SOLID MECHANICS MINI PROJECT

TEAM MEMBERS: YASH KONDKAR (M1910027)(batch B)
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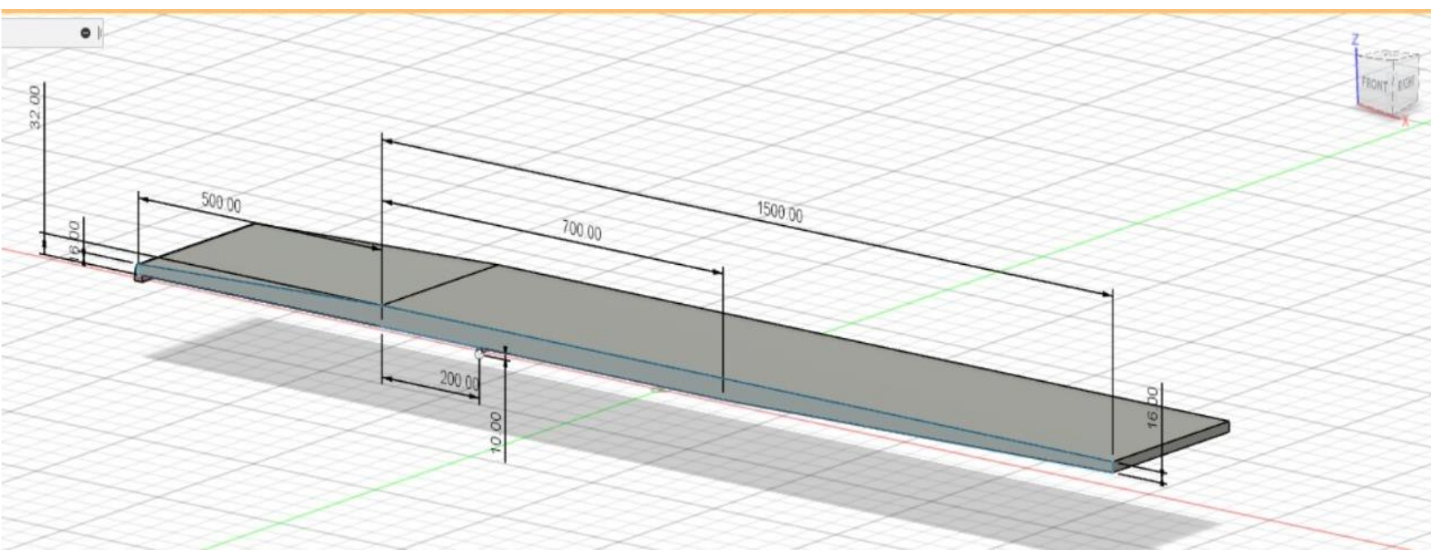
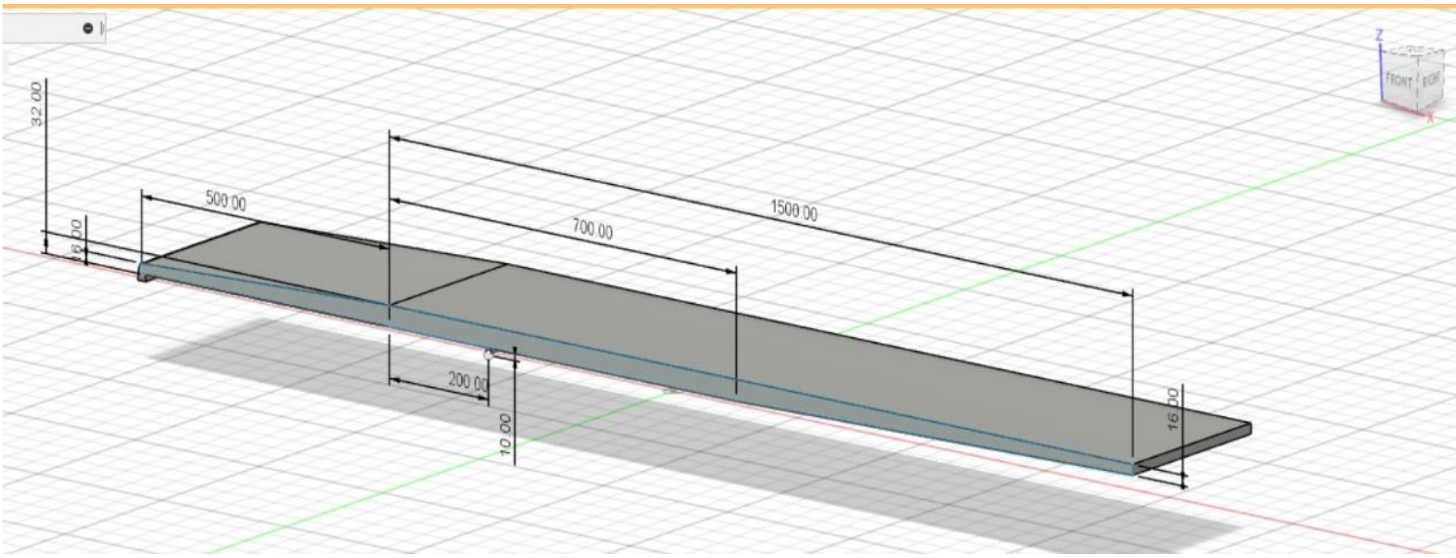
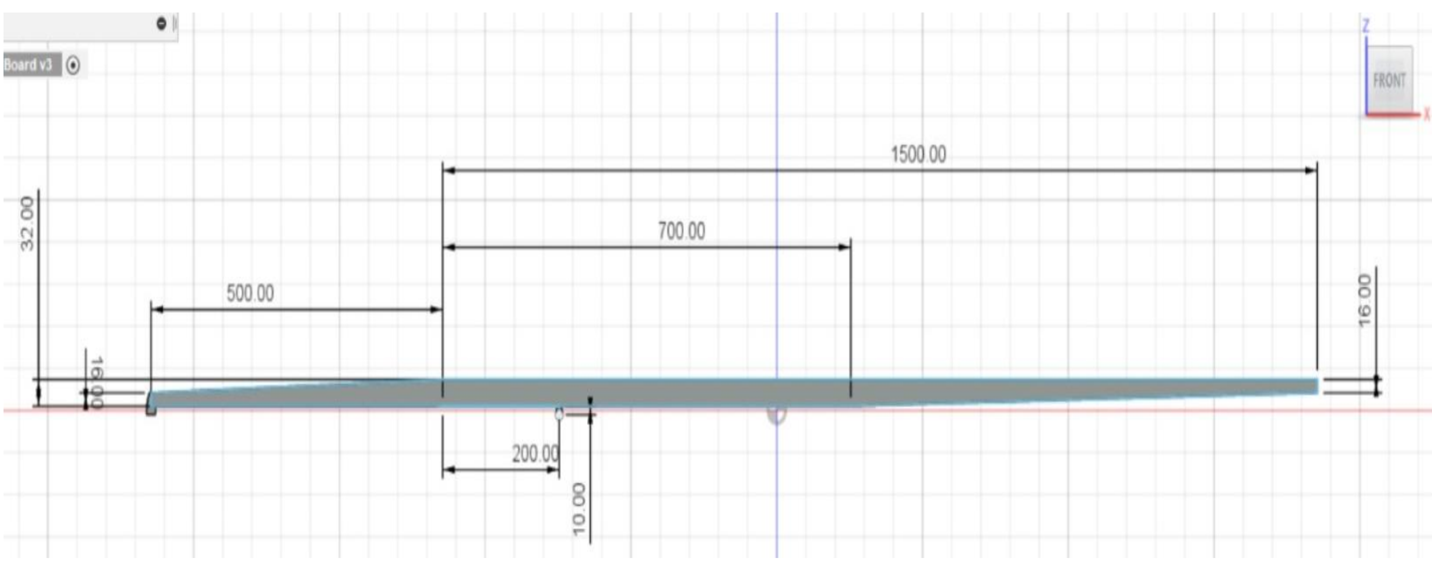
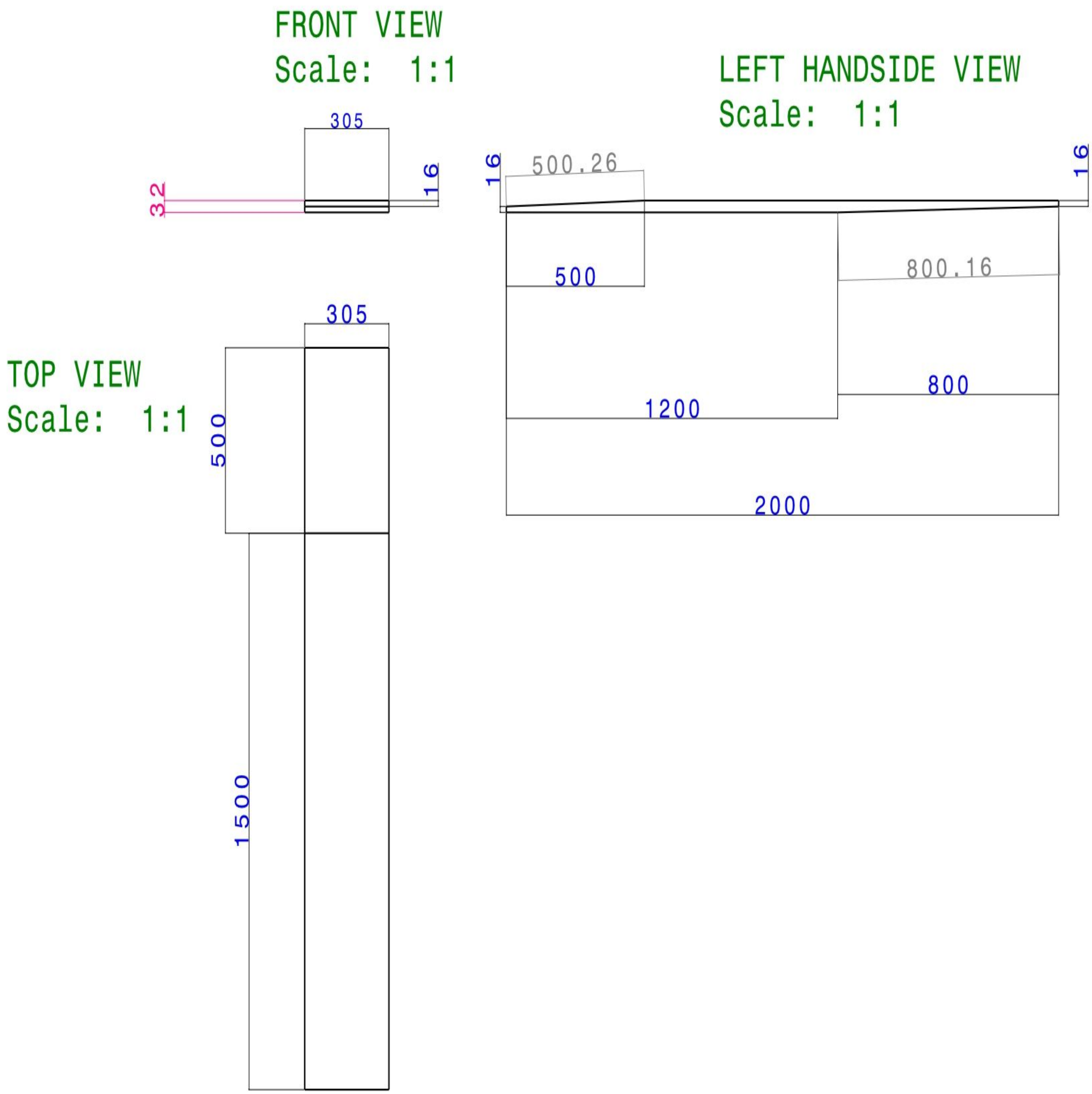
Final Design

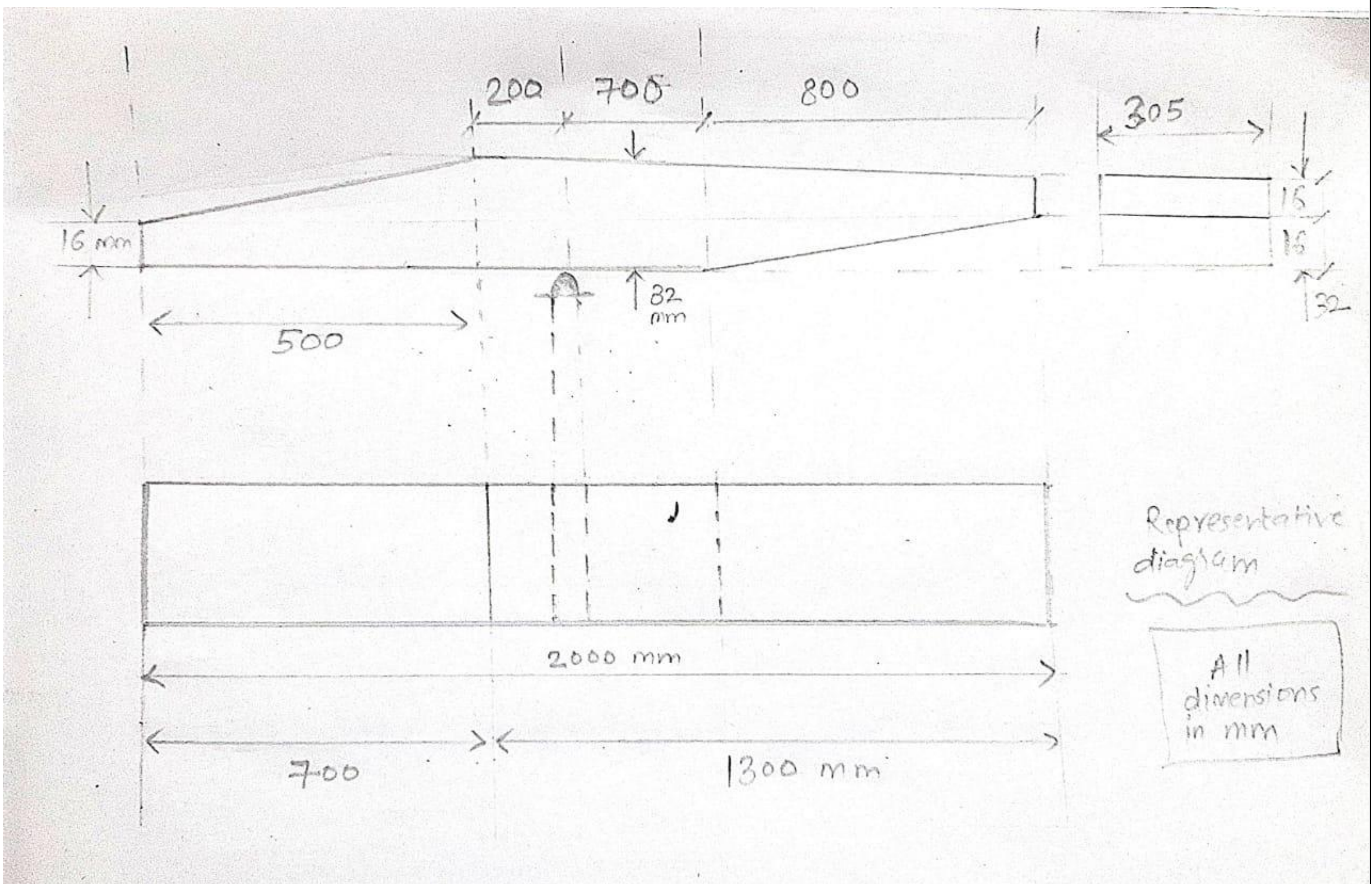


(the old board calculations are at end of the project)


32.00
500.00
1500.00
700.00
16.00
200.00
10.00
16.00
FRONT
RIGHT


Board v3
FRONT
1500.00
700.00
32.00
500.00
16.00
16.00
200.00
10.00


FRONT VIEW
Scale: 1:1
305
16
32
LEFT HANDSIDE VIEW
Scale: 1:1
16
500.26
16
500
800.16
800
1200
2000
TOP VIEW
Scale: 1:1
305
500
1500



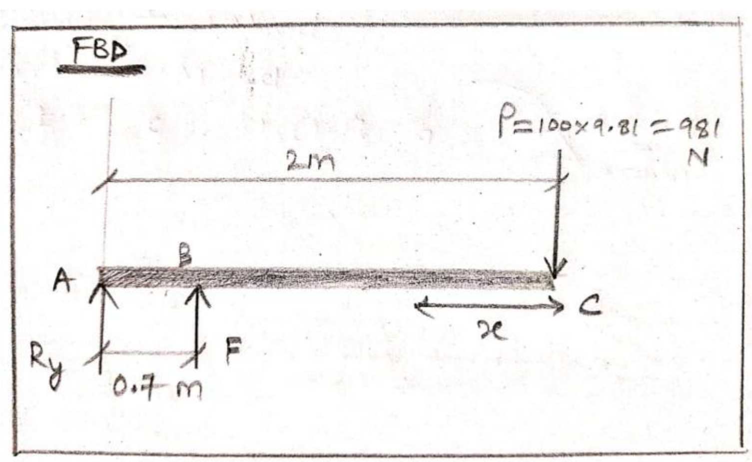
200
700
800
305
16 mm
16
16
32
82
mm
500
Representative
diagram
2000 mm
All
dimensions
in mm
700
1300 mm

Total weight reduction by the design:
%weight. reduced=%volume reduced
Volume reduced=(0.8*0.016*0.5*0.305)+(0.5*0.016*0.5*0.305)
Original before design volume=0.305*0.032*2
Thus %volume reduced=(volume reduced/original volume)*100
=16.25%

Weight reduced=16.25 %

A. *Calculation of reaction forces and shear and bending moment diagrams for the board with a 100 kg person standing at the free end.*

1) FBD



By equation of equilibrium,

1) $$\Sigma Mc = 0 \quad (positive)$$

$$Ry \times 2 + F \times (2 - 0.7) = 0$$

$$2Ry = -1.3F \quad -------------I$$

$$2) \Sigma Fy = 0$$

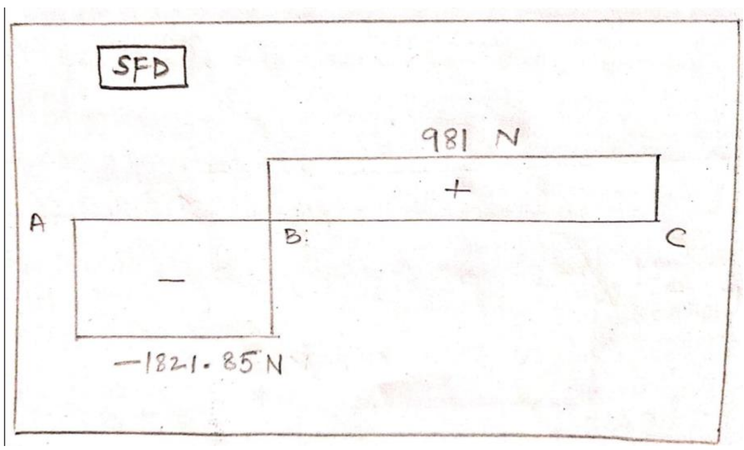
$$Ry + F = 981 \quad ---------------II$$

$$Thus\ from\ I\ \&\ II,$$
$$(-1.3F) \div 2) + F = 981$$
$$F(1 - 1.8/2) = 981$$
$$Reaction\ \ forces,$$
$$F = 2802.85 \quad N\ (\uparrow)$$
$$Ry = 1821.85 \quad N\ (\downarrow)$$

$$For\ SFD \qquad : At\ C \Rightarrow Fc = 0$$
$$Between\ B\ to\ C : F_{bc} = 981\ N$$
$$At\ F \qquad : F_b = -(1821.85)\ N$$

$$Between\ A\ \&\ B\ : F_{AB} = -1821.85$$
$$At\ \ A \qquad : F_A \Rightarrow -1821.85 + 1821.85 = 0$$

SFD :

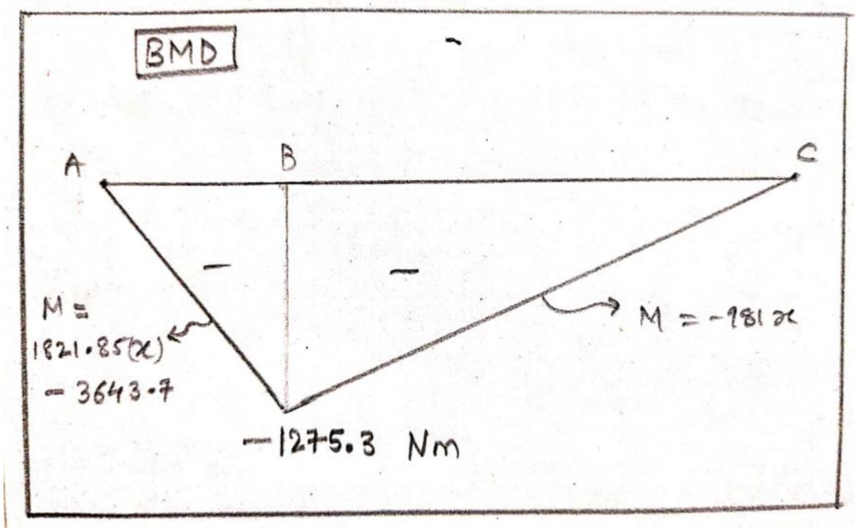


$$BMD \Rightarrow At\ C : M_C = 0$$
$$Between\ \ B\&C : M_C = -981(x) \qquad [Linear]$$
$$At\ B \qquad : M_B = -981 \times 1.3$$
$$M_B = -1275.3\ Nm$$

$$Between\ A\ \&\ B : M_{AB} = -981x + 2802.85(x - 1.3)$$
$$M_{AB} = 1821.85x - 3643.705 \qquad (linear)$$

$$At\ \ A\ (x = 2) \qquad : M_A = 0 \quad (substituting\ x = 2\ in\ M_{AB}\ equation)$$

BMD :



(The diving board is mostly used with a jump always.The board is designed for the jump case of 250mm.the self weight of the board is negligible compared to the load that arises due to the jump.Hence we have not considered the self weight of the board.)

$B)$ $Maximum\ deflection\ occurs\ at\ free\ end\ of\ cantilever\ which\ is\ given\ as,$

We are using,

Constalgiano's method { reference SS RATTAN }

$$MI\ of\ section\ A\ to\ B\ = I2\ = 0.032 \times (0.016 + 0.032x)^3/12$$

$$MI\ of\ section\ B\ to\ C\ = I = 832853.33 \times 10^{-12}\ m^4$$

$$MI\ of\ section\ C\ to\ D\ = I = 0.032 \times (0.016 + 0.032x)^3/12$$

$$MI\ of\ section\ D\ to\ E\ = I1\ = 0.032 \times (0.016 + 0.02x)^3/12$$

$$Deflection\ at\ free\ end\ E,$$

$$By\ , Constalgiano\ method$$

$$\delta e = \int_0^l (M/EI)(\partial M/\partial W)\, dx$$

$$In\ AC:\ M_{AC} = -1821.85x\ \Rightarrow (-1.8562W)x \qquad |\ x\ from\ A$$

$$In\ CE:\ M_{CE} = -981x = -Wx$$

$$In\ AC: (\partial M/\partial W) = -1.8562x$$

$$In\ CE: (\partial M_{CE}/\partial W) = -x$$

$$\delta e = \int_0^{0.5} (M_{AC}((\partial M/\partial W)_{AC}/EI_2 + \int_0^{0.5} (M_{AC}((\partial M/\partial W)_{AC}/EI +$$

$$+\int_0^{0.5} (M_{CE}((\partial M/\partial W)_{CE}/EI + \int_0^{0.5} (M_{CE}((\partial M/\partial W)_{CE}/EI_1$$

$$= J_1 + J_2 + J_3 + J_4$$

$$J_1 = 1/E \int_0^{0.5} (132793.15x^2/(0.016 + 0.032x)^3 \, dx$$

$$J_2 = 1/EI \int_0^{0.7} (3375.15x^2) \, dx = 0.02864$$

$$J_3 = (1/EI) \int_{0.8}^{1.3} (981x^2) \, dx = 0.06423$$

$$J_4 = 1/E \int_0^{0.8} (981)x^2/(0.016 + 0.02x)^3 \, dx = 0.03192$$

$deflection \quad is \quad \delta e = 0.1516m \Rightarrow 15.16cm$

B. Assuming cross-sectional dimensions of 305 mm x 32 mm and with material E = 10.3 GPa, find the largest principal stress at any location of the board when a 100 kg person is standing at the free end.

$Largest\ principal\ stress\ on\ the\ beam:$

$Stress\ on\ points\ of\ the\ diving\ board;$

$I)\ Bending\ Stress. \quad [There\ is\ no\ direct\ tensile/compressive\ stress\ applied]$

$II)\ Shear\ Stress.$

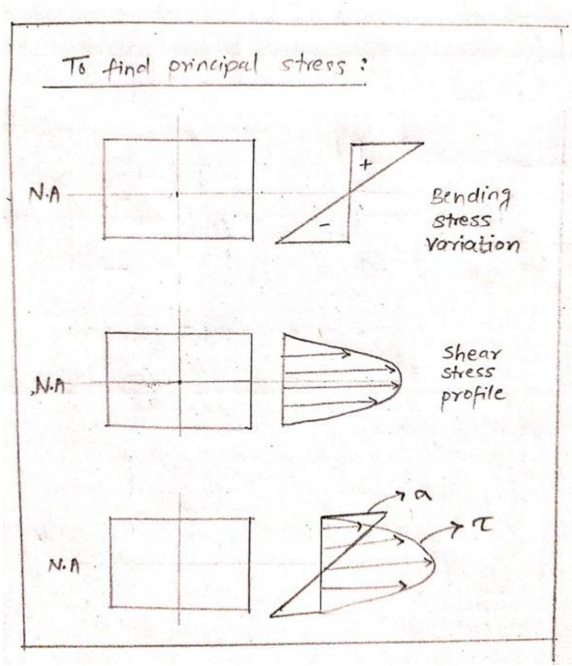
$To\ find\ exact\ location\ of\ principal\ stress:$

$Methodology:\ I)\ Determine\ section\ having\ maximum\ bending\ stress.$

$:\ II)\ Along\ the\ section\ check\ location, where\ shear\ stress\ is\ zero.$

$From\ BMD, BM_{MAX}\ is\ at\ section\ above\ fulcrum\ at\ B$

$$and \quad BM_{MAX} = 1275.3\ Nm.$$
$$At\ same\ section, checking\ shear\ profile,$$
$$\tau = Fa\underline{y}/IZ$$



$$\tau = 0 \quad at\ B\ and\ B'$$
$$A\ variation\ along\ the\ section\ is\ as\ follows:$$
$$As, \quad a\ \alpha\ y\ (\ and\ \ y\ is\ -ve\ below\ N.A)$$

$$Where\ , \ \sigma_{max} = BM_{max}(y)/I$$
$$= 1275.3 \times 16 \times 10^{-3}/832853.33 \times 10^{-12}$$
$$\sigma_{max} = 24.5\ MPa$$

$As\ shear\ stress\ is\ zero\ at\ B,$ BM $of\ the\ section\ above\ the\ fulcrum\ and\ bending\ stress\ is\ max\ thus\ itself.$

$Thus\ it\ is\ principal\ stress.$

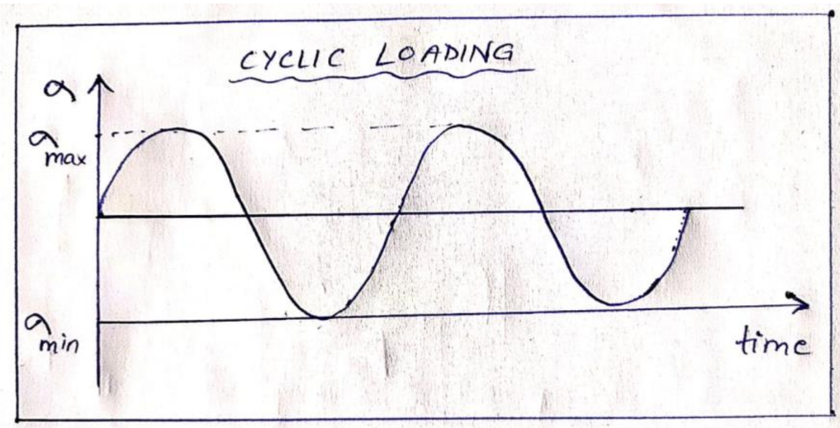
$Principal\ stress = \pm 24.5 MPa$

C. Calculate factor of safety if the material is brittle fiberglass with UTS = 130 MPa in longitudinal direction.(Sd)

$$For\ UTS = 130\ MPa\ (bitter\ fiberglass)$$

$$F.O.S\ = (UTS \backslash principal\ stress)$$
$$= (\frac{130}{24.5})$$
$$F.O.S = 5.306$$

D. Calculate factor of safety against fatigue* for infinite life, if endurance strength of material is 39 MPa



σ_{max}=$\sigma_{principal}$ ⇒24.5 Mpa (when man jumps 250 mm)

σ_{min}=0 Mpa (when no one is standing on the board)

Amplitude of cyclic stress= $(\sigma_{max}$- $\sigma_{min})/2$= 12.5 Mpa

$\sigma_{endurance}$= 12.5 MPa

Endurance limit = 39 Mpa

F.OS.=edurance limit/$\sigma_{endurance}$=39/12.5= 3.12

F.O.S against fatigue for infinite life =3.12

E. The board sits on a fulcrum that has cylindrical contact surface of 5 mm radius. What are contact stresses at fulcrum if board is fiberglass (E = 10.3 GPa, ν = 0.3) and fulcrum is aluminium.

$$Contact\ stress\ at\ fulcrum:$$

1)$Fulcrum:\quad Aluminium \rightarrow \gamma = 5mm(cylindrical)$

$i.e\quad d_1 = 10mm$

$$l_1 = width\ of\ board = 305mm$$

$$E_1 = 69GPa\ , v_1 = 0.334$$

$$Board:\ fiberglass\quad E_2 = 10.3GPa, v_2 = 0.3, d_2 = (\infty)$$

$$(flat\ surface)$$

$$Force\quad F' = 2802.85\ is\ getting\ applied\ on\ he\ system$$

$$\sigma_x = 2v_{pmax} \Rightarrow -45.6325N/mm^{2}$$

$$\sigma_y = -p_{max} \Rightarrow -26.054N/mm^2$$

$$\sigma_z = -p_{max} \Rightarrow -76.054N/mm^2$$

$$Half\ width\ of\ contact\ surface:$$

$$b = \sqrt{\frac{2F}{\pi l}[\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}]}$$

$$b = \sqrt{(2 \times 2802.85)/(\pi \times 305)[(1-0.334^2)/(69 \times 10^3) + (1-0.3^2)/(10.3 \times 10^3)]}$$

$$b = 8.684527334 \times 10^{-3}/[(1/10) + (1/\infty)]$$

$$b = .0769mm$$

$$Maximum\ Contact\ stress\ of\ fulcrum\ is,$$

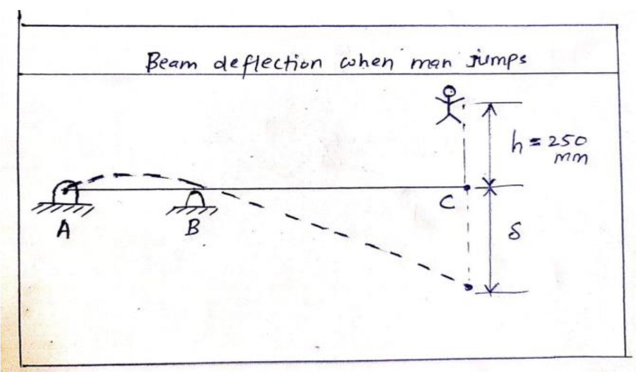
$$P_{max} = 2F/\pi hl \Rightarrow 76.077MPa$$

$$P_{max} = 76.077\ MPa$$

Thus Ƭmax= 0.301 * Pmax= 22.899

This value is well within the shear strength of the fiberglass chosen i.e 45Mpa

F. Assume that the 100 kg person jumps up 250 mm and lands back on board. What are stresses in board and those at contact in this case?



To find energy stored ,we assume the board to be like a vertical spring

$When\ \ 100\ Kg\ man\ stands\ at\ the\ tip\ of\ the\ board,$

$deflection\ \delta = 0.1288m$

$From\ spring\ equation,$

$$F\ =\ k\delta$$

$$100 \times 9.81 = k \times 0.1516$$

$$k\ = 6470.97N/m \qquad [stiffness]$$

$When\ man\ jumps\ from\ 250\ mm\ above\ board\ and\ lands, the\ board\ deflects\ by\ some\ value\ x,$

$By\ Energy\ conservation,$

$P.E\ of\ man = Energy\ stored\ in\ board\ after\ x\ - deflection$

$Assuming\ spring - like, the\ U_{diving\ board} = 1/2kx^2$

$$mg(h + x) = 1/2kx^2$$

$$981(0.25 + x) = 0.5 \times 6470.97 \times x^2$$

$$245.25 + 981x = 3235.485(x^2)$$

$$3535.485(x^2) - 981(x) - 245.25 = 0$$

$$x = 0.4658m$$

$Thus\ the\ equivalent\ load\ acting\ at\ tip\ of\ the\ board\ (say\ P)\ is,$

$$P = kx$$

$$P = 6470.97 \times 0.4658$$

$$P = 3014.17N$$

$Now\ principal\ stress\ is\ again\ the\ bending\ stress\ at\ fulcrum,$

$$M\ = BM_{max} = BM_B = P \times (2 - 0.7)$$

$$BM_B = P \times 1.3$$

$$BM_B = 3014.17 \times 1.3$$

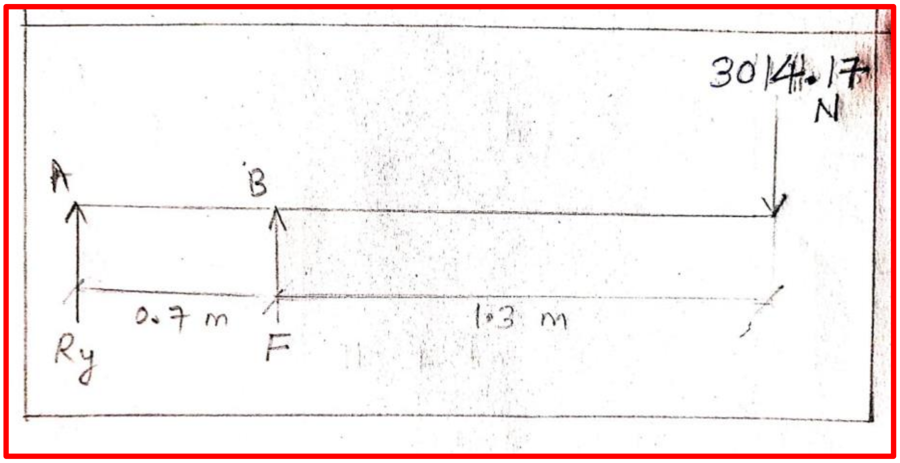
$BM_B\ = 3918.4Nm$

$$\sigma_{max} = My_{max}/I = M \times 0.016/832853.33 \times 10^{-12}$$

$$= 3918.4 \times 0.016/832853.33 \times 10^{-12}$$

σ_{max}
$= 75.27MPa \quad [Is\ the\ principal\ stress\ in\ this\ case\ and\ occurs\ above\ fulcrum]$

$Now\ due\ to\ change\ in\ load, reaction\ forces\ will\ also\ change$



add new

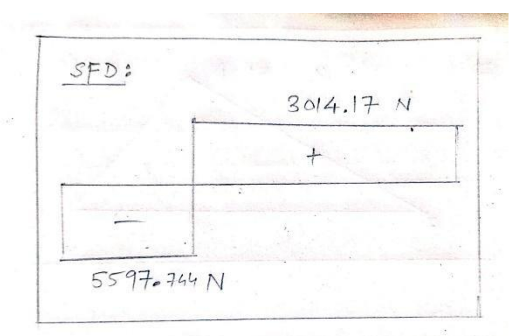
$$By\ equation\ of\ equilibrium\ ,$$
$$\Sigma F_y = 0$$
$$R_y + F = 3014.17 \qquad -------(1)$$

$$\Sigma M_c = 0$$
$$R_y \times 2 + F \times 1.3 = 0$$
$$R_y = -0.65F \qquad --------- (2)$$

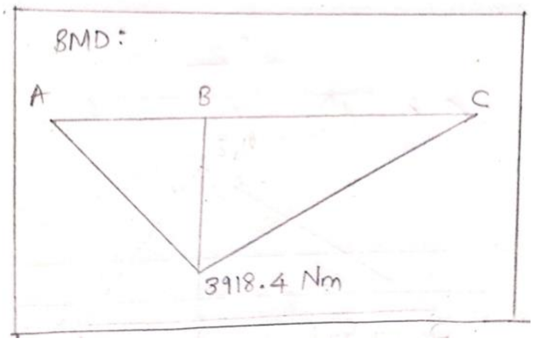
$$from\ (1)\ and\ (2)$$
$$-0.65F + F = 3014.17$$
$$F = 3014.17/0.35 \Rightarrow 8611.914N$$
$$R_y = -5597.74N$$

$$F = 8611.914N\ \{\uparrow\}$$
$$R_y = 5597.74N\ \{\downarrow\}$$

SFD:



BMD:



$Contact\ stress\ at\ fulcrum:$

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$i.e \quad d_1 = 10mm$

$l_1 = width\ of\ board = 305mm$

$E_1 = 69GPa\ , v_1 = 0.334$

$Board: \ fiberglass \quad E_2 = 10.3GPa, v_2 = 0.3, d_2 = (\infty)$

$(flat\ surface)$

$Force \quad F' = \ 8611.914\ is\ getting\ applied\ on\ the\ system$

$\sigma_x = -2v \times p_{max} \Rightarrow -79.956N/mm^{2}$

$\sigma_y = -p_{max} \Rightarrow -133.26N/mm^2$

$\sigma_z = -p_{max} \Rightarrow -133.26N/mm^2$

$Half\ width\ of\ contact\ surface:$

$$b = \sqrt{\frac{2F}{\pi l}[\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}]}$$

$$b = \sqrt{\{(2 \times 8611.914)/(\pi \times 305)[(1-0.334^2)\}/(69 \times 10^3) + (1-0.3^2)/(10.3 \times 10^3)]\}}$$

$$b = 8.684527334 \times 10^{-3}/[(1/10) + (1/\infty)]$$

$$b = 0.13489mm$$

$Maximum\ Contact\ stress\ of\ fulcrum\ is,$

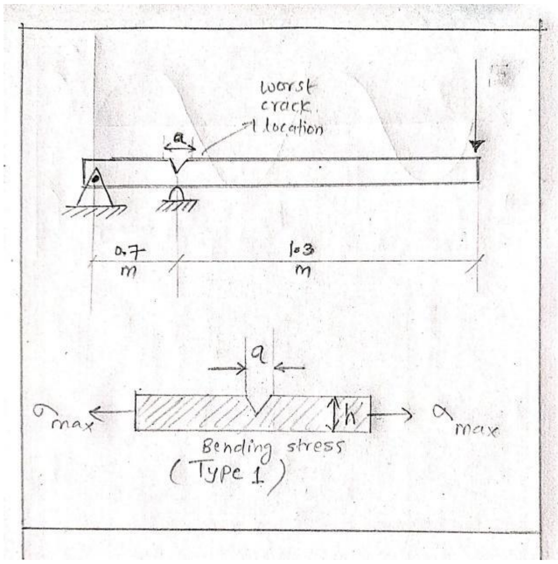
$P_{max} = 2F/\pi hl \Rightarrow 133.26MPa$

$P_{max} = 133.26\ MPa$

Thus τ_{max}= 0.301 * Pmax= 40.11Mpa

This is stress is within shear strength of 45 MPa

G. Discuss the worst location and orientation of crack on this board. Calculate maximum size of such crack using fracture mechanics approach.



$Worst\ location\ and\ orientation\ of\ crack\ is, just\ above\ fulcrum\ along\ the\ width\ of\ the\ board\ on\ upper\ face\ of\ the\ board\ i.e\ 0.7\ m\ from\ left\ end\ of\ the\ board.$

$Assuming\ a\ man\ jumps\ to\ height\ of\ 250\ mm\ above\ board\ and\ lands, the\ maximum\ bending\ stress\ is\ at\ the\ crack\ location.$

$$\sigma_{max} = 75.27\ MP\quad a$$

$$from\ \ BM_{max} = M \Rightarrow 3198.4\ Nm$$

$$Let\ the\ size\ of\ the\ crack\ be\ "a"$$

$$Fracture\ toughness\ of\ the\ board\ be\ (k_{IC}) = 0.5MPa\sqrt{m}$$

$$(K_I)_m = [6M/BH^2]\sqrt{\pi a}\ Y_m$$

$$Neglecting\ the\ shear\ force\ made\ effect\ at\ the\ crack\ tip,$$

$$(k_I\quad)_M = [6M/Bh^2]\sqrt{\pi a}Y_m$$

$$Where\ Y_m = 1.122 - 1.4\alpha + 7.33\alpha^2 - 13.08\alpha^3 + 14\alpha^4$$

$where\ \alpha = a/h\quad [from\ design\ handbook]$

$considering\ average - h\ \ for\ this\ designed\ beam\ as\ beam\ depth\ is\ variable\ .$

$$2 \times h = (0.032) + (0.032 \times 0.5) + \int\ h\ dx + \int\ h\ dx$$

$$= 0.7h + \int_0^{0.5} (0.016 + 0.032)\, dx \ + \int_0^{0.8} (0.016 + 0.02x)\ dx$$

$$0.7h \times 0.032 + 0.016x + 0.032/2x^2 \ + 0.016x + 0.02x^2/2$$

$$= 0.0224 + 0.0312$$

$$= 0.0536m$$

$$\boxed{h = 0.0268m}$$

$$\alpha = a/h \Rightarrow a/0.0268$$

$$a = 0.0268\alpha$$

$$(k_I)_M = [6 \times 3918.4/0.305 \times 0.0268^2]\sqrt{\pi \times 0.0268}\ [\sqrt{\alpha}.Y_m]$$

$$(k_{I)M} = 31.14[\sqrt{\alpha}.Y_m]\text{MPa}\sqrt{m}$$

$But\ for\ the\ diving\ board\ to\ sustain\ loads\ despite\ the\ crack,$

$(k_I)_M \leq (k_I\ \ )_C$

$31.14\sqrt{\alpha}.Y_m \leq 0.5$

$\sqrt{\alpha}.Y_m = 0.5/31.14$

$\sqrt{\alpha}.Y_m = 0.01605$

$$\alpha^{1/2}(1.122 - 1.4\alpha + 7.33\alpha^2 - 13.08\alpha^3 + 14\alpha^4\ \ ) = 0.01605$$

$$\alpha = 2.047 \times 10^{-4}$$

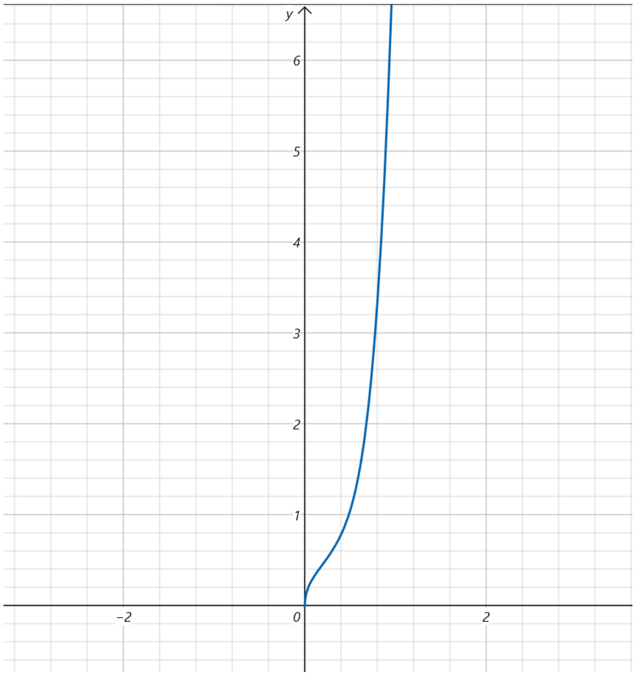
$$a/0.032 = 2.047 \times 10^{-4} \qquad \{\,\alpha = a/0.032\}$$

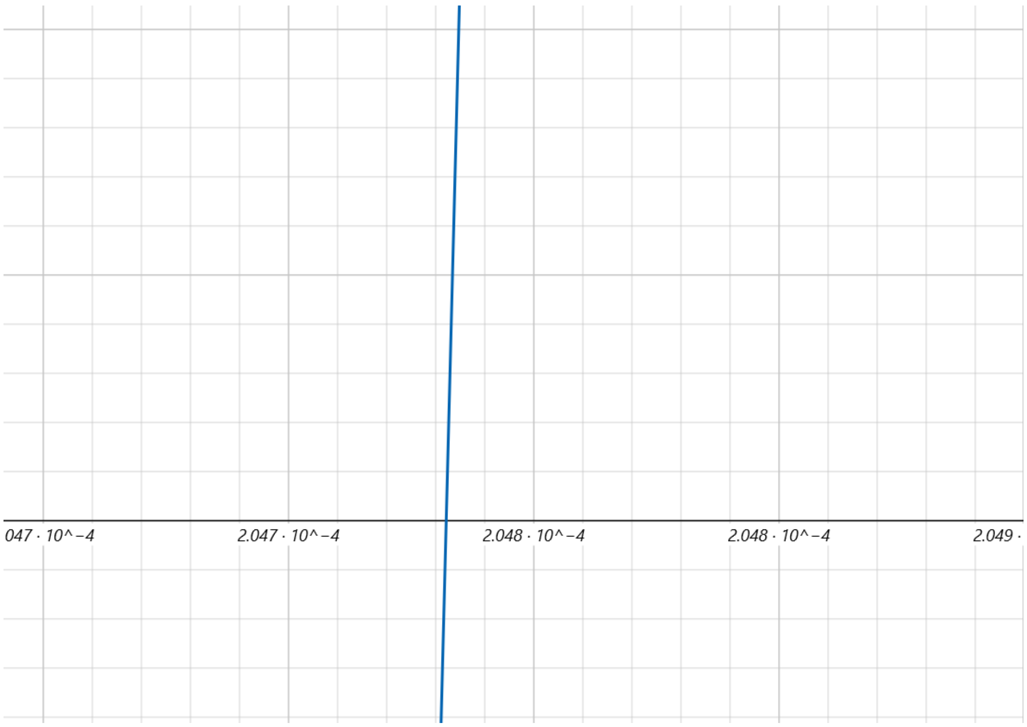
$$a = 0.0268 \times 2.047 \times 10^{-4}$$

a
$= 5.488 microns\ \ is\ the\ maximum\ crack\ size\ for\ which\ the\ diving\ board\ will\ be\ able$
$sustain\ the\ man\ jump\ to\ 250mm\ and\ land\ back.$

We are finding critical size for worst crack.we have considered man jumps case for the calculation.however there maybe multiple cracks which would make the actually allowable crack-sizes much smaller

Graphical plot of : $f(x) = 1.122x^{0.5} - 1.4x^{1.5} + 7.33x^{2.5} - 13.08x^{3.5} + 14x^{4.5} - 0.01605$





H. Discuss possible real life scenarios such as what will happen if the person does not stand in the middle of board's width and rebounds at an angle. Wherever possible, include calculations for such situations.

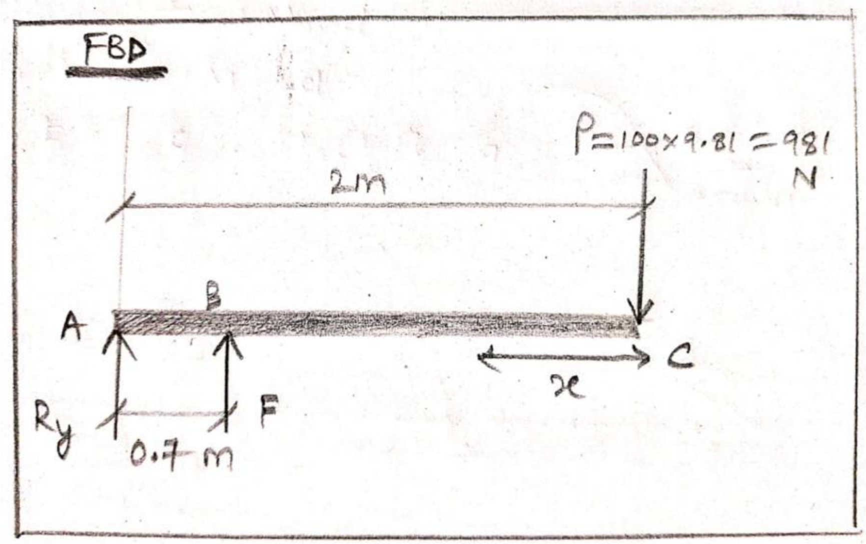
- The man jumps and lands on a corner and not middle of the board. The strength of fiberglass composite will vary in the direction. Thus result cannot be estimated.The stresses will arise due to torsion of non-circular sections case.However the calculations couldn't be done as the cross-section varies throughout the length of the diving board
- More than 1 person stand on the board simultaneously in a line
- Resonance occurs as the man's jump reaches back on the board just as the board has oscillated up and is reverting down.This would be worst scenario as the strains would multiply enormously resulting in much higher stresses than when man jumps simply.
- The man jumps more than 250 mm and also is heavier than 100Kg.

Diving board calculations for already given board

Diving board calculations for already given board

A. *Calculation of reaction forces and shear and bending moment diagrams for the board with a 100 kg person standing at the free end.*

2) FBD



By equation of equilibrium,

$$1)\ \Sigma Mc = 0 \quad (positive)$$
$$Ry \times 2 + F \times (2 - 0.7) = 0$$
$$2Ry = -1.3F \quad -------------I$$

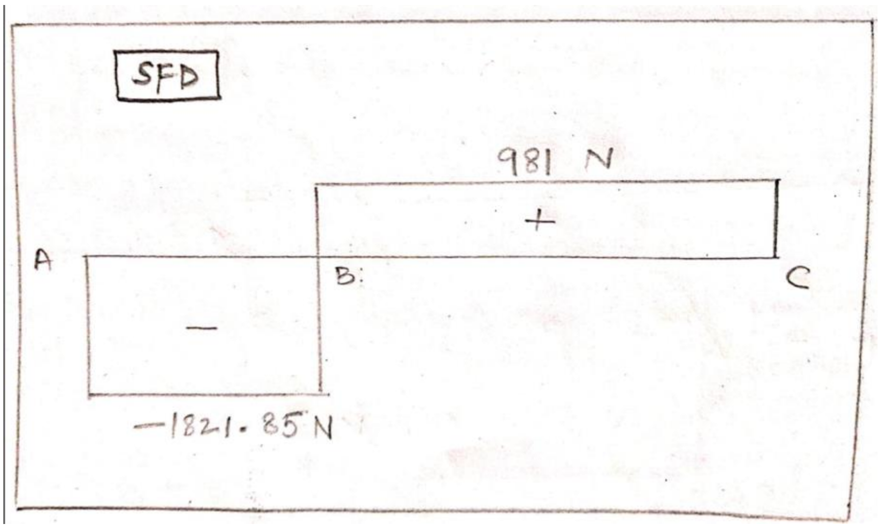
$$2)\Sigma Fy = 0$$
$$Ry + F = 981 \quad ---------------II$$

$$Thus\ from\ I\ \&\ II,$$
$$(-1.3F) \div 2) + F = 981$$
$$F(1 - 1.8/2) = 981$$
$$Reaction\ \ forces,$$
$$F = 2802.85 \quad N\ (\uparrow)$$
$$Ry = 1821.85 \quad N\ (\downarrow)$$

$$For\ SFD \quad : At\ C \Rightarrow Fc = 0$$
$$Between\ B\ to\ C : F_{bc} = 981\ N$$
$$At\ F \quad : F_b = -(1821.85)\ N$$

$$Between\ A\ \&\ B : F_{AB} = -1821.85$$
$$At\ A \quad : F_A \Rightarrow -1821.85 + 1821.85 = 0$$

<u>*SFD :*</u>

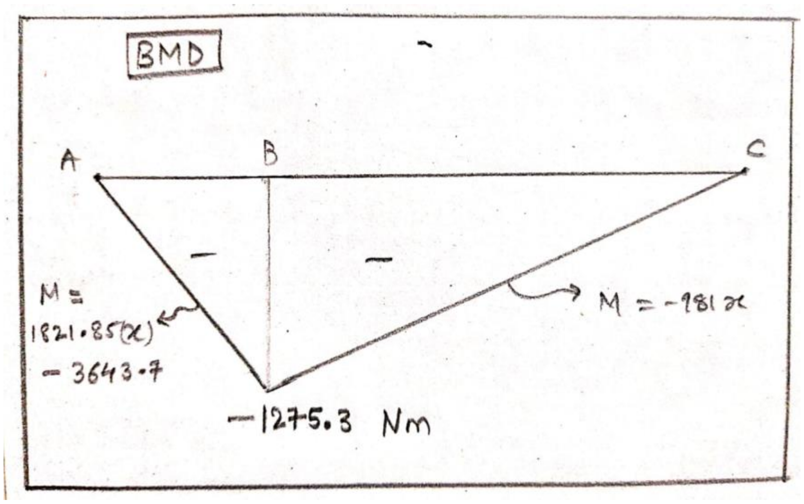


$$BMD \Rightarrow At\ C : M_C = 0$$
$$Between\ B\&C : M_C = -981(x) \qquad [Linear]$$
$$At\ B \quad : M_B = -981 \times 1.3$$
$$M_B = -1275.3\ Nm$$

$$Between\ A\ \&\ B : M_{AB} = -981x + 2802.85(x - 1.3)$$
$$M_{AB} = 1821.85x - 3643.705 \qquad (linear)$$

$$At\ A\ (x = 2) \quad : M_A = 0 \quad (substituting\ x = 2\ in\ M_{AB}\ equation)$$

BMD :



B. Assuming cross-sectional dimensions of 305 mm x 32 mm and with material E = 10.3 GPa, find the largest principal stress at any location of the board when a 100 kg person is standing at the free end. Calculate maximum deflection.

$$Largest\ principal\ stress\ on\ the\ beam:$$

$$Stress\ on\ points\ of\ the\ diving\ board;$$

$$I)\ \ Bending\ Stress.\quad [\,There\ is\ no\ direct\ tensile/compressive\ stress\ applied\,]$$

$$II)\ Shear\ Stress.$$

$$To\ find\ exact\ location\ of\ principal\ stress:$$

$$Methodology:\ I)\ Determine\ section\ having\ maximum\ bending\ stress.$$

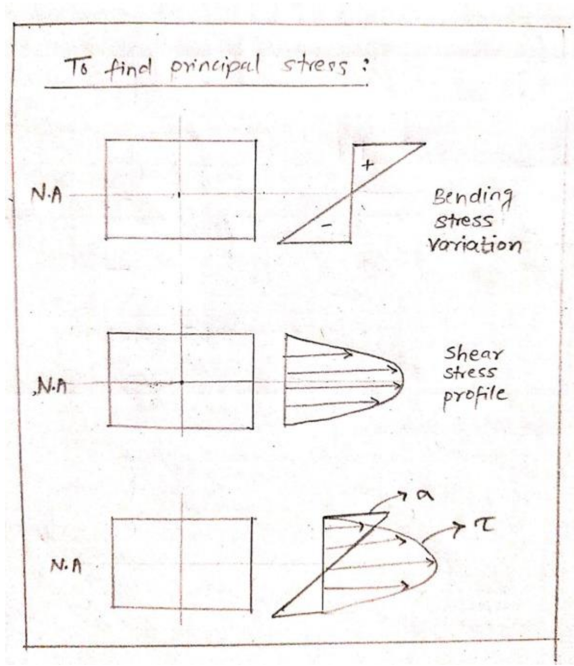
$$:\ II)\ Along\ the\ section\ check\ location, where\ shear\ stress\ is\ zero.$$

$$From\ BMD, BM_{MAX}\ is\ at\ section\ above\ fulcrum\ at\ B$$

$$and\quad BM_{MAX} = 1275.3\ Nm.$$

$$At\ same\ section, checking\ shear\ profile,$$

$$\tau = F\underline{y}/IZ$$



$$\tau = 0 \quad at\ B\ and\ B'$$

A variation along the section is as follows:

As, σ proportional to y (and y is $-ve$ below N.A)

$$Where\ ,\sigma_{max} = BM_{max}(y)/I$$

$$= 1275.3 \times 16 \times 10^{-3}/832853.33 \times 10^{-12}$$

$$\sigma_{max} = 24.5\ MPa$$

As shear stress is zero at B, BM of the section above the fulcrum and bending stress is max thus itself.

Thus it is principal stress.

$$Principal\ stress = \pm 24.5MPa$$

Bending moment of any section at 'x' distance from support 'a' is,

$$EI(d^2y/dx^2) = -1821.85x$$

Integrating above equation, we get,

$$EI(dy/dx) = [-1821.85/2]x^2 + c_1$$

Integrating the equation again we get,

$$EI \times y = [-1821.85/(2 \times 3)]x^3 + xc_1 + c_2$$

Boundary condition : (1) No deflection at hinge

$$hence\ y = 0\ for\ x = 0$$
$$0 = 0 + c_2$$
$$c_2 = 0$$
$$Boundary\ condition : (2)\ No\ deflection\ at\ fulcrum\ i.e\ y = 0\ for\ x = 0.7$$
$$0 = -1821.85 \times (0.7)^3/6 + c_1 \times 0.7 + 0$$
$$c_1 = 148.78$$
$$The\ deflection\ equation\ is$$
$$EI \times y = [-303.64]x^3 + [14.78]x + c_2$$
$$Here\ E = 1.3\ GPa$$
$$I = (832853.33 \times 10^{-12})\ m^4$$
$$EI = (8578.38) Nm^2$$
$$y = 1/8578(-303.64x^3 + 148.78x)$$

$$Now\ the\ deflection\ at\ end\ C\ i.e\ tip\ of\ diving\ board\ is ,$$
$$Substituting\ x = 2\ in\ above\ equation ,$$
$$y = 1/8578(-303.64(2)^3 + 148.78(2))$$
$$y = -0.1288m$$

$$Maximum\ deflection = 128.8mm\ at\ the\ tip\ end\ of\ diving\ board.$$

C. Calculate factor of safety if the material is brittle fiberglass with UTS = 130 MPa in longitudinal direction.

$For\ UTS = 130\ MPa\ (bitter\ fiberglass)$

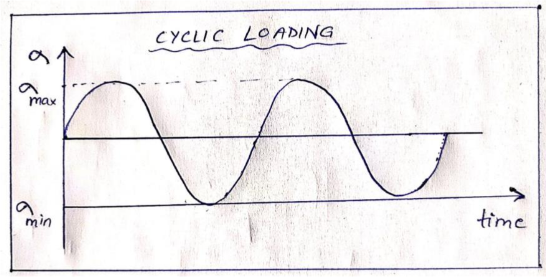
$F.O.S = (UTS \backslash principal\ stress)$

$= (130/24.5)$

$F.O.S = 5.306$

D. Calculate factor of safety against fatigue* for infinite life, if endurance strength of material is 39 MPa.

Note: We have considered the case when man jumps 250 mm for this calculation. Otherwise for static man standing F.O.S is much higher.



σ_{max}=24.5 Mpa (when man stands at free end)

σ_{max}=0 Mpa (when no one is standing on the board)

Amplitude of cyclic stress= (σ_{max}-σ_{max})/2= 12.5Mpa

σendurance= 12.5 MPa

Endurance limit = 39 Mpa

F.OS.=endurance limit/σendurance=39/12.5=3.12

F.O.S=3.12

E. The board sits on a fulcrum that has cylindrical contact surface of 5 mm radius. What are contact stresses at fulcrum if board is fiberglass (E = 10.3 GPa, ν = 0.3) and fulcrum is aluminium

$Contact\ stress\ at\ fulcrum:$

$$1)Fulcrum: \quad Aluminium \rightarrow \gamma = 5mm(cylindrical)$$
$$i.e \quad d_1 = 10mm$$
$$l_1 = width\ of\ board = 305mm$$
$$E_1 = 69GPa\ , v_1 = 0.334$$
$$Board: \ fiberglass \quad E_2 = 10.3GPa, v_2 = 0.3, d_2 = (\infty)$$
$$(flat\ surface)$$

$$Force \quad F' = \ 2802.85\ is\ getting\ applied\ on\ he\ system$$
$$\sigma_x = 2v_{pmax} \Rightarrow -45.6325N/mm^{\ 2}$$
$$\sigma_y = -p_{max} \Rightarrow -26.054N/mm^2$$
$$\sigma_z = -p_{max} \Rightarrow -76.054N/mm^2$$

$$Half\ width\ of\ contact\ surface:$$
$$b = \sqrt{\frac{2F}{\pi l}[\frac{(1-\nu_1{}^2)/E_1 + (1-\nu_2{}^2)/E_2}{1/d_1 + 1/d_2}]}$$

$$b = \sqrt{(2 \times 2802.85)/(\pi \times 305)[(1 - 0.334^2)/(69 \times 10^3) + (1 - 0.3^2)/(10.3 \times 10^3)]}$$

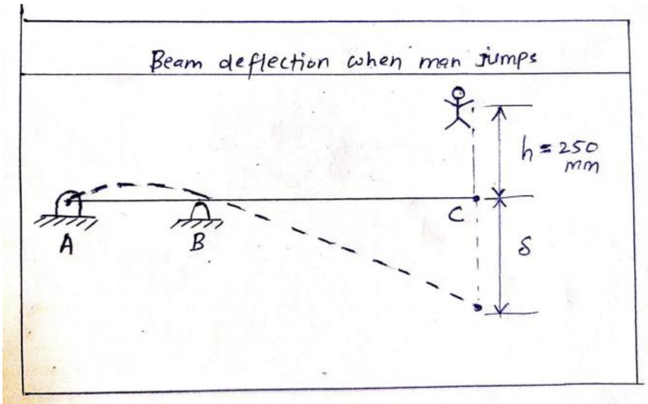
$$b = 8.684527334 \times 10^{-3}/[(1/10) + (1/\infty)]$$
$$b = .0769mm$$

$$Maximum\ Contact\ stress\ of\ fulcrum\ is,$$
$$P_{max} = 2F/\pi hl \Rightarrow 76.077MPa$$
$$P_{max} = 76.077\ MPa$$

Thus Ƭmax= 0.301 * Pmax= 22.899

Well within shear strength of 45 MPa

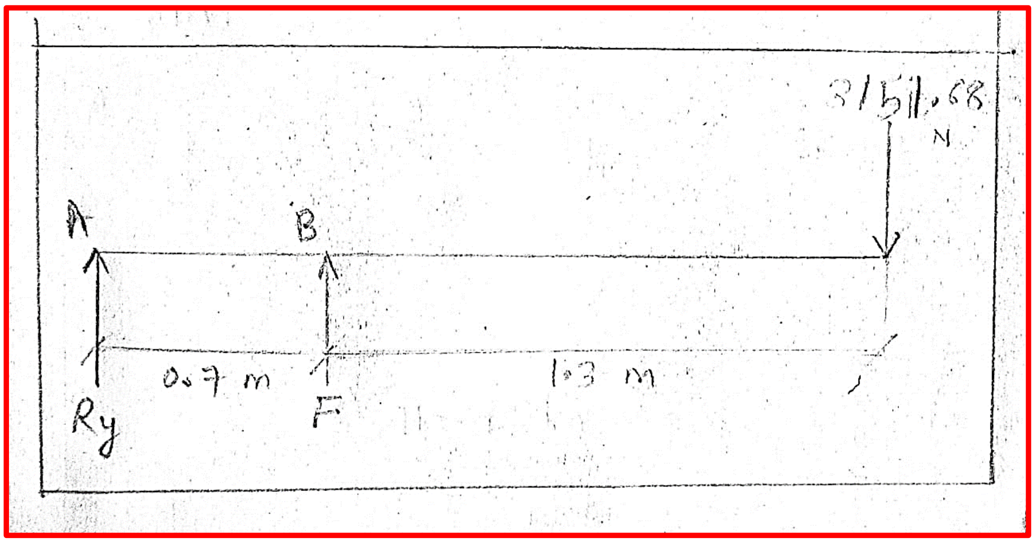
F. Assume that the 100 kg person jumps up 250 mm and lands back on board. What are stresses in board and those at contact in this case?



$$To\ find\ energy\ stored\ , we\ assume\ the\ board\ \ to\ be\ like\ a\ vertical\ spring$$
$$When\ \ 100\ Kg\ man\ stands\ at\ the\ tip\ of\ the\ board,$$
$$deflection\ \delta = 0.1288m$$
$$From\ spring\ equation,$$
$$F = k\delta$$
$$100 \times 9.81 = k \times 0.1288$$
$$k = 7616.45N/m \qquad [stiffness]$$

$$When\ man\ jumps\ from\ 250\ mm\ above\ board\ and\ lands, the\ board\ deflects\ by\ some\ value\ x,$$
$$By\ Energy\ conservation,$$
$$P.E\ of\ man = Energy\ stored\ in\ board\ after\ x\ - deflection$$
$$Assuming\ spring - like, the\ U_{diving\ board} = 1/2kx^2$$
$$mg(h + x) = 1/2kx^2$$
$$981(0.25 + x) = 0.5 \times 7616.45 \times x^2$$

$$245.25 + 981x = 3808.255(x^2)$$
$$3808.255(x^2) - 981(x) - 245.25 = 0$$
$$x = 0.4138\ m$$
$$Thus\ the\ equivalent\ load\ acting\ at\ tip\ of\ the\ board\ (say\ P)\ is,$$
$$P = kx$$
$$P = 7616.45 \times 0.4138$$
$$P = 3151.68N$$
$$Now\ principal\ stress\ is\ again\ the\ bending\ stress\ at\ fulcrum,$$
$$M = BM_{max} = BM_B = P \times (2 - 0.7)$$
$$BM_B = P \times 1.3$$
$$BM_B = 3151.68 \times 1.3$$
$$BM_B = 4097.18Nm$$
$$\sigma_{max} = My_{max}/I = M \times 0.016/832853.33 \times 10^{-12}$$
$$4097.18 \times 0.016/832853.33 \times 10^{-12}$$
$$\sigma_{max} = 78.7MPa \quad [Is\ the\ principal\ stress\ in\ this\ case\ and\ occurs\ above\ fulcrum]$$
$$Now\ due\ to\ change\ in\ load, reaction\ forces\ will\ also\ change$$



change me

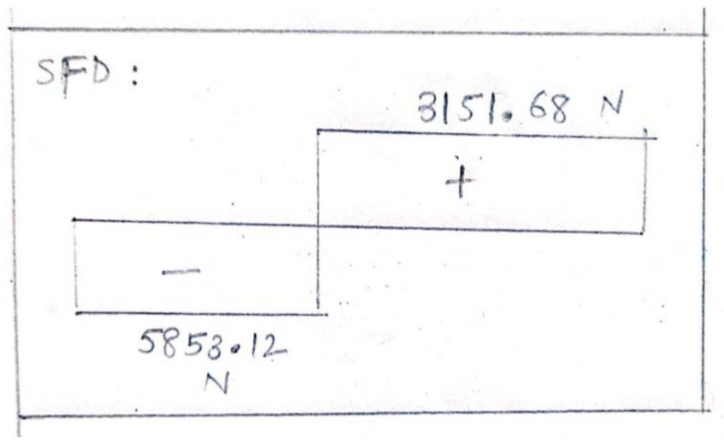
$$By\ equation\ of\ equilibrium\ ,$$
$$\Sigma F_y = 0$$
$$R_y + F = 3151.68 \qquad ------(1)$$

$$\Sigma M_c = 0$$
$$R_y \times 2 + F \times 1.3 = 0$$
$$R_y = -0.65F \qquad --------(2)$$

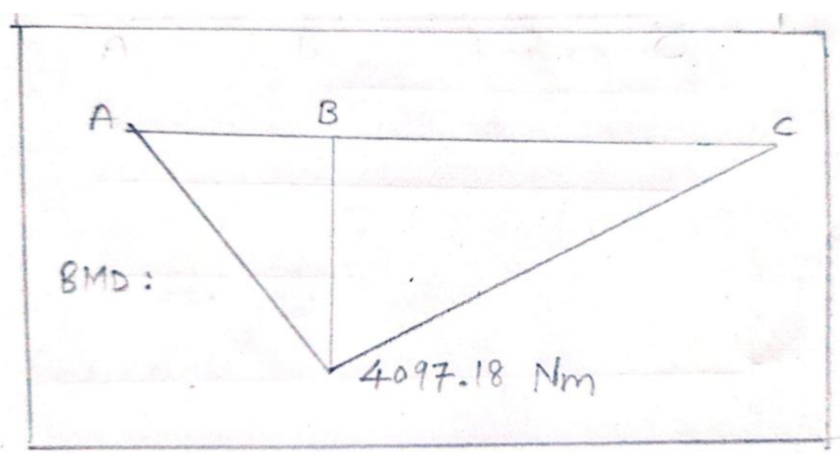
$$from\ (1)\ and\ (2)$$
$$-0.65F + F = 3151.68$$
$$F\ = 3151.68/0.35 \Rightarrow\ 9004.8N$$
$$R_y = -5853.12\ N$$

$$F\ =\ 9004.8\ N\ \{\uparrow\}$$
$$R_y =\ -5853.12\ N\ \{\downarrow\}$$

SFD:



BMD:



$Contact\ stress\ at\ fulcrum$:

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$$Force\quad F' = 9004.8\ is\ getting\ applied\ on\ he\ system$$
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$$Half\ width\ of\ contact\ surface:$$

$$b = \sqrt{\frac{2F}{\pi l}[\frac{(1-\nu_1^{2})/E_1 + (1-\nu_2^{2})/E_2}{1/d_1 + 1/d_2}]}$$

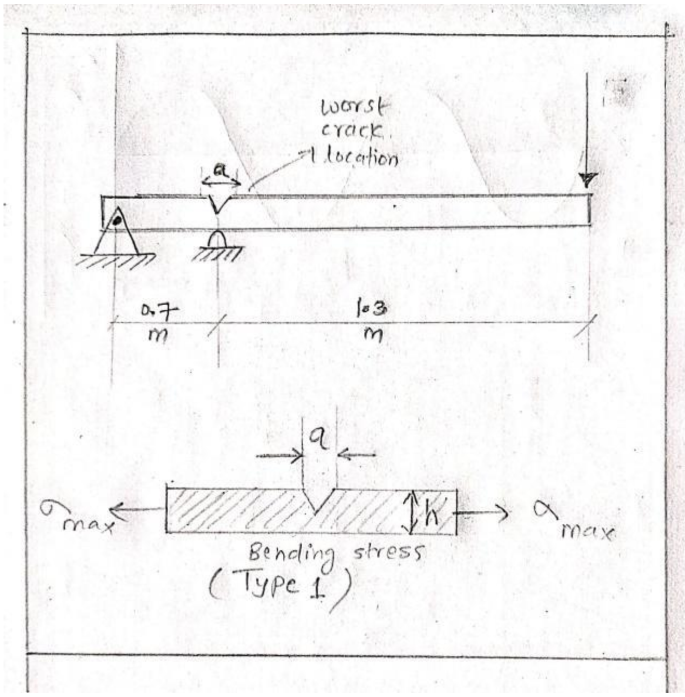
$$b = \sqrt{(2 \times 9004.8)/(\pi \times 305)[(1-0.334^2)/(69 \times 10^3) + (1-0.3^2)/(10.3 \times 10^3)]/[(1/10}$$
$$b = 8.684527334 \times 10^{-3}/[(1/10) + (1/\infty)]$$
$$b = 0.137mm$$

$$Maximum\ Contact\ stress\ of\ fulcrum\ is,$$
$$P_{max} = 2F/\pi bl \Rightarrow 137.19MPa$$
$$P_{max} = 137.19\ MPa$$

Thus τmax= 0.301 * Pmax= 41.29

G. Discuss the worst location and orientation of crack on this board. Calculate maximum size of such crack using fracture mechanics approach.

Worst location and orientation of crack is, just above fulcrum along the width of the board on upper face of the board i.e 0.7 m from left end of the board.



Assuming a man jumps to height of 250 mm above board and lands, the maximum bending stress is at the crack location.

$$\sigma_{max} = 78.7\ MP \quad a$$

$$from\ \ BM_{max} = M \Rightarrow 4097.18\ Nm$$

Let the size of the crack be "a"

Fracture toughness of the board be $(k_{IC}) = 0.5MPa$

Neglecting the shear force made effect at the crack tip,

$(k_I\ \ )_M = \sigma M/Bh^2\sqrt{\pi a}Y_m$

$$Where\ Y_m = 1.122 - 1.4\sigma + 7.33\sigma^2 - 13.08\sigma^3 + 14\sigma^4$$

$where\ \alpha = a/h$ $\quad$ *[from design handbook]*

$$\alpha = a/h \Rightarrow a/0.032$$

$$a = 0.032\alpha$$

$$(k_I)_M = 6 \times 4097.18/0.305 \times 0.032^2\sqrt{\pi \times 0.032}\ \ [\sqrt{\alpha}.Y_m]$$

$$(k_{I)M} = 24.95 \times \sqrt{\alpha}.Y_m$$

But for the diving board to sustain loads despite the crack,

$(k_I)_M \leq kic$

$$24.95\sqrt{\alpha}.Y_m \leq 0.5$$
$$\sqrt{\alpha}.Y_m = 0.5/24.95$$
$$\sqrt{\alpha}.Y_m = 0.02004$$

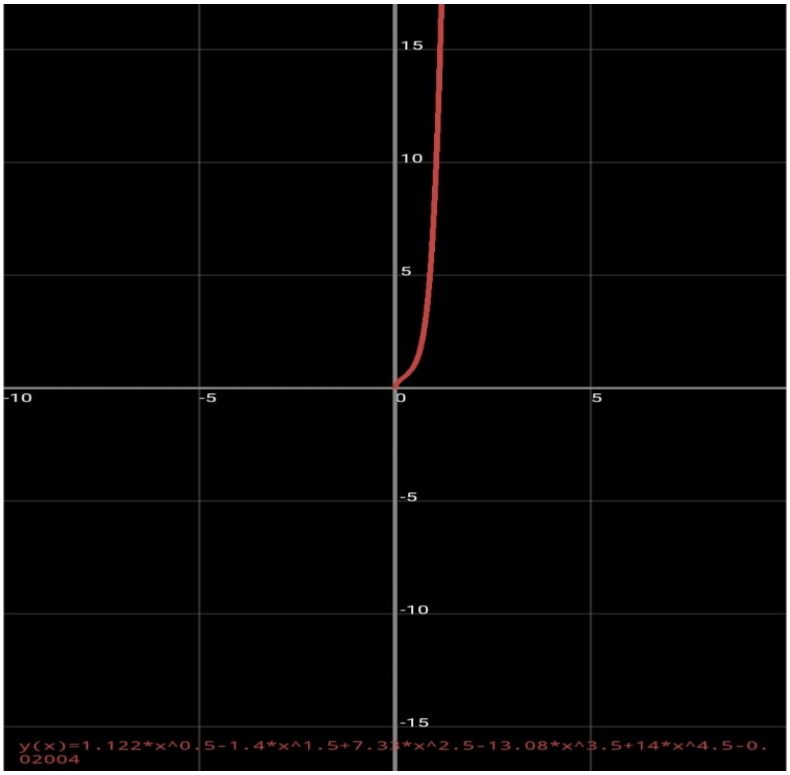
$$\alpha^{1/2}(1.122 - 1.4\alpha + 7.33\alpha^2 - 14.08\alpha^3 + 14\alpha^4 \quad) = 0.02004$$

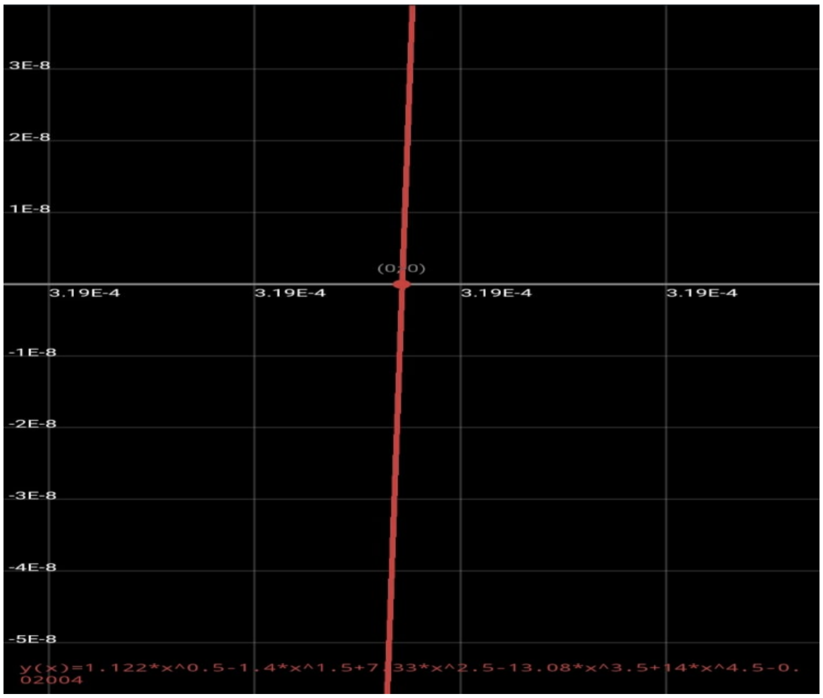
$\alpha = 3.19 *$
10^{-4} $[graphical\ based\ solution\ (plotted\ graphs\ to\ find\ roots\ \ on\ next\ page)]$

$$a/0.032 = 3.19 \times 10^{-4} \qquad\qquad \{\ \alpha = a/0.032\}$$
$$a = 1.0208 \times 10^{-5}m \Rightarrow 10.208 \times 10^{-6}m$$

a
$= 10.208\ micrometers\ \ is\ the\ maximum\ crack\ size\ for\ which\ the\ diving\ board\ will\ be$
$sustain\ the\ man\ jump\ to\ 250mm\ and\ land\ back.$

Graph plot of $\alpha^{\frac{1}{2}}\left(1.122 - 1.4\alpha + 7.33\alpha^{2} - 14.08\alpha^{3} + 14\alpha^{4}\right) - 0.02004$





H. Discuss possible real life scenarios such as what will happen if the person does not stand in the middle of board's width and rebounds at an angle. Wherever possible, include calculations for such situations.

- The man jumps and lands on corner and not middle of the board. The strength of fiberglass composite will vary in the direction. Thus result cannot be estimated.The stresses will arise due to torsion of non-circular sections case.
- More than 1 person stand on the board simultaneously in a line
- Resonance occurs as the man's jump reaches back on the board just as the board has oscillated up and is reverting down.This would be worst scenario as the strains would multiply enormously resulting in much higher stresses than when man jumps simply.
- The man jumps more than 250 mm and also is heavier than 100Kg.