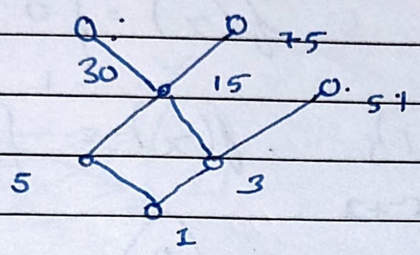
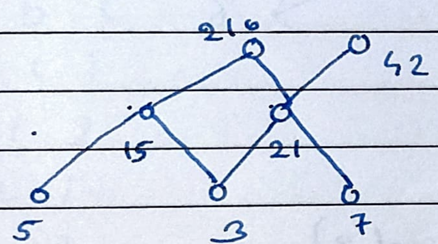
Tutorial - 7.

Q.1 Draw the hasse diagrams of the following sets under the partial ordering relation: divides.

1) {1, 3, 5, 15, 30, 75, 51}



2) {3, 5, 7, 21, 15, 210, 42}.



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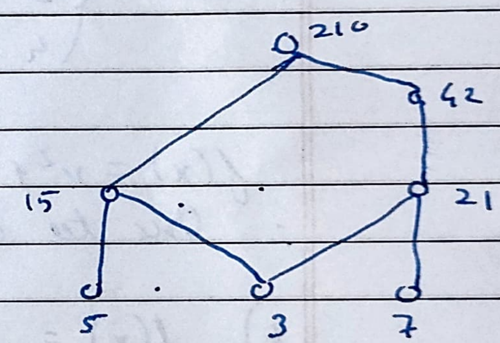


Q.2 Give a relation which is both a partial ordering relation and an equivalence relation on a set (Both symmetric and anti-symmetric)

→

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Q.3 Let, n be set of natural nos. determine which of the following function are one to one which are onto which are one to one onto

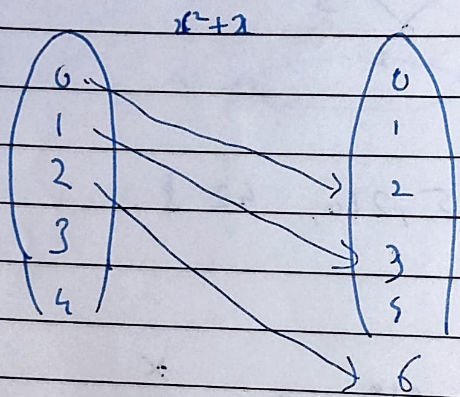
a) $f: N \to N$. $f(x) = x^2 + 2$.

b) $f: N \to N$ $f(x) = x \pmod 3$.

c) $f: N \to N$ $f(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ 1 & \text{otherwise.} \end{cases}$

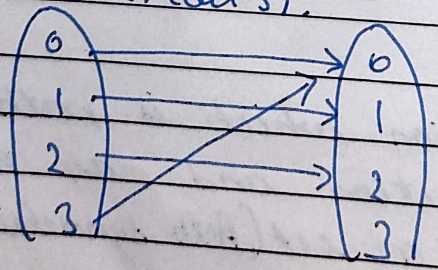
d) $f: N \to \{0, 1\}$. $f(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ 1 & \text{otherwise.} \end{cases}$

→ a)



$f(x) = x^2 + 2$
∴ One to one

b) $f(x) = x \pmod 3$.



Neither one to one, nor onto.

c) nor One to one. nor onto.

d) Neither one to one ~~nor~~, onto.

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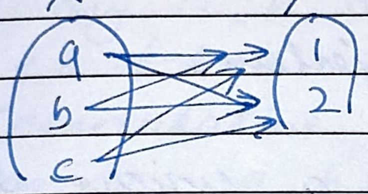
Q.4 Define all possible functions. $F: X \to Y$ also, mention which of them are one to one, onto one to one onto. $X = \{a, b, c\}$. $Y = \{1, 2\}$

→ Possible functions: a b c
$2 \times 2 \times 2 = 8$



$X . Y = \{\langle a, 1\rangle, \langle a, 2\rangle, \langle b, 1\rangle, \langle b, 2\rangle, \langle c, 1\rangle, \langle c, 2\rangle\}$

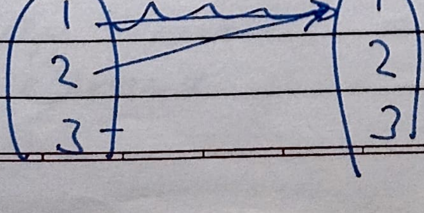
a	b	c
1	1	1
1	1	2
1	2	1
1	2	2
⋮		

Q.5 Let, I be the set of integers. I+ be the set of positive integers and $I_p = \{0, 1, 2, 3, \dots p-1\}$ determine which of the following functions are injective, surjective, bijective.

$f: I_+ \to I_+$ $f(x)$ = greatest integer $\leq \sqrt{x}$.

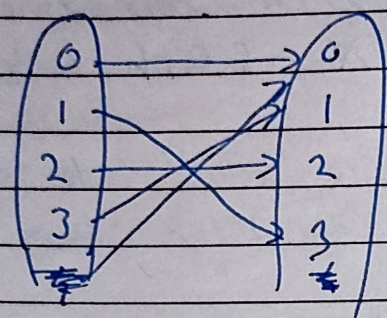
$f: I_4 \to I_4$ $f(x) = 3x \pmod 4$.

→



Onto.

2)



injective and surjective.

Q. Prove that there is a unique inverse of an invertible function.

→ For a function to have an inverse, it must be bijective.

∴

Consider g inverse of f.

$\therefore f \cdot g = I_y$ and $g \cdot f = I_x$.

Let, h be another inverse of f.

$\therefore f \cdot h = I_y$ and $h \cdot f = I_x$.

$\therefore g \cdot f \cdot h = g \cdot (f \cdot h) = g \cdot (I_y) = g$.

$g \cdot f \cdot h = (g \cdot f) \cdot h = I_x \cdot h = h$.

$\therefore g = h$.

∴ Assumption was wrong

There is only one unique inverse