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Unit - 5

* Two basic types of counting mechanism.
  1] Product rule (If task is divided into subtasks (i.e. t_1, t_2))
  2] Sum rule. (If task can be done either in one of n_1 ways or in one of n_2 ways.)

* Subtraction Rule :- ~~Eta~~ If a task can be done in either n_1 ways or n_2 ways, then ther no. of ways to do the task is n_1+n_2 minus the no. of ways to do the task that are common to the two different ways.
  Subtractn rule is also known as principle of inclusion-exclusion.
  $$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

* Division Rule :- If the finite set A is the union of n pairwise disjoint subsets each with d elements, then $n = |A|/d$".

* Pigenhole Principle :- If k is +ve integer & k+1 or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.
  also called Dirichlet drawer principle.

* Generalized Principle :-
  - if N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

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* Permutation & combinatⁿ :-

A permutatⁿ of a set of distinct objects is an ordered arrangement of these objects.

- an arrangement of r elements of a set is called an r-permutation.

- $P(n,r) = n(n-1)(n-2) \dots (n-r+1)$.

* If n & r are integers with $0 \leq r \leq n$, then $P(n,r) = \frac{n!}{(n-r)!}$

⇒ Combinatⁿ :- unordered selectⁿ of objects.
r-combinatⁿ is a set of unordered selectⁿ of r elements from the set.

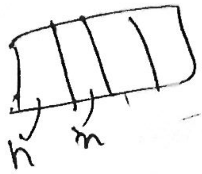
→ denoted by $C(n,r)$ or $\binom{n}{r}$ and is called a binomial coeff.

→ $C(n,r) = \frac{n!}{r!\,(n-r)!}$

→ let n & r be nonnegative integers with $r \leq n$. Then
$C(n,r) = C(n, n-r)$. $\frac{n!}{(n-r)!\,r!} = \frac{n!}{r!\,(n-r)!}$

* Binomial theorem :-

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \dots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n .$$



* Generalized Permutatⁿ & combinatⁿ :-

1) Permutation with repeatition :-

Thm :- The no. of r-permutatⁿs of a set of n objects with repetition allowed is n^r.

2) Combinatⁿ with repetitⁿ :- ~~n!~~ $\frac{(n+r-1)!}{(n-1)!\,n!}$

* Permutatⁿ with indisinguishable objects :-

The no. of different permutatⁿs of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2, -- & n_k indistinguishable objects of type k, is $\frac{n!}{n_1!\,n_2!\ldots n_k!}$

* n distinguishable objects into k distinguishable boxes so that n_i objects are placed into box i, $i = 1, 2, \ldots k$, equals $\frac{n!}{n_1!\,n_2!\,n_3!\ldots n_k!}$

* Indistinguishable objects & distinguishable boxes :-

no. of ways $= C(n+r-1, n-1)$ ways to place r indistinguishable objects into n distinguishable boxes.

$k \to n_1, \; k-1 \; n_2$

$$\frac{n!}{n_1! \, n_2!}$$

* Probability Distribution :- The function p from the set of all outcomes of the sample space S is called a probability distribution.

* Conditional Probability :-

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$ where (E & F are events) with $P(F) > 0$.

* The events E & F are independent iff $P(E \cap F) = P(E) \cdot P(F)$

* Baye S' Theorem :-

$$P(F/E) = \frac{P(F \cap E)}{P(E)}$$

Suppose that E & F are events from a sample space S such that $P(E) \neq 0$ & $P(F) \neq 0$. Then

$$P(F/E) = \frac{P(E/F)P(F)}{P(E/F) \cdot P(F) + P(E/\bar{F})P(\bar{F})}$$

$$P(F/E) = \frac{P(E/F)P(F)}{P(E)}$$

* Bayesian Spam Filters :-

By Bayes' thm, the probability that the msg. is spam, given that it contains the word w, is

$$P(S/E) = \frac{P(E/S)P(S)}{P(E/S)P(S) + P(E/\bar{S}) \cdot P(\bar{S})}$$

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- Permutations of a distinct objects is an ordered arrangement of these objects.
in $S = \{1, 2, 3\}$ 2 is 2-permutation, 1 is 1-permutatn

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!\,r!}$$

* $P(n,n) = n!$
* $C(n,n) = 1$.

$P(n,r) = n^r$

→ For 15 objects → $P(n,r) = 4+15-1 = 18$.
$\therefore {}^{18}C_3$ (Repeatatn are allowed)
$= C(18,3)$

$$\underset{\text{with repetn}}{C(n,r)} = \frac{(n+r-1)!}{r!\,(n-r)!}$$

Q. SUCCESS How many distict words can form the given word & (Repetitn ~~not~~ allowed).

$1 \times (7 \times 5) \times$

→ $S \to 3$, $C = 2$, $U = 1$, $E = 1$

$$\text{ways} = {}^7C_3 \times {}^4C_2 \times {}^2C_1 \times {}^1C_1$$
$$= \frac{7!}{3!\,4!} \times \frac{4!}{2!\,2!} \times \frac{2!}{1!\,1!} \times \frac{1!}{1!\,0!} = \frac{7!}{3!\,2!} = \frac{7 \times 6 \times 5 \times 4}{2!}$$
$$= \boxed{420}$$

Q.2] How many ways ~~#~~ are there to distribute 5 cards to each for 4 player in set of standard set of 52 cards?

→ no. of ways = ${}^{52}C_5 \times {}^{47}C_5 \times {}^{42}C_5 \times {}^{37}C_5$.

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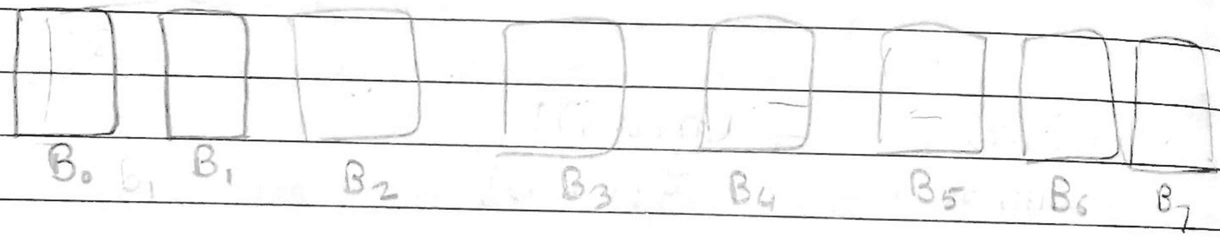
•] ~~Q.2]~~ The no. of ways to distribute n distinguishable objects into k distinguishable

$n!$

Q. How many ways are there to place 10 indistinguishable balls into eight distinguishable bins.



∴ no. of ways = ${}^{17}C_{10}$

$C(8+10-1, 10) = C(17,10)$.

Q.4] How many ways are there to put four different employees into three indistinguishable offices,

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when each office can contain any no. of employees:

→ _ _ _

0	0	4
3	1	0
2	2	0
2	1	1

~~0 1 3~~ (covered) ~~1 1 2~~ (covered)

★ $P(E_1 + E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

$P(E_2) = 3$

$P(E_1 \cup E_2) = 7/8 + 3/8 - 3/8 = 7/8$.

$P(E_1) =$ MSB is 1 $= 4/8$

$P(E_2) =$ Exact two 1's $= 3/8$

$P(E_1 \cap E_2) = 2/8$

$P(E_1 \cup E_2) = 4/8 + 3/8 - 2/8 = 5/8$

0	0	0
0	1	0
0	0	1
0	1	1
1	0	0
1	1	0
1	0	1
1	1	1

★ probability of compliment :-

$P(\bar{E_1}) = 1 - P(E_1)$

Q.1] Suppose that a die is biased so that 3 appears twice as often as each other no. but that the other five ~~components~~ outcomes are equally likely. What is the probability that an odd no. appears when we roll this die?

$P(1) = P(2) = P(4) = P(5) = P(6) = 1/7 = x$

$P(3) = 2P(1) = 2x$.

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$P(\text{odd no.}) = P(1) + P(3) + P(5)$
$= 1/7 + 2/7 + 1/7 = \boxed{4/7}$

* Conditional Probability :-

$$\boxed{P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}.}$$

* $P(E_1 \cap E_2) = P(E_1) . P(E_2/E_1)$

Q. → 3 bits string so that 8 bits strings of length three is equality, what is the prob. that it contains at least two consecutive 0's, given that its first bit is a 0 ?

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$\therefore P(E_2/E_1) = \frac{2}{4} = \frac{2/8}{4/8} = \frac{P(E_1 \cap E_2)}{P(E_1)}.$

* if $P(E_2/E_1) = P(E_2)$ then $P(E_2) / E_2$ is independent.

$\therefore \boxed{P(E_1 \cap E_2) = P(E_1) . P(E_2)}.$

* $P(E_1/E_2) = P(E_1)$

$P(E_2|E_1) = P(E_2)$: $P(E_1 \cap$

* Baye's Theorem :-

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \qquad \Big| \qquad P(F/E) = \frac{P(E \cap F)}{P(E)}$$

$$P(E/F) \cdot P(F) = P(F/E) \cdot P(E).$$

$$\boxed{P(E/F) = \frac{P(F/E) \cdot P(E)}{P(F)}}$$

* $P(TP) = \dfrac{P(TP/I) \cdot P(I)}{P(I/TP)}$

$= \dfrac{0.92 \times (0.2)}{P(I/TP)}$

$=$

$\therefore P(TP) = P(TP \cap I) \cup P(TP \cap \bar{I})$

$= 0.184 + 0.120$

1] $P(TP) = 0.304$

$P(I) = 0.2$

$P(TP/I) = 0.92$

$P(TP/\bar{I}) = 0.15$

$P(TP \cap I) = P(TP/I) \cdot P(I)$

$= 0.92 \times 0.2$

$= 0.184$

$P(TP \cap \bar{I}) = P(TP/\bar{I}) \, P(\bar{I})$

$= 0.15 \times 0.8$

$= 0.120$

2] $P(I/TP) = \dfrac{0.2 \times 0.92}{0.304}$

$= \dfrac{0.184}{0.304} = 0.605$

10
I 5 → 3
TP 5 → 1

$P(TP) \neq P(TP/I) + P(TP/\bar{I})$

$\neq 3/5 + 1/5$

$P(TP) = P(TP \cap I) + P(TP \cap \bar{I})$

$= 3/10 + 1/10 = 0.4$

* $$\boxed{P(F/E) = \frac{P(E/F) \cdot P(F)}{[P(E \cap F) + P(E \cap \bar{F})]}}$$

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Q. 1]

Box-A	Box-B
3R + 5G	5R + 9G

$P(R/F) = \frac{P(R \cap F)}{P(F)} = 3/8$

$P(F) = 1/2$

$P(R/F) = 3/8$

$P(F/R) = \frac{P(F) \cdot P(R/F)}{P(R)}$

$P(R \cap F) \cup P(R \cap \bar{F})$

$P(R/F) = \frac{P(R \cap F)}{P(F)}$

$\therefore P(R \cap F) = P(R/F) \cdot P(F)$

$= \frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$

$P(R \cap \bar{F}) = P(R/\bar{F}) \cdot P(\bar{F})$

$= 5/14 \cdot 1/2$

$= 5/28$

$\therefore P(R) = \frac{3}{16} + \frac{5}{28} = \cancel{\frac{8}{16}} = \cancel{\frac{1}{2}} = \cancel{\frac{21}{41}} = \frac{41}{112}$

$\therefore \boxed{P(F/R) = \frac{1/2 \cdot 3/8}{41/112} = \frac{21}{41}}$.

Q. 2] Show that if there are 30 students in a class, then at least two have last names that begin with the same latter.

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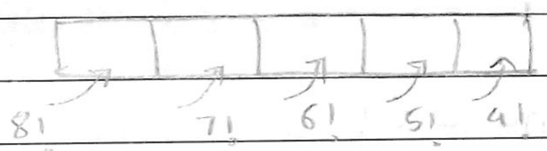
No. of letters = 26 = no. of boxes
and no. of students = 30.
Hence, we must put the each name (out of 30) in any one of these 26 boxes.
So, It is obvious that atleast ~~one~~ two have last name begins with the same letter.

Q.3] Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.

1 2 3 4 5 6 7 8



9 = 1+8
= 2+7
= 3+6
= 5+4

if we select one from each
let if we take 1, 7, 3, 5. In this elements no any pair such that their sum is 9.
Now any one selected from remaining 4 then for all cases we have a pair whose sum is 9.

Q.4] How many different permutations are there of the set {a, b, c, d, e, f, g}?

$$P(n,r) = P(7,0) + P(7,1) + P(7,2) + P(7,3) + P(7,4) + P(7,5) + P(7,6) + P(7,7)$$

$P(7,1) = 7^1 = 7$

$P(7,0) = 7^0 = 1$

$P(7,2) =$

$\dots =$

$7! =$ different permutions possible.

Q. 5] How many permutations of $\{a,b,c,d,e,f,g\}$ end with a?

$\underset{6}{\_}\ \underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}\ \underset{}{a}$

$\therefore$ no. of permutations $= 6!$

Q. 6] Let $S = \{1,2,3,4,5\}$

i) List all the 3-permutations of S.

$\rightarrow$ $\boxed{}\boxed{}\boxed{}$

$5 \quad 4 \quad 3$

No. of 3-permutations $= 5 \times 4 \times 3$
$= \underline{\underline{60}}$.

$$P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60.$$

ii) List all the 3-combinations of S

$$C(5,3) = {}^5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = \underline{\underline{10}}$$

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$\therefore$ No. of 3-combinations = 10 .

Q.7] How many bits strings of length 10 contain
a) exactly four 1's ?
b) an equal no. of 0's & 1's ?

→

1	2	3	4	5	6	7	8	9	10

a) exactly 4 1's

$\therefore C(10,4) = {}^{10}C_4 = \frac{10!}{4!\,(6)!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} = 210$.

b) no. of 1's = no. of 0's

$\therefore C(10,5) = {}^{10}C_5$

$= \frac{10!}{5!\,5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$

Q.8] In how many ways can a set of two positive integers less than 100 be chosen ?

→ no. of ways $= {}^{99}C_2$.

$= \frac{99!}{97! \times 2!} = \frac{99 \times 98}{2} = 49 \times 99$.

Q.9] A group of contains n men & n women. How many ways are there to arrange these people in a row if the men and women alternate ?

O M O M O M O M O M O

$\cancel{n!\,{}^{n}C_0\,(n+1)!}$

$n! \times {}^{n+1}C_n$

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For ~~A~~ row starts with male / men.

$\therefore$ no. of ways = $4! \times 4!$

(Repeatation or 4 mens can seat with 4! arrangements)

no. of ways = $4! \times 4!$

total no. of ways = $2 \times 4! \times 4!$

Q. 10] How many letters contains permutations ~~contains~~ of the letters ABCDEFG contains,

a) the string BCD ? (Repeatation not allowed)

b) the strings: CBA and BED ?

$\rightarrow$ a) $\boxed{B \quad C \quad D}$

$= \cancel{7 \times 6 \times 5 \times 4!}$

$= 5!$

?

b) $\boxed{C \quad B \quad A}$ $\boxed{B \quad E \quad D}$

7

3 2 1

no. of ways = $7 \times 3!$