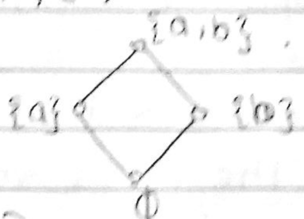
Unit 4 :

GLB → Greatest lower bound.

Page No.			
Date	15	11	22

* POSET :- $\langle P, \leq \rangle$

$A = \{a, b\}$

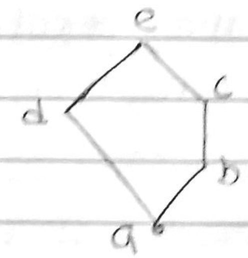


GLB $(\{a\}, \{b\}) = \phi$

LUB ~~{a,b}~~ $(\{a\},\{b\}) = \{a\},\{b\}$

In a POSET if

1)

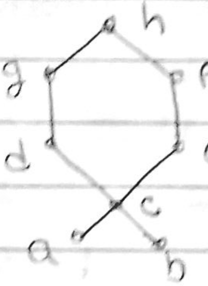


GLB $(\{d\},\{c\}) = a$

LUB $(\{d\},\{c\}) = e$

✓ It is a Lattice.

2)

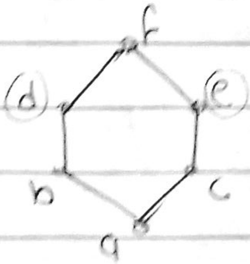


It is not a lattice

GLB $(\{b\}, \{a\})$ = ~~ar~~ undefined

or no GLB of $(\{a\}, \{b\})$.

3)

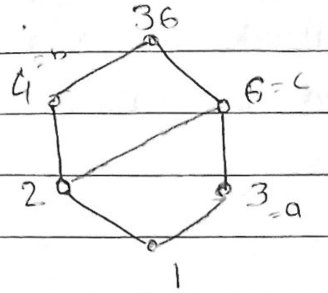


there is no GLB of $(\{d\}, \{e\})$

Page No.
Date

notations :-
GLB = $*$
LUB = $\oplus$

4)



GLB(4,6) = 2
LUB($\cancel{2}$,$\cancel{3}$) = 6 .

GLB(4,4) = 4
GLB(3,3) = 3

\# $\therefore \boxed{a * a = a}$ — (I)

\# $\boxed{a \oplus a = a}$ — (II)

\# $a * b = b * a$

\# $a \oplus b = b \oplus a$

\# $a * (b * c) = (a * b) * c$ ---(associative)

$3 * (4 * 6) = (3 * 4) * 6$

$= 3 * 2 \quad = 1 * 6$

$= 1 \quad = 1$

$\therefore$ associative ✓

\# $a \oplus b = b \oplus a$

✓ $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

$3 \oplus 36 \qquad 36 \oplus 6$

$36 = 36$

\# $a * (a \oplus b) = a$ ... (absorptivity)

& $a \oplus (a * b) = a$

$3 * (3 \oplus 4)$

$= 3 * 36$

$= 3 = \underline{a}$.

Page No.
Date

* Partially Ordered set :- Relation which is reflexive, transitive & antisymmetric.

Q. $<L, *, \oplus>$

→ it satisfies Idempotency
commutativity
associativity
absorptivity

then we called it as Lattice.

- Sub-Lattice :- If $S \subset L$ & S is closed under the operations '$*$ & $\oplus$ then $<S, *, \oplus>$ is called sub-lattice of L.

where, $L = <L, *, \oplus>$

For any $a, b \in S$ it must,

$a \oplus b \in S$

& $a * b \in S$.

* Direct-Product of two lattices :-

$<L_1, *, \oplus>, <L_2, \wedge, \vee>$

$<L_1 \times L_2, \circ, +>$

$\because$ For any $l_{11}, l_{12} \in L_1$

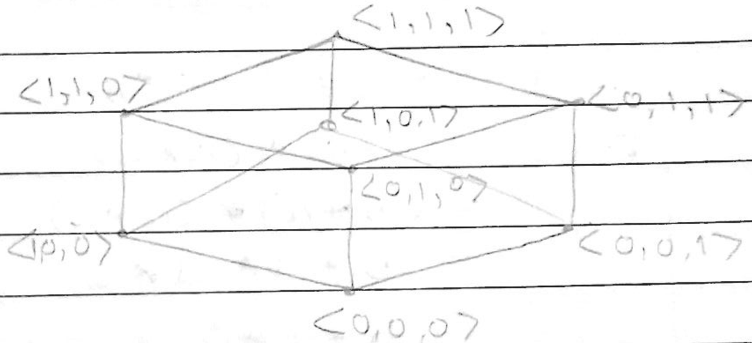
$l_{21}, l_{22} \in L_2$

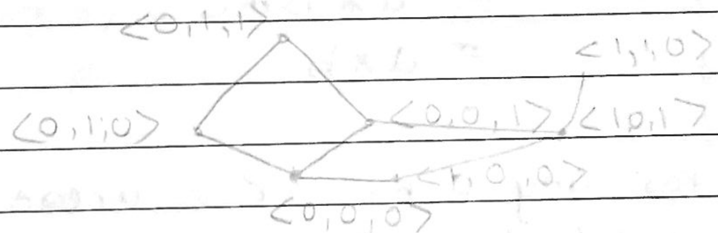
$\langle l_{11}, l_{21}\rangle \cdot \langle l_{12}, l_{22}\rangle = \underline{\underline{\langle l_{11} * l_{12}, l_{21} \wedge l_{22}\rangle}}$

$\langle l_{11}, l_{21}\rangle + \langle l_{12}, l_{22}\rangle = \langle l_{11} \oplus l_{12}, l_{21} \vee l_{22}\rangle$.

$\therefore$ $\langle L_1 \times L_2, \cdot, + \rangle$ is also <u>Lattice</u>.

000
001
010
011
100
101
110
111





* morphism - $\langle L_1, *_1, \oplus_1 \rangle$, $\langle L_2, *_2, \oplus_2 \rangle$

~~ψ (x~~

$\psi(x *_1 y) = \psi(x) *_2 \psi(y)$

\# Lattice can be defined as a POSET Algebraic system. $\langle P, \leq \rangle$

POSET is a lattice, if for any $a, b \in P$ & GLB & LUB for $\{a, b\}$

Q. $\langle P, \leq \rangle$

For any $a, b, c \in P$ if,

$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$

$(a * b) * (a * c) = a * b$

L.H.S $= (a * b) * (a * c)$

$= (b * a) * (a * c)$ ...(operatn is commutati

$= b * a * a * c$

$= b * a * c$

$= a * b * c$

$= a * b$

Q. For any $a, b, c \in P$ where P is $\langle P, \leq \rangle$. P.T.

$a \oplus (b * c) \leq (a \oplus b) + (a \oplus c)$

$a * (b \oplus c) \leq (a * b) \oplus (a * c)$

$a \leq b$

- Hasse diagrams -

$\le$: $\ldots, 4, 3, 2, 1, 0, -1, -2, -3, \ldots$ — no least & greatest members.

$\ge$: $0, 1, 2, 3, \ldots$ — greatest element = 0, no least member.

$\le$: $3, 2, 1, 0$ (converse of $\le$ is $\ge$). Greatest element = (not defined). least member = 0.

These type of lattices are called unbounded lattices.

8] $\langle P, \le \rangle$

$a, b, c \in P$

$a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$

& $a * (b \oplus c) \le (a * b) \oplus (a * c)$

if $a \le a \oplus b$ & $a \le a \oplus c$

then $a \le (a \oplus b) * (a \oplus c)$ — (I)

Now, we have to prove that

$(b * c) \le (a \oplus b) * (a \oplus c)$

$b * c \le b$ ~~$b \le b * c$~~

$b * c \le c \le a \oplus b$

& $c \le a \oplus c$ $\therefore \boxed{b * c \le a \oplus c}$ --- (associativity). — (II)

$\boxed{b * c \le a \oplus b}$ — (III)

from (II) & (III),

$b * c \le (a \oplus b) * (a \oplus c)$. — (IV)

From (I) & (IV),

$a \oplus (b * c) \le (a \oplus b) * (a \oplus c)$.

* $\langle L, *, \oplus \rangle$

For any $a, b, c \in L$,

if $a * b = a * c \cap (a \oplus b = a \oplus c \Rightarrow b = c$

$(a * b) \oplus c = (a * c) \oplus c = c$.

$(a * c) \oplus b = (a * b) \oplus b$.

$(a \oplus c) * b = (a \oplus b) * b = b$?

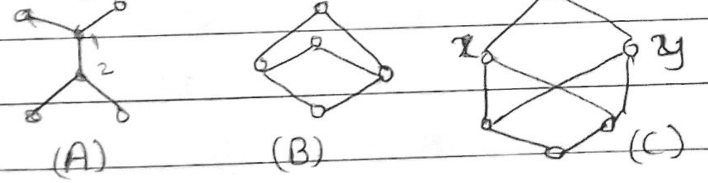
* Lattices as partially ordered sets :- A lattice is a partially ordered set $<L, \leq>$ in which every pair of elements $a, b \in L$ has a greatest lower bound and a least upper bound.

e.g. The GLB of a subset $\{a,b\} \subseteq L$ will be denoted by $a*b$ and the least upper bound by $a \oplus b$. It is customary to call the GLB $\{a,b\} = a*b$ the meet or product of a and b, & the LUB $\{a,b\} = a \oplus b$ the join or sum of a & b.

— Any statement about lattices involving the operations $*$ & $\oplus$ and the relations $\leq$ & $\geq$ remains true if $*$ is replaced by $\oplus$, $\oplus$ by $*$; $\leq$ by $\geq$ & $\geq$ by $\leq$.

The operations $*$ & $\oplus$ are called duals of each other as are the relations $\geq$ and $\leq$. similarly, the lattices $<L, \leq>$ & $<L, \geq>$ are called duals of each other

Q.1 Why below POSETS are not a lattices ?



(A) (B) (C)

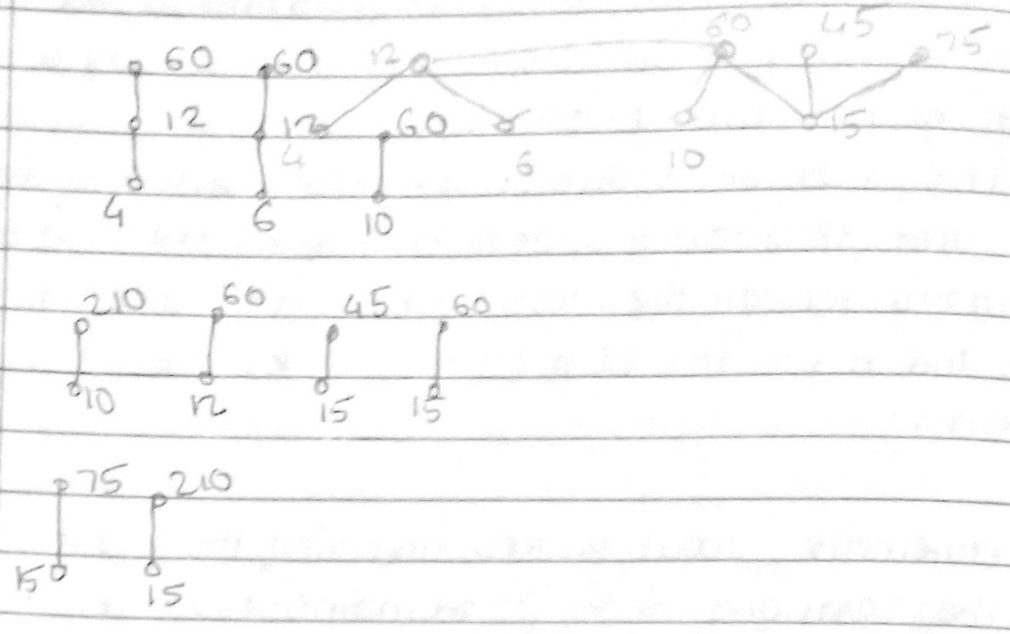
In case A) there is no GLB & LUB for (1 & 2)

B)

C) NO GLB for x & y.

Q.2 $\langle S_n, D \rangle$ $n = 4, 6, 10, 12, 15, 45, 60, 75, 210$.

Chain: lattice:



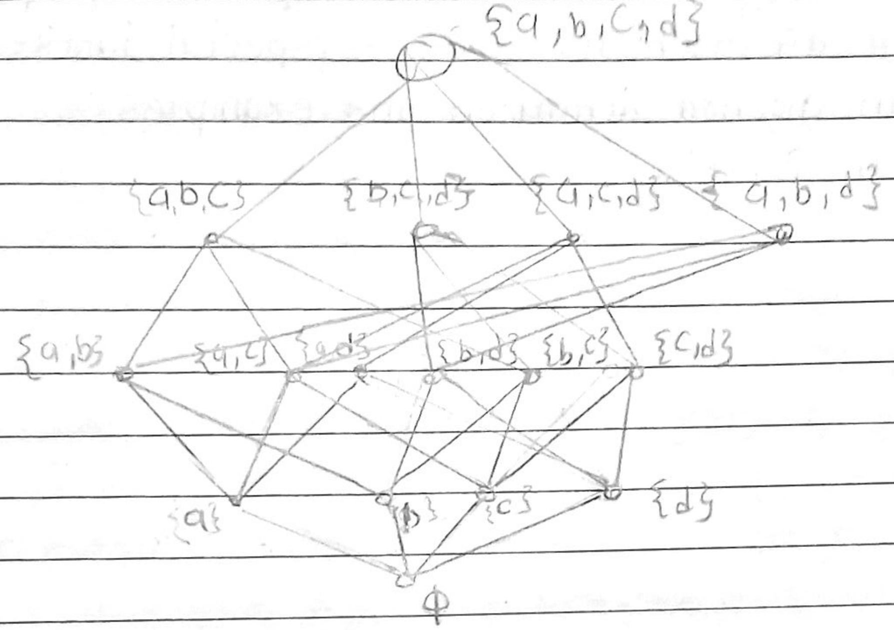
Q.3] $S \subseteq P(A)$ is called a partition of A. if 1] $\forall X \in S$, $X \neq \phi$ 2] $\forall X, Y \in S$, $X = Y$ or $X \cap Y = \phi$
3] $\bigcup_{x \in X} X = A$.

let, $A = \{a, b, c, d\}$
Partition sets are $\Rightarrow$ $\{\{a\}, \{b, c, d\}\}$, $\{\{b\}, \{a, c, d\}\}$,
$\{\{c\}, \{a, b, d\}\}$, $\{\{d\}, \{a, b, c\}\}$, $\{\{a, b\}, \{c, d\}\}$,
$\{\{a, c\}, \{b, d\}\}$, $\{\{a, d\}, \{b, c\}\}$, $\{\{a\}, \{b\}, \{c\}, \{d\}\}$
$\{\{a\}, \{b\}, \{c, d\}\}$, $\{\{a\}, \{c\}, \{b, d\}\}$, $\{\{a\}, \{d\}, \{b, c\}\}$
$\{\{b\}, \{c\}, \{a, d\}\}$, $\{\{b\}, \{d\}, \{a, c\}\}$, $\{\{c\}, \{d\}, \{a, b\}\}$
$\{\{a, b, c, d\}\}$

ϕ

Page No.
Date

Q.3] Hasse diagram :-



* Properties of Lattices :- Properties of the two binary operations of meet & join denoted by $*$ & $\oplus$ on a lattice $<L, \leq>$. For any $a, b, c \in L$, we have

L-1] $a * a = a$. L-1'] $a \oplus a = a$ — (Idempotent)
2] $a * b = b * a$ L-2'] $a \oplus b = b \oplus a$ — (commutative)
3] $(a * b) * c = a * (b * c)$ L-3'] $(a \oplus b) \oplus c = a \oplus (b \oplus c)$ — (associative)

4] $a * (a \oplus b) = a$ L-4'] $a \oplus (a * b) = a$ — (abasorptive).

* Theorem - 1 : let $<L, \leq>$ be a lattice in which $*$ & $\oplus$ denote the operations of meet & join respectively. For any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

Page No.
Date

Theorem-2: let $\langle L, \leq \rangle$ be a lattice, for any $a, b, c \in L$, the following properties called isotonicity, hold:

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

Theorem 3: let $\langle L, \leq \rangle$ be a lattice. for any $a, b, c \in L$, the following inequalities, called the distributive inequalities, hold:

$$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$$
$$a * (b \oplus c) \geq (a * b) \oplus (a * c).$$

Theorem 4: let $\langle L, \leq \rangle$ be a lattice. for any $a, b, c \in L$ the following holds:

$$a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$$

* Lattices as Algebraic Systems :- A lattice is an algebraic system $\langle L, *, \oplus \rangle$ with two binary operations $*$ & $\oplus$ on L which are both 1) commutative 2) associative & 3) satisfy absorption laws.

\# The advantage of defining a lattice as an algebraic system is that we can introduce the concept of sublattices in a natural way.

⇒ Sublattice :- let $\langle L, *, \oplus \rangle$ be a lattice & let $S \subseteq L$ be a subset of L. The algebra

$\langle S, *, \oplus \rangle$ is a sublattice of $\langle L, *, \oplus \rangle$ iff S is closed under both operations $*$ & $\oplus$.

Sublattice itself is a lattice.

- If $\langle P, \leq \rangle$ is a partially ordered set & $Q \subseteq P$ then $\langle Q, \leq \rangle$ is also partially ordered set.

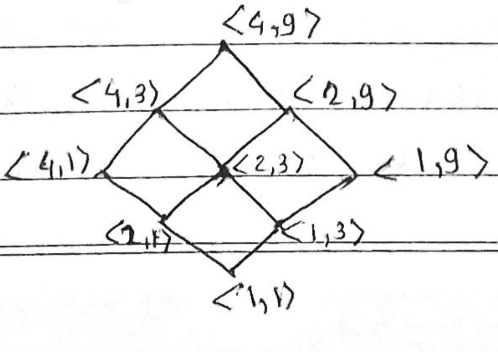
* direct product :- let $\langle L, *, \oplus \rangle$ & $\langle S, \wedge, \vee \rangle$ be two lattices. The algebraic system $\langle L \times S, \cdot, + \rangle$ in which the binary operations $\oplus$ & $\cdot$ on $L \times S$ are such that for any $\langle a_1, b_1 \rangle$ & $\langle a_2, b_2 \rangle$ in $L \times S$.

$$\langle a_1, b_1 \rangle \cdot \langle a_2, b_2 \rangle = \langle a_1 * a_2, b_1 \wedge b_2 \rangle$$
$$\langle a_1, b_1 \rangle + \langle a_2, b_2 \rangle = \langle a_1 \oplus a_2, b_1 \vee b_2 \rangle$$

is called direct product of the lattices $\langle L, *, \oplus \rangle$ and $\langle S, \wedge, \vee \rangle$

- The operations $+$ and $\cdot$ on $L \times S$ are commutative and associative & satisfy the absorption laws because they are defined in terms of the operations $*, \oplus$ and $\wedge, \vee$.
- $\therefore$ direct product is itself a lattice.

e.g. : let chains of divisors of 4 & 9, that is, $L_1 = \{1, 2, 4\}$ & $L_2 = \{1, 3, 9\}$, the partial ordering relation is "division". The lattice $L_1 \times L_2$ is ? (diagram of the lattice ?)



Page No.
Date

* **Lattice Homomorphism :-** Let $\langle L, *, \oplus \rangle$ and $\langle S, \wedge, \vee \rangle$ be two lattices. A mapping $g : L \to S$ is called a lattice homomorphism from the lattice $\langle L, *, \oplus \rangle$ to $\langle S, \wedge, \vee \rangle$ if for any $a, b \in L$

$$g(a * b) = g(a) \wedge g(b) \quad \text{and}$$
$$g(a \oplus b) = g(a) \vee g(b) .$$

- In homomorphism both the operations of meet & join are preserved. There may be mappings which preserve only one of the two operations. Such mappings are not lattice homomorphisms.

* **Special Lattices :-**

1] A lattice is called complete if each of its nonempty subsets has a least upper bound & a greatest lower bound.

2] In a bounded lattice $\langle L, *, \oplus, 0, 1 \rangle$, an element $b \in L$ is called a complement of an element $a \in L$ if
$a * b = 0$ & $a \oplus b = 1$.

⇒ **Complemented Lattice :-** A lattice $\langle L, *, \oplus, 0, 1 \rangle$ is said to be a complemented lattice if every element of L has at least one complement.

Page No.
Date

⇒ Distributive Lattice :- A lattice $\langle L, *, \oplus \rangle$ is called a distributive lattice if for any $a, b, c \in L$,

$$a * (b \oplus c) = (a * b) \oplus (a * c)$$
&
$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$

the operations $*$ & $\oplus$ distributive over each other.

Theorem: Every chain is a distributive lattice (395).

Thm 2: The direct product of any two distributive lattices is a distributive lattice. (396).

Thm 3: IMP Let $\langle L, *, \oplus \rangle$ be a distributive lattice. For any $a, b, c \in L$,

$$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c.$$

Proof → Page no. (396).

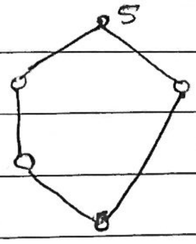
$$(a * b) \oplus c = (a * c) \oplus c = c$$
$$(a * b) \oplus c = (a \oplus c) * (b \oplus c)$$
$$= (a \oplus b) * (b \oplus c)$$
$$= b \oplus (a * c) = b \oplus (a * b) = b.$$

* Boolean Algebra :- Boolean Algebra is a complemented, distributive lattice.

- Generally denoted by $\langle B, *, \oplus, ', 0, 1 \rangle$ in which $\langle B, *, \oplus \rangle$ is a lattice with two binary operations $*$ and $\oplus$.

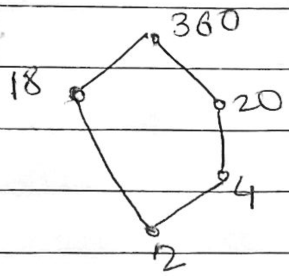
* Special Lattices :-

1) Complete Lattice :



every complete lattice must have finite lattice & vise versa.
If lattice have ~~always~~ both least & greatest element then it is bounded lattice.

2) Complimented lattice :- if every element in lattice have atleast one ~~to~~ complement.



3) Distributive Lattice :-

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$

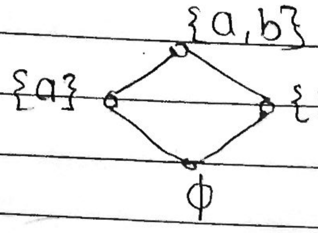
$(\because \; a * (b \oplus c) \neq (a * b) \oplus (a * c))$

If the lattice is distributive as well as complemented lattice.

a) 

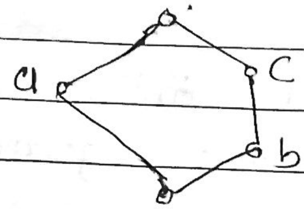
simplest distributive complemented lattice.

b) $\langle P(A), \underset{\cap}{*}, \underset{\cup}{\oplus}, ', \underset{\phi}{0}, \underset{A}{1} \rangle$.



$<S, \wedge, \vee,$

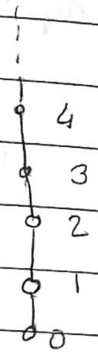
$<S, \wedge, \vee, \neg, f, T>$ (distributive complemented lattice)



* Every chain lattice is distributive.

$<L, *, \oplus>$

For any $a, b, c \in L$



$a \leq b$	or	$a \leq c$
F	$\vee$	F
F	$\vee$	T
T	$\vee$	F
T	$\vee$	T

→ For this 3 observat^s ≤ then for ... ∨ B (1st case).

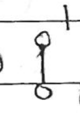
- In a distributive lattice $\langle L, *, \oplus \rangle$ for any $a, b, c \in L$
$(a * b) = (a * c) \cap (a \oplus b) = (a \oplus c)$
$\therefore \boxed{b = c}$

Proof: $(a * b) \oplus c = (a \oplus c) * (b \oplus c)$

$L.H.S. = (a * b) \oplus c$
$= (a * c) \oplus c$
$= c$

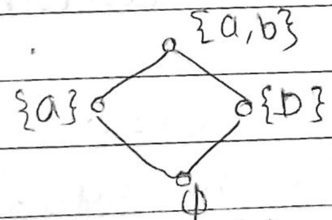
$R.H.S. = (a \oplus c) * (b \oplus c)$
$= (\cancel{b \oplus c}) * \oplus c * (b \oplus c)$
$= (a \oplus b) * (b \oplus c)$
$= b + (a * c)$
$= b \oplus (a * b)$
$= b$

$L.H.S. = R.H.S.$
$\therefore b = c$

- A distributive & complemented lattice is a Boolean Algebra.
e.g. simplest boolean algebra $\rightarrow$ 

$\langle B, *, \oplus, ', 0, 1 \rangle$

Q. $A = \{a, b\}$

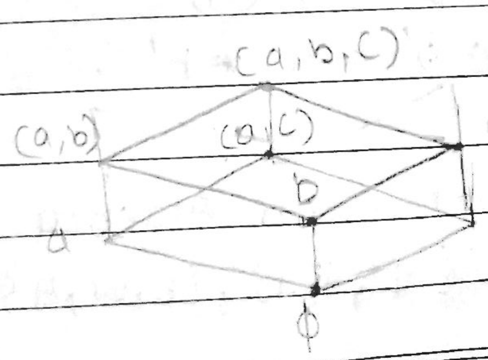


representation : $\langle P^{(A)}, \cap, \cup, -, \phi, A \rangle$

Q. $A = \{a, b, c\}$

$a * a' = 0$

$a \oplus a' = 1$



$\therefore a' = \{b, c\}, \; b' = \{a, c\}$

Page No.
Date

Q. $\langle B, *, \oplus, ', 0, 1 \rangle$ as an algebraic system what properties Boolean algebra can satisfy?

1) 4 properties ~~of~~ for lattices
2) distributive.

$a * (b \oplus c) = (a * b) \oplus (a * c)$

$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

L.H.S. $= (a * b) \oplus (b * c) \oplus (c * a)$
$= (b * (a + c)) \oplus (a * c)$
$= ((a * c) \oplus b) * ((a * c) \oplus (a \oplus c))$
$= (a + b) * (b \oplus c) * ((a \oplus c \oplus a) * (c \oplus a \oplus c))$
$= (a \oplus b) * (b \oplus c) * ((a \oplus c) * (a \oplus c))$
$= (a \oplus b) * (b \oplus c) * (a \oplus c)$
$=$ R.H.S.

3) For any $a \in B$ $a * a' = 0$ & $a \oplus a' = 1$
(every element has a complement)

- $a * 0 = 0$ $a \oplus 1 = 1$
  $a \oplus 0 = a$ $a * 1 = a$.

- $(a * b)' = a' \oplus b'$
- $(a \oplus b)' = a' * b'$.

* Sub-Boolean Algebra :-

$\{\phi, \{a\}, \{b, c\}, A\}$ it is perfect sub boolean algebra of A.

$\{\phi, \{a\}, \{b\}, \{b, c\}, \{a, c\}, A\}$ = perfect Sub-Algebra.

- POSET becomes a lattice iff each pair in set have GLB & LUB. (e.g. $<P, \leq>$ $\hookrightarrow a, b \in N$)

* - Boolean algebra is lattice if it is distributive & complemented. $< B, *, \oplus, ', 0, 1 >$.
0 → least element, 1 → greatest element.

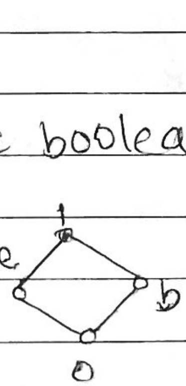
$\therefore$ It is finite. (not infinite)

* Boolean Expressions :- A Boolean expression in n is $x_1, x_2, \dots x_n$.

1)
2) every x_i $1 \leq x_i \leq n$ is boolean exp^n.
3) if α & β are boolean exp^n (BE), $(\alpha) * (\beta)$, $(\alpha) \oplus \beta$, $(\alpha)'$.
4) e.g. $(x_1 * x_2') \oplus x_3$

- statement & ~~ste~~ set algebra are boolean algebra.

* $(x_1 * x_2') \oplus (x_2)$: evaluate over the algebra → a



$x_1 = 0$, $x_2 = a$.

$= (0 * b) \oplus a$

$= 0 \oplus a = a$.

* Given,
$x_1 = a$, $x_2 = 0$, $x_3 = a$, $x_4 = b$ evaluate,

$(x_1 * x_4) \oplus (x_2' * x_3) \oplus (x_4')$

$= (a * b) \oplus (1 * a) \oplus (a)$

$= a \oplus (a) \oplus a$

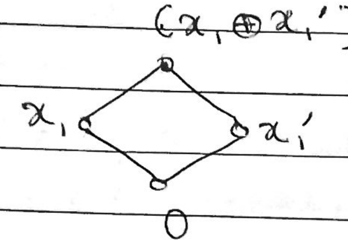
$= a \oplus a$

$= a$

* $2^{(2^3)}$ distinck PDNF's possible for 3 variables same for 2, 4, 5 variables.

$x_1 \equiv x_1 * (x_2 \oplus x_2')$

→ for 1 variable,



x_1	x_1'	
1	1	$x_1 \oplus x_1'$
0	1	x_1'
1	0	x_1
0	0	ϕ / F

in ~~this~~ all boolean algebra each node represents PDNF's.

⇒ $B^n \rightarrow B$

↳ It is a boolean function.

* Boolean Algebra :- "Boolean Algebra is a complemented, distributive lattice.

- Generally denoted by $\langle B, *, \oplus, ', 0, 1 \rangle$ in which $\langle B, *, \oplus \rangle$ is a lattice with two binary operations $*$ and $\oplus$.

- Properties of boolean algebra in which $a, b, c \in B$ & B is set $\langle B, *, \oplus \rangle$ is a lattice.

1] $\langle B, *, \oplus \rangle$ is a lattice in which $*$ and $\oplus$ satisfy the following identities –

L-1 : $a * a = a$ L-1' ; $a \oplus a = a$

L-2 : $a * b = b * a$ L-2' : $a \oplus b = b \oplus a$

L-3 ; $(a * b) * c = a * (b * c)$ L-3' ; $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

L-4 : $a * (a \oplus b) = a$ L-4' ; $a \oplus (a * b) = a$.

2] $\langle B, *, \oplus \rangle$ is a distributive lattice & satisfies these identities.

D-1] $a * (b \oplus c) = (a * b) \oplus (a * c)$

D-2] $a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

D-3] $(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

D-4] $a * b = a * c$ & $a \oplus b = a \oplus c \Rightarrow b = c$.

3] $\langle B, *, \oplus, 0, 1 \rangle$ is a bounded lattices in which for any $a \in B$, the following hold:

B-1] $0 \leq a \leq 1$

B-2] $a * 0 = 0$ B-2]' $a \oplus 1 = 1$

B-3] $a * 1 = a$ B-3]' $a \oplus 0 = a$

4] $\langle B, *, \oplus, ', 0, 1 \rangle$ is a uniquely complemented lattice in which the complement of any element $a \in B$ is denoted by $a' \in B$ & satisfies the follo. identities :-

C1] $a * a' = 0$ C1]' $a \oplus a' = 1$

C2] $a' = 1$ C2]' $1' = 0$

C3] $(a * b)' = a' \oplus b'$ C3]' $(a \oplus b)' = a' * b'$.

5] There exist a partial ordering ~~such~~ ~~tha~~ relatⁿ $\leq$ on B such that,

P1] $a * b = GLB\{a, b\}$ P1)' $a \oplus b = LUB\{a, b\}$

P2] $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

P3] $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$

★ **Sub boolean algebra** :- Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a boolean algebra & $S \subseteq B$. If S contains the elements 0 & 1 and is closed under the operations $*$, $\oplus$ & $'$, then $\langle S, *, \oplus, ', 0, 1 \rangle$ is called a sub-Boolean algebra.

For any $a, b \in B$

$$a \oplus b = (a' * b')' \quad \& \quad 1 = (a * a') \quad \& \quad 0 = a * a'.$$

★ **Boolean Homomorphism** :- Let $\langle B, *, \oplus, ', 0, 1 \rangle$ & $\langle P, \cap, \cup, -, \alpha, \beta \rangle$ be two boolean algebras. A mapping $f : B \rightarrow P$ is called a Boolean homomorphism if all the operations of the boolean algebra are preserved. i.e., for any $a, b \in B$

$$f(a * b) = f(a) \cap f(b)$$

$$f(a \oplus b) = f(a) \cup f(b).$$

$$f(a') = \overline{f(a)}, \quad f(0) = \alpha \quad \& \quad f(1) = \beta.$$

★ **Boolean functions** :-

Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean Algebra. A funcⁿ $f : B^n \rightarrow B$ which is associated with a Boolean expression in n variables is called a Boolean function.