4-2023
Thursday

4. Graph Algorithm.

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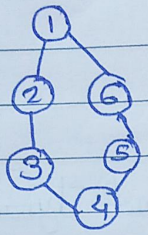
Minimum Cost spanning Tree

It follows the concept of greedy method.



$G = (V, E)$

$|V| = 6 = \{1, 2, 3, 4, 5, 6\}$.

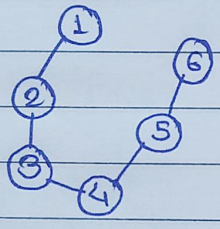
$|E| = 6 = \{(1,2), (1,6), (2,3), (3,4), (4,5), (5,6)\}$.

$|E| = |V| - 1$

$= 6 - 1$

$|E| = 5$

∴ There 5 edges of Subgraph.



So this is spanning Tree.

To represent spanning tree or subgraph

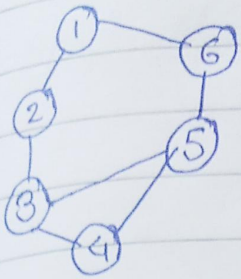
$G = (V', E')$

∴ $\boxed{\begin{aligned} V' &= V \\ E' &= |V| - 1 \end{aligned}}$

No. of spanning Tree = $|V| C_{E = |V|-1}$

$= {}^{6}C_{5} = \boxed{6}$

∴ There are 6 spanning tree we can generate from above original graph.

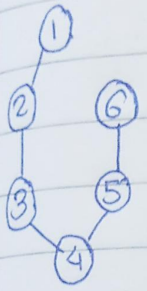


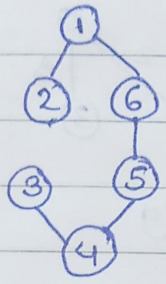
$|V| = 6$

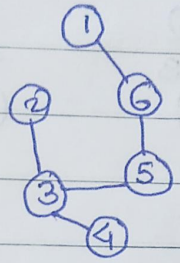
$|E| = 7$

$\therefore$ No. of spanning tree $= {}^{6}C_5$

$= 5$







Here we can't determine the exact size of spanning tree.
So change the formula.

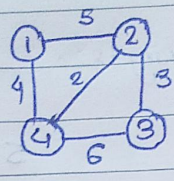
$\therefore {}^{|V|}C_{|V|-1}$ - No of cycle.

- If In a graph there are more than one cycle are present then use new formula of spanning tree
otherwise use $|V|C_{E=|V|-1}$.

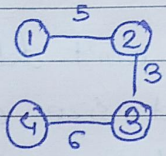
$\therefore {}^{|V|}C_{|V|-1}$ - No of cycle.

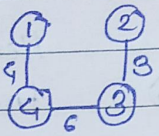
$\therefore {}^{6}C_5 - 2$

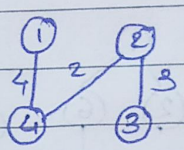
$\therefore 4$

2)  = cost = 20

Spanning tree :-







Cost = 14 = 13 = 9

Constraints :-

- Avoid

objective :-

- Find minimum cost of subgraph of original graph.

There are two method suggested from greedy method to find minimum cost spanning tree.

i) Prim's Algorithm
ii) Kruscle

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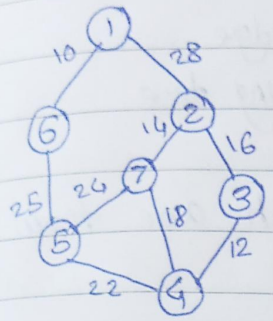
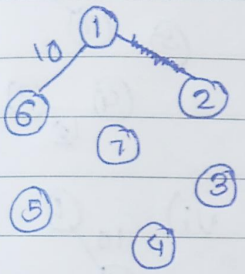
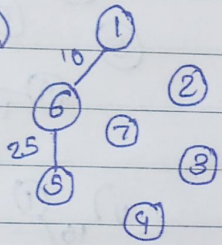
i) Prim's Algorithm :-

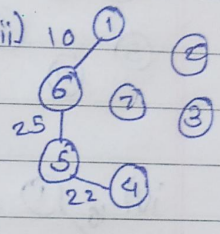
Rules:-

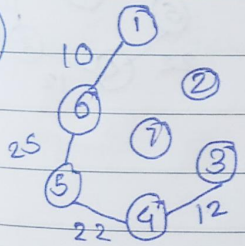


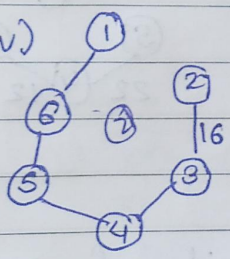
i) Select smallest edge.
ii) Take this vertices and also draws the minimum cost of edge from both vertex.
iv) Can't consider that edges which are form a cycle.

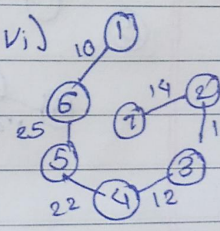
i)


ii)


iii)


iv)


v)


vi)


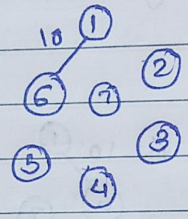
Rules of prim's Algorithm.

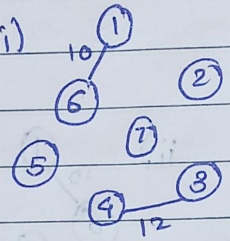
i) Consider a smallest edge from a given graph.
ii) Consider all the connected or adjacent edges from the present vertices and choose the minimum one.
iii) Repeat the same process until get minimum cost spanning tree [until and unless we cover all the vertices of the given graph and [Edges = |V| - 1]].

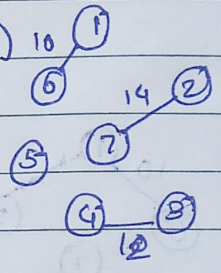
ii) Kruskal Algorithm

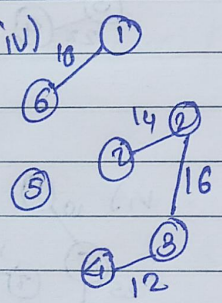
i) Get minimum cost edge
ii) Get second minimum cost edge
iii) Repeat until we get spanning tree.

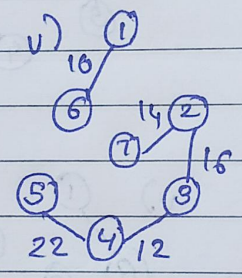
Not consider that edges which are form a cycle.

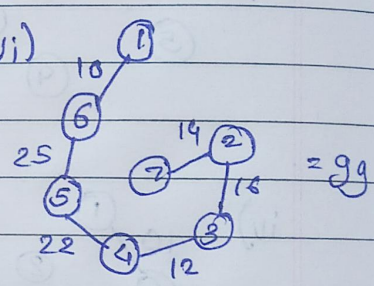
i) 

ii) 

iii) 

iv) 

v) 

vi) 

= gg

- **Time Complexity of Minimum cost Spanning Tree**

V = n
E = n-1

∴ O(|V| · |E|)
O(n · (n-1)) = O(n²).

Min Heap
T = log n

O(V · E)
O(n · log n)
O(log n)

Min Heap is used to reduce the time complexity. There don't need of comparison.

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Note:-

MinHeap :-

Basically minHeap is data structure it use to give you a retrive minimum cost edge from a given Graph without making a comparison of remaining edges of a given graph.

logn = time taken by minHeap to retrive minimum cost edge.

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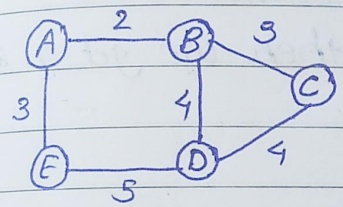
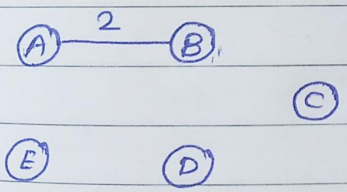
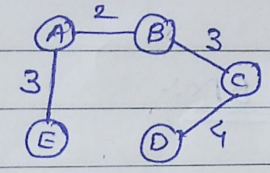


Prims Algorithm.

i)



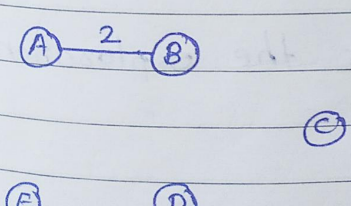
ii)



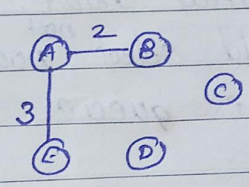
cost = 12.

Kriscale Algorithm.

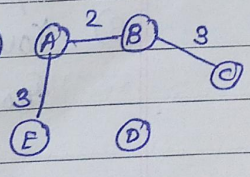
i)



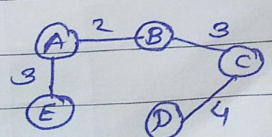
ii)



iii)

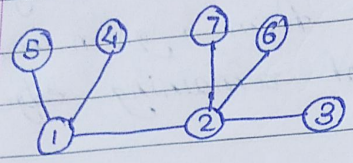


iv)



Cost = 12.

- Deapth first search Algorithm (DFS)
- Breadth first search Algorithm (BFS)

1) 

BFS:

1, 2, 4, 5, 7, 6, 3. or 1, 4, 2, 5, 7, 6, 3

Here first we start from vertex 1 and explore them then we get 5, 4, 2 vertex.
Now we explore 2 vertex then we get 7, 6, 3.
Here we explore all vertex

DFS:

1, 2, 3, 6, 7, 4, 5

Rule of BFS:-

1) You may start a graph traversing from any node
2) Onee you select the vertex try to completely explore that node.
3) You should select next node for the exploration fforom the queue.

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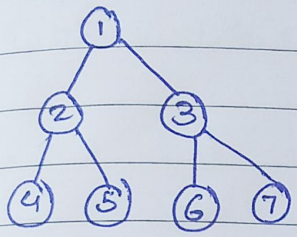
Rule of DFS:

1) You may start a Graph traversing from any node.
2) Instead of exploring all the vertex considering only one adjacent node.
3) You should select one node from the stack.

2)



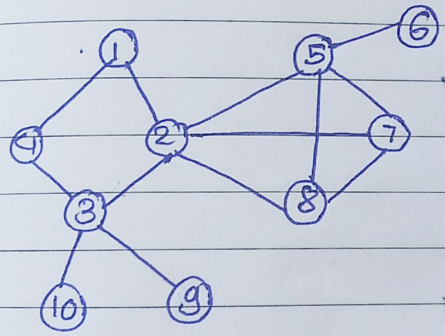
BFS:

1, 2, 3, 4, 5, 6, 7 } levelorder

DFS:

1, 2, 4, 5, 3, 6, 7 } Preorder

3)



BFS:

i) BFS: 1

Q | 1 | | | |

ii) BFS: 1, 4, 2

Q: | 1 | 4 | 2 | |

iii) BFS: 1, 4, 2, 3

Q: | x | 4 | 2 | 3 |

iv) BFS: 1, 4, 2, 3

Q: | x | 4 | 2 | 3 |

v) BFS: 1, 4, 2, 3, 5, 7, 8

Q: | x | x | 2 | 3 | 5 | 7 | 8 |

vi) BFS: 1, 4, 2, 3, 5, 7, 8, 9, 10

Q: | x | x | x | 3 | 5 | 7 | 8 | 9 | 10 |

vii) BFS: 1, 4, 2, 3, 5, 7, 8, 9, 10, 6

Q: | x | 4 | x | 3 | 5 | 7 | 8 | 9 | 10 |

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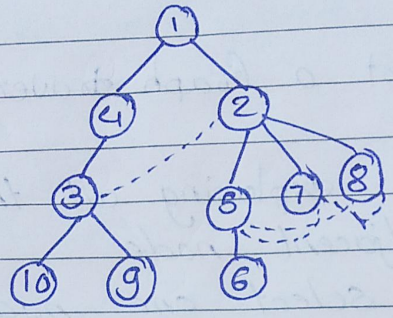
Tree :-



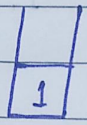
dotted line :-
already present is so/?

DFS :-

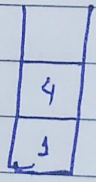
i) DFS : 1

Stack :

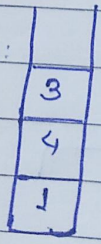
ii) DFS :- 1, 4 S :



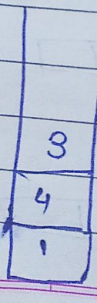
iii) DFS :- 1, 4, 3 S :



iv) DFS :- 1, 4, 3, 10 S :



v) DFS :- 1, 4, 3, 10, 9, 5 S :



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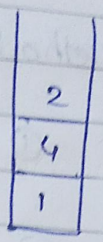
v.) DFS: 1, 4, 3, 10, 9, 2, 8

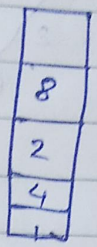
S:



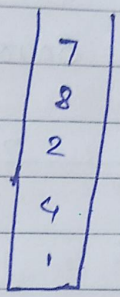
Here POP 3.

vi.) DFS: 1, 4, 3, 10, 9, 2, 8, 7.

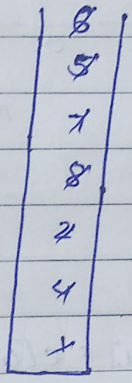
S:



viii) DFS: 1, 4, 3, 10, 9, 2, 8, 7, 5 : S:

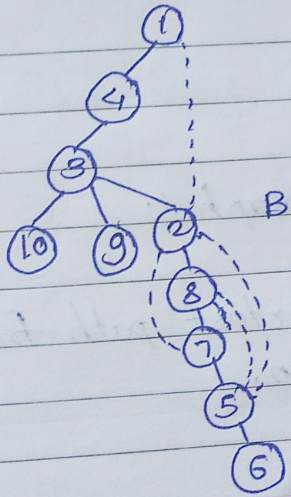


ix) DFS: 1, 4, 3, 10, 9, 2, 8, 7, 5, 6 S:



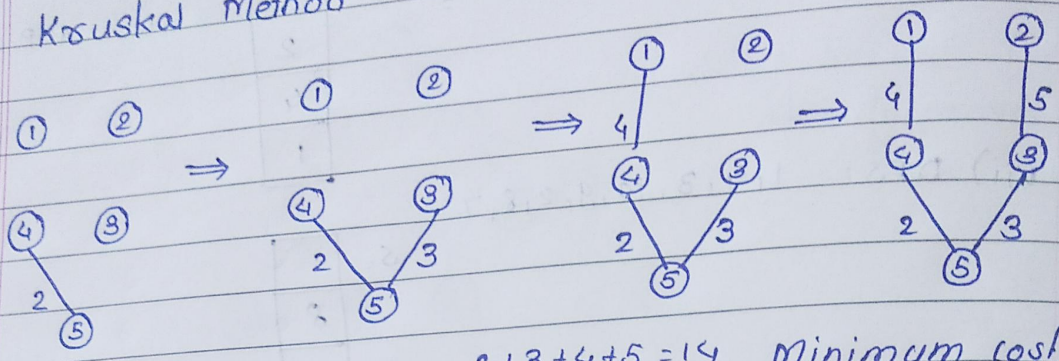
Here explore all so pop all vertex.

Tree :-



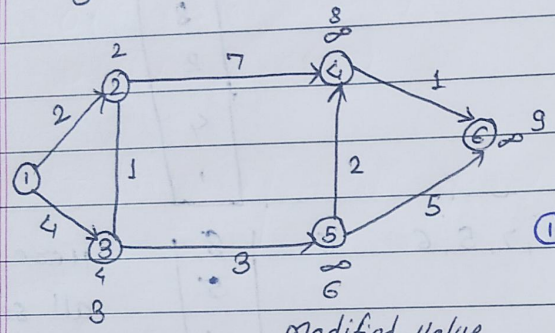
Back edges.

- Kruskal Method



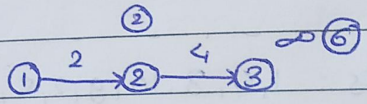
2+3+4+5 = 14 Minimum cost

- Single Source shortest Path (Dijekstra's Algorithm):



Modified value

Here 2 to 3 there is a no direct edge.



6 is less than ∞ so, we can change / modified value of vertex - 3 it is called Relaxation.

if (d[u] + c[u,v] ≤ d[v])

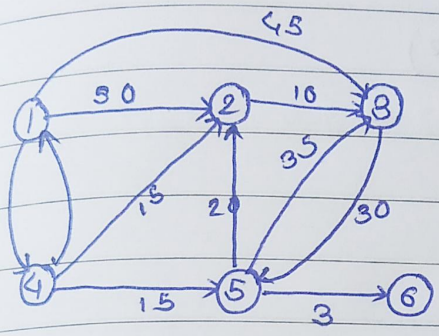
2 + 4 = 6 ≤ ∞

d[v] = d[u] + c[u,v]

Applied on weighted graph :-

Objective :- Find shortest path from source to destination.

V	d[v]
2	2
3	3
4	8
5	6
6	9



	2	3	4	5	6	
From Node						
[4]	50	45	10	∞	∞	
smallest cost	50	45	10	25	∞	Vertex 4 cost 10+15=25
[5]	50	45	(10)	(25)	∞	
[2]	45	45	(10)	(25)	∞	From 5th node value of 5 vertex is 25+20=45 45<50 so New value of vertex 2 is 45
[3]	(45)	(45)	(10)	(25)	∞	
[6]	(45)	(45)	(10)	(25)	(∞)	

Draw separate table for each stage.

V	d[V]
2	45
3	45
4	10
5	25
6	∞

- Time Complexity : $O(n^2)$

$O(E \cdot No\ of\ Relax)$

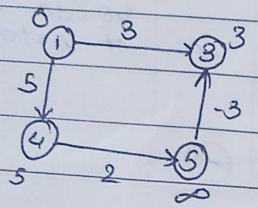
$O(n \cdot n) = O(n^2)$

$E = \left[\frac{n(n-1)}{2}\right]$ - if graph is complete graph.

$O\left(\left(\frac{n(n-1)}{2}\right) \cdot n\right) = O(n^2 \cdot n)$

$= O(n^3)$

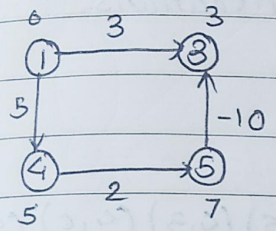
- Drawback / Limitation of dijstra's algorithm



start from vertex (1)

	3	4	5
3	③	5	∞
4	③	⑤	∞
5	③	⑤	⑦
	③	⑤	⑦

v	d[v]
3	3
4	5
5	7

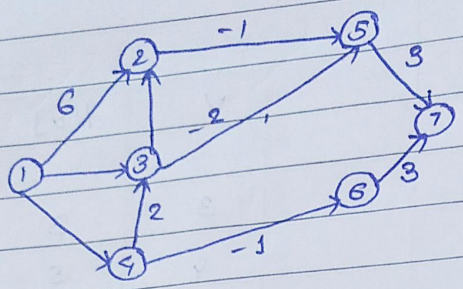


With this example with we got -3 value of vertex 8 so we can't apply dijkstra's algorithm on this example.

3	4	5	v	d[v]
			3	-3
③	5	∞	4	5
③	⑤	∞	5	7
③	⑤	⑦		
-3	⑤	⑦		

Dijkstra's algorithm does not allows negative cost of vertex so in above example dijkstra's algorithm does not work.

* Bellman - Ford Algorithm:



if (d[u] + c[u,v] < d[v])
d[v] = d[u] + c[u,v]

Relaxation of Edges = |V| - 1 = 7 - 1 = 6 iterations.
↓
No. of vertex

Edges : (1,2) (1,3) (1,4) (2,5) (3,2) (3,5) (4,3) (4,6) (5,7) (6,7)

4. May. 2023
Thursday

Time Complexity of Bellman - Ford

R = |V| - 1
O(|E| · (|V| - 1)) ... O(no. of edges x no of relaxation)
O(n · (n-1))
O(n²)

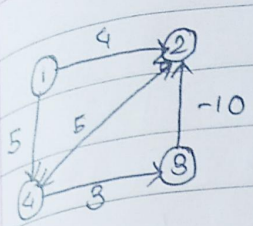
* Complete Graph = There is edge present between every pair of vertices. That is pretend as complete Graph.

$E = \frac{n(n-1)}{2}$

$O\left(\frac{n(n-1)}{2} \cdot (n-1)\right)$

$O(n^3)$

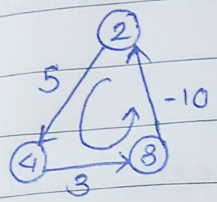
Drawaback - doesn't handle negative weight cycle.



Apply Bellman-Ford Algorithm.

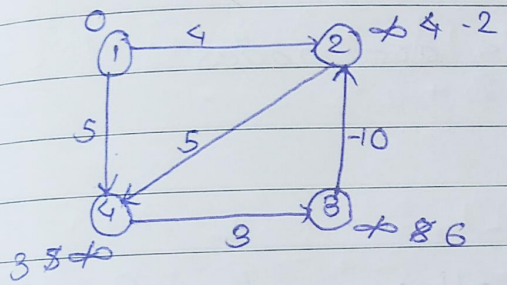
$E = \{(3,2), (4,3), (1,4), (1,2), (2,4)\}$.

$|V| - 1 = 4 - 1 = 3$



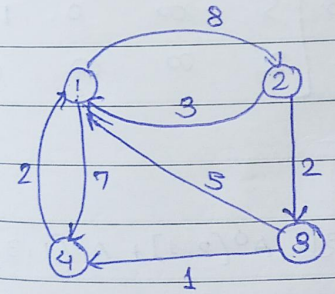
$5 + 3 + (-10) = -2$

∴ Bellman Ford doesn't handle negative weight cycle.



8 time Relaxation.

- All pair shortest path Algorithm. (Floyd warshell's Algo)
  - Determine or find shortest path we determine all of the edges.

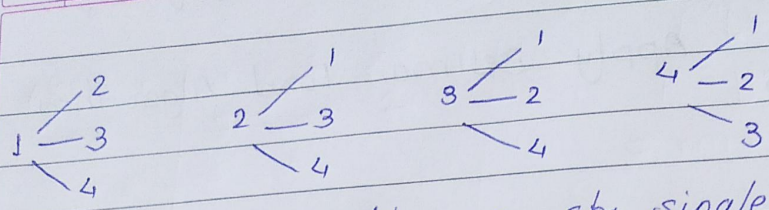


Form matrix.

A^0 :	1	2	3	4
1	0	8	∞	7
2	3	0	2	∞
3	5	∞	0	1
4	2	∞	∞	0

∴ Self loop are not present so diagonal elements are 0.

∴ ∞ - not directly connected.



This is look like as ~~the~~ single shortest path Algorithm

Time complexity of single ~~the~~ source shortest path Algorithm is :- $o(n^2)$

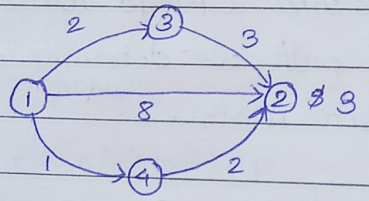
∴ How we apply single source shortest path on all ~~path~~ shortest path algorithm.

$o(4n^2)$ - Here 4 are nodes

$o(n \cdot n^2)$

$o(n^3)$

∴ Time complexity of all pair shortest path algorithm is $\underline{\underline{o(n^3)}}$



$A^0 =$

	1	2	3	4
1	0	8	∞	7
2	3	0	2	∞
3	5	∞	0	1
4	2	∞	∞	0

$A^1 =$

	1	2	3	4
1	0	8	∞	7
2	3	0	2	15
3	5	8	0	1
4	2	5	∞	0

$\therefore A^0[2,3] = A^0[2,1] + A^0[1,3]$

$2 = 8 + \infty$

$2 < \infty$

$\therefore A^0[2,4] = A^0[2,1] + A^0[1,4]$

$= 8 + 7$

$\infty, 7 = 15$

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$A^0[3,2] = A^0[3,1] + A^0[1,2]$
$= 5 + 3$
$\infty > = 8$

$A^0[3,4] = A^0[3,1] + A^0[1,4]$
$1 < 5 + 7$
$1 < 12$

$A^0[4,2] = A^0[4,1] + A^0[1,2]$
$\infty = 2 + 3$
$\infty > 5$

$A^0[4,3] = A^0[4,1] + A^0[1,3]$
$\infty > 2 + \infty$
∞

$A^2 =$

	1	2	3	4
1	0	3	5	7
2	8	0	2	15
3	5	8	0	1
4	2	5	7	0

$A^1[1,3] = A^1[1,2] + A^1[2,3]$
$\infty = 3 + 2$
$\infty > = 5$

$A^1[1,4] = A^1[1,2] + [2,4]$
$7 < 3 + 15$
$7 < 18$

$A^1[3,4] = A^1[3,2] + A^1[2,4]$
$= 1 < 8 + 15 \quad = 1 < 24$

$A^1[4,1] = A^1[4,2] + A^1[2,1]$
$2 < 5 + 8$
$2 < 13$

$A^1[3,1] = A^1[3,2] + A^1[2,1]$
$5 < 8 + 5$
$5 < 13$

$A^1[4,3] = A^1[4,2] + [2,3]$
$= 5 + 2$
$\infty > 7$

$A^3 =$

	1	2	3	4
1	0	3	5	6
2	7	0	2	8
3	5	8	0	1
4	2	5	7	0

$A^2[1,2] = A^2[1,3] + A^2[3,2]$
$3 = 5 + 8$
$3 <= 13$

$A^3[1,4] = A^2[1,3] + A^2[3,4]$
$7 > 5 + 1$
> 6

$A^2[2,1] = A^2[2,3] + A^2[3,1]$
$8 > 2 + 5 = 8 > 7$

$A^2[2,4] = A^2[2,3] + A^2[3,4]$
$15 > 2 + 1$
$15 > 3$

$A^2[4,1] = A^2[4,3] + A^2[3,1]$
$2 < 7 + 5$
$2 < 12$

$A^2[4,2] = A^2[4,3] + A^2[3,2]$

$A^4 =$

	1	2	3	4
1	0	3	5	6
2	5	0	2	3
3	3	6	0	1
4	2	5	7	0

$A^4[3,1] = \min\{A^3[3,1], A^3[3,4] + A^3[4,1]\}$
$= \min\{5, 1+2\} = 3$

$A^4[3,2] = \min\{A^3[3,2], A^3[3,4] + A^3[4,2]\}$
$= \min\{8, 1+5\} = 6$

$A^3(1,2) = A^3(1,4) + A^3(4,3)$
$3 = 6 + 7$
$3 <= 12$

$A^4(1,3) = A^3(\min\{A^3[1,3], A^3[1,4] + A^3[4,3]\})$
$= \min\{5, 6+7\} = 5$

$A^4(2,1) = \min\{A^3[2,1], A^3[2,4] + A^3[4,1]\}$
$= \min\{7, 3+2\} = 5$

$A^4[2,3] = \min\{A^3[2,3], A^3[2,4] + A^3[4,3]\}$
$= \min\{2, 3+7\} = 2$

Formula:-

$$A^k[i,j] = \min\{A^{k-1}[i,j], A^{k-1}[i,k] + A^{k-1}[k,j]\}$$

Code:-

```
for (k=1; k<=n; k++)
{
    for (i=0; i<=n; i++)
    {
        for (j=1; j<=n; j++)
        {
            A^k[i,j] = min{A^(k-1)[i,j], A^(k-1)[i,k] + A^(k-1)[k,j]}
        }
    }
}
```

used to generate matrix.