Tutorial No - 6
Statistical Techniques

23/11/22

10/10

Q.1 Fit a straight line to the following data.

x	0	5	10	15	20	25
y	12	15	17	22	24	30

→ Let, $y = a + bx$ be the equation of straight line
∴ Normal equations are,

$$\Sigma y = Na + b\Sigma x.$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

x	y	xy	x^2
0	12	0	0
5	15	75	25
10	17	170	100
15	22	330	225
20	24	480	400
25	30	750	625

$\therefore \Sigma x = 75$
$\Sigma xy = 1805$
$\Sigma y = 120$
$\Sigma x^2 = 1375$
$N = 6$

$\therefore 120 = 6a + 75b$ — (1)
$1805 = 75a + 1375b$ — (2)
Eqn (2) / 5.
$361 = 15a + 275b$ — (3).

Put $a = \frac{120 - 75b}{6}$ in eqn (2)

$\therefore 361 = 15\left(\frac{120 - 75b}{6}\right) + 275b$

$\therefore 361 = 300 - 187.5b + 275b$

$\therefore 61 = 87.5b$

$\therefore \underline{b = 0.69}$

Put $b = 0.69$ in eqn (1)

$\therefore 120 = 6a + 75(0.69)$

$6a = 67.71$

$\underline{a = 11.28}$

$\therefore y = 11.28 + 0.69x$

Q.2 Fit a straight line to the following data $(x,y) = (-1,-5), (1,1), (2,4), (3,7), (4,10)$. Estimate y when $x = 7$.

→ Let, $y = a + bx$ be the equation of straight line. Normal equations are,

$\Sigma y = Na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	xy	x^2
-1	-5	5	1
1	1	1	1
2	4	8	4
3	7	21	9
4	10	40	16

$\therefore \Sigma x = 9$

$\Sigma y = 17$

$\Sigma xy = 75$

$\Sigma x^2 = 31$

$N = 5$

$\therefore 17 = 5a + 9b$ — (1)

$75 = 9a + 31b$ — (2)

Substitute $a = \frac{17 - 9b}{5}$ in eqn (2).

$\therefore 75 = 9\left(\frac{17 - 9b}{5}\right) + 31b.$

$375 = 153 - 81b + \cancel{31b}\ 155b.$

$\therefore 74b = 222$

$b = 3.$

Substitute $b = 3$ in equation (1).

$\therefore 17 = 5a + 27$

$\therefore 5a = -10$

$a = -2.$

$\therefore y = -2 + 3x$

When $x = 7$, $y = -2 + 21$ $\therefore y = 19.$

Q.3 Fit a second degree parabolla to following data.

x	0	1	2	3	4
y	7.1	2.4	2.6	2.7	3.4

$\longrightarrow$ Let, $y = a + bx + cx^2$ be the eq^n of parabola

$\therefore$ Normal equations are ;

$$\Sigma y = Na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	xy	x^2	x^3	x^4	x^2y
0	7.1	0	0	0	0	0
1	2.4	2.4	1	1	1	2.4
2	2.6	5.2	4	8	16	10.4
3	2.7	8.1	9	27	81	24.3
4	3.4	13.6	16	64	256	54.4

$\therefore \Sigma x = 10 \quad \Sigma y = 18.2 \quad \Sigma xy = 29.3 \quad \Sigma x^2 = 30$

$\Sigma x^3 = 100 \quad \Sigma x^4 = 354 \quad \Sigma x^2 y = 91.5 \quad N = 5$

$\therefore 18.2 = 5a + 10b + 30c \quad —— (1)$

$29.3 = 10a + 30b + 100c \quad —— (2)$

$91.3 = 30a + 100b + 354c \quad —— (3)$

2 x eqn (1) − eqn (2).

$$36.4 = 10a + 20b + 60c$$
$$-\quad 24.3 = 10a + 30b + 100c$$
$$7.1 = -10b - 40c.$$
$$\therefore 10b + 40c = -7.1 \quad \text{---} \quad (4)$$

3 x eqn (2) − eqn (3).

$$87.9 = 30a + 90b + 300c$$
$$-\quad 91.5 = -30a + 100b + 354c$$
$$-3.6 = -10b - 54c$$
$$10b + 54c = +3.6 \quad \text{---} \quad (5)$$

eqn (4) − eqn (5).

$$10b + 40c = -7.1$$
$$-\quad 10b + 54c = -3.6$$
$$-14c = -10.7$$
$$c = 0.76.$$

Put $c = 0.76$ in eqn (5).

$$\therefore 10b + 54(0.76) = 3.6.$$
$$10b + 41.27 = 3.6$$
$$10b = 3.6 - 41.27$$
$$\therefore b = -3.76.$$

Put value of b and c in eqn (1)

$\therefore 18.2 = 5a + 10(-3.76) + 30(0.764)$

$\therefore 18.2 = 5a - 37.6 + 22.92$.

$\therefore 5a = 32.88$

$\underline{a = 6.57}$.

Equation of parabola :

$y = 6.57 - 3.76x + 0.764x^2$

Q.4 Fit a second degree curve to the following data and estimate the value of y when $x = 80$.

x	10	20	30	40	50	60	70
y	20	60	70	80	90	100	100

→ Let, $y = a + bx + cx^2$ be the equation of parabola. Normal equations are :

$\Sigma y = Na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

x	y	xy	x^2	x^2y	x^3	x^4
10	20	200	100	2000	1000	10000
20	60	1200	400	24000	8000	160000
30	70	2100	900	63000	27000	810000
40	80	3200	1600	128000	64000	2560000

x	y	xy	x^2	x^2y	x^3	x^4
50	90	4500	2500	225000	125000	6250000
60	100	6000	3600	360000	216000	12960000
70	100	7000	4900	490000	343000	24010000

$\Sigma x = 280$ $\quad \Sigma y = 520$ $\quad \Sigma xy = 24200$ $\quad \Sigma x^2 = 14000$

$\Sigma x^2 y = 1292000$ $\quad \Sigma x^3 = 784000$ $\quad \Sigma x^4 = 46760000$ $\quad N = 7$

$\therefore 520 = 7a + 280b + 14000c$ — (1)

$24200 = 280a + 14000b + 784000c$ — (2)

$1292000 = 14000a + 784000b + 46760000c$ — (3)

$40 \times$ eqn (1) − eqn (2).

$20800 = 280a + 11200b + 560000c$

$-\ 24200 = -280a \pm 14000b \pm 784000c$

$-3400 = -2800b - 224000c$ — (4).

$\therefore 34 = 28b + 2240c$

$50 \times$ eqn (2) − eqn (3)

$1210000 = 14000a + 700000b + 39200000c$

$-\ 1292000 = 14000a + 784000b + 46760000c$

$-82000 = -84000b - 7560000c$ — (5)

$\therefore 82 = 84 + 7560c$

3 × eqn (4) - eqn (5).

$$102 = 84b + 6720c$$
$$82 = 84b + 7560c$$
$$20 = -840c$$
$$\therefore \underline{c = -0.023}$$

Substitute $c = -0.023$ in equation (4).

$$\therefore 34 = 28b + 2240(-0.023)$$
$$\therefore 34 = 28b - 51.52$$
$$\therefore \underline{b = 3.054}$$

Put value of b and c in equation (1).

$$\therefore 520 = 7a + 280b + 14000c$$
$$\therefore 520 = 7a + 280(3.054) + 14000(-0.023)$$
$$\therefore 520 = 7a + 855.12 - 322$$
$$\therefore 7a = -13.12$$
$$\therefore \underline{a = -1.87}$$

$\therefore$ Equation of second degree curve

$$y = -1.87 + 3.054x - 0.023x^2$$

When $x = 80$,

$$y = -1.87 + 3.054(80) - 0.023(80)^2$$
$$\therefore y = -1.87 + 244.32 - 147.2$$
$$\therefore y = 95.25$$

Q. Fit a least square geometric curve $y = ax^b$ to the following data.

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

→ Consider, $y = ax^b$

Taking log on both sides,

$\log y = \log a + b \log x$

Let, $\log y = Y$, $\log a = A$, $\log x = X$

$\therefore Y = A + bX$

$\therefore$ Normal equations are,

$\Sigma Y = nA + b\Sigma X$

$\Sigma XY = A\Sigma X + b\Sigma X^2$

x	y	X	Y	X^2	XY
1	0.5	0	-0.693	0	0
2	2	0.693	0.693	0.480	0.480
3	4.5	1.098	1.504	1.205	1.651
4	8	1.386	2.079	1.920	2.881
5	12.5	1.609	2.525	2.588	4.062

$\Sigma X = 4.786$

$\Sigma Y = 6.108$

$\Sigma XY = 9.074$

$\Sigma X^2 = 6.193$

$\therefore 6.108 = 5A + 4.786b$ — (1)

$9.074 = 4.786A + 6.193b$ — (2).

$4.786 \times$ equation (1) $- 5 \times$ equation (2).

$$\therefore \quad 29.232 = 23.\cancel{9}3A + 22.905b$$
$$45.370 = 23.93A + 30.965b$$
$$-16.138 = -8.06b$$
$$b = 2.002$$

Put $b = 2.002$ in eqn (1).

$$6.108 = 5A + 9.581$$
$$5A = -3.473$$
$$A = -0.694.$$

$$A = \log a.$$
$$\therefore a = e^{A} = e^{-0.694} = 0.499.$$

$$\therefore y = 0.499\, x^{2.002}.$$

Q. 6 If growth of certain kind of bacteria follows law $y = ab^x$. Then find best fitting values of a and b using following data.

x	1	2	3	4	5
y	233.2	253.4	282.3	302.4	332.8

→ Consider, $y = ab^x$

Taking log on both sides,

$\log y = \log a + x \log b$.

Put, $Y = \log y$, $A = \log a$, $B = \log b$.

$\therefore Y = A + Bx$

Normal equations are,

$\Sigma Y = nA + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

x	y	Y	xY	x^2
1	233.2	5.451	5.451	1
2	253.4	5.534	11.068	4
3	282.3	5.642	16.926	9
4	302.4	5.711	22.844	16
5	332.8	5.807	24.035	25

$\Sigma x = 15$

$\Sigma Y = 28.145$

$\Sigma xY = 85.324$

$\Sigma x^2 = 55.$

$\therefore 28.145 = 5A + 15B \quad - \quad (1)$

$85.324 = 15A + 55B \quad - \quad (2).$

$\therefore 3\times$ eqn (1) $-$ eqn (2).

$84.435 = \cancel{15A} + 45B$

$85.324 = \cancel{15A} + 55B$

$-0.889 = -10B$

$B = 0.088.$

Put $B = 0.088$ in eqn (1).

$\therefore 28.145 = 5A + 15(0.088).$

$\therefore 5A = 26.825$

$A = 5.365$

$\therefore A = \log a \quad \therefore a = e^{5.365} = 213.79$

$B = \log b \quad b = e^{0.088} = 1.091$

$\therefore y = (213.79)(1.091)^x$

Q.7 Fit a least square geometric curve $y = ae^{bx}$ to following data:

x	2	3	4	5	6
y	34.385	79.0855	181.9	418.36	962.23

→ Consider,

$y = ae^{bx}$

Taking log on both sides,

$\log y = \log a + bx \log e$

$\therefore \log y = \log a + bx$.

Let, $\log y = Y$, $\log a = A$.

$\therefore Y = A + bx$.

$\therefore$ Normal equations are,

$\Sigma Y = nA + b\Sigma x$

$\Sigma xY = A\Sigma x + b\Sigma x^2$

x	y	Y	xY	x^2
2	34.385	3.537	7.074	4
3	79.085	4.370	13.11	9
4	181.90	5.203	20.812	16

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x	y	Y	xY	x^2
5	418.36	6.036	30.18	25
6	962.23	6.869	41.214	36.

$\therefore \Sigma x = 20$ $\qquad \Sigma Y = 26.015$ $\qquad \Sigma xY = 112.39$

$\Sigma x^2 = 90$

$$26.015 = 5A + 20b \quad — \quad (1)$$

$$112.39 = 20A + 90b \quad — \quad (2).$$

4 x equation (1) − equation (2).

$$\therefore 104.06 = 20A + 80b.$$

$$\underline{112.39 = 20A + 90b}$$

$$-8.33 = -10b.$$

$$\therefore b = 0.833.$$

Put $b = 0.833$ in equation (1).

$$\therefore 26.015 = 5A + 20(0.833)$$

$$5A = 9.355$$

$$A = 1.871.$$

$\log a = A.$

$$a = e^{A}$$

$$a = e^{1.871} = 6.494.$$

$$\therefore y = (6.494)e^{0.833x}$$