Date
Page

Tutorial - 9.

1) If $\langle G, * \rangle$ is an abelian group then for $a, b \in G$ show that $(a*b)^n = a^n * b^n$.

$\rightarrow$ $(a*b)^n = a^n * b^n$

LHS $= (a*b)^n$

$= (a*b)*(a*b)*(a*b) \ldots * n$ times.

$= a*b*a*b.*b*b*\ldots a*b$. (Associative)

$= a^n * b^n$

2) Write the composition table for $\langle Z_5, +_5 \rangle$.

$+_5$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

$[R]_0 = \{5, 10, 15, 20, \ldots\}$

$[R]_1 = \{6, 11, 16, 21, \ldots\}$

$[R]_2 = \{7, 12, 17, 22, \ldots\}$

$[R]_3 = \{8, 13, 18, 23, \ldots\}$

$[R]_4 = \{9, 14, 19, 24, \ldots\}$

$[R]_5 = [R]_0$

2) $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$

Find $\alpha \Delta \beta$ (Right composition.

$\rightarrow$ $(\alpha \Delta \beta)(x) = \beta(\alpha(x))$

$$\alpha \Delta \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

3) Find subgroups of $\langle Z_5, +_5 \rangle$

→ $Z_5 \pm \{ \langle 0, 5 \rangle, \langle 0,$

$[R]_0 = \{ 5, 10, 15, 20, 25, \ldots \}$

~~$[R]_0 = \{$~~

For any $X \subseteq Z_5$,

$X = \{ [0], [2], [3] \}$.

$X = \{ [0] \}$.

$X = \{ [0], [1], [4] \}$.

4)

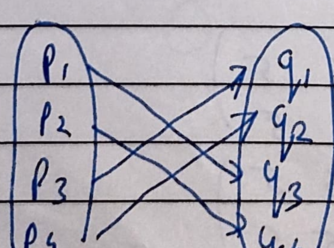
$*$	P_1	P_2	P_3	P_4
P_1	P_1	P_2	P_3	P_4
P_2	P_2	P_1	P_4	P_3
P_3	P_3	P_4	P_1	P_2
P_4	P_4	P_3	P_2	P_1

Δ	q_1	q_2	q_3	q_4
q_1	q_3	q_4	q_1	q_2
q_2	q_4	q_3	q_2	q_1
q_3	q_1	q_2	q_3	q_4
q_4	q_2	q_1	q_4	q_3

Show that $\langle G, * \rangle$ and $\langle S, \Delta \rangle$ are isomorphic.

→ Let, $g : G \to S$.

$\therefore$



For any $a, b \in G$, $g(a * b) = g(a) \Delta g(b)$.

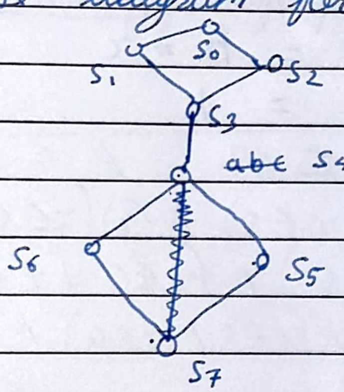
$\therefore g(P_2 * P_3) = g(P_2) \Delta g(P_3)$

$\therefore g(P_4) = q_4 \Delta q_1$

$q_2 = q_2.$

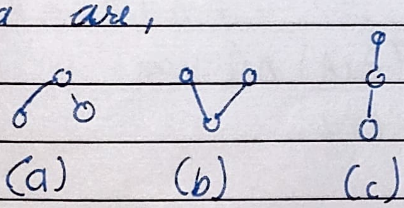
5) $S_0 = \{a, b, c, d, e, f\}.$
$S_1 = \{a, b, c, d, e\}.$
$S_2 = \{a, b, c, e, f\}.$
$S_3 = \{a, b, c, e\}.$
$S_4 = \{a, b, c\}$
$S_5 = \{a, b\}.$
$S_6 = \{a, c\}.$
$S_7 = \{a\}.$

Draw hasse diagram for $\langle L, \subseteq \rangle$, $L = \{S_0, S_1, \ldots S_7\}$.



6) Show that a lattice with 3 elements is a chain.

→ Possible hasse diagrams for a 3 variable algebra are,



The hasse diagrams a and b don't have LUB and GUB hence, a lattice with 3 elements is chain

7) Show that in lattice if $a \leq b$, $c \leq d$ then,
$a * c \leq b * d.$

$\rightarrow$ $a * c \leq a \leq b$

$a * c \leq c \leq d.$

$\therefore a * c \leq b$

$a * c \leq d$

$\therefore a * c \leq b * d.$

8) Show that in lattice $a \leq b \leq c$ then,
$a \oplus b = b * c$ and $(a * b) \oplus (b * c) = b = (a + b) * (a \oplus c)$

$\rightarrow$

$$(a * b) \oplus (b * c) = a \oplus b$$
$$= b.$$

$$(a + b) * (a \oplus c) = b * c$$
$$= b.$$

d $(R \subseteq S) \wedge (S \subseteq Q)$ $R \subset Q$

2 17 $(x)(x \in R \Longleftrightarrow x \in S) \wedge (x)(x \in S \rightarrow x \in Q \wedge (x \in S \wedge x \notin Q$

2 8 - 0 $(x \notin R \vee x \in S) \wedge (x \notin S \vee x \in Q) \wedge (x \in S \wedge x \notin Q)$

2 4 - 0 $(x \notin R \wedge x \in Q) \vee (x \in S \wedge x \in Q) \wedge (x \in S \wedge x \notin Q)$

2 2 - 0

2 1 - 1 255 $B_{17} = B_{010001} = B_{00010001} = \{a_4, a_8\}$

0

$x \in A \Longleftrightarrow x \in \{x \mid (x \in A) \vee (x \in B)\}.$

$(x \in A) \Longleftrightarrow (x \in A) \vee (x \in B)$

$(A \wedge B) \vee (A \wedge \sim B) \Rightarrow A \wedge (B \vee \sim B) \Rightarrow A$

$(A \vee A) \wedge (A \vee \sim B) \wedge (B \vee A) \wedge (B \vee \sim B)$

$A \wedge (A \vee \sim B) \wedge (A \vee B)$