GoodLuck Page No. Date

Tutorial No. 4

Q.1 Explain the various types of Grammars.

→ According to Noam Chomsky there are four types of grammars - Type 0, Type 1, Type 2 and Type 3.

Grammar Type	Grammar Accepted	Language Accepted	Automation
Type 0	Unrestricted grammar	Recursively Enumerable	Turing machine
Type 1	Context-sensitive	Context-sensitive	Linear-bounded machine
Type 2	Context-free	Context free language.	Push down automata
Type 3	Regular grammar	Regular language	Finite state automation

i) Type-0 Grammar:

Type-0 grammar generate recursively enumerable languages. The languages are recognized by a Turing machine. The productions can be in the form $\alpha \to \beta$ where α is a string of terminals and nonterminals with at least one non-terminal and α cannot be null. β is a string of terminals and non-terminals.

2) **Type - 1 Grammar:**

Type-1 grammars generate context-sensitive languages. The productions must be is the form $\alpha A \beta \rightarrow \alpha \gamma \beta$ where, $A \in N$ (Non-terminals) and $\alpha, \beta, \gamma \in (T \cup N)^*$ (string of terminals and non-terminals). The strings α and β can be empty, but γ must be non-empty.

3) **Type - 2 Grammar:**

Type-2 grammar generate context-free languages. The languages generated by these grammars are recognized by a non-deterministic pushdown automation. The productions must be in the form $A \rightarrow \gamma$ where, $A \in N$ (Non terminal) and $\gamma \in (T \cup N)^*$ (string of terminals and non-terminals).

4) **Type - 3 Grammar:**

Type-3 grammar generate regular languages. The languages generated by this grammar are recognized by finite state automation. The productions must be in the form $X \rightarrow a$ or $X \rightarrow aY$ where, $X, Y \in N$ (Non terminal) and $a \in T$ (Terminal). The rule $S \rightarrow \varepsilon$ is allowed if S does not appear on the right side of any rule.

Q.2 **Give the formal definition of CFG, CFL and CNF.**

→ 1) **Context free Grammar (CFG):**

A context free grammar is defined by 4 tuples (V, T, P, S) where,

V = Set of variables.

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T = set of terminal symbols
P = set of rules and productions
S = start symbol and $S \in V$

2) Context Free Language (CFL):

Let, $G = (V, T, P, S)$ is a context free grammar. Then the language G denoted by $L(G)$ is the set of terminal strings that have derivation from the start symbol in G. The language generated by a CFG is called context free language.

3) Chomsky Normal Form (CNF):

A context free grammar $G = (V, T, P, S)$ is said to be in Chomsky Normal Form (CNF) if, every production in G are in one of the two forms:

$A \rightarrow BC$ and
$A \rightarrow a$ where, $A, B, C \in V$ and $a \in T$.

Thus, a grammar in CNF is one which should not have ϵ productions, unit productions and useless symbols.

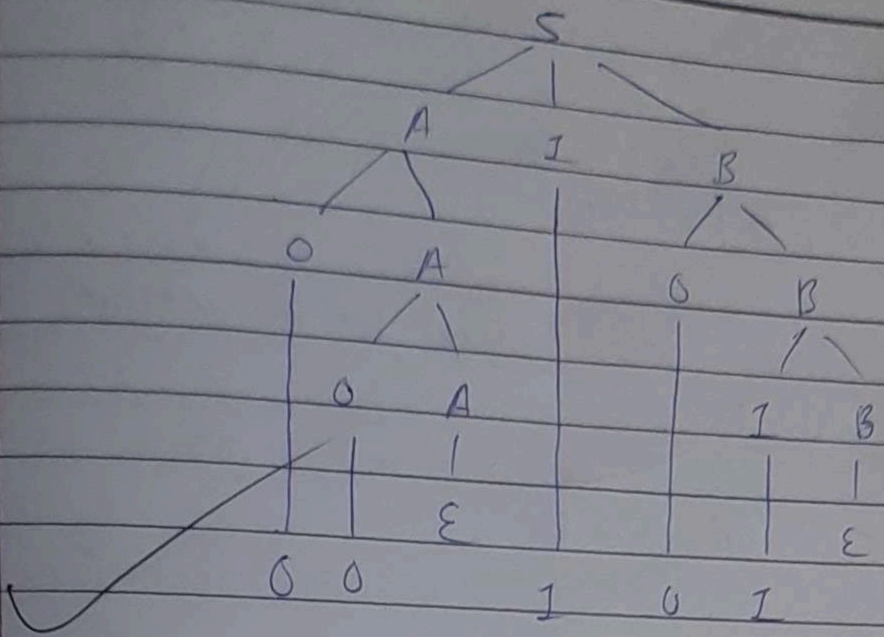
Q.3 Consider the grammar G:

$S \rightarrow A1B$
$A \rightarrow 0A \mid \epsilon$
$B \rightarrow 0B \mid 1B \mid \epsilon$

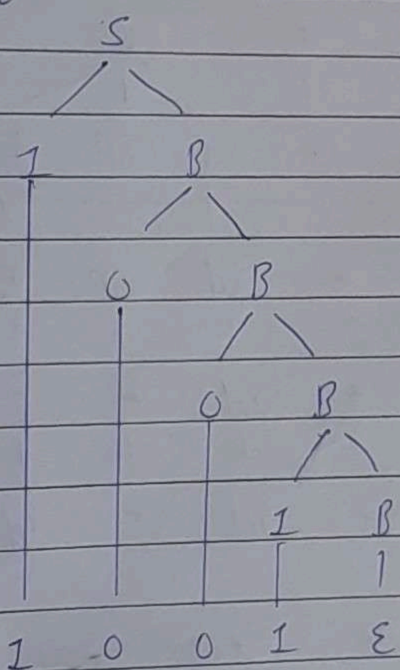
1) Construct parse tree for 00101
2) Construct parse tree for 1001
3) Construct parse tree for 00011

→

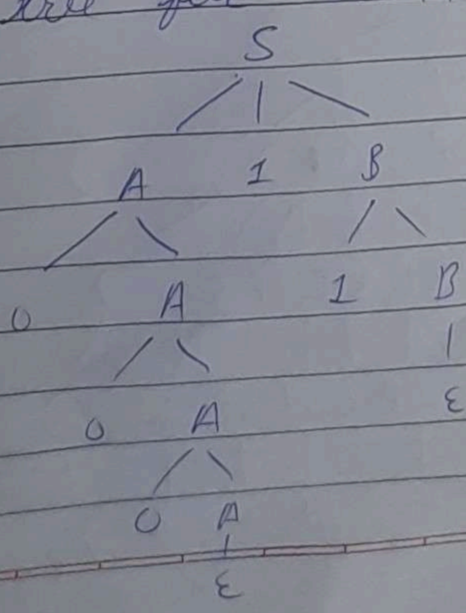
1) Parse tree for 00101



2) Parse tree for 1001



3) Parse tree for 00011 :

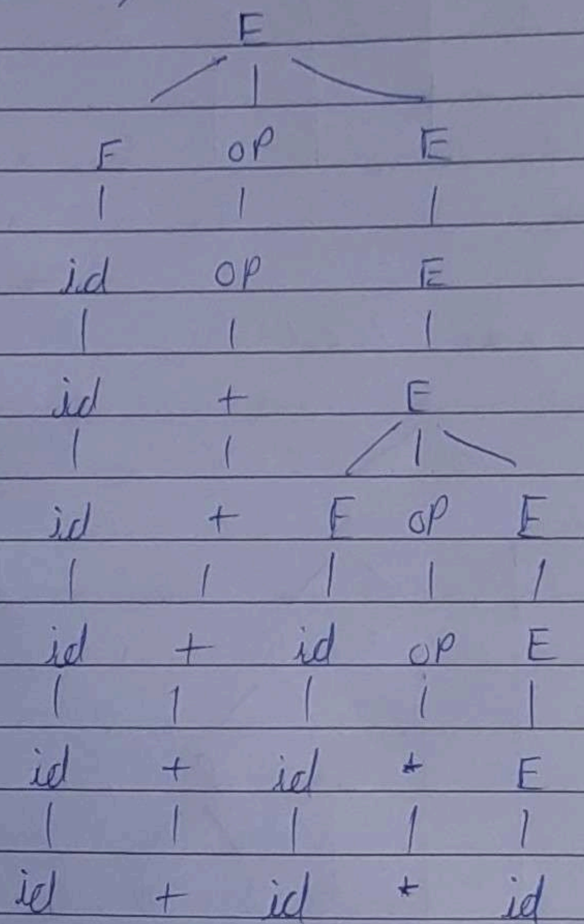


Q.4 a) Consider the grammar for arithmetic expression.

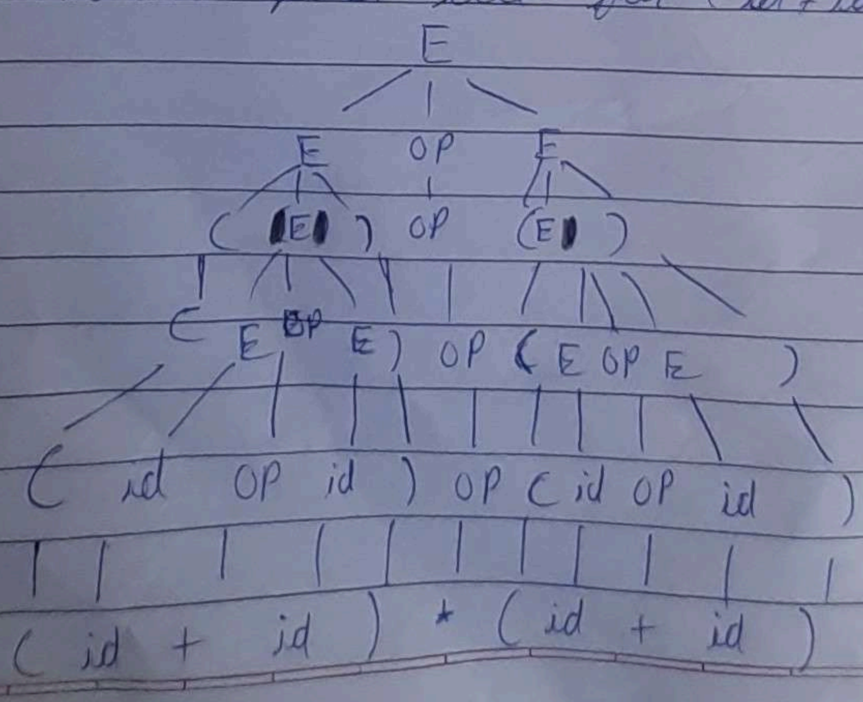
$E \rightarrow E \; OP \; E \mid (E) \mid id$

$OP \rightarrow + \mid - \mid * \mid / \mid \uparrow$

1) Construct parse tree for id + id * id.



2) Construct parse tree for (id + id) * (id + id)



Q.5 b) Construct a CFG that generates language of balanced paranthesis. Show the parse tree computation for

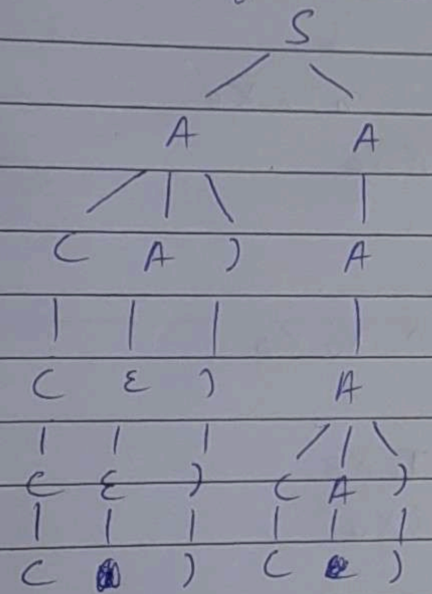
1) ()()
2) ((()))()

→ CFG to generate language of balanced paranthesis:

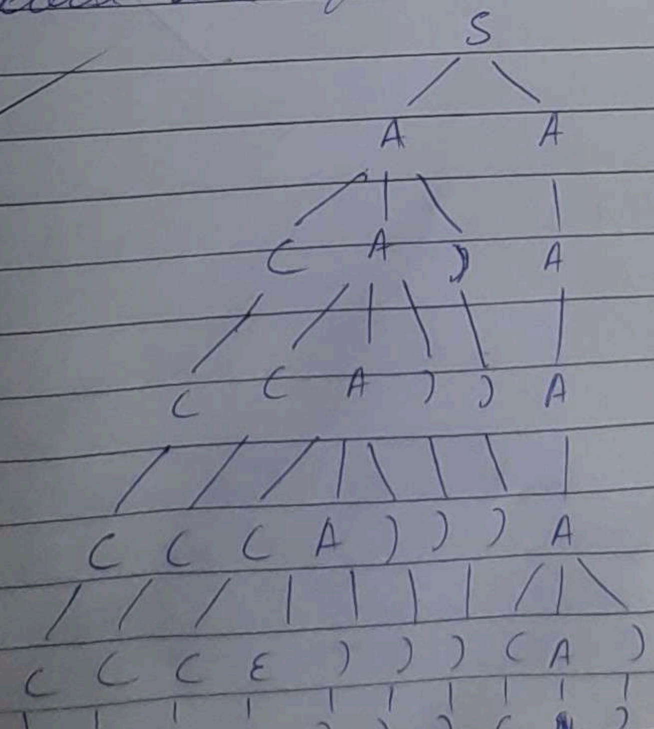
$S \rightarrow AA$

$A \rightarrow (A) \mid \varepsilon$

1) Parse tree for ()():



2) Parse tree for ((()))():



Q.5 Convert the following CFG to CNF

a) $S \rightarrow abA \mid bB \mid aba$

$A \rightarrow b \mid aB \mid Ba$

$B \rightarrow aB \mid aA$

$\rightarrow$ The given CFG doesn't have any useless symbols, null productions and unit productions.

Consider,

$X_a \rightarrow a$

$X_b \rightarrow b.$

∴ New rules are,

$S \rightarrow X_a X_b A \mid X_b B \mid X_a X_b X_a$

$A \rightarrow X_b \mid X_a B \mid B X_a$

$B \rightarrow X_a B \mid X_a A$

Consider,

$T_1 \rightarrow X_a X_b$

∴ CNF is,

∴ $S \rightarrow T_1 A \mid X_b B \mid T_1 X_a$

$A \rightarrow X_b \mid X_a B \mid B X_a$

$B \rightarrow X_a B \mid X_a A.$

$T_1 \rightarrow X_a X_b$

$X_a \rightarrow a$

$X_b \rightarrow b$

b) $S \rightarrow bS \mid aA \mid \Lambda$

$A \rightarrow aA \mid bB \mid b$

$B \rightarrow bS$

$\rightarrow$

1) There are no useless symbols in given CFG.

2) Nullable variable in given CFG is S.

∴ Eliminating null production, using substitu

$\therefore S \rightarrow bS \mid aA \mid a \mid b \mid \cancel{A}$

$A \rightarrow aA \mid bB \mid b$

$B \rightarrow bS$

3) ~~Eliminating unit productions~~

~~Here, $S \rightarrow A$ is unit production.~~

~~$\therefore S \rightarrow bS \mid aA \mid a \mid b \mid aA \mid bB \mid b$~~

~~$A \rightarrow aA \mid bB \mid b$~~

~~$B \rightarrow bS.$~~

4) Consider new rules,

$X_a \rightarrow a$

$X_b \rightarrow b.$

$\therefore$ CNF is,

$S \rightarrow X_bS \mid X_aA \mid X_a \mid X_b \mid \cancel{X_aA \mid X_bB \mid X_b}$

$A \rightarrow X_aA \mid X_bB \mid X_b$

$B \rightarrow X_bS$

$X_a \rightarrow a$

$X_b \rightarrow b.$

c) $S \rightarrow AabB$

$A \rightarrow aA \mid bA \mid \Lambda$

$B \rightarrow Bab \mid Bb \mid ab \mid b.$

1) Eliminating null production. Here, A is a nullable variable.

$\therefore S \rightarrow AabB$

$A \rightarrow aA \mid bA \mid a \mid b.$

$B \rightarrow Bab \mid Bb \mid ab \mid b.$

2) There are no useless symbols or unit production in the CFG.

c) Consider,

$X_a \rightarrow a$

$X_b \rightarrow b$

$T_1 \rightarrow X_a X_b$

$T_2 \rightarrow A T_1$

$\therefore$ New rules are,

$S \rightarrow T_2 B$

$A \rightarrow X_a A \mid X_b A \mid X_a \mid X_b$

$B \rightarrow B T_1 \mid B X_b \mid X_a X_b \mid X_b$

$T_2 \rightarrow A T_1$

$T_1 \rightarrow X_a X_b$

$X_a \rightarrow a$

$X_b \rightarrow b$.

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d) $S \rightarrow AB$

$A \rightarrow aAa \mid bAb \mid a \mid b$

$B \rightarrow aB \mid bB \mid \Lambda$

$\rightarrow$ 1) Removing null productions using subset method.
Here, variable B is nullable.

$\therefore S \rightarrow AB$

$A \rightarrow aAa \mid bAb \mid a \mid b$

$B \rightarrow aB \mid bB \mid a \mid b$

2) There are no useless symbols or unit productions

3) Consider,

$X_a \rightarrow a$

$X_b \rightarrow b$.

$T_1 \rightarrow X_a A$

$T_2 \rightarrow X_b B$

$\therefore$ New rules are,

$S \rightarrow AB$
$A \rightarrow T_1 X_a \mid T_2 X_b \mid X_a \mid X_b$
$B \rightarrow X_a B \mid X_b B \mid X_a \mid X_b$
$T_1 \rightarrow X_a A$
$T_2 \rightarrow X_b B$
$X_a \rightarrow a$
$X_b \rightarrow b$

e) $S \rightarrow AAS \mid ab \mid aab$
$A \rightarrow ab \mid ba \mid \Lambda$

→ 1) Eliminating null productions using subsets. Here, A is nullable variable.

$S \rightarrow AAS \mid ab \mid aab$
$A \rightarrow ab \mid ba \mid a \mid b$

2) There are no useless symbols or unit productions.

3) Consider,

$X_a \rightarrow a$
$X_b \rightarrow b$.

$\therefore S \rightarrow AAS \mid X_a X_b \mid X_a X_a X_b$
$A \rightarrow X_a X_b \mid X_b X_a \mid X_a \mid X_b$.

Consider,

$T_1 \rightarrow X_a X_b$
$T_2 \rightarrow AA$

$\therefore$ New rules are,

$S \rightarrow T_2 S \mid T_1 \mid X_a T_1$
$A \rightarrow T_1 \mid X_b X_a \mid X_a \mid X_b$
$T_1 \rightarrow X_a X_b$
$T_2 \rightarrow AA$
$X_a \rightarrow a$
$X_b \rightarrow b$.

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