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Tutorial – 1

Q1) construct a truth table for:-

1) $(P \land Q) \lor (\neg P \land Q) \lor (P \land \neg Q) \lor (\neg P \land \neg Q)$

Ans:- $(P \land Q) \lor (\neg P \land Q) \lor (P \land \neg Q) \lor (\neg P \land \neg Q) \Rightarrow X$

P	Q	¬P	¬Q	P∧Q	¬P∧¬Q	¬P∧Q	P∧¬Q	X
T	T	F	F	T	F	F	F	T
T	F	F	T	F	F	F	T	T
F	T	T	F	F	F	T	F	T
F	F	T	T	F	T	F	T	T

2) $(Q \land (P \rightarrow Q)) \rightarrow P$

Ans:-

P	Q	P→Q	Q∧(P→Q)	(Q∧(P→Q))→P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

3) $\neg(P \lor (Q \land R)) \rightleftarrows ((P \lor Q) \land (P \lor R))$

Ans:- $\neg(P \lor (Q \land R)) \Rightarrow A$, $((P \lor Q) \land (P \lor R)) \Rightarrow B$

P	Q	R	Q∧R	P∨Q	P∨R	¬(P∨(Q∧R))	B	A⇄B
F	F	F	F	F	F	T	F	F
F	F	T	F	F	T	T	F	F
F	T	F	F	T	F	T	F	F
F	T	T	T	T	T	F	T	F
T	F	F	F	T	T	F	T	F
T	F	T	F	T	T	F	T	F
T	T	F	F	T	T	F	T	F
T	T	T	F	T	T	F	T	F

Q2) Represent the statements in form of symbolic

1) The crop will destroyed if there is flood.

Ans: P: If there is flood Q: The crop will destroyed

$\therefore P \rightarrow Q$

2) If Sun is shining today then 2+7 is greater than 5.

Ans: P: If Sun is shining today Q: then 2+7 is greater than 5

$\therefore P \rightarrow Q$

3) If either Rajan takes calculus or John takes Sociology then Smith will take English.

Ans: P: If either Rajan takes calculus
Q: John takes Sociology
R: Smill will take English

$\therefore (P \vee Q) \rightarrow R$

Q3) Given Truth values of P and Q as T and those of R and S as F. Find truth values of following.

1) $(\neg(P \wedge Q) \vee \neg R) \vee (Q \rightleftarrows \neg P) \longrightarrow (R \vee \neg S)$

Ans: $(\neg(T \wedge T) \vee \neg F) \vee (P \rightleftarrows \neg T) \longrightarrow (F \vee \neg F)$

$(F \vee T) \vee T \quad (F \rightleftarrows F) \longrightarrow (F \vee T)$

$T \vee T \rightarrow T$

$T \quad \therefore$ Truth values is T

2) $(P \rightleftarrows R) \land (\neg Q \longrightarrow S)$

Ans: $(T \rightleftarrows F) \land (\neg T \longrightarrow F)$

$F \land T$

F $\therefore$ Truth value is F

3) $(P \lor (Q \longrightarrow R \land \neg P))$

Ans: $(T \lor (T \longrightarrow F \land \neg T))$

$T \lor (T \rightarrow (F \land F))$

$T \lor (T \rightarrow F)$

$T \lor T$

T $\therefore$ Truth value is T

4) $P \lor (Q \land R) \rightleftarrows (Q \lor \neg Q)$

Ans: $T \lor (T \land F) \rightleftarrows (T \lor \neg T)$

$T \lor (T \land F) \rightleftarrows (T \lor F)$

$T \lor (F) \rightleftarrows T$

$T \rightleftarrows T$

T $\therefore$ Truth value is T

5) $(\neg(P \land Q) \lor \neg R) \lor (((\neg P \land Q) \lor \neg R) \land S)$

Ans: $(\neg(T \land T) \lor \neg F) \lor (((\neg T \land T) \lor \neg F) \land F)$

$\neg(T \land T) \lor T \lor (((F \land T) \lor T) \land F)$

$F \lor T \lor (F \lor T \land F)$

$T \lor F \lor F$

$T \lor F$

T $\therefore$ Truth value is T

Q4) construct the truth table

a) $(Q \wedge (P \rightarrow Q)) \rightarrow P$

Ans:

P	Q	$P \rightarrow Q$	$Q \wedge (P \rightarrow Q)$	$(Q \wedge (P \rightarrow Q)) \rightarrow$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

b) $\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee$

Ans:

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee$
T	T	T	T	T	T	F
T	T	F	F	T	T	F
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	T	F
F	T	F	F	T	F	F
F	F	T	F	T	T	F
F	F	F	F	F	F	F

Q5) show the following equivalences by using Truth Table.

a) $P \rightarrow (Q \rightarrow P) \Leftrightarrow \neg P \rightarrow (P \rightarrow Q)$

Ans:

P	Q	$\neg P$	$Q \to P$	$P \to Q$	$P \to (Q \to P)$	$\neg P \to (P \to Q)$
T	T	F	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

$\therefore P \to (Q \to P) \Longleftrightarrow \neg P \to (P \to Q)$

$A \Longleftrightarrow B$

b) $P \to (Q \vee R) \Longleftrightarrow (P \to Q) \vee (P \to R)$

Ans:

P	Q	R	$Q \vee R$	$P \to Q$	$P \to R$	$P \to (Q \vee R)$	$(P \to Q) \vee (P \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

$\therefore P \to (Q \vee R) \Longleftrightarrow (P \to Q) \vee (P \to R)$

$A \Longleftrightarrow B$

c) $\neg (P \rightleftarrows (P \vee Q) \wedge \neg (P \wedge Q)$

Ans:

P	Q	$P \vee Q$	$\neg P \wedge Q$	$\neg P$	$P \vee Q \wedge \neg (P$
T	T	T	F	F	F
T	F	T	T	F	T
F	T	T	T	T	T
F	F	T	T	T	

$\therefore \neg (P \not\Longleftrightarrow (P \vee Q) \wedge \neg (P \wedge Q)$

$\therefore A \not\Longleftrightarrow B$

Q6) From the formulas given below select which of these are wff according to defn

a) $(P \rightarrow (P \vee Q))$

It is well formed formulae

Ans:-

P	Q	$P \vee Q$	$(P \rightarrow (P \vee Q))$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$\therefore$ It is a Tautology

b) $((P \rightarrow (\neg P)) \rightarrow \neg P)$

It is well formed formulae

Ans:-

P	$\neg P$	$P \rightarrow \neg P$	$P \rightarrow (\neg P) \rightarrow \neg P$
T	F	F	T
T	F	F	T
F	T	T	T
F	T	T	T

$\therefore$ It is a Tautology

c) $((\neg Q \wedge P) \wedge P)$

It is well formed formulae

P	Q	$\neg Q$	$\neg Q \wedge P$	$\neg Q \wedge P \wedge P$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	F

It is neither contradiction nor Taut

d) $((P \to (Q \to R)) \to ((P \to Q) \to (P \to R)))$

It is well formed formulae

Ans:

P	Q	$P \to Q$	$Q \to R$	$P \to R$	$((P \to (Q \to R)) \to ((P \to Q) \to P \to R$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

d)

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$	$((P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

It is a tautology

e) $((\neg P \to Q) \to (Q \to P))$

It is not a well formed formulae

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f) $((P \wedge Q) \rightleftarrows P)$

It is a well formed formulae

P	Q	$P \wedge Q$	$((P \wedge Q) \rightleftarrows P)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

It is neither Tautology nor contradiction

Q7) What is preposition? Give example

Ans: A preposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A propositional consists of propositional variables and connectives. We denote propositional variables by capital letters (A, B) etc.

Eg:- "Grass is green", Truth value is True

"2 + 5 = 5" => Truth value is False

Q 8) Explain basic connections with Truth Table

1) Negation

Ans: The Negation of a statement is generally formed by introducing the word "not" at a proper place in the statement or by prefixing the statement with the phrase. If 'P' denotes a statement, then the negation of "p" is written as "7P" and read as "not P". If the truth value of "p" is T, then the truth value of "7P" is F. Also if the truth value of "p" is T;

Eg: P: London is a city => 7P: Lodon is not a city

P	7P
T	F
F	T

2) Conjunction

Ans:- The conjunction of two statements P and Q is the statement $P \wedge Q$ which is read as "P and Q". The statement $P \wedge Q$ has the truth value T whenever both P and Q have the truth value T otherwise it has the truth value F. The conjunction is defined.

eg:- P: It is raining today Q: There are 20 tables in room.
=> It is raining today and there are 20 tables in this room.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction

Ans:- The disjunction of two statements P and Q is the statement $P \vee Q$ which is read as "P or Q". The statement $P \vee Q$ has the truth value F only when both P and Q have the truth value F; otherwise is true. The disjunction is defined.

eg: P: I shall watch the game on television
Q: I shall go to the game.
=> I shall watch the game on television or I go to the game.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Q 9) Define well formed formulae with example

Ans: String consisting of variables Statement formula connected by connectives & enclosed with parenthe
① Primitive statement is WFF (eg:- P, Q)
② Two statements connected by basic connectivities and conditional, biconditional is WFF (eg:- P ∧ Q, P →
③ Equal Number of left & right parenthesis ((P → Q (R ∨ S))
④ Balanced Formula

Eg: ① (P → Q) → WFF
② P ∨ ¬P → R → (Not WFF
③ ((P ∨ Q ∧ ((Q ∨ ¬P)) → (R ∨ ¬P))) → WFF
④ (¬P → R ∨ P → Not WFF

Q10) Define

1) Tautology

Ans: A statement formulae is true regards of truth values of statement which replace v in it is called universally valid formul eg:

P	¬P	P ∨ ¬P	P ∧ ¬P
T	F	T	F
F	T	T	F

2) Contradiction

Ans:- A statement formulae which is false negations of the truth values of the statement which replace the variable in it, is called identically false.

P	¬P	P ∧ ¬P
T	F	F
F	T	F

3) Tautological implications

Ans:- A statement is said to be tautological implied implications when 'A' implied 'B' if and only if $A \rightarrow B$ is a Tautology.

Q11) What are the equivalent formulae.

Ans:- Let A and B are two statement formulae if the truth values of 'A' equal to truth values of 'B' for every one of the 2^n possible set of truth values then A and B are equivalent.

In the definition of equivalence of two formulae, it is not necessary to assume that they both contain the same variables. This point is illustrated in the examples given 3 and 5.

It may, however, be noted that if two formulae are equivalent and a particular variable occurs in only one of them, then the truth value of the formula is independent of this variable.

"$A \Leftrightarrow B$"

Eg:- ① $\neg\neg P$ is equivalent to P

② $P \vee P$ is equivalent to P

③ $(P \wedge \neg P) \vee Q$ is equivalent to Q

⑤ $P \vee \neg P$ is equivalent to $Q \vee \neg Q$

$(P \vee \neg P) \Leftrightarrow (Q \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee \neg P$	$Q \vee \neg Q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore A \Leftrightarrow B$

Q) Write a note on conditional and Biconditional

Ans:- If P and Q are any two Statements, then the statement $P \rightarrow Q$ which is read as "If P, then Q" is called a conditional statement. The statement $P \rightarrow Q$ has a truth value F when Q has the truth value F and P the truth value T, otherwise it has the truth value T. The conditional is defined.

The Statement P is called antecedent and Q the consequent in $P \rightarrow Q$. Again, according to the defination, it is not necessary that there by any kind of relation between P and Q in order to form $P \rightarrow Q$.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

eg:- If either Jerry takes calculus or Ken takes sociology then Larry will take English.

J: Jerry takes calculus

K: Ken takes sociology.

L: Larry takes English

$$(J \vee K) \rightarrow L$$

If P and Q are any two statements, then the statement $P \rightleftarrows Q$, which is read as "P if and only if Q" and abbreviated as "P iff Q" is called a biconditional statement. The statement $P \rightleftarrows Q$ has the truth value T whenever both P and Q have identical truth values.

P	Q	$P \rightleftarrows Q$
T	T	T
T	F	F
F	T	F
F	F	T

eg:- The result was displayed if only if Match was played

$$P \rightleftarrows Q$$

Q13) Explain Duality law with examples.

Ans: In this section we shall consider formulas which contain the connectives $\wedge, \vee$, and $\neg$. There is no loss of generally in restricting our consideration to these connectives since we shall see later that any formula containing any other connective can be replaced by an equivalent formula containing only these three connectives.

Two formulas A and A^* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$. The connectives $\wedge$ and $\vee$ are also called duals of each other. If the formula A contains the special variable T or F, then A^*, its dual is obtained by replacing T by F and F by T in addition to the above-mentioned interchanges.

eg: 1) $(P \vee Q) \wedge R \Rightarrow (P \wedge Q) \vee R$

2) $(P \wedge Q) \vee T \Rightarrow (P \vee Q) \wedge F$

3) $\neg(P \vee Q) \wedge (P \vee \neg(Q \wedge \neg S)) \Rightarrow (P \wedge Q) \vee (P \wedge (Q \vee S)$

Q15) Explain other connectives?

Ans: It was shown earlier that not all connectives de... thus far are necessary for the description of the statement calculus. For any formula of the state calculus, there exists an equivalent formula in which appear only those connectives belonging to one of functionally complete sets.

Let P and Q be any two formulas. Then the formula $P \bar{\nabla} Q$ in which the connective $\bar{\nabla}$ is called an exclusive OR, is true whenever either P or Q, but not both is true. The exclusive OR is also called the exclusive disjunction.

1) $P \bar{\nabla} Q \Leftrightarrow Q \bar{\nabla} P$ (symmetric)
2) $(P \bar{\nabla} Q) \bar{\nabla} R \Leftrightarrow P \vee (Q \bar{\nabla} R)$ (associative)
3) $P \wedge (Q \bar{\nabla} R) \Leftrightarrow (P \wedge Q) \bar{\nabla} (P \wedge R)$ (distributive)
4) $(P \bar{\nabla} Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$
5) $(P \bar{\nabla} Q) \Leftrightarrow \neg (P \rightleftarrows Q)$

eg. $P \uparrow Q \Leftrightarrow \neg (P \wedge Q)$ — (NAND) gate
$P \downarrow Q \Leftrightarrow \neg (P \vee Q)$ — (NOR) gate

P	Q	$P \bar{\nabla} Q$
T	T	F
T	F	T
F	T	T
F	F	F

Q 1 S.T. i) $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$

$\Leftrightarrow (P \to (\neg Q \vee R))$
$\Leftrightarrow \neg P \vee (\neg Q \vee R)$
$(\neg (P \wedge Q)) \vee R$
$\therefore (P \wedge Q) \to R$

2) S.T. $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow$

$\neg P \wedge (\neg Q \wedge R) \vee (Q \vee P) \wedge R$

$(\neg P \wedge \neg Q) \wedge R \vee (Q \vee P) \wedge R$

$\neg(P \vee Q) \wedge R \vee (Q \vee P) \wedge R$

$P \vee \neg P \wedge R$

$\therefore T \wedge R \Leftrightarrow R$

3) $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q))$.

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$

$\Leftrightarrow (P \vee \neg P) \wedge (Q \vee Q) \wedge (Q \vee \neg P)$

$\Leftrightarrow (Q \vee \neg P) \vee Q$.

4) $(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is tautology.

$(\neg P \wedge \neg Q) (\neg P \wedge \neg R)$

$\Leftrightarrow \neg(P \vee Q) \Leftrightarrow \neg(P \vee R)$

$\neg((P \vee Q) \vee \neg(P \vee R))$

$\Leftrightarrow (\neg((P \vee Q) \wedge (P \vee R))$

$\neg\neg P \wedge (\neg Q \vee \neg R)$

$\neg(\neg P \wedge \neg(Q \wedge R))$

$\Leftrightarrow P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

$\neg(P \vee Q) \vee (P \vee R)$

$P \vee \neg P \Leftrightarrow T$

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