

Tutorial 7.

Q1) calculate the coefficient of correlation from the following data.

~~66~~
~~4/1/22~~
~~10~~
~~10~~

x	y	x^2	y^2	xy	
65	67	4225	4489	4355	
66	68	4356	4624	4488	
67	66	4489	4356	4422	
68	69	4624	4761	4692	
69	72	4761	5184	4968	
70	72	4900	5184	5040	
71	69	5041	4761	4899	
Σ	476	483	32,396	33,359	32,866

$$\bar{X} = \frac{\sum x_i}{n} = \frac{476}{7} = 68$$

$$\bar{Y} = \frac{\sum y_i}{n} = \frac{483}{7} = 69$$

$$r = \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

$$r = \frac{7 \times (32,864) - (476) \times (483)}{\sqrt{7(32,396) - (476)^2} \sqrt{7(33,359) - (483)^2}}$$

$$r = \frac{230068 - 229908}{\sqrt{226772 - 226576} \sqrt{233513 - 233289}}$$

$$= \frac{140}{\sqrt{196} \sqrt{224}} = \frac{140}{14 \times 14.966}$$

$$\boxed{r = 0.668}$$

coefficient of correlation $r = 0.668$

Q.2)

Calculate the coefficient of correlation from the following data.

x	y	x ²	y ²	xy
10	18	100	324	180
14	12	196	144	168
18	24	324	576	432
22	6	484	36	132
26	30	676	900	780
30	36	900	1296	1080
Σ 120	126	2680	3276	2772

$$\bar{X} = \frac{\sum x_i}{n} = \frac{120}{6} = 20$$

$$\bar{Y} = \frac{\sum y_i}{n} = \frac{126}{6} = 21$$

$$\begin{aligned}
 r &= \frac{n \sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}} \\
 &= \frac{6(2772) - 120(126)}{\sqrt{6(2680) - (120)^2} \sqrt{6(3276) - (126)^2}} \\
 &= \frac{1512}{\sqrt{1680} \sqrt{3780}} \\
 &= \frac{1512}{40.987 \times 61.481} = \frac{1512}{2519.950} \\
 &= 0.6000
 \end{aligned}$$

coefficient of correlation $r = 0.6000$

Q3) Given, Covariance = 12.5 $r = 0.5$

Variance of $x = 25$ find σ_y

where, variance of $x = \sigma_x^2$
variance of $y = \sigma_y^2$

We know,

$$r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$

$$\sigma_y = \frac{\text{Cov}(x, y)}{r \sigma_x}$$

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$$\sigma x = \sqrt{\text{var}(x)}$$

$$= \sqrt{25}$$

$$\sigma(x) = 5$$

$$\therefore \sigma(y) = \frac{12.5}{0.5 \times 5} = 5$$

Q.4) Use the following data obtain the regression equation of y on x and x on y.

x	y	xy	x ²	y ²
6	9	54	36	81
2	11	22	4	121
10	5	50	100	25
4	8	32	16	64
8	7	56	64	49
Σ 30	40	214	220	360

$$\bar{X} = \frac{\Sigma x}{N} = \frac{30}{5} = 6$$

$$\bar{Y} = \frac{\Sigma y}{N} = \frac{40}{5} = 8$$

$$b_{yx} = \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{\Sigma x^2 / N - (\Sigma x / N)^2}$$

$$= \frac{214}{5} - \frac{6 \times 8}{25}$$

$$\frac{220}{5} - \frac{36}{25}$$

$$b_{yx} = \frac{-26}{40} = -0.65$$

$$b_{xy} = \frac{\sum xy/N - (\sum x/N)(\sum y/N)}{\sum y^2/N - (\sum y)^2/N}$$

$$= \frac{214/5 - (8 \times 6)}{340/5 - 64}$$

$$b_{xy} = \frac{214 - 240}{340 - 320} = -1.3$$

Equation of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 8 = -0.65(x - 6)$$

$$y - 8 = -0.65x + 3.9$$

$$y = -0.65x + 11.9$$

Equation of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 6 = -1.3(y - 8)$$

$$x - 6 = -1.3y + 10.4$$

$$x = -1.3y + 16.4$$

9.) Obtain two regressive equations from the following data.

x	y	xy	x ²	y ²
2	4	8	4	16
4	2	8	16	4
6	5	30	36	25
8	10	80	64	100
10	3	30	100	9
12	6	72	144	36
Σ	42	30	228	364

$$\bar{x} = \Sigma x / N = 42 / 6 = 7$$

$$\bar{y} = \Sigma y / N = 30 / 6 = 5$$

$$b_{yx} = \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{(\Sigma x^2 / N) - (\Sigma x / N)^2}$$

$$= \frac{228 / 6 - (7 \times 5)}{36}$$

$$= \frac{364 / 6 - \frac{49}{36}}{36}$$

$$= \frac{18}{70} = 0.257$$

$$b_{xy} = \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{\Sigma y^2 / N - (\Sigma y / N)^2}$$

$$= \frac{228 / 6 - \frac{35}{36}}{190 / 6 - \frac{25}{36}}$$

$$= \frac{18}{40} = 0.45$$

equation of y on x :

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 5 = 0.257(x - 7)$$

$$y - 5 = 0.257x - 1.799$$

$$y = 0.257x + 3.201$$

equation of x on y :

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - 7 = 0.45(y - 5)$$

$$x - 7 = 0.45y - 2.25$$

$$x = 0.45y + 4.75$$

8) obtain the two regression equations from the following data & predict the value of x when $y=40$ & value of y when $x=50$

x	y	xy	x^2	y^2
16	60	960	256	3600
22	65	1430	484	4225
28	63	1764	784	3969
29	66	1914	841	4356
27	68	1836	729	4624
25	71	1775	625	5041
18	70	1260	324	4900
Σ 149	453	8585	3275	29395

$$\bar{x} = \sum x/n = \frac{149}{7} = 21.28$$

$$\bar{y} = \sum y/n = \frac{453}{7} = 64.714$$

$$b_{yx} = \frac{\sum xy/n - (\sum x/n)(\sum y/n)}{\sum x^2/n - (\sum x/n)^2}$$

$$= \frac{8585.4 - (149/7 \times 453/7)}{3275/7 - (149/7)^2}$$

$$= \frac{-1056.77}{105.136}$$

$$= -10.032$$

$$= -10.032$$

$$b_{xy} = \frac{\sum xy/n - (\sum x/n)(\sum y/n)}{\sum y^2/n - (\sum y/n)^2}$$

$$= \frac{1926.42 - 1377.113}{4199.28 - 4187.90} = \frac{-250.693}{11.38}$$

$$b_{xy} = -22.029$$

equation of y on x,

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 21.28 = -22.029(y - 64.714)$$

$$x = -22.029y + 1446.866$$

put $y = 40$

$$x = -22.029 \times 40 + 1446.866$$

$$x = 565.704$$

$$\text{Correlation coefficient } (r) = -\sqrt{b_{xy} \cdot b_{yx}}$$

$$r = -15.865$$

Q2) Given, Variance of $x = 9$. The regression equation are $8x - 10y + 66 = 0$, $40x - 18y + 214 = 0$ find,

1) average value of x & y

2) correlation coefficient betⁿ two variable.

3) standard deviation of y .

⇒ To get \bar{x} & \bar{y} , solving the given equations simultaneously.

equations ① - equations ②

$$40x - 50y = -330$$

$$- \quad 40x + 18y = -214$$

$$-32y = 544$$

$$y = -544/32 = 17$$

putting $y = 17$ in equation ②

$$40x - 18 \times 17 = 214$$

$$40x - 126 = 214$$

$$40x = 340$$

$$\boxed{x = 8.5}$$

∴ mean of x (\bar{x}) = 8.5

∴ mean of y (\bar{y}) = 17

let $8x - 10y + 66 = 0$ be a line of regression of y on x .

$$8x - 10y = -66$$

$$-10y = -66 - 8x$$

$$y = \frac{8x}{10} + \frac{66}{10}$$

$$b_{yx} = \frac{8}{10} = \frac{4}{5}$$

let $40x - 18y + 216 = 0$ be a line of regression of x on y .

$$40x = -216 + 18y$$

$$x = -\frac{216}{40} + \frac{18}{40}y$$

$$b_{xy} = \frac{9}{20}$$

$$r = \pm \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{\frac{9}{25}} = 0.6$$

Now we know,

$$b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

$$\text{var}(x) = 9 \quad \sigma_x = 3$$

$$\sigma_y = r \frac{\sigma_x}{b_{xy}} = 0.6 \times \frac{3}{9/20}$$

$$= 0.6 \times \frac{20}{3}$$

$$\boxed{\sigma_y = 4}$$