ ifrs ≤ 9 || 16 ≤ ifrs ≤ 18],
    Boole[1 ≤ ifrs ≤ 3 || 10 ≤ ifrs ≤ 12]}, { $\frac{3}{2}$ ,  $\frac{3}{2}$ ,  $\frac{3}{2}$ }}, {ifrd, Length[lst]};
sq];
PermutateMultiply[Srtst_, List_] := Module[{Cntr = Srtst, ndSt = {}},
  For[i = 1, i ≤ Length[List], i++, Cntr = Cntr /. prmt[[List[[i]]]];
  ndSt = Exch[{StartSet, Cntr}]; (*Srtst:StartSet, List: AntTrace[...]*)
TImdtl[sqr_, prmttn_] := Module[{sq = sqr, keep = {}, xtrctSt = {}, i = 1, j = 1},
  xtrctSt = IntegerDigits[ExtractSet[StartSet, prmttn]];
  For[i = 1, i ≤ Length[xtrctSt[[1]]], i++,
    AppendTo[keep, sq[[xtrctSt[[1, i, 1]], xtrctSt[[1, i, 2]], 1]]];
  For[i = 1, i ≤ Length[xtrctSt[[1]]], i++,
    sq[[xtrctSt[[2, i, 1]], xtrctSt[[2, i, 2]], 1] = keep[[i]]];
  sq];
TCntnS[sqr_, lst_, nm_] :=
  Module[{sq = sqr, ifrTImdtl = Floor[nm], nmfrTS = nm - Floor[nm], prmttn = {}},
    prmttn = PermutateMultiply[StartSet, lst[[1 ;; ifrTImdtl]]];
    sq = TImdtl[sq, prmttn];
    sq = TS[lst[[Min[ifrTImdtl + 1, Length[lst]]]]][sq, nmfrTS];
    sq
  ];
Dltlst[lst_] := Module[
  {ls = lst, ilst = Select[DeleteDuplicates[lst, Mod[#1 - #2, 9] == 0 &, # ≤ 9 &], ifr = 1},
  While[ifr ≤ Length[ilst],
    If[Position[ls, ilst[[ifr]]] ≠ {} && Position[ls, ilst[[ifr]] + 9] ≠ {},
      ls = Delete[ls,
        {First[Position[ls, ilst[[ifr]]]], First[Position[ls, ilst[[ifr]] + 9]]}], ifr++
    ];
  ];
  ls
];
Dlt[lst_] := Module[{ifrDlt = 1, ls =
  Flatten[Dltlst /@ Sort /@ Split[lst, IntegerPart[If[#1 > 9, (#1 - 10) / 3, (#1 - 1) / 3]] ==
    IntegerPart[If[#2 > 9, (#2 - 10) / 3, (#2 - 1) / 3]] &]]],
  While[ifrDlt < Length[ls],
    If[Abs[ls[[ifrDlt]] - ls[[ifrDlt + 1]]] == 9,
      ls = Delete[ls, {{ifrDlt}, {ifrDlt + 1}}]; ifrDlt--;
    ifrDlt++;
  ];
  ifrDlt = 1;
  While[ifrDlt < Length[ls] - 1,
    If[ls[[ifrDlt]] == ls[[ifrDlt + 1]] == ls[[ifrDlt + 2]],
      ls = Delete[ls, {{ifrDlt + 1}, {ifrDlt + 2}}];
    If[ls[[ifrDlt]] ≤ 9, ls[[ifrDlt]] += 9, ls[[ifrDlt]] -= 9]; ifrDlt--;
    ifrDlt++;
  ];
  ls
];

```

### 3. Main Process



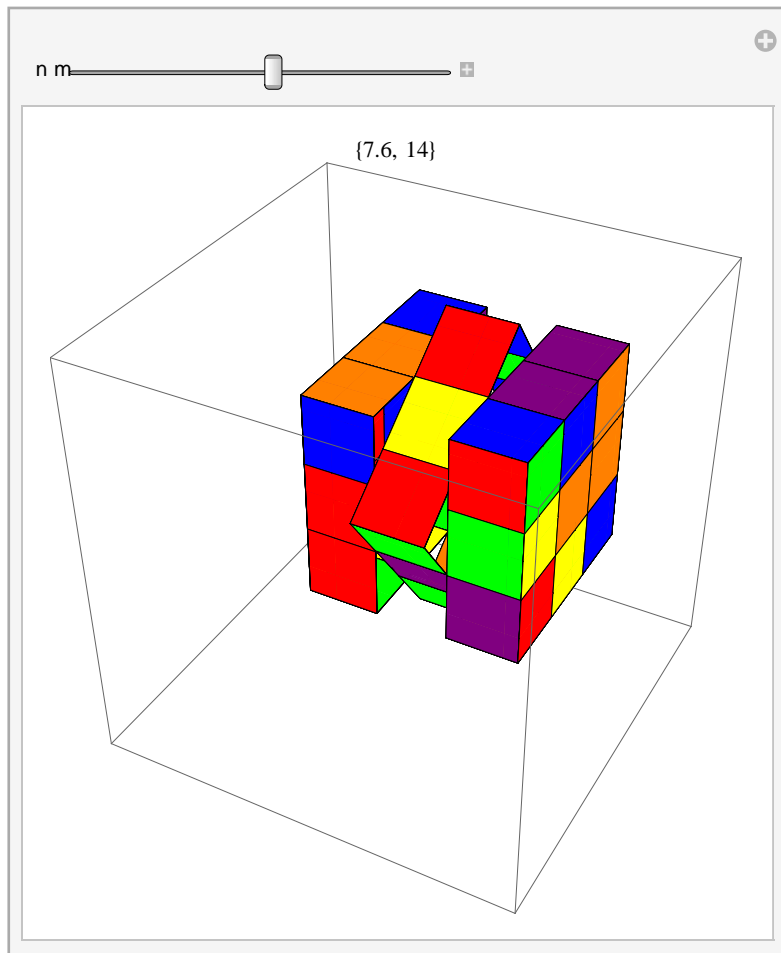
## First Process for each step

```

In[92]:= StartP = Exch[{StartSet, StartSet /. S8 /. S5 /. S1 /. S2}];
list = Vsl[StartP];
While[list != Dlt[list], list = Dlt[list]];
Manipulate[Pause[0.1]; Graphics3D[
  TChnS[TImdtl[square, StartP], list, nm], PlotRange → {{-2, 4}, {-2, 4}, {-2, 4}},
  PlotLabel → {nm, Length[list]}], {nm, 0, Length[list], 0.1}]

```

Out[95]=



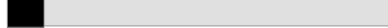
## Second Process which is Automatic

```

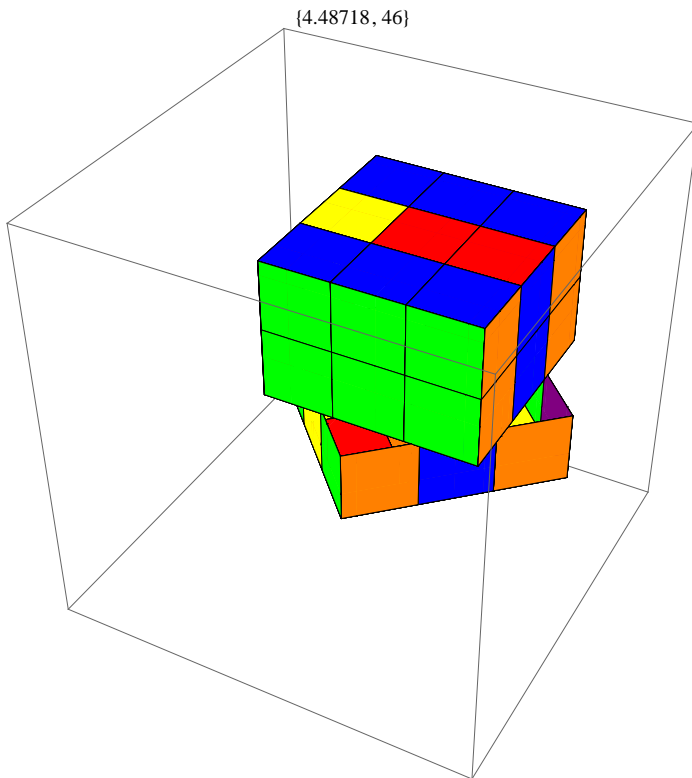
In[103]:= StartP = Exch[{StartSet, StartSet /. s8 /. s3 /. s8 /. s2 /. s5 /. s7 /. s8 /. s1 /. s9 /. s6}];
nm = 0;
list = Vsl[StartP];
While[list != Dlt[list], list = Dlt[list]];
ProgressIndicator[Dynamic[nm / Length[list]]]
Dynamic[Graphics3D[TCntnS[TImdtl[square, StartP], list, nm],
  PlotRange -> {{-2, 4}, {-2, 4}, {-2, 4}}, PlotLabel -> {nm, Length[list]}]]
Mouseover[stop,
  Table[With[{i = i}, nm = i; Pause[0.01]], {i, 0, Length[list], 0.1}]]

```

Out[107]=



Out[108]=



Out[109]= \$Aborted

## Third Process which is too much Manipulate

```

In[117]:= Dynamic[t = MousePosition["WindowScaled"]]
Dynamic[t]

```

Out[117]= {0.506429, 0.460905}

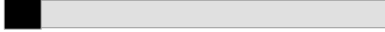
Out[118]= {0.506429, 0.460905}

```

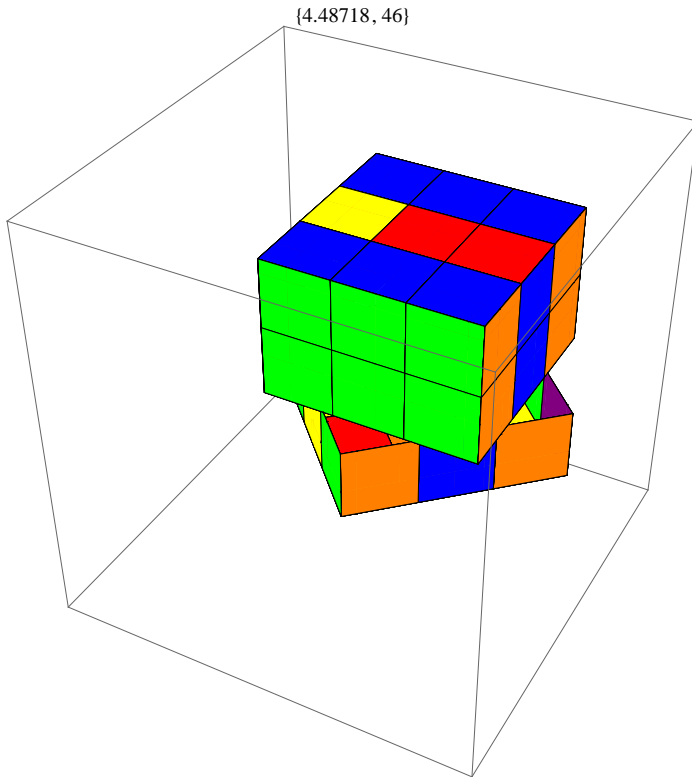
In[119]:= StartP = Exch[{StartSet, StartSet /. S8 /. S3 /. IS8 /. S3}];
nm = 0;
list = Vsl[StartP];
While[list != Dlt[list], list = Dlt[list]];
ProgressIndicator[Dynamic[nm / Length[list]]]
Dynamic[Graphics3D[TCntnS[TImdtl[square, StartP], list, nm],
  PlotRange → {{-2, 4}, {-2, 4}, {-2, 4}}, PlotLabel → {nm, Length[list]}]]
Mouseover[Button[Dynamic[nm = Min[Max[nm + (2 t[[1]] - 1), 0], Length[list]]],
  Button["Stop"]]

```

Out[123]=



Out[124]=



Out[125]=

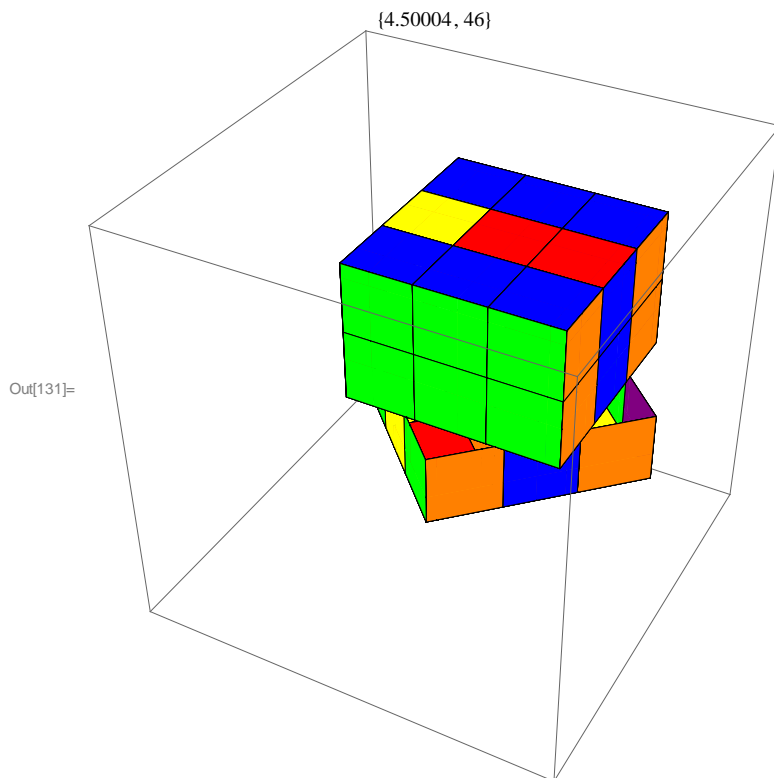
4 . 5 0
---------

## Forth Process which is updated from last one

```
In[126]:= StartP = Exch[{StartSet, StartSet /. S8 /. S3 /. IS8 /. S3 /. S6 /. S1}];
nm = 0;
list = pdtVsl[StartP];
Print[Length[Vsl[StartP]], "->", Length[list]];
ProgressIndicator[Dynamic[nm / Length[list]]]
Dynamic[Graphics3D[TCntnS[TImdtl[square, StartP], list, nm],
  PlotRange -> {{-2, 4}, {-2, 4}, {-2, 4}}, PlotLabel -> {nm, Length[list]}]
Mouseover[Button[Dynamic[nm = Min[Max[nm + (2 t[[1]] - 1), 0], Length[list]]],
  Button["Stop"]]
```

66->46

Out[130]= 



Out[132]= 

```
Dynamic[t = MousePosition["WindowScaled"]]
Dynamic[t]
{0.839763, 0.394663}
{0.839763, 0.394663}
```