Tutorial No. 3
Probability Distribution

1) The probability density function of a random variable x zero except at $x=0,1,2$. At these points.
$p(0) = 3c^3$, $p(1) = 4c - 10c^2$, $p(2) = 5c - 1$
Find i) The value of c ii) $p(0 < x \leq 2)$

→ Given:-

$$f(x) = \begin{cases} 0 & x \in I - [0,2] \\ 3c^3 & x = 0 \\ 4c - 10c^2 & x = 1 \\ 5c - 1 & x = 2 \end{cases}$$

$\therefore \quad P(0) = 3c^3, \quad P(1) = 4c - 10c^2, \quad P(2) = 5c - 1$

Now, $P(X \leq x) = \int_{-\infty}^{\infty} f(x) \cdot dx$

$$\therefore \int_{-\infty}^{\infty} 3c^3 \cdot dx + \int_{-\infty}^{\infty} 4c - 10c^2 \cdot dx + \int_{-\infty}^{\infty} 5c - 1 \cdot dx + \int_{-\infty} 0 \cdot dx = 1$$

i) $p(0) + p(1) + p(2) = 1$

$3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

$$
\begin{array}{r}
3c^2-7c+2 \\
c-1 \,\big)\, 3c^3-10c^2+9c-2 \\
-\;3c^3-3c^2 \\
\hline
-7c^2+9c \\
-\;-7c^2+7c \\
\hline
2c-2 \\
-\;2c-2 \\
\hline
0\;-0
\end{array}
$$

$\therefore\ (c-1)(3c^2-7c+2)=0$

$\therefore\ c=1$ & $3c^2-7c+2=0$

not possible $\quad 3c^2-6c-c+2=0$

$3c(c-2)-1(c-2)=0$

$(3c-1)(c-2)=0$

$\therefore\ c=1/3$ & $c=2$.

Not possible.

$\therefore\ \underline{c=1/3}$.

ii) $P(0<x\leq 2) = p(1)+p(2)$.

$= \frac{4}{3}-\frac{10}{9}+\frac{5}{3}-1$

$P(0<x\leq 2) = \frac{8}{9}$

2) A random variable x has the following probability distributions

X	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Find (i) the value of k (ii) mean (iii) variance (iv) $P(X\geq 1)$ (v) $P(X<1)$ (vi) $P(-2<X<2)$

X_i	$P(x_i)$	$x_i\,P(X_i)$	$x_i^2\,P(X_i)$
-2	0.1	-0.2	0.4
-1	$k=0.1$	-0.1	0.1
0	0.2	0	0
1	$2k=0.2$	0.2	0.2
2	0.3	0.6	1.2
3	$k=0.1$	0.3	0.9
		$\Sigma x_i P(x_i)=0.8$	$\Sigma x_i^2 P(x_i)=2.8$

i) The value of k.

$\therefore\ \Sigma P(x_i)=1$

$\therefore\ P(-2)+P(-1)+P(0)+P(1)+P(2)+P(3)=1$

$0.1+k+0.2+2k+0.3+k=1$

$4k+0.6=1$

$k=0.4/4$

$\boxed{k=0.1}$

No.:
Date:

ii) mean, $m = E(x_i) = \sum P(x_i) x_i$

$$\boxed{E(x) = 0.8}$$

iii) variance, $\sigma^2 = E(x^2) - (E(x))^2$

$= 2.8 - (0.8)^2$

$= 2.8 - 0.64$

$$\boxed{\sigma^2 = 2.16}$$

iv) $P(X \geq 1) = P(1) + P(2) + P(3)$

$= 0.2 + 0.3 + 0.1$

$= \underline{0.6}$

v) $P(X < 1) = 1 - P(X \geq 1)$

$= 1 - 0.6$

$= \underline{0.4}$

vi) $P(-2 < X < 2) = P(-1) + P(0) + P(1)$

$= 0.1 + 0.2 + 0.2$

$= \underline{0.5}$

No.:
Date:

3) Following is the probability density function of random variable x. $f(x) = ke^{-2x}$, $x > 0$. Find the value of k.

→ Given,

$$f(x) = \begin{cases} ke^{-2x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$\therefore$ $f(x)$ is the probability density function.

$\therefore \displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\displaystyle\int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{\infty} ke^{-2x}\,dx = 1$

$k\displaystyle\int_{0}^{\infty} e^{-2x}\,dx = 1$

put $2x = t$

$dt = 2dx$

$x \to 0 \quad \therefore t \to 0$

$x \to \infty \quad t \to \infty$

$\therefore \dfrac{k}{2}\displaystyle\int_{0}^{\infty} e^{-t}\,dt = 1$

$\therefore \dfrac{k}{2}\left[-e^{-t}\right]_0^{\infty} = 1$

$\therefore \dfrac{k}{2} = 1 \Rightarrow \boxed{k = 2}$

No. :
Date :

4) Following is the probability density function of a random variable x.

$f(x) = kx^4 e^{-x/2} \quad 0 \le x < \infty$

→ i) value of k.

Given:-

$$f(x) = \begin{cases} kx^4 e^{-x/2} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}$$

is p.d.f.

$\therefore \int_{-\infty}^{\infty} f(x) \cdot dx = 1$

$\int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{\infty} kx^4 \cdot e^{-x/2} \cdot dx = 1$

$\int_{0}^{\infty} ke^{-x/2} \cdot x^4 \cdot dx = 1$

put $x/2 = t \Rightarrow x = 2t$.

$dt = \frac{dx}{2} \Rightarrow dx = 2dt$

x	0	∞
t	0	∞

No. :
Date :

$\therefore \int_{0}^{\infty} k\, 16t^4\, e^{-t}\, 2 \cdot dt = 1$

$\therefore 32k \int_{0}^{\infty} t^4 e^{-t} = 1$

$32k \sqrt{5} = 1$

$32k \cdot 4! = 1$

$32k \cdot 24 = 1$

$\boxed{k = 0.0013 = 1/768}$

ii) mean & variance.

mean, $E(x) = \int_{-\infty}^{\infty} x f(x) \cdot dx$

$= \int_{0}^{\infty} k x^5 e^{-x/2} \cdot dx$

put $x/2 = t \Rightarrow 2t = x$

$dx = 2dt$

x	0	∞
t	0	∞

$= \int_{0}^{\infty} 0.001 \times 32 t^5 e^{-t}\, 2 \cdot dt$

$= \cancel{0.064} \sqrt{6} \times 0.0013 \times 64$

$\cancel{= 0.064 \times 5! = 7.68}$

No.:
Date:

$= 0.0832 \times 120$

$= 9.984 \approx 10.0$

variance, $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \cdot dx.$

$\therefore \sigma^2 = \int_0^{\infty} (x-10)^2 \cdot \cancel{0.0013} \frac{1}{768} e^{-x/2} x^4 \cdot dx$

$= \frac{1}{768} \int_0^{\infty} (x^2 - 20x + 100) x^4 \cdot e^{-x/2} \cdot dx$

$= \frac{1}{768} \left[\int_0^{\infty} x^6 \cdot e^{-x/2} \cdot dx - 20 \int_0^{\infty} x^5 \cdot e^{-x/2} \cdot dx + 100 \int_0^{\infty} x^4 \cdot e^{-x/2} \cdot dx \right]$

$= \frac{1}{768} \left[2^7 \sqrt{7} - 20 \cdot 2^6 \cdot \sqrt{6} + 100 \cdot 2^5 \cdot \sqrt{5} \right]$

$= \frac{1}{768} \left[2^7 \times 6! - 20 \times 2^6 \times 5! + 100 \times 2^5 \times 4! \right]$

$\boxed{\sigma^2 = 20}$

5) If the sum of mean and variance of a Binomial distribution for 5 trials is 4.8. find the distribution.

$\longrightarrow$ Given:-

$n = 5$

$np + npq = 4.8$

$\therefore 5[p + pq] = 4.8$

No.:
Date:

$\therefore p + p - p^2 = 0.96$

$2p - p^2 = 0.96$

$p^2 - 2p + 0.96 = 0$, i.e. $a = 1$, $b = -2$, $c = 0.96$

$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 - 3.84}}{2}$

$p = \frac{2 + 0.4}{2}$ & $p = \frac{2 - 0.4}{2}$

$p = 1.2$ not possible

$\boxed{p = 0.8}$

$\therefore q = 1 - p$

$= 1 - 0.8$

$\boxed{q = 0.2}$

$\therefore$ By binomial distribution,

$P(X = r) = {}^nC_r \, p^r \, q^{n-r}$

$\boxed{P(X=r) = {}^5C_r (0.8)^r (0.2)^{5-r}}$

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6) Let X be a binomial random variable with parameter 05 & 0.7 compute

1) $P(X = 2)$ 2) $P(X \leq \cancel{8}\,3)$ 3) $P(X > 2)$

→ Given :-

$n = 5$

$p = 0.7$

$q = 1 - p = 1 - 0.7 = 1 - 0.7 = \underline{0.3}$

Now, $P(X=r) = {}^nC_r \, p^r q^{n-r}$

1) $P(X=2) = {}^5C_2 \, p^2 q^3$

$= {}^5C_2 (0.7)^2 (0.3)^3$

$= \underline{0.1323}$

2) $P(X \leq \cancel{8}\,3) = P(0) + P(1) + P(2) + P(3)$

$= {}^5C_0 \, \cancel{p}\, 0.7^0 \cdot 0.3^5 + {}^5C_1 \cdot 0.7 \cdot 0.3^4 + 0.1323$

$+ {}^5C_3 \; 0.7^3 \, 0.3^2$

$= \underline{0.47178}$

3) $P(X > 2) = 1 - [P(2) + P(1) + P(0)]$

$= 1 - 0.16308$

$= \underline{0.83692}$