1. Mathematical Logic

Connectives -

1) Negation ($\neg$) - NOT
2) conjunction ($\land$) - AND
3) Disjunction ($\lor$) - OR

P	Q	$P \land Q$	$P \lor Q$	$\neg P$	$\neg Q$	$P \rightarrow Q$	$P \rightleftarrows Q$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	F	F
F	T	F	T	T	F	T	F
F	F	F	F	T	T	T	T

Q.1) Convert the given English statements into its statemet formulas.

P : Today it's a very sunny day.
Q : Ram is smarter than sham.
R : Vinay don't like tea.

1) Ram is smarter than sham but vinay don't like tea
2) Today is very sunny day and vinay likes tea.
3) vinay likes a tea & Ram is smarter than sham.
4) Today is a rainy day & vinay don't like tea
5) Today is rainy day and vinay don't like tea or Ram is not smarter than sham.

→
1) $Q \land R$
2) $P \land \neg R$
3) $\neg R \land Q$
4) $\neg P \land \cancel{\neg} R$
5) $\neg P \land R \lor \neg Q$

Q.2) Using the foll^n^ statements convert it into symbolic form

R : Mark is rich

H : Mark is happy.

M : I shall watch the game on television.

N : There is something wrong with bulb.

1) Mark is poor but happy.
2) Mark is rich & happy but there is something wrong with bulb.
3) Mark is neither rich nor happy & I shall watch the game on television.
4) Mark is poor or he is both rich & unhappy or the bulb works fine.
5) I am not interested to watch the game but the bulb works fine or mark is rich & unhappy.

→ 1) $\neg R \wedge H$

2) $R \wedge H \wedge N$

3) $\neg R \wedge \neg H \wedge M$

4) $\neg R \vee R \wedge \neg H \vee \neg N$

5) $\neg M \wedge \neg N \vee R \wedge \neg H$

Q.3) construct the truth table for a given statements

1) $\neg(\neg P \vee \neg Q)$ 2) $\neg(\neg P \vee \neg Q \wedge R)$ 3) $P \wedge (\neg P \vee Q)$

4) $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge \neg R) \vee (P \wedge R)$

5) $P \vee (Q \wedge \neg R)$

→ 1) $\neg(\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

2) ¬(¬P ∨ ¬Q ∧ R)

P	Q	R	¬P	¬Q	¬P ∨ ¬Q = A	A ∧ R	¬(A ∧ R)
T	T	T	F	F	F	F	T
T	T	F	F	F	F	F	T
T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	T
F	T	T	T	F	T	T	F
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	F
F	F	F	T	T	T	F	T

3) P ∧ (¬P ∨ Q)

P	Q	¬P	¬P ∨ Q	P ∧ (¬P ∨ Q)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	F
F	F	T	T	F

5) P ∨ (Q ∧ ¬R)

P	Q	R	¬R	Q ∧ ¬R	P ∨ (Q ∧ ¬R)
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

4) $\neg P \wedge ((\neg Q \wedge R) \vee (Q \wedge \neg R) \vee (P \wedge R))$

→

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$\neg Q \wedge R = A$	$Q \wedge \neg R = B$	$P \wedge R = C$	$A \vee B \vee C = D$	$\neg P \wedge$
T	T	T	F	F	F	F	F	T	T	F
T	T	F	F	F	T	F	T	F	T	F
T	F	T	F	T	F	T	F	T	T	F
T	F	F	F	T	T	F	F	F	F	F
F	T	T	T	F	F	F	F	F	F	F
F	T	F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	T	F	F	T	T
F	F	F	T	T	T	F	F	F	F	F

6) $\neg P \wedge [(P \vee Q \wedge R) \vee (\neg P \vee Q)]$

→

P	Q	R	$\neg P$	$P \vee Q \wedge R = A$	$B = \neg P \vee Q$	$A \vee B$	$\neg P \wedge (A \vee B)$
T	T	T	F	T	T	T	F
T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

7) $(\neg P \wedge \neg Q) \wedge \neg (Q \wedge R)$

→

P	Q	R	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q = A$	$Q \wedge R$	$\neg (Q \wedge R) = B$	$A \wedge B$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	T	F
T	F	T	F	T	F	F	T	F
T	F	F	F	T	F	F	T	F
F	T	T	T	F	F	T	F	F
F	T	F	T	F	F	F	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

8) $P \wedge (Q \wedge R) \vee \neg(P \vee Q \wedge S)$

P	Q	R	S	$Q \wedge R$	$P \wedge (Q \wedge R) = A$	$P \vee Q \wedge S = B$	$\neg B$	$A \vee \neg B$
T	T	T	T	T	T	T	F	T
T	T	T	F	T	T	F	T	T
T	T	F	T	F	F	T	F	F
T	T	F	F	F	F	F	T	T
T	F	T	T	F	F	T	F	F
T	F	T	F	F	F	F	T	T
T	F	F	T	F	F	T	F	F
T	F	F	F	F	F	F	T	T
F	T	T	T	T	F	T	F	F
F	T	T	F	T	F	F	T	T
F	T	F	T	F	F	T	F	F
F	T	F	F	F	F	F	T	T
F	F	T	T	F	F	F	T	T
F	F	T	F	F	F	F	T	T
F	F	F	T	F	F	F	T	T
F	F	F	F	F	F	F	T	T

9) $(\neg Q \wedge \neg R) \vee (\neg P \vee Q) \vee \neg R$

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$\neg Q \wedge \neg R = A$	$\neg P \vee Q = B$	$A \vee B$	$(A \vee B) \vee \neg R$
T	T	T	F	F	F	F	T	T	T
T	T	F	F	F	T	F	T	T	T
T	F	T	F	T	F	F	F	F	F
T	F	F	F	T	T	T	F	T	T
F	T	T	T	F	F	F	T	T	T
F	T	F	T	F	T	F	T	T	T
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T

Q. 3) Let the truth value of P and Q is 'T' and R & S is 'F'. then what is the truth value of folln.

1) $(P \wedge Q \vee \neg R) \vee (\neg Q \vee \neg R) \wedge (R \vee S)$

→ $(P \wedge Q \vee \neg R) \vee (\neg Q \vee \neg R) \wedge (R \vee S)$

$= (T \wedge T \vee T) \vee (F \vee T) \wedge (F \vee F)$

$= T \vee T \wedge F$

$= T \wedge F$

$= F$

2) $((\neg P \vee \neg Q) \vee \neg R) \wedge (\neg S \vee Q \wedge R) \vee (\neg Q \vee \neg S \vee P)$

→ $= ((F \vee F) \vee T) \wedge (T \vee T \wedge F) \vee (F \vee T \vee T)$

$= (F \vee T) \wedge (T \wedge F) \vee (T \vee T)$

$= T \wedge F \vee T$

$= F \vee T$

$= T$

3) $\neg(P \vee Q) \wedge \neg(Q \vee S) \wedge (P \wedge \neg Q) \vee (R \vee S \vee \neg Q)$.

→ $\neg(P \vee Q) \wedge \neg(Q \vee S) \wedge (P \wedge \neg Q) \vee (R \vee S \vee \neg Q)$

$= \neg(T \vee T) \wedge \neg(F \vee F) \wedge (T \wedge F) \vee (F \vee F \vee F)$

$= F \wedge T \wedge F \vee F$

$= F \wedge F \vee F$

$= F \vee F$

$= F$

- conditional $(\longrightarrow)$

It represents If - then

- Biconditional $(\leftrightarrows)$

It represents if & only if (iff)

e.g. P = It is very sunny day

Q : There is no rainfall today.

$P \rightarrow Q$ = If it is very sunny day then there is no rainfall today.

$P \rightleftarrows Q$ = It is very sunny day if & only if there is no rainfall today.

Q.1) Write a foll^n statements in symbolic form.

1) If either Jerry takes maths or Ken takes History then Larry takes English.

→ Let, P = Jerry takes maths

Q = Ken takes History.

R = Larry takes English

$(P \vee Q) \longrightarrow R$

2) The crop will be destroyed if there is a flood & there is no production.

→ Let, P = The crop will be destroyed.

Q = There is a flood.

R = There is no production.

$(Q \longrightarrow P) \wedge R$

2) Construct the Truth table for

$(\neg P \vee Q) \rightleftarrows (P \rightarrow Q)$

P	Q	$\neg P$	$\neg P \vee Q = A$	$P \rightarrow Q = B$	$A \rightleftarrows B$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

3) construct the TT for:

$\neg(P \wedge Q) \rightleftarrows (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q) = A$	$\neg P \vee \neg Q = B$	$A \rightleftarrows B$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

4) $((P \to Q) \wedge (Q \to R)) \to (P \to R)$

P	Q	R	$P \to Q$	$Q \to R$	$(P \to Q) \wedge (Q \to R) = A$	$P \to R = B$	$\mapsto B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

5) $(\neg P \vee \neg Q) \rightleftarrows \neg(P \wedge Q)$

$\to$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q = A$	$P \wedge Q$	$\neg(P \wedge Q) = B$	$A \rightleftarrows B$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

6) $(P \rightleftarrows Q) \rightleftarrows (P \to Q) \wedge (Q \to P)$

$\to$

P	Q	$P \rightleftarrows Q = A$	$P \to Q$	$Q \to P$	$(P \to Q) \wedge Q \to P = C$	$A \rightleftarrows C$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

* Well formed formula

If A & B are two statements & A & B are wff then

i) $\neg A$ & $\neg B$ is wff.

ii) $(A \wedge B)$, $(A \vee B)$ is wff

iii) $(A \rightarrow B)$, $(B \rightarrow A)$, $(A \rightleftarrows B)$ & $(B \rightleftarrows A)$ is wff.

iv) Except above all other is not wff.

- $P \rightarrow q = \neg P \vee q$
- $P \rightleftarrows q = (P \rightarrow q) \wedge (q \rightarrow P)$

$= (\neg P \vee q) \wedge (\neg q \vee P)$

Q. Show the foll^n equivalence with Truth Table.

1] $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

→

P	Q	R	¬P	¬Q	¬Q∧R=A	Q∧R=B	P∧R=C	¬P∧A=D	D∨B	(D∨B)∨C
T	T	T	F	F	F	T	T	F	T	T
T	T	F	F	F	F	F	F	F	F	F
T	F	T	F	T	T	F	T	F	F	T
T	F	F	F	T	F	F	F	F	F	F
F	T	T	T	F	F	T	F	F	T	T
F	T	F	T	F	F	F	F	F	F	F
F	F	T	T	T	T	F	F	T	T	T
F	F	F	T	T	F	F	F	F	F	F

* Formulae -

1) $P \wedge P \Leftrightarrow P$, $P \vee P \Leftrightarrow P$

2) $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

3) $P \wedge Q \Leftrightarrow Q \wedge P$ $\quad$ $P \vee Q \Leftrightarrow Q \vee P$

4) $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ } Distributive law

5) i) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ } De Morgan's law
ii) $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

6) i) $P \wedge T \Leftrightarrow P$ iv) $P \vee T \Leftrightarrow T$
ii) $P \wedge F \Leftrightarrow F$ v) $P \vee F \Leftrightarrow P$
iii) $P \wedge \neg P \Leftrightarrow F$ vi) $P \vee \neg P \Leftrightarrow T$

7) $P \wedge (P \vee Q) \Leftrightarrow P$ } Absorption law
$P \vee (P \wedge Q) \Leftrightarrow P$

1) $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (Q \wedge R) \Leftrightarrow R$

→ $\neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$

$= (\neg P \wedge \neg Q \vee Q \vee P) \wedge R$

$= (\neg P \wedge T \vee P) \wedge R$

$= (T \wedge T) \wedge R$

$= T \wedge R$

$= R$

2) $P \rightleftarrows Q \Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

→ LHS $= P \rightleftarrows Q$

$= (P \to q) \wedge (q \to P)$

$= (\neg P \vee Q) \wedge (\neg q \vee P)$

$=$ RHS

3) $P \rightleftarrows Q \Leftrightarrow (\neg P \wedge \neg Q) \vee (P \wedge Q)$

→ LHS $= P \rightleftarrows Q$

$= (P \to Q) \wedge (Q \to P)$

$= (\neg P \vee Q) \wedge (\neg q \vee P)$

$= \neg P \wedge (\neg q \vee P) \vee Q \wedge (\neg Q \vee P)$

$= (\neg P \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge \neg q) \wedge (P \wedge q)$

$= (\neg P \wedge \neg Q) \vee F \vee F \wedge (P \wedge q)$

$= (\neg P \vee \neg Q) \vee (P \wedge Q)$

Q. Show that the given equivalences without constructing Truth Table.

1] $P \vee (P \wedge Q) \Leftrightarrow P / P \vee Q$

→ $P \vee (P \wedge Q)$	$P \vee (P \wedge Q)$
$= (P \vee P) \wedge (P \vee Q)$	$= (P \wedge T) \vee (P \wedge Q)$
$= P \wedge P \vee Q$	$= [P \wedge (Q \vee \neg Q)] \vee (P \wedge Q)$
$= P \vee Q$	$= (P \wedge Q) \vee (P \wedge \neg Q) \vee P \wedge Q)$
	$= (P \wedge Q) \vee (P \wedge \neg Q)$
	$= P \wedge (Q \vee \neg Q) = P \wedge T = P$

2] $P \vee (\neg P \wedge Q) \Leftrightarrow P \vee Q$

→ $LHS = P \vee (\neg P \wedge Q)$
$= (P \vee \neg P) \wedge (P \vee Q)$
$= T \wedge (P \vee Q)$
$= P \vee Q$

3] $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$

→ $LHS = (P \rightarrow Q) \wedge (R \rightarrow Q)$
$= (\neg P \vee Q) \wedge (\neg R \vee Q)$
$= (\neg P \wedge \neg R) \vee Q$
$= \neg (P \vee R) \vee Q$
$= (P \vee R) \rightarrow Q$
$= RHS$

4] $\neg (P \rightleftarrows Q) \Leftrightarrow (P \vee Q) \wedge \neg (P \wedge Q)$

→ $LHS = \neg (P \rightleftarrows Q)$
$= \neg [(P \rightarrow Q) \wedge (Q \rightarrow P)]$
$= \neg [(\neg P \vee Q) \wedge (\neg Q \vee P)]$
$= (P \wedge \neg Q) \vee (Q \wedge \neg P)$ $= P \vee (Q \wedge \neg P) \wedge \neg Q \vee (Q \wedge \neg P)$
$= (P \vee Q) \wedge (\neg Q \vee \neg P)$ ~~$= (P \vee Q) \wedge T \wedge T (\neg Q \wedge \neg P)$~~
$= (P \vee Q) \wedge \neg (P \wedge Q)$
$= RHS$

5] $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

$\rightarrow$ LHS $= \neg(P \wedge Q)$

$= \neg P \vee \neg Q$

$=$ RHS

Q. Convert the given formula in $\neg$ & $\vee$.

1) $(P \rightleftarrows Q) \wedge \neg R$

$\rightarrow$ $(P \rightleftarrows Q) \wedge \neg R$

$(\neg P \vee Q) \wedge (\neg Q \vee P) \wedge \neg R$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge \neg Q) \vee (Q \wedge P) \wedge \neg R$

$\neg(P \vee Q) \vee F \vee F \vee (Q \wedge P) \wedge \neg R$

$\neg(P \vee Q) \vee F \vee (Q \wedge P) \wedge \neg R$

$\neg(P \vee Q) \vee Q \wedge P \wedge \neg R$

$\neg(P \vee Q) \vee \neg(\neg Q \vee \neg P \vee R)$

* Tautological implication.

$P \rightleftarrows Q \Rightarrow P \rightleftarrows Q$ is true.

implication.

$P \Rightarrow Q = P \rightarrow Q$ is T

1] Show the foll^n implication.

i] $P \Rightarrow (Q \rightarrow P)$

P	Q	$Q \rightarrow P$	$P \rightarrow (Q \rightarrow P)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

ii] $(P \land Q) \Rightarrow P \to Q$

→

P	Q	$P \land Q$	$P \to Q$	$(P \land Q) \to (P \to Q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

iii] $(P \to Q) \to Q \Rightarrow P \lor Q$

→ $(P \to Q) \to Q \Rightarrow P \lor Q$

$= (\neg P \lor Q) \to Q \Rightarrow P \lor Q$

$= \neg(\neg P \lor Q) \lor Q \Rightarrow P \lor Q$

$= (P \land \neg Q \lor Q) \Rightarrow P \lor Q$

$= (P \land \neg Q \lor Q) \to P \lor Q$

$= \neg(P \land \neg Q \lor Q) \lor (P \lor Q)$

$= (\neg P \lor Q \land \neg Q) \lor (P \lor Q)$

$= (\neg P \land \neg Q \lor P) \lor Q$

$= (T \land \neg Q) \lor Q$

$= \neg Q \lor Q$

$= T$

$P \land Q \land R$

$P \land R \land Q$

$\neg($ $R \land Q \land P$

$\cancel{P \land Q}$ $P \land Q \lor R$

$(P \lor R) \land (Q \lor R)$

$(P \to Q) \to Q \Rightarrow P \lor Q$

$= (\neg P \lor Q) \to Q \Rightarrow P \lor Q$

$= \neg(\neg P \lor Q) \lor Q \Rightarrow P \lor Q$

$= P \land \neg Q \lor Q \Rightarrow P \lor Q$

$= P \land T \Rightarrow P \lor Q$

$= P \to P \lor Q$

$= \neg P \lor P \lor Q$

$= T \lor Q$

$= T$

* **Duality law**

$\wedge, \vee$ ~~are~~ duals of each other.

T, F are duals of each other.

e.g. $(\neg P \wedge Q) \vee F \Rightarrow (\neg P \vee Q) \wedge T$

* **other connectives-**

$\overline{\vee}$ (EXOR)

$\uparrow$ (Nand)

$\downarrow$ (Nor)

$P \uparrow P$ $\quad\quad$ $P \downarrow P$

$= \neg(P \wedge P)$ $\quad\quad$ $\neg(P \vee P)$

$= \neg P$ $\quad\quad$ $\neg P$

$(P \uparrow P) \uparrow (Q \uparrow Q)$

$\rightarrow \neg(P \wedge P) \uparrow \neg(Q \wedge Q)$

$\neg P \uparrow \neg Q$

$\neg(\neg P \wedge \neg Q)$

$P \vee Q$

$(P \uparrow Q) \uparrow (P \uparrow Q)$

$\rightarrow \neg(P \wedge Q) \uparrow \neg(P \wedge Q)$

$\neg(\neg(P \wedge Q) \wedge \neg(P \wedge Q))$

$(P \wedge Q) \vee (P \wedge Q)$

$P \wedge Q$

Q. Show the folln equivalence with T.T or without T.T

1> $\neg(P \wedge Q) \rightarrow [\neg P \wedge (\neg P \vee Q)] \Leftrightarrow \neg P \vee Q$

2] $(P \vee Q) \wedge (\neg P \wedge (\neg P \vee Q)) \Leftrightarrow \neg P \wedge Q$

3] $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$

4] $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$.

→ 1)

P	Q	$\neg P$	$P \wedge Q$	$\neg(P \wedge Q) = A$	$\neg P \vee Q$	$\neg P \vee (\neg P \vee Q) = B$	$A \rightarrow B$
T	T	F	T	F	T	T	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	T
F	F	T	F	T	T	T	T

2] $(P \vee Q) \wedge (\neg P \wedge (\neg P \vee Q)) \Leftrightarrow \neg P \wedge Q$

→

P	Q	$\neg P$	$P \vee Q$ (A)	$\neg P \vee Q$	$\neg P \wedge (\neg P \vee Q)$ (B)	$A \wedge B$	$\neg P \wedge Q$
T	T	F	T	T	F	F	F
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	F	T	F	T	T	F	F

3]
→

P	Q	$\neg Q$	$P \rightarrow Q$	$\neg(P \rightarrow Q)$	$P \wedge \neg Q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

∴ $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$ $\qquad \neg(P \wedge Q)$

→ $= \neg(\neg P \vee Q) \Leftrightarrow P \wedge \neg Q$

$= (P \wedge \neg Q) \rightarrow (P \wedge \neg Q)$

$= \neg(P \wedge \neg Q) \vee (P \wedge \neg Q)$

$= (\neg P \vee Q) \vee (P \wedge \neg Q)$

$= (\neg P \vee P) . (\neg P \quad \neg Q) \vee (Q \vee P) \wedge (Q \vee \neg Q)$

$= T \wedge \neg(P \wedge Q) \vee (P \vee Q) \wedge T$

4] $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$

$\rightarrow$

			A	B			
P	Q	R	$P \rightarrow Q$	$R \rightarrow Q$	$A \wedge B$	$P \vee R$	$(P \vee R) \rightarrow Q$
T	T	T	T	T	T	T	T
T	F	F	T	T	T	T	F
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

] $\neg(P \wedge Q) \rightarrow [\neg P \vee (\neg P \vee Q)] \Rightarrow \neg P \vee Q$

$\rightarrow$ $\neg(P \wedge Q) \rightarrow [\neg P \vee (\neg P \vee Q)] \Rightarrow \neg P \vee Q$

$= \neg(\neg(P \wedge Q) \vee (\neg P \vee (\neg P \vee Q)) \rightarrow \neg P \vee Q$

$= P \wedge Q \vee (\neg P \vee \neg P) \vee Q \rightarrow \neg P \vee Q$

$= (P \wedge Q) \vee (\neg P \vee Q) \rightarrow \neg P \vee Q$

$= (P \vee \neg P) \vee (P \vee Q) \wedge (\neg P \vee Q) \wedge (Q \vee Q) \rightarrow \neg P \vee Q$

$= T \vee (P \vee Q) \wedge (\neg P \vee Q) \wedge Q \rightarrow \neg P \vee Q$

$= T \wedge (\neg P \vee Q) \wedge Q \rightarrow \neg P \vee Q$

$= (\neg P \vee Q) \wedge Q \rightarrow \neg P \vee Q$

$= \neg[(\neg P \vee Q) \wedge Q] \vee \neg P \vee Q$

$= (P \wedge \neg Q \vee \neg Q) \vee \neg P \vee Q$

$= (P \wedge \neg Q) \vee (\neg P \vee Q)$

$= \neg(\neg P \vee Q) \vee (\neg P \vee Q)$

$= T$

• $\neg(P \wedge Q) \rightarrow [\neg P \vee (\neg P \vee Q)] \Leftrightarrow \neg P \vee Q$

$\rightarrow$ $= \neg(P \wedge Q) \rightarrow (\neg P \vee Q)$

$= (P \wedge Q) \vee (\neg P \vee Q)$

$= P \vee (\neg P \vee Q) \wedge Q \vee (\neg P \vee Q)$

$= (P \vee \neg P \vee Q) \wedge (Q \vee \neg P \vee Q)$

$= T \wedge (\neg P \vee Q)$

$= \neg P \vee Q$

2] $(P \vee Q) \wedge (\neg P \wedge (\neg P \vee Q)) \Leftrightarrow \neg P \wedge Q$

→ $LHS = (P \vee Q) \wedge (\neg P \wedge (\neg P \vee Q))$

$= (P \vee Q) \wedge \neg P$ By Absorption law

$= (P \wedge \neg P) \vee (P \wedge \neg P)$

$= F \vee (\neg P \wedge Q)$

$= \neg P \wedge Q$

3] $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$

→ $LHS = \neg(P \rightarrow Q)$

$= \neg(\neg P \vee Q)$

$= P \wedge \neg Q$

4) $(P \rightarrow Q) \wedge (R \rightarrow Q) \Leftrightarrow (P \vee R) \rightarrow Q$

→ $LHS = (P \rightarrow Q) \wedge (R \rightarrow Q)$

$= (\neg P \vee Q) \wedge (\neg R \vee Q)$

$= (\neg P \wedge \neg R) \vee Q$

$= \neg(P \vee R) \vee Q$

$= (P \vee R) \rightarrow Q$

Q. Show the folln implication with /without T.T.

1) $(P \rightarrow Q) \rightarrow Q \Rightarrow P \vee Q$

2) $((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R) \Rightarrow (Q \rightarrow R)$

3) $(P \rightarrow (Q \rightarrow R)) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$

1) $((P \rightarrow Q) \rightarrow Q) \rightarrow P \vee Q$

→

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow Q = A$	$P \vee Q$	$A \rightarrow P \vee Q$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	F	T

2) $((P \vee \neg P) \to Q) \to ((P \vee \neg P) \to R) \to Q \to R$

$\to$

					A	B	C	D	
P	Q	R	$\neg P$	$P \vee \neg P$	$P \vee \neg P \to Q$	$(P \vee \neg P) \to R$	$A \to B$	$Q \to R$	$C \to D$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	T	F	F	F	T
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	F	T
F	F	T	T	T	T	T	T	T	T
F	F	F	T	T	T	F	F	T	T

3) $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

$\to$

				A			B	
P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \to Q$	$P \to R$	$(P \to Q) \to (P \to R)$	$A \to B$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

$(P \to Q) \to Q \Rightarrow P \vee Q$

$\to$ $(P \to Q) \to Q \to P \vee Q$

$(\neg P \vee Q) \to Q \to P \vee Q$

$\neg(\neg P \vee Q) \vee Q \to P \vee Q$

$(P \wedge \neg Q) \vee Q \to P \vee Q$

$\neg(\ P \wedge \neg Q \vee Q) \vee (P \vee Q)$

$\neg P \vee \underbrace{Q \wedge \neg Q} \vee P \vee Q$

$(\neg P \vee\ F) \vee (P \vee Q)$

$\underbrace{\neg P \vee P} \vee Q \Longrightarrow T \vee Q \Rightarrow T$

2] $((P \vee \neg P) \to Q) \to ((P \vee \neg P) \to R) \Rightarrow (Q \to R)$

$\to$ $(T \to Q) \to (T \to R) \to (Q \to R)$

$(F \vee Q) \to (F \vee R) \to Q \to R$

$(Q \to R) \to (Q \to R)$

$\neg(Q \to R) \vee (Q \to R)$

$\neg A \vee A$ where, $A = Q \to R$

T

3] $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

$\to$ $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

$= \neg P \vee (Q \to R) \Rightarrow \neg(P \to Q) \vee (P \to R)$

$= \neg P \vee (\neg Q \vee R) \Rightarrow \neg(\neg P \vee Q) \vee (\neg P \vee R)$

$= (\neg P \vee \neg Q \vee R) \Rightarrow (P \wedge \neg Q) \vee (\neg P \vee R)$

$= (\neg P \vee \neg Q \vee R) \Rightarrow P \vee (\neg P \vee R) \wedge \neg Q \vee (\neg P \vee R)$

$= (\neg P \vee \neg Q \vee R) \Rightarrow T \wedge (\neg Q \vee \neg P \vee R)$

$= (\neg P \vee \neg Q \vee R) \Rightarrow (\neg Q \vee \neg P \vee R)$

$= (\neg P \vee \neg Q \vee R) \to (\neg Q \vee \neg P \vee R)$

$= \neg(\neg P \vee \neg Q \vee R) \vee (\neg Q \vee \neg P \vee R)$

$= (P \wedge Q \wedge \neg R) \vee (\neg Q \vee \neg P \vee R)$

$= P \vee (\neg Q \vee \neg P \vee R) \wedge Q \vee (\neg Q \vee \neg P \vee R) \wedge \neg R \vee (\neg Q \vee \neg P \vee R)$

$= T \wedge T \wedge T$

$= T$

- Normalization- converting any statement formula with a standard form.

Normal forms

CNF (conjuctive normal form)

PCNF (principel conductive form)

DNF (Disjuctive normal form)

PDNF (principel disjuctive form)

* CNF

$(\vee) \wedge (\vee) \wedge (\vee)$

DNF

$(\wedge) \vee (\wedge) \vee (\wedge)$

PDNF -

minterms - Either formula.

(minterm) V

(minterm) V (minterm)

- minterm-

P	Q	R	Minterm
T	T	T	$P \wedge Q \wedge R$
T	T	F	$P \wedge Q \wedge \neg R$
T	F	T	$P \wedge \neg Q \wedge R$
T	F	F	$P \wedge \neg Q \wedge \neg R$
F	T	T	$\neg P \wedge Q \wedge R$
F	T	F	$\neg P \wedge Q \wedge \neg R$
F	F	T	$\neg P \wedge \neg Q \wedge R$
F	F	F	$\neg P \wedge \neg Q \wedge \neg R$

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Q.1) Find the PDNF of given formula.

$\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

→

P	Q	$P \vee Q$	$P \wedge Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \rightleftarrows (P \wedge Q)$
T	T	T	T	F	F
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	F	T	F

The pdnf of given is $(P \wedge \neg Q) \vee (\neg P \wedge Q)$

2) $\neg P \rightarrow (P \rightleftarrows \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \rightleftarrows \neg Q$	$\neg P \rightarrow (P \rightleftarrows \neg Q)$
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

PDNF $= (P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

3) $(P \rightarrow (Q \wedge R)) \wedge (\neg P \wedge \neg R)$

→

P	Q	R	$\neg P$	$\neg R$	$Q \wedge R$	$P \rightarrow (Q \wedge R) = A$	$\neg P \wedge \neg R = B$	$A \wedge B$
T	T	T	F	F	T	T	F	F
T	T	F	F	T	F	F	F	F
T	F	T	F	F	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	T	T	F	F
F	T	F	T	T	F	T	T	T
F	F	T	T	F	F	T	F	F
F	F	F	T	T	F	T	T	T

PDNF $= (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

4) Find PDNF $(\neg P \vee \neg Q) \rightarrow \neg R \rightarrow (P \rightleftarrows \neg R)$

→

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$\neg P \vee \neg Q = A$	$A \rightarrow \neg R = B$	$P \rightleftarrows \neg R = C$	$B \rightarrow$
T	T	T	F	F	F	F	T	F	F
T	T	F	F	F	T	F	T	T	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	T	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	T	T	T	F	F
F	F	T	T	T	F	T	F	T	T
F	F	F	T	T	T	T	T	F	F

PDNF $= (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R)$
$\vee (\neg P \wedge \neg Q \wedge R)$

- $P = P \wedge T$
  $= P \wedge (Q \vee \neg Q)$
  $= (P \wedge Q) \vee (P \wedge \neg Q)$ [PDNF]

- $P = P \wedge T \wedge T$
  $= P \wedge (Q \vee \neg Q) \wedge (R \vee \neg R)$
  $= [(P \wedge Q) \vee (P \wedge \neg Q)] \wedge (R \vee \neg R)$
  $= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$

- $Q = Q \wedge T \wedge T$
  $= Q \wedge (P \vee \neg P) \wedge (R \vee \neg R)$
  $= [(Q \wedge P) \vee (Q \wedge \neg P)] \wedge (R \vee \neg R)$
  $= (Q \wedge R \wedge R) \vee (Q \wedge P \wedge \neg R) \vee (Q \wedge \neg P \wedge R) \vee (Q \wedge \neg P \wedge \neg R)$

Q. obtain the PDNF without T.T

1] $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

→ $[\neg(P \vee Q) \longrightarrow (P \wedge Q)] \wedge [(P \wedge Q) \longrightarrow \neg(P \vee Q)]$

$[\neg(\neg(P \vee Q)) \vee (P \wedge Q)] \wedge [\neg(P \wedge Q) \vee \neg(P \vee Q)]$

$[(P \vee Q) \vee (P \wedge Q)] \wedge [(\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q)]$

$[P \vee (P \wedge Q) \vee Q \vee (P \wedge Q)] \wedge [\neg P \vee (\neg P \wedge \neg Q) \vee \neg Q \vee (\neg P \wedge \neg Q)]$

$(P \vee Q) \wedge (\neg P \vee \neg Q)$

$[P \wedge (\neg P \vee \neg Q)] \vee [Q \wedge (\neg P \vee \neg Q)]$

$[(P \wedge \neg P) \vee (P \wedge \neg Q)] \vee [(Q \wedge \neg P) \vee (Q \wedge \neg Q)]$

$[F \vee (P \wedge \neg Q)] \vee [(Q \wedge \neg P) \vee F]$

$(P \wedge \neg Q) \vee (Q \wedge \neg P)$

$(P \wedge \neg Q) \vee (\neg P \wedge Q)$

2] $\neg P \longrightarrow (P \rightleftarrows \neg Q)$.

→ $\neg P \rightarrow [(P \rightarrow \neg Q) \wedge (\neg Q \rightarrow P)]$

$\neg P \rightarrow [(\neg P \vee \neg Q) \wedge (Q \vee P)]$

$\neg P \rightarrow [\neg P \wedge (P \vee Q) \vee \neg Q \wedge (P \vee Q)]$

$\neg P \rightarrow [F \vee (\neg P \vee Q) \vee (P \wedge \neg Q) \vee F]$

$\neg P \rightarrow [(\neg P \vee Q) \vee (P \wedge \neg Q)]$

$P \vee [(\neg P \vee Q) \vee (P \wedge \neg Q)]$

$(P \wedge T) \vee [\neg P \vee (P \wedge \neg Q) \wedge Q \vee (P \wedge \neg Q)]$

$P \wedge (Q \vee \neg Q) \vee [T \wedge (\neg P \vee \neg Q) \wedge (P \wedge Q) \wedge T]$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee [(\neg P \vee \neg Q) \wedge (P \wedge Q)]$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee [(\neg P \wedge P \wedge Q) \vee (\neg P \vee \neg Q \wedge Q)]$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee [(F \wedge Q) \vee (F \vee P)]$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee [P \vee Q]$

$(P \wedge Q) \vee (P \wedge \neg Q \vee (\neg P \wedge Q)$

3] $(\neg P \vee \neg Q) \to \neg R \to (P \rightleftarrows \neg R)$

$\to$ $(\neg P \vee \neg Q) \to \neg R \to (P \rightleftarrows \neg R)$

$[\neg(\neg P \vee \neg Q) \vee \neg R] \to [(P \to \neg R) \wedge (\neg R \to P)]$

$(P \wedge Q) \vee \neg R \to [(\neg P \vee \neg R) \wedge (R \vee P)]$

$[(P \vee \neg R) \wedge (Q \vee \neg R)] \to [(\neg P \vee \neg R) \wedge (P \vee R)]$

$\neg[(P \vee \neg R) \wedge (Q \vee \neg R)] \vee [(\neg P \vee \neg R) \wedge (P \vee R)]$

$[\neg(P \vee \neg R) \vee \neg(Q \vee \neg R)] \vee [(\neg P \vee \neg R) \wedge (P \vee R)]$

$(\neg P \wedge R) \vee (\neg Q \wedge R) \vee [$

$[(\neg P \wedge R) \wedge (Q \vee \neg Q)] \vee [(\neg Q \wedge R) \wedge (P \vee \neg P)]$

$\vee [(P \wedge \neg R) \wedge (Q \vee \neg Q)] \vee [R \wedge (Q \vee \neg Q) \wedge (P \wedge \neg P)$

* PCNF

P	Q	Maxterm
T	T	$\neg P \vee \neg Q$
T	F	$\neg P \vee Q$
F	T	$P \vee \neg Q$
F	F	$P \vee Q$

Q. Obtain PDNF & PCNF with T.T.

1) $\neg(P \to Q) \vee R$

$\to$

P	Q	R	$P \to Q$	$\neg(P \to Q)$	$\neg(P \to Q) \vee R$
T	T	T	T	F	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	T

F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	T	F	F

PDNF - $(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee$
$(\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

PCNF - $(\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

2) $\neg(P \wedge Q) \rightarrow (\neg R \vee P)$

→

P	Q	R	$\neg R$	$P \wedge Q$	$\neg(P \wedge Q) = A$	$\neg R \vee P = B$	$A \rightarrow B$
T	T	T	F	T	F	T	T
T	T	F	T	T	F	T	T
T	F	T	F	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	F	F
F	T	F	T	F	T	T	T
F	F	T	F	F	T	F	F
F	F	F	T	F	T	T	T

PDNF = $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
$\vee (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

PCNF = $(P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$

1) $\neg(P \rightarrow Q) \vee R$

$\rightarrow$ $\neg(P \rightarrow Q) \vee R$

$= \neg(\neg P \vee Q) \vee R$

$= (P \wedge \neg Q) \vee R$

$= (P \vee R) \wedge (\neg Q \vee R)$

$= [(P \vee R) \vee F] \wedge [(\neg Q \vee R) \vee F]$

$= [(P \vee R) \vee (Q \wedge \neg Q)] \wedge [(\neg Q \vee R) \vee (P \wedge \neg P)]$

$= [(P \vee R \vee Q) \wedge (P \vee R \vee \neg Q)] \wedge [(\neg Q \vee R \vee P) \wedge (\neg Q \vee R \vee \neg P)]$

$= [(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)] \wedge [(P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R)]$

$= (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$

Missing max term -

$(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R)$
$\wedge (\neg P \vee Q \vee \neg R)$

Appling negation, PDNF =

$(\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$
$\vee (P \wedge \neg Q \wedge R)$

* Infix & Polish Notations

(Infix, postfix & prefix notation)

Infix - └ left → right └ right to left

e.g. $\neg P \wedge Q$, $(\neg P \wedge \neg R) \rightleftarrows Q$

infix	Postfix	Prefix
$A \wedge B$	$A B \wedge$	$\wedge A B$
$\neg A$	$A \neg$	$\neg A$
$A \wedge B \wedge C$	$A B C \wedge \wedge$	$\wedge \wedge A B C$
$(A \wedge B) \vee C$	$A B \wedge C \vee$	$\wedge A B \vee C$
$(\neg P \wedge Q) \rightarrow \neg R$	$P \neg Q \wedge R \neg \rightarrow$	$\rightarrow \wedge \neg P Q \neg R$

$P \uparrow P = \neg(P \wedge P)$

$P \downarrow Q = \neg(P \vee Q)$, $P \downarrow P = \neg P$, $Q \downarrow Q = \neg Q$

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1) $(\neg P \to Q) \to R \rightleftarrows (\neg R \vee \neg P \wedge Q)$

$\to$ Postfix - $P \neg Q \to R \to R \neg P \neg Q \wedge \vee \rightleftarrows$

Prefix: $\rightleftarrows \vee \to \to \neg P Q R \neg R \wedge \neg P Q$

2) $(P \to Q) \to R \to (\neg R \rightleftarrows \neg P)$

$\to$ Postfix - $PQ \to R \to R \neg P \neg \rightleftarrows \to$

Prefix- $\to \to \to P Q R \rightleftarrows \neg R \neg P$

3) $\neg(P \rightleftarrows Q \vee \neg R) \wedge (\neg Q \vee R \wedge \neg S)$

$\to$ Postfix - $PQR\neg\vee\rightleftarrows\neg\ Q\neg RS\neg\wedge\vee\wedge$

Prefix- $\wedge\neg\vee\rightleftarrows PQ\neg R\wedge\vee\neg QR\neg S$

4) $(P \to Q) \to R \to S \to P$

$\to$ Postfix - $PQ \to R \to S \to P \to$

Prefix- $\to \to \to \to PQRSP$

Q. Convert given formula interms of $\uparrow$ only.

1) $(\neg P \to Q) \vee R$

$\to$ $(\neg P \to Q) \vee R$

$= (P \vee Q) \vee R$

$= \neg[(\neg P \wedge \neg Q)] \vee R$

$= \neg P \uparrow \neg Q \vee R$

$= \neg[\neg(\neg P \uparrow \neg Q) \wedge \neg R]$

$= \neg(\neg P \uparrow \neg Q) \uparrow \neg R$

$= (\neg P \uparrow \neg Q) \uparrow (\neg P \uparrow \neg Q) \uparrow R \uparrow R$

$= P \uparrow P \uparrow Q \uparrow Q \uparrow P \uparrow P \uparrow Q \uparrow Q \uparrow R \uparrow R$

2) $\neg P \vee Q$

$\rightarrow \quad \neg P \vee Q$

$= \neg(P \wedge \neg Q)$

$= P \uparrow \neg Q$

$= P \uparrow Q \uparrow Q$

3) $\neg(P \rightleftarrows Q) \vee (R \rightarrow \neg P)$

$\rightarrow \quad \neg(P \rightleftarrows Q) \vee (R \rightarrow \neg P)$

$= \neg[(P \rightarrow Q) \wedge (Q \rightarrow P)] \vee (\neg R \vee \neg P)$

$= \neg[(\neg P \vee Q) \wedge (\neg Q \vee P)] \vee \neg(P \wedge R)$

$= (\neg P \vee Q) \uparrow (\neg Q \vee P) \vee \neg(P \wedge R)$

$= \neg[(P \wedge \neg Q) \uparrow \neg(Q \wedge \neg P)] \wedge \neg(P \wedge R)]$

$= (P \wedge \neg Q) \uparrow (Q \wedge \neg P) \uparrow (P \wedge R) \uparrow (P \wedge R)$

$= P \uparrow Q \uparrow Q \uparrow Q \uparrow P \uparrow P \uparrow P \wedge R \uparrow P \wedge R$

Q convert given formula in terms of ↓ only.

1) $(P \wedge \neg Q) \wedge R$

$\rightarrow \quad \neg(\neg P \vee Q) \wedge R$

$= \neg P \downarrow Q \wedge R$

$\neg(P \vee Q) = \neg[(P \downarrow \neg Q) \vee (\neg R)]$

$\neg(P \downarrow Q) = \neg(P \downarrow \neg Q) \downarrow \neg R$

$= (P \downarrow Q \downarrow Q) \downarrow (P \downarrow Q \downarrow R) \downarrow R \downarrow R$

2) $\neg(P \rightarrow R) \wedge Q$

$\rightarrow \quad \neg(P \rightarrow R) \wedge Q$

$= \neg(\neg P \vee R) \wedge Q$

$= \neg P \downarrow R \wedge Q$

$= \neg(P \downarrow P \downarrow R) \wedge Q$

$= \neg[\neg(P \downarrow P \downarrow R) \vee \neg Q]$

$= \neg(P \downarrow P \downarrow R) \downarrow \neg Q$

$= (P \downarrow P \downarrow R) \downarrow (P \downarrow P \downarrow R) \downarrow Q \downarrow Q$

Prefix - Right to left
Postfix - left to right

Q. Convert the given formula in completely parenthesised form.

1) $\rightarrow \rightarrow PQ \rightarrow \rightarrow RST$

$\rightarrow$ $\rightarrow \rightarrow PQ \rightarrow \rightarrow RST$

$(P \rightarrow Q) \rightarrow (R \rightarrow S) \rightarrow T$

2) $((AB\neg \rightarrow) \neg)B A(B \wedge \vee \neg) \rightarrow$

$\rightarrow$ $\neg(A \rightarrow \neg B) \rightarrow \neg(.B \vee A \wedge B)$

3) $((P(Q(RS\vee)\vee) \rightarrow) \neg)Q(R(\neg S)\vee)\vee)\wedge$

$\rightarrow$ $\neg(P \rightarrow Q \vee (R \vee S) \wedge (Q \vee \neg R \vee S)$

4) $\wedge \wedge \neg P \rightarrow Q R \rightarrow \rightarrow S T \vee \vee P S Q$

$\rightarrow$ $(\neg P \wedge Q \rightarrow R \rightarrow (S \rightarrow T)) \wedge P \vee S \vee Q$

Generating new knowledge from existing knowledge.

- Inference Theory

If $H_1, H_2, \text{-----} H_n$ be any hypothesis & C be the conclusion then C is valid conclusion if

$H_1 \wedge H_2 \wedge \text{-----} \wedge H_n \Rightarrow C$ is True.

$H_1 \wedge H_2 \wedge \text{-----} \wedge H_n \rightarrow C$ is True.

$H_1 = \neg P$, $H_2 = Q \rightarrow \neg P$, $H_3 = Q$, $C: \neg P$

$H_1 \wedge H_2 \wedge H_3 \Rightarrow C$

$\neg P \wedge (Q \rightarrow \neg P) \wedge Q \rightarrow \neg P$

P	Q	$\neg P$	$Q \rightarrow \neg P$	$\neg P \wedge (Q \rightarrow \neg P) \wedge Q \Rightarrow A$	$A \rightarrow \neg P$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	T	F	T

Rule P.- Existing premise / Existing Rule
Rule T - Generating new rule from existing rule

1) Show the foll^n premises without using Truth table.

1) $C \vee D$
2) $C \vee D \rightarrow \neg H$
3) $\neg H \rightarrow (A \wedge \neg B)$
4) $(A \wedge \neg B) \rightarrow (R \vee S)$

Then P.T. $R \vee S$ is valid conclusion.

→
$C \vee D$ → Rule P
$C \vee D \rightarrow \neg H$ → Rule P
$\neg H \rightarrow (A \wedge \neg B)$ → Rule P
$C \vee D \rightarrow (A \wedge \neg B)$ → Rule T
$\neg H \rightarrow (A \wedge \neg B)$ → Rule P
$(A \wedge \neg B) \rightarrow (R \vee S)$ → Rule P
$\neg H \rightarrow (R \vee S)$ → Rule T
$C \vee D$ → Rule P
$C \vee D \rightarrow \neg H$ → Rule P
$\neg H$ → Rule T
$R \vee S$ — Rule T

2) S.T. $\neg P$ is valid conclusion 1) $\neg Q$
2) $P \rightarrow Q$

→
$\neg Q$ - Rule P
$P \rightarrow Q$ - Rule P
$\neg Q \rightarrow \neg P$ - Rule T
$\neg Q$ - Rule P
$\neg P$ - Rule T

3) S.T. given premise is valid conclusion based on the given ($S \vee R$ is valid)

1) $P \vee Q$ 2) $P \rightarrow R$ 3) $Q \rightarrow S$.

→ $P \vee Q$ - Rule P
$\neg P \rightarrow Q$ - Rule T
$Q \rightarrow S$ - Rule P

$\neg P \rightarrow S$ - Rule T

$P \rightarrow R$ - Rule P
$\neg R \rightarrow \neg P$. - Rule T
$\neg R \rightarrow S$ - Rule T
$R \vee S$ - Rule T.

2. Set Theory

set - collection of objects where aject is real world entity.

* Mathematical general set representation

$R = \{ x \mid x \in \text{all rivers in the world} \}$

$E = \{ x \mid x \in 2n, \text{ where } n = \{0, 1, 2, 3 ---- n\} \}$

$O = \{ x \mid x \in 2n+1, \text{ where } n = \{0, 1, 2, 3, ---- n\} \}$.

- To represent setname $\Rightarrow$ A - Z
- To " elements of set $\Rightarrow$ a - z

* Types of set & operations of set.

ϕ 1) Empty set / Null $\rightarrow$ only one element which is ϕ

E 2) Universal set $\rightarrow$ Every real world entity is in universal set.

$\subseteq \rightarrow$ 3) Subset - partial / subpart of a given set.

$\rightleftarrows$ 4) Equal set

- B is subset of A

if $x \in B$ then $x \in A$

$(B \subseteq A) = \{ x \mid \cancel{x \in} \; x \in B \rightarrow x \in A \}$

5) Negation of set ($\sim$)

$\sim A = \{ x \mid x \notin A \}$

$\sim A = E - A$

- $S = \{1, 3, 5, 7, 9, ---\}$

$S = \{ x \mid x \in 1+2n \; ; \; n = \{0, 1, 2, 3, ---\} \}$

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- Operations on set

i) Union (U) - If A & B are any 2 sets then the $A \cup B$ is the union of A & B.

$A \cup B = \{x \mid x \in A \vee x \in B\}$ The element is in A or in B.

$A = \{2,3,4,5\}$, $B = \{2,4,6,8,10\}$

$A \cup B = \{2,3,4,5,6,8,10\}$

ii) Intersection ($\cap$) - If A & B are any 2 sets then $A \cap B$ is the intersection of A & B

$A \cap B = \{x \mid x \in A \wedge x \in B\}$ The element is in A & B

$A = \{2,3,4,8\}$, $B = \{2,4,6,8,10\}$

$A \cap B = \{2,4,8\}$

iii) Relative complement - If A & B are any 2 sets then A-B is relative complement of each other.

$A - B = \{x \mid x \in A \wedge x \notin B\}$

$B - A = \{x \mid x \in B \wedge x \notin A\}$

$A = \{2,3,4,5\}$, $B = \{2,4,6,8,10\}$

$A - B = \{3,5\}$

$B - A = \{6,8,10\}$

iv) Symmetric difference - If A & B are any 2 sets then A+B is said to be

$A+B = \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$

$A+B = (A-B) \cup (B-A)$

1) Let, the set given is $A = \{x \mid 0 < x < 10, x \text{ is +ve integer}\}$

$B = \{x \mid 0 < x < 20, x \text{ is even +ve integers}\}$

$C = \{1, 2, 3, 4, 5, 6, 9, 11, 13, 15, 17, 19, 20\}$ then Find

i) $A - B$ ii) $A \cup (B \cap C)$ iii) $(A+B)+C$

iv) $(A \cap C) \cup B$

→ $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$C = \{1, 2, 3, 4, 5, 6, 9, 11, 13, 15, 17, 19, 20\}$

$A - B = \{1, 3, 5, 7, 9\}$

$(B \cap C) = \{2, 4, 6\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A + B = (A-B) \cup (B-A)$

$= \{1, 3, 5, 7, 9\} \cup \{10, 12, 14, 16, 18\}$

$\therefore A+B = \{1, 3, 5, 7, 9, 10, 12, 14, 16, 18\}$

$(A+B)+C = [(A+B)-C] \cup [C-(A+B)]$

$= \{7, 10, 12, 14, 16, 18\} \cup \{2, 4, 6, 11, 13, 15, 17, 19, 20\}$

$\therefore (A+B)+C = \{2, 4, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A \cap C = \{1, 2, 3, 4, 5, 6, 9\}$

$(A \cap C) \cup B = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 16, 16\}$

• Note

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3) $\sim(A \cap B) = \sim A \cup \sim B$

4) $A \cup \sim A = E$

5) $A \cap \sim A = \phi$

6) i) $A \cup \Phi = A$ ii) $A \cap \phi = \phi$

iii) $A \cup E = E$ iv) $A \cap E = A$

* $A \subseteq B = \{ x \in A \rightarrow x \in B \}$ $A \subseteq B = \sim A \cup B$

$= \{ x \notin A \cup x \in B \}$

- ordered pairs

$A = \{ x \mid 0 < x \le 2n+1, n = \{0,1,2,----\} \}$

- pair - $<2,3>$
- Triple - $<<2,3>,4>$

*• Let A & B are any 2 sets & $x \in A$, $y \in B$ then $A \times B$ (cartesian product) can be written as

$A \times B = \{ <x,y> \mid x \in A \wedge y \in B \}$

$B \times A = \{ <y,x> \mid y \in B \wedge x \in A \}$

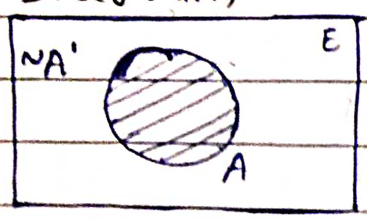
$\therefore A \times B \neq B \times A$

* $A \cup B = B \cup A$, $A \cap B = B \cap A$

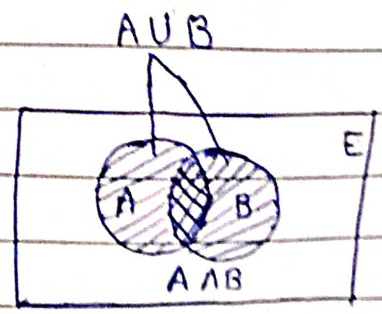
- Let $A = \{1,2,3,4\}$, $B = \{$ john, Tim, Tom, Jerry $\}$

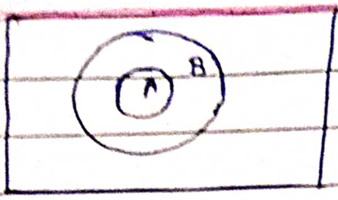
→ $A \times B = \{ <1,john>, <1,Tim>, <1,Tom>, <1,jerry>,$
$<2,john>, <2,Tim>, <2,Tom>, <2,jerry>,$
$<3,john>, <3,Tim>, <3,Tom>, <3,jerry>,$
$<4,john>, <4,Tim>, <4,Tom>, <4,jerry> \}$

* Venn Diagram

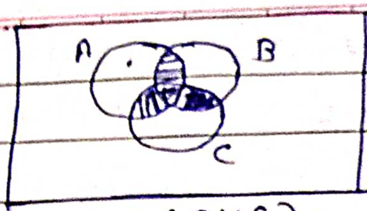


Universal set

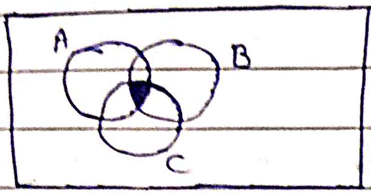




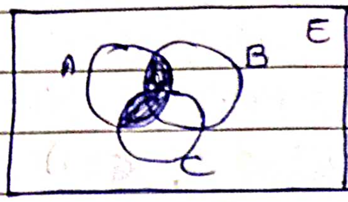
$A \subseteq B$



$A \cap (B \cup C)$



$A \cap (B \cap C)$



$A \cap (B \cup C)$

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  $= A \cap (B \cup C)$

  $= \{x \mid x \in A \wedge x \in (B \cup C)\}$

  $= \{x \mid x \in A \cap (x \in B) \cup x \in C)\}$

  $= \{x \mid (x \in A \cap x \in B) \cup (x \in A \cap x \in C\}$

  $= (A \cap B) \cup (A \cap C)$

  $=$ RHS

- $A \subseteq A \cup B$

  $= \{x \mid x \in (A \subseteq A \cup B)\}$

  $= \{x \mid x \in A \rightarrow x \in (A \cup B)\}$

  $= \{x \mid x \in A \rightarrow (x \in A \cup x \in B)\}$

  $= \{x \mid x \in A \rightarrow (x \in A \vee x \in B)\}$

  $= \{x \mid \neg x \in A \vee (x \in A \vee x \in B)\}$

  $= \{\underline{\sim A \cup A} \cup B\}$

  $= E \cup B \quad T$

  $= E$

1. Binary Relation

A relation of any 2 element in a given set

$R = \{\langle x,y\rangle \mid xRy,\ x,y \in X\}$

$X = \{0,1,2,3,4,5,6,7,8,9,10\}$

$R = \{\langle x,y\rangle \mid x,y \in X$ & $(x-y)$ is integral multiple of 2

$R = \{\langle 0,0\rangle, \langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 5,5\rangle,$
$\langle 6,6\rangle, \langle 7,7\rangle, \langle 8,8\rangle, \langle 9,9\rangle, \langle 10,10\rangle,$
$\langle 3,1\rangle, \langle 1,3\rangle, \langle 2,4\rangle, \langle 4,2\rangle, \langle 5,1\rangle, \langle 1,5\rangle$
$\langle 6,0\rangle, \langle 6,2\rangle, \langle 6,4\rangle, \langle 8,0\rangle, \langle 8,2\rangle, \langle 2,8\rangle,$
$\langle 2,10\rangle, \langle 10,2\rangle, \langle 9,1\rangle, \langle 1,9\rangle, \langle 10,0\rangle, \langle 10,2\rangle,$
$\langle 10,4\rangle, \langle 4,10\rangle, \langle 6,8\rangle, \langle 8,6\rangle, \langle 10,6\rangle, \langle 10,1\rangle$
$\}$

- A Binary relation is relation bet^n 2 elements x & y
  It is an ordered pair.
  $R = \{\langle x,y\rangle \mid x,y \in X\}$

Binary Relation representation.

1) set Representation
2) Graph —"—
3) Matrix —"—

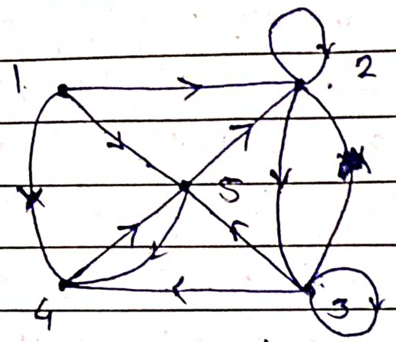
$G(V,E)$ - V = Vertex or Node
E = Edges / Arc

$X = \{1,2,3,4,5\}$

$R = \{\langle x,y\rangle \mid (x-y)$ is integral multiple of 3 $\}$

→ $R = \{(1,1), (2,2), (3,3), (4,4), (5,5);$
$(4,1), \langle 1,4\rangle, \langle 5,2\rangle, \langle 2,5\rangle\}$

$$
M_R = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & [illegible] & 0 & 0 & 1 & 0 \\
2 & 0 & 1 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 1 & 0 & 0 & 1 & 0 \\
5 & 0 & 1 & 0 & 0 & 1
\end{array}
\qquad
M_S = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$



$R = \{(1,2), (1,4), (1,5), (4,5), (3,4), (3,2), (5,2), (5,4), (3,5), (2,3), (2,2), (3,3)\}$

$$
M_R = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 1 & 0 & 1 & 1 \\
2 & 0 & 1 & 1 & 0 & 0 \\
3 & 0 & 0 & 1 & 1 & 1 \\
4 & 0 & 0 & 0 & 0 & 1 \\
5 & 0 & 1 & 0 & 1 & 0
\end{array}
$$

- **Properties of Binary Relation**

1) Reflexive - If R be an binary Relation over a set X then R is said to be reflexive if for all x then $\langle x, x\rangle \; \forall \; x \in X$

$R = \{\langle x, x\rangle \mid \text{for all } x \in X\}$

$X = \{0, 1, 2, 3, 4, 5\}$

$R = \{\langle x, y\rangle \mid x \text{ is less than or equal to } y\}$

$S = \{\langle x, y\rangle \mid (x-y) \text{ is integral multiple of } 2\}$

- $R = \{\langle 0,1\rangle, \langle 0,2\rangle, \langle 0,3\rangle, \langle 0,4\rangle, \langle 0,5\rangle, \langle 0,0\rangle\}$ for all $x \in X$

$S = \{\langle 2,0\rangle, \langle 0,2\rangle, \langle 3,1\rangle, \langle 1,3\rangle, \langle 2,4\rangle, \langle 4,2\rangle, \langle 1,5\rangle, (5,1\rangle, \langle 0,4\rangle, \langle 4,0\rangle\}$

2) Symmetric - If R is binary relation R is symmetric if all $x, y \in X$

$R = \{ \langle x, y \rangle \langle y, x \rangle \mid x, y \in X \}$

$xRy \quad yRx$

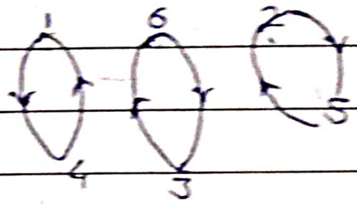
$X = \{1, 2, 3, 4, 5, 6\}$

$R = \{ \langle x, y \rangle \mid (x-y)$ is intergral multiple of $3 \}$

$R = \{ \langle 1,4 \rangle, \langle 4,1 \rangle, \langle 2,5 \rangle, \langle 5,2 \rangle, \langle 3,6 \rangle,$
$\langle 6,3 \rangle \}$

3) Transitive: If R be any relation & x, y & $z \in X$ then R is said to be transitive if xRy, yRz then xRz

less than Reln $C = \{ \langle 1,2 \rangle, \langle 2,3 \rangle, \langle 1,3 \rangle$
$\langle 2,3 \rangle, \langle 3,4 \rangle, \langle 2,4 \rangle$
$\langle 3,4 \rangle, \langle 4,5 \rangle, \langle 3,5 \rangle \}$



greater then Reln Let, $X = \{0, 2, 4, 6, 8, 10\}$

$C = \{ \langle 4,2 \rangle, \langle 2,0 \rangle, \langle 4,0 \rangle, \langle 6,4 \rangle, \langle 4,2 \rangle, \langle 6,$
$\langle 8,6 \rangle, \langle 6,4 \rangle, \langle 8,4 \rangle, \langle 10,8 \rangle, \langle 8,6 \rangle, \langle 10,$
$\langle 10,2 \rangle, \langle 2,0 \rangle, \langle 10,0 \rangle$
$\langle 8,4 \rangle, \langle 4,2 \rangle, \langle 8,2 \rangle \}$

4) Irreflexive - If for all $x \in X$ $\langle x, x \rangle \notin R$ then it is irreflexive.

$X = \{1, 2, 3, 4\}$

5) Antisymmetric - If R is an Antisymmetric relation then for all $x, y \in X$ if xRy, yRx, $x = y$

- $X = \{0, 1, 2, 3, 4\}$ $\quad$ $x-y$ is integral multiple of 2

$C = \{ \langle 0,2 \rangle, \langle 2,0 \rangle, \langle 2,4 \rangle, \langle 4,2 \rangle, \langle 1,3 \rangle, \langle 3,1 \rangle$
$\langle 4,0 \rangle, \langle 0,4 \rangle, \} \langle 0,0 \rangle, \langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle,$
$\langle 4,4 \rangle \}$

- If R be any binary relation on set X then if R.
  1) Reflexive
  2) Symmetric
  3) Transitive the equivalence relation.

- Reflexive, Antisymmetric, transitive $\Rightarrow$ partial order relation.

- $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
  & $R = \{\langle x, y\rangle \mid (x-y)$ is divisible by 2, $x, y \in X\}$

$R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 5,5\rangle, \langle 6,6\rangle, \langle 7,7\rangle,$
$\langle 8,8\rangle, \langle 9,9\rangle, \langle 1,3\rangle, \langle 3,1\rangle, \langle 2,4\rangle, \langle 4,2\rangle, \langle 1,9\rangle,$
$\langle 9,1\rangle, \langle 1,5\rangle, \langle 5,1\rangle, \langle 1,7\rangle, \langle 7,1\rangle, \langle 2,6\rangle, \langle 6,2\rangle,$
$\langle 2,8\rangle, \langle 8,2\rangle, \langle 3,9\rangle, \langle 9,3\rangle, \langle 3,7\rangle, \langle 7,3\rangle, \langle 5,9\rangle,$
$\langle 9,5\rangle, \langle 8,6\rangle, \langle 6,8\rangle, \langle 9,7\rangle, \langle 7,9\rangle, \langle 5,7\rangle, \langle 7,5\rangle,$
$\langle 3,5\rangle, \langle 5,3\rangle, \langle 8,4\rangle, \langle 4,8\rangle$

1) R is reflexive for all $x \in X$ $\langle x, x\rangle \in R$
$\{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \ldots, \langle 9,9\rangle\} \in R$
$\therefore$ R is Reflexive

2) R is symmetric for all $\langle x, y\rangle$ $\langle y, x\rangle \in R$

3) R is Transititive for all $x, y, z \in X$
$\langle x, y\rangle \langle y, z\rangle$ $\langle x, z\rangle$

$\therefore$ R is equivalence relation.

2) $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

& $S = \{\langle x,y\rangle \mid (x-y)$ is divisible by $3,\ x,y \in X\}$

→ $S = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 5,5\rangle, \langle 6,6\rangle, \langle 7,7\rangle, \langle 8,8\rangle, \langle 9,9\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 5,2\rangle, \langle 2,5\rangle, \langle 6,3\rangle, \langle 3,6\rangle, \langle 7,1\rangle, \langle 1,7\rangle, \langle 7,4\rangle, \langle 4,7\rangle, \langle 8,2\rangle, \langle 2,8\rangle, \langle 8,5\rangle\ \langle 5,8\rangle, \langle 9,3\rangle, \langle 3,9\rangle, \langle 9,6\rangle, \langle 6,9\rangle\}$

i) R is Reflexive for all $x \in X$ $\langle x,x\rangle \in R$

ii) R is symmetric for all $\langle x,y\rangle \langle y,x\rangle \in R$

iii) R is transititive for all $x,y,z \in X$

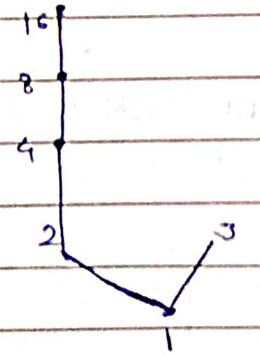
∴ R is equivalence relation.

- Partial order relation

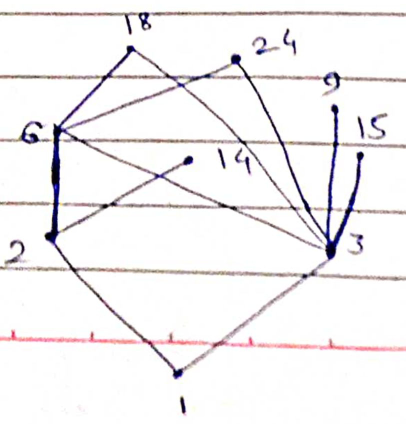
It is denoted by Hasse diagram.

$X = \{1, 2, 3, 4, 8, 16\}$

$ID_2 = \{\langle 1,2\rangle, \langle 2,4\rangle, \langle 2,8\rangle, \langle 2,16\rangle, \langle 4,8\rangle, \langle 4,16\rangle, \langle 1,4\rangle, \langle 1,8\rangle, \langle 1,16\rangle\}$



$X = \{1, 2, 3, 6, 9, 14, 15, 18, 24\}$



LB - lower Bond. UB = upper bond.

[illegible] - [illegible]

G.D = sigle (- linked list)

[illegible]://googleform

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⊙ Power set - Power Set of any given set is set of all possible subsets of given set.

- It is denoted by $\wp$
- 2^n elements in Power set.

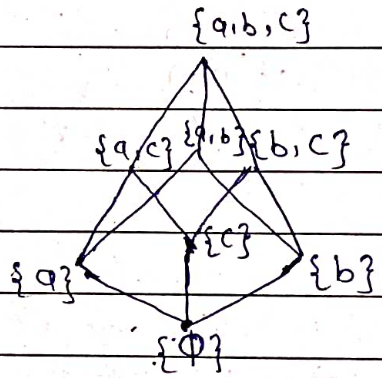
$n = 3 \Rightarrow 2^n = 8$

$\wp(A) = \{\phi, (1), (2), (3), (1,2), (1,3), (2,3), (1,2,3)\}$

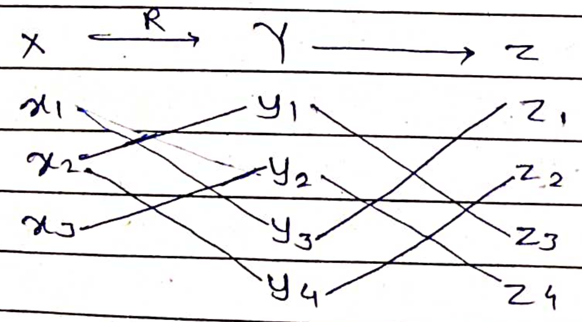
~~{1,1}~~

$B = \{a, b, c\}$

$\wp(B) = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$



* composition of Relations :



$R \cdot S = \{\langle x_1, z_1\rangle, \langle x_2, z_3\rangle, \langle x_3, z_4\rangle, \langle x_2, z_2\rangle\}$

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$R = \{\langle 1,2\rangle, \langle 2,3\rangle \langle 3,3\rangle \langle 3,4\rangle\}$

$S = \{\langle 2,2\rangle, \langle 1,3\rangle, \langle 3,5\rangle, \langle 2,3\rangle\}$

$M = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,4\rangle \langle 3,2\rangle\}$

$R.S = \{\langle 1,2\rangle \langle 1,3\rangle \langle 2,5\rangle, \langle 3,5\rangle\}$

$S.R = \{ \cancel{\langle 1,4\rangle} \langle 1,2\rangle, \langle 3,5\rangle, \langle 2,5\rangle, \langle 1,3\rangle\}$

$R.M = \{\langle 1,2\rangle, \langle 2,4\rangle \langle 2,2\rangle, \langle 3,4\rangle, \langle 3,2\rangle, \}$

$M.R = \{\langle 3,3\rangle, \langle 1,2\rangle, \langle 2,3\rangle\}$

converse of Relation

$S = \{\langle x, 3x\rangle \mid x \in I \quad I = \{0,1,2,3, ---- 10\}\}$

$S = \{\langle 0,0\rangle, \langle 1,3\rangle, \langle 2,6\rangle, \langle 3,9\rangle, \langle 4,12\rangle, \langle 5,15\rangle,$
$\langle 6,18\rangle, \langle 7,21\rangle, \langle 8,24\rangle, \langle 9,27\rangle, \langle 10,30\rangle\}$

$\tilde{S} = \{\langle 0,0\rangle, \langle 3,1\rangle, \langle 6,2\rangle, \langle 9,3\rangle, \langle 12,4\rangle, \langle 15,5\rangle,$
$\langle 18,6\rangle, \langle 21,7\rangle, \langle 24,8\rangle, \langle 27,9\rangle, \langle 30,10\rangle\}$

$R.S = \{\langle 0,0\rangle, \langle 1,2\rangle, \langle 2,4\rangle, \langle 3,6\rangle, \langle 4,8\rangle, \langle 5,10\rangle,$
$\langle 6,12\rangle, \langle 7,14\rangle, \langle 8,16\rangle, \langle 9,18\rangle, \langle 10,20\rangle\}$

$R.S = \{\langle 3,6\rangle, \langle 4,24\rangle, \langle 5,30\rangle, \langle 3,18\rangle, \langle 1,6\rangle, \langle 2,12\rangle\}$

$$* \quad M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$M_s = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

① convert the matrix into Relation

$M_{\tilde{R}}$, $M_{\tilde{S}}$ $M_{R \circ S}$ $M_{\tilde{R} \circ \tilde{S}}$

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$R = \{ \langle 1,1 \rangle \langle 1,3 \rangle \langle 1,4 \rangle \langle 2,1 \rangle \langle 2,2 \rangle \langle 2,3 \rangle \langle 3,2 \rangle \langle 3,4$
$\langle 3,5 \rangle \}$

$S = \{ \langle 1,1 \rangle \langle 1,2 \rangle \langle 1,3 \rangle \langle 2,1 \rangle \langle 2,3 \rangle \langle 2,4 \rangle \langle 3,1 \rangle \langle 3,2 \rangle$
$\langle 3,3 \rangle \langle 4,3 \rangle \}$

$M_{\tilde{R}} = \{ \langle 1,1 \rangle \langle 3,1 \rangle \langle 4,1 \rangle \langle 1,2 \rangle \langle 2,2 \rangle \langle 3,2 \rangle \langle 2,3 \rangle \langle 4,3 \rangle \langle$

$M_{\tilde{S}} = \{ \langle 1,1 \rangle \langle 2,1 \rangle \langle 3,1 \rangle \langle 1,2 \rangle \langle 3,2 \rangle \langle 4,2 \rangle \langle 1,3 \rangle \langle 2,3 \rangle$
$\langle 3,3 \rangle \langle 3,4 \rangle \}$

$R \circ S = \{ \langle 1,1 \rangle \langle 1,2 \rangle \langle 1,3 \rangle \langle 2,1 \rangle \langle 2,2 \rangle \langle 2,3 \rangle \langle 2,4 \rangle \langle 3,1 \rangle$
$\langle 3,3 \rangle \langle 3,4 \rangle \}$

$\tilde{R} \circ \tilde{S} = \{ \langle 1,1 \rangle \langle 2,1 \rangle \langle 3,1 \rangle \langle 1,2 \rangle \langle 2,2 \rangle \langle 3,2 \rangle \langle 4,2 \rangle \langle 1,3 \rangle$
$\langle 3,3 \rangle \langle 4,3 \rangle \}$

$$M_{\tilde{R}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_{\tilde{S}} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_{R \circ S} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$M_{\tilde{R} \circ \tilde{S}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

- Function -

It is a special type of relation which is reln between Input - Domain & output - Range.

It is Represented by 'f'

Types of function

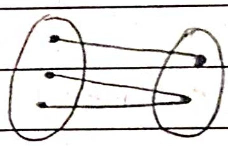
1) onto function (injective) surjective
2) into function
3) one-to-one function (surjective) injective
4) one-to-one-onto function (Bijective).

onto funn -

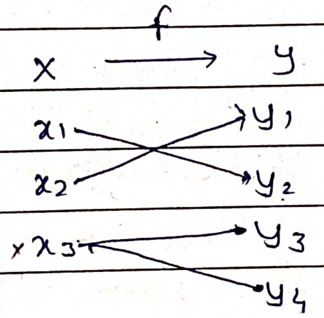
If $x \to y$ such that, $x \in X$ & $y \in Y$

$D_f = X$

$R_f = y$

→ onto funn 

$X \geq Y$



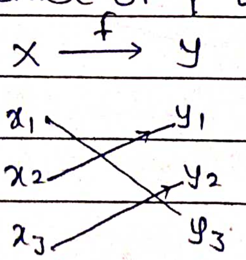
$f = \{ \langle x_1, x_2 \rangle \langle x_2, y_1 \rangle, \langle x_3, y_3 \rangle \langle x_2, x_3 \rangle \langle x_2, y_2 \rangle \}$

A funn f from set 'x' to set 'Y' is onto if each element of Y is mapped to atleast ~~one element of X~~.

one-to-one function

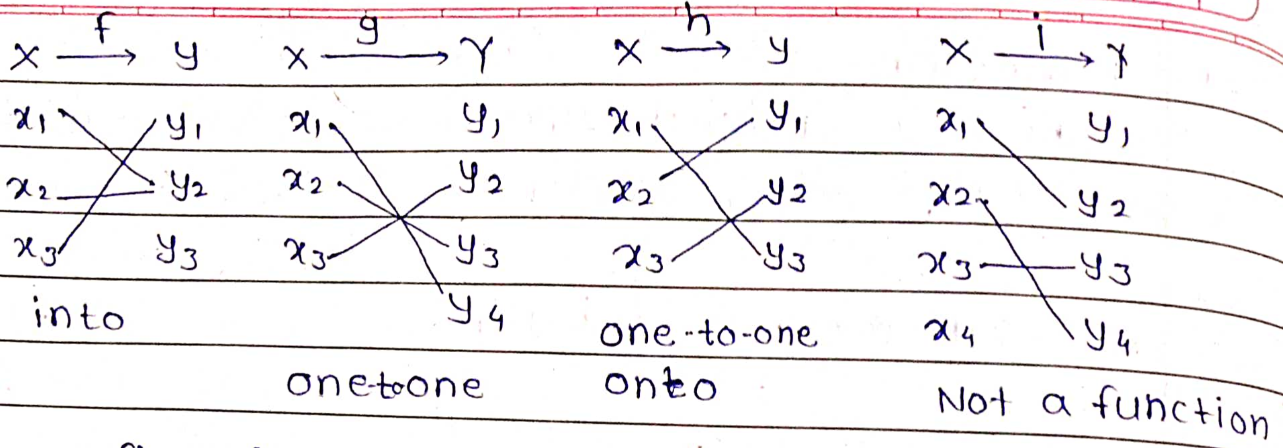
If $f: x \to y$ is a function over an $x \in X$, $y \in Y$ &

$X \leq Y$ districtelement of set x are mapped with district elemet of Y then one-to-one funn.



$f = \{ \langle x_1, y_3 \rangle \langle x_2, y_1 \rangle, \langle x_3, y_2 \rangle \}$

A funn f from set 'x' to set 'y' is one to one if no 2 elements in 'x' are mapped to same element in B



$x \rightarrow x$

$g(x) = x^2$ $\quad x \in \{0, 1, 2, 3, ---- n\}$

$\Rightarrow$ one to one function

$h(x) = 2x+3$ $\quad x \in \{0, 1, 2, 3, ----- n\}$

$\hookrightarrow$ one to one function

$h(x) = 2x - 3 + 3 - 2x$

$\hookrightarrow$ into function

* Composition of Function

$x \xrightarrow{f} y \qquad y \xrightarrow{g} z$

$f = \{\langle 1,2\rangle \langle 2,3\rangle \langle 3,3\rangle\}$

$g = \{\langle 1,1\rangle \langle 2,2\rangle \langle 3,1\rangle\}$

$h = \{\langle 1,2\rangle \langle 2,3\rangle \langle 3,3\rangle\}$

$fog = \{\langle 1,2\rangle \langle 2,3\rangle \langle 3,2\rangle\}$

$gof = \{\langle 1,2\rangle \langle 2,1\rangle \langle 3,1\rangle\}$

$goh = \{\langle 1,2\rangle \langle 2,1\rangle \langle 3,1\rangle$

$hof = \{(1,3) \langle 2,3) \langle 3,3\rangle\}$

3. Algebraic structures

An algebraic structure / system is an a special system which is a combination of set & operations

It is denoted as $<S, +>$, $<M, *>$, $<Z_4, +_4>$
where S, M, Z_4 are sets & $+, *, +_4$ are operations

- Algebraic system → semigroup → monoid → Group
Algebraic system → semigroup (AS+1) →
monoid (semigroup+1) → Group (monoid+1)

Properties of Algebraic structure

$<A, +>$, $<M, *>$ be any 2 algebraic system

1) Associative - If a, b, c be the element of set A then under the operation + ~~then~~ if
$a+(b+c) = (a+b)+c$ then A is associative under + operation.

e.g. Let, $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$a = 2, \; b = 0, \; c = 7$

$a+(b+c) = 2+(0+7) = 9$

$(a+b)+c = (2+0)+7 = 9$

$\therefore a+(b+c) = (a+b)+c$

2) Commutative - If a, b be the element of set A then if $a+b = b+a$ under the operation + then A be the commutative.

3) Identity element - If a be the element of set A & 'e' be any element which satisfies.

$a+e = e+a = a$

then 'e' be the identity element of set A under operation '+'

0 is the identity element for '+'.

4) Inverse element - Let, a & b are any 2 elements of given set A & b is said to be inverse element of a if it satisfies under the given operation.

$$a + (b) = e \quad \text{where } b = -a$$

⊙ $\langle M, * \rangle$

1) Associative- If a, b, c be the element of set M then under the operation $*$ then if

$$a * (b * c) = (a * b) * c$$

2) commutative- $a * b = b * a$

3) Identity element- 1 is the identity element for $*$

4) Inverse element- $a * (b) = e$ where, $b = \frac{1}{a} \Rightarrow a^{-1}$

- $a +_m b = (a+b) + (\text{mol}$

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- If Every algebraic system $\langle X, \circ \rangle$ satisfies associative then it is semigroup.

- If every semigroup $\langle X, \circ \rangle$ has an identity element 'e' then it is monoid.

1) Show that / check for m=5 the given algebraic structure is monoid ?

→ $\langle Z_5, +_5 \rangle$, $\langle Z_5, *_5 \rangle$

$Z_5 = \{[0], [1], [2], [3], [4]\}$

$+_5$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

$[2] +_5 ([3] +_5 [4]) = ([2] +_5 [3]) +_5 [4]$

$[2] +_5 [2] = [0] +_5 [4]$

$[4] = [4]$

∴ It is Associative so, it is semigroup.

There exist identity element in semigroup i.e. 0

∴ It is monoid.

$*_5$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]
[2]	[0]	[2]	[4]	[1]	[3]
[3]	[0]	[3]	[1]	[4]	[2]
[4]	[0]	[4]	[3]	[2]	[1]

$[1] *_5 ([3] *_5 [3]) = ([1] *_5 [3]) *_5 [3]$

$[1] *_5 [4] = [3] *_5 [3]$

$[4] = [4]$

It satisfies associativity so, it is semigroup

Identity element is [1]

∴ Given algebraic structure monoid.

- Homomorphism

If $<S, *>$ & $<T, \Delta>$ be any 2 semigroups & $g : S \to T$ then it is under homomorphic semigroup.

$$\boxed{g(a*b) = g(a) \Delta g(b)}$$

1) * For any $a, b \in I$ then there is element $0 \in I$ such that $a \in I$ $a+0 = 0+a = a$ then a is called as <u>Identity element</u>

2) In the algebraic system $<N, +, \times>$ where N is the set of natural no. & the operations + & × have usual meanings

→ Inverse element.

1) S.T. Algebraic structure is monoid.

→ $<Z_7, +_7>$, $<Z_7, *_7>$

$Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$+_7$	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[0]	[1]
[3]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[4]	[4]	[5]	[6]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[0]	[1]	[2]	[3]	[4]	[5]

$[1] +_7 ([5] +_7 [3]) = ([1] +_7 [5]) +_7 [3]$

$[1] +_7 [1] = [6] +_7 [3]$

$[2] = [2]$

It satisfies associativity so, it is semigroup

Identity element is $[0]$

$\therefore$ Given AS is monoid.

$x \circledast y = \min(x,y)$

2) Given a set of +ve Natural numbers The operation $x \circledast y = \max(x,y)$. check it is monoid

$\circledast$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[1]	[2]	[3]	[4]	[5]
[2]	[2]	[2]	[2]	[3]	[4]	[5]
[3]	[3]	[3]	[3]	[3]	[4]	[5]
[4]	[4]	[4]	[4]	[4]	[4]	[5]
[5]	[5]	[5]	[5]	[5]	[5]	[5]

$[1] \circledast ([4] \circledast [5]) = ([1] \circledast [4]) \circledast [5]$

$[1] \circledast ([5]) = [4] \circledast [5]$

$[5] = [5]$

It satisfies associativity. So, it is semigroup.

Identity element is 0

$\therefore$ Given AS is monoid.

$\circledast$	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[1]	[1]	[1]
[2]	[0]	[1]	[2]	[2]	[2]
[3]	[0]	[1]	[2]	[3]	[3]
[4]	[0]	[1]	[2]	[3]	[4]

$[0] \circledast ([2] \circledast [3]) = ([0] \circledast [2]) \circledast [3]$

$[0] \circledast [2] = [0] \circledast [3]$

$[0] = [0]$

Identity element $[4]$

⊙ Groups -

- An Algebraic system with associative property is semigroup
- A semigroup with identity element is monoid.
- A Monoid which satisfies an inverse element is group.
- A Group which satisfies commutative property is a commutative group.

$\langle Z_4, +_4 \rangle$

$+_4$	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]

Identity element = [0]

$a \oplus b = e$

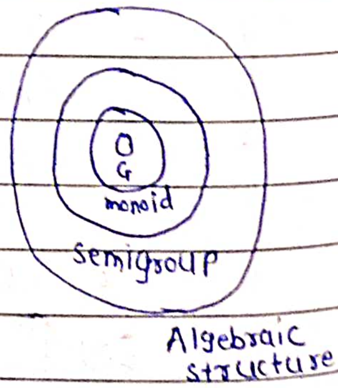
Inverse element -

$[0] +_4 [0] = 0$

$[1] +_4 [3] = 0$

$[2] +_4 [2] = 0$

$[3] +_4 [1] = 0$



$\langle Z_4, *_4 \rangle$

$*_4$	[0]	[1]	[2]	[3]
[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]
[2]	[0]	[2]	[0]	[2]
[3]	[0]	[3]	[2]	[1]

Identity element = [1]

* ## Abelion Group

An group which satisfy commutative property is an abelion group

$$\text{A Group} \begin{cases} \text{Inverse} \\ \\ \text{commutative.} \end{cases}$$

Q. check the given mapping is an abelion group or not

$F = \{ f^0, f^1, f^2, f^3 \}$ when $c \in f = f^0$. (Identity element)

o	f^0	f^1	f^2	f^3
f^0	f^0	f^1	f^2	f^3
f^1	f^1	f^2	f^3	f^0
f^2	f^2	f^3	f^0	f^1
f^3	f^3	f^0	f^1	f^2

Associativity -

$$f^1 o (f^2 o f^3) = (f^1 o f^2) o f^3$$

$$f^1 o f^1 = f^2 o f^3$$

$$f^2 = f^2$$

Commutative Property -

$$f^2 o f^3 = f^3 o f^2$$

$$f^1 = f^1$$

1) If G is a group $(G, *)$ - If it satisfies associative, Identity, Inverse

1) Let, $P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$, $P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

$P_3 \Diamond P_4$ (a)

$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \Diamond \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \end{pmatrix} = P_4$

$P_4 \Diamond P_5$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \Diamond \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \Diamond \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = P_2$

$P_2 \Diamond P_6$

$\begin{pmatrix} 1 & 2 & 3 \\ \cancel{2} & 1 & 3 \end{pmatrix} \Diamond \begin{pmatrix} 1 & \cancel{2} & 3 \\ 3 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = P_4$

$P_1 \Diamond P_3$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \Diamond \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = P_3$

$P_3 \Diamond P_4$

$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \Diamond \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = P_5$

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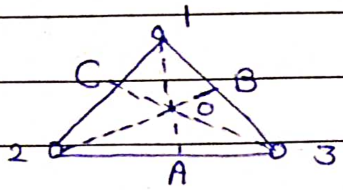
- A symmetric group which satisfies symmetries of regular polygon is called dihedral group.

Triangle

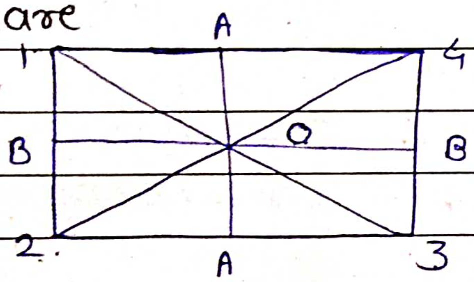
120°

$$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$



Δ	P_1	P_5	P_6
P_1	P_1	P_5	P_6
P_5	P_5	P_5	P_1
P_6	P_6	P_1	P_5

square



$$r_1 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad r_2 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

$$r_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix}, \quad r_4 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

At AA,

$$P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

At 13

$$P_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$$

At BB,

$$P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

At 24.

$$P_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

	r_1	r_2	r_3	r_4	r_5	p_1	p_2	p_3	p_4
r_1	r_1	r_2	r_3	r_4		p_1	p_2	p_3	p_4
r_2	r_2	r_3	r_4	r_1					
r_3									
r_4									
p_1									
p_2									
p_3									
p_4									

$\leq$ - notation for Partial ordering

4. Lattice & Boolean Algebra

An lattice is an partial order set with algebraic structure for every element there is an GLB & LUB $\langle L, \leq \rangle$

GLB - Greatest lower bound

LUB - least upper bound.

- A Relation L be an partial order relation if
  1) Reflexive
  2) Antisymmetric
  3) Transitive.

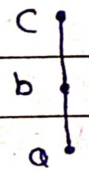
- properties of lattice

$\langle L, *, \oplus \rangle$

$*$ - meet - GLB $\{a, b\}$

$\oplus$ - join - LUB $\{a, b\}$

$a \leq b \leq c$ =



$a * b = a$, $b * c = b$

$a \oplus b = b$, $b \oplus c = c$

- Let, the given set $L = \{a, b, c\}$ & $a \leq b \leq c$, $\langle L, *, \oplus \rangle$ be an lattice under $*$ & $\oplus$

i) $a * a = a$ $\qquad$ $a \oplus a = a$

ii) $a * b = b * a$ $\qquad$ $a \oplus b = b \oplus a$

iii) $(a * b) * c = a * (b * c)$ $\qquad$ $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

iv) $a * (a \oplus b) = a$ $\qquad$ $a \oplus (a * b) = a$

v) $b \oplus (a * b) = b$ $\qquad$ $(a * b) * (a * c) = a$

if $a \leq b \leq c$

i) $(a * b) \oplus (b * c) = b$

ii) $(a \oplus b) * (a \oplus c) = b$

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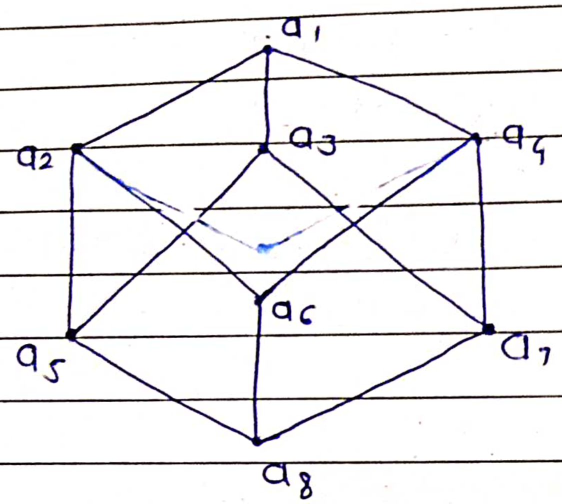
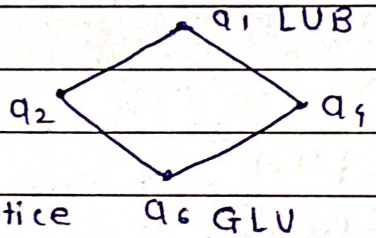
- If $\langle L, *, \oplus \rangle$ & S be any set, $S \subseteq L$ then $\langle S, *, \oplus \rangle$ is an sublattice iff S is closed under $*$ & $\oplus$



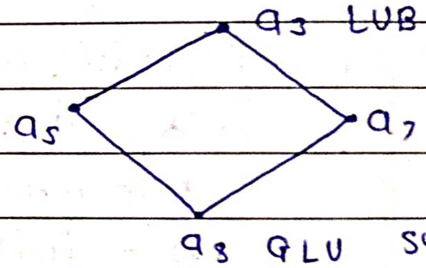
$S_1 = \{a_1, a_2, a_4, a_6\}$
$S_2 = \{a_3, a_5, a_7, a_8\}$
$S_3 = \{a_1, a_2, a_4, a_8\}$

S_1 =



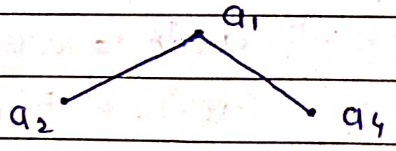
Sublattice

S_2 =



Sublattice

S_3 =



Not sublattice

$S_4 = \{a_2, a_7, a_4\}$
Not sublattice

⊙ Special lattice

If $\langle L, *, + \rangle$ be an lattice under $*$ & $\oplus$ operations

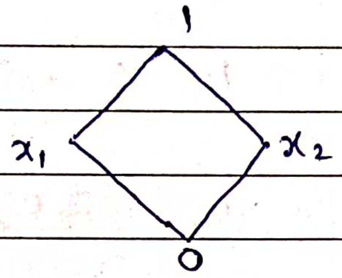
$a * b = GLB(a, b) = a$
$a \oplus b = LUB(a, b) = b$

1) complete lattice
2) Bounded lattice
3) Complemented lattice
4) Distributive lattice

complete lattice - An lattice which has non-empty GLU & LUB $G \leq L$

2) Bounded lattice - It is an complete lattice which has an boundry defined as 0 & 1.

$\langle L, *, \oplus \rangle$ but there is unique GLU is 0 & unique LUB is 1



⊙ $a \oplus 0 = a$ $\quad a * 0 = 0$

$a \oplus 1 = 1$ $\quad a * 1 = a$

3) complemented lattice -

An lattice $\langle L, *, \oplus, 0, 1 \rangle$ be an ~~boundary~~ lattice have an complement such that

$a * b = 0$; $a \oplus b = 1$

where a is complement of b & vice versa then it is complemented lattice.

$(*)' = \oplus$ $\quad (a * b)' = a' \oplus b'$

$(\oplus)' = *$ $\quad (a \oplus b)' = a' * b'$

4) Distributive lattice -

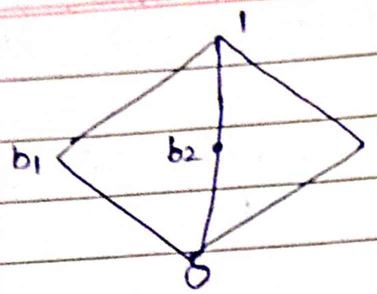
$\langle L, *, \oplus \rangle$ be an distributive lattice if it satisfies

$a * (b \oplus c) = (a * b) \oplus (a * c)$

$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$

D = divides relation

S_n = set of all multiples of n



$b_1 * (b_2 \oplus b_3) = (b_1 * b_2) \oplus (b_1 * b_3)$

$b_1 * 1 = 0 \oplus 0$

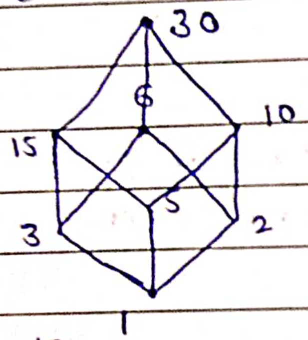
$b_1 = 0$

$\therefore$ LHS $\neq$ RHS

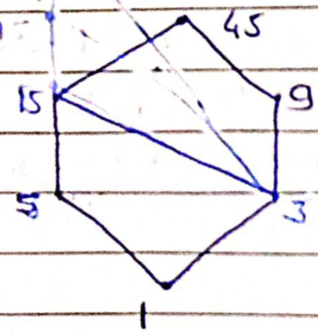
$\therefore$ Given lattice is not distributive lattice.

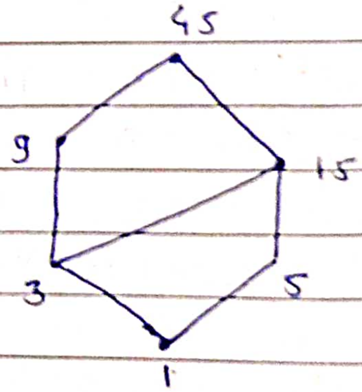
1) If on $\langle S_n, D \rangle$ for $n=30$ & $n=45$ which of them is complemented.

$\rightarrow$ $S_n = S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$



$S_{45} = \{1, 3, 5, 9, 15, 45\}$





2) If $a \leq b \leq c$ then 1) $(a*b) \oplus (b*c) \oplus (c*a)$

2) $(a \oplus b) * (b \oplus c) * (c+a)$

3) $(a*b) \oplus (b*c) \oplus (c*a) \oplus (c*d)$

$\rightarrow$ i) $(a*b) \oplus (b*c) \oplus (c*a)$

$= (a \oplus b) \oplus a$

$= b \oplus a$

$= b$

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ii) $(a \oplus b) * (b \oplus c) * (c \oplus a)$

$= (b * b) * c$

$= b * c$

$= b$

iii) $(a * b) \oplus (b * c) \oplus (c * a) \oplus (c * d)$

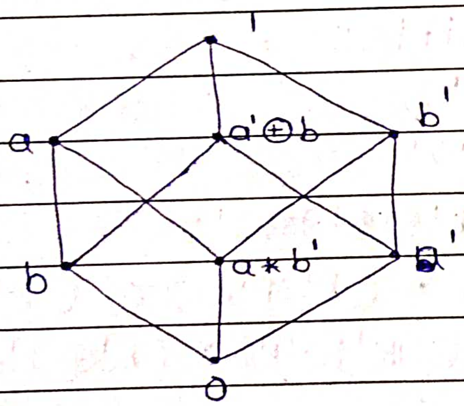
$= a \oplus b \oplus a \oplus c$

$= b \oplus c$

$= c$

* **Boolean Algebra**

An Boolean Algebra $\langle B, *, \oplus, 0, 1 \rangle$ is an bounded distributed & complemented lattice.



$B = \{ a, b, a', b', 0, 1, a' \oplus b, a * b' \}$

$S_1 = \{ a, a', 0, 1 \}$ } subboolean Algebra

$S_2 = \{ a' \oplus b, a * b', 0, 1 \}$

$S_3 = \{ b', a * b', 0 \}$ } Not subboolean Algebra.

$S_4 = \{ a, a', 1 \}$

1) Show that the foll^n boolean identities

$a \oplus (a' * b) = a \oplus b$

→ LHS $= a \oplus (a' * b)$

$= (a \oplus a') * (a \oplus b)$

$= 1 * (a \oplus b)$

$= a \oplus b$

$=$ RHS

2) S.T. $a*(a'\oplus b) = a*b$

$\rightarrow$ LHS $= a*(a'\oplus b)$

$= (a*a')\oplus(a'*b)$

$= 0\oplus(a'*b)$

$= a*b$

$=$ RHS

3) S.T. $(a*b*c)\oplus(a*b) = a*b$

$\rightarrow$ LHS $= (a*b*c)\oplus(a*b)$

$= a*b*(c\oplus 1)$

$= (a*b)*1$

$= a*b$

Q. solve the foll^{n} identities

1) $(a*b)'\oplus(a\oplus b)'$

$\rightarrow$ $(a*b')'\oplus(a\oplus b')'$

$= (a'\oplus(b')\oplus) * (b'*a)\oplus b')$

$= [(a'\oplus(a'*b'))\oplus b'\oplus(a'*b)\oplus(b'\oplus b')$

$= (a'\oplus a')*(a'\oplus b')\oplus(b'\oplus a')*(b'\oplus b')$

$= (a'*b')\oplus(b'*b') \quad \Rightarrow (a'\oplus b')\oplus(a'*b')$

$= (a'*b')\oplus b' \quad a'\oplus(a'*b')\oplus b'\oplus(a'*b')$

$= (a'\oplus b')*(b' \quad (a'\oplus a')*(a'\oplus b')\oplus(b'\oplus a')*(b'\oplus b')$

$a'*(a'\oplus b')\oplus(b'\oplus a')*b'$

$a'\oplus b'$

2) $(1*a)\oplus(0*a')$

$\rightarrow$ $(1*a)\oplus(0*a')$

$= 1\oplus(0*a')*a\oplus(0*a')$

$= (1\oplus 0)*(1\oplus a')*(a\oplus 0)*(a\oplus a')$

$= 1*1*a*1$

$= 1*a$

$= a$

3) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

$\rightarrow$ $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

$= \cancel{b' * (c * (a' \oplus a)) \oplus (a * b' * c')}$

$= \cancel{[(b' * c) * 1] \oplus (a * b' * a') \ldots}$

$= \cancel{b' \oplus (a * b' * c')}$ $\quad (a' * b' * c) \oplus (a * b') * (c \oplus c')$

$= \cancel{b' [1 \oplus (a * c')]}$ $\quad (a' * b' * c) \oplus (a * b') * 1$

$= \cancel{(b' \oplus a) * (b' \oplus b'}$ $\quad (a' * b' * c) * (a * b')$

$(a' + a * b') * (b' * a * b') * (c * a * b')$

* Boolean functions

It is an set of boolean expressions.

1) 0 & 1 is boolean expression.
2) $x_1, x_2, \ldots, x_n$ be an boolean expression.
3) If α_1 & α_2 be an boolean expression then $\alpha_1 * \alpha_2$, $\alpha_1 \oplus \alpha_2$ be an boolean expression.
4) α is boolean expression then α' is also an boolean expression.

product of sum $\rightarrow$ CNF (meet of join)

$*$ $\rightarrow$ product

$\oplus$ $\rightarrow$ sum

$(\oplus) * (\oplus) * (\oplus)$

sum of product $\rightarrow$ DNF (join of meet)

$(*) \oplus (*) \oplus (*)$

canonical product of sum $\rightarrow$

$(x_1' * x_2) \oplus (x_1 * x_2)$	x_1	x_2	minterm $(*)$	maxterm $(\oplus)$
min1 $\oplus$ min3	0	0	$x_1' * x_2'$ 0	$x_1 \oplus x_2$ 3
$\oplus$ min 1, 3	0	1	$x_1' * x_2$ 1	$x_1 \oplus x_2'$ 2
	1	0	$x_1 * x_2'$ 2	$x_1' \oplus x_2$ 1
	1	1	$x_1 * x_2$ 3	$x_1' \oplus x_2'$ 0

sum of Product → $x_1 * 1$
$x_1 * (x_2 \oplus x_2')$

Product of sum → $x_1 = x_1 \oplus 0$
$x_1 \oplus (x_2 * x_2')$

canonical sum of Product →

Q. obtain Product of sum & sum of Product canonical form

1) $x_1 * (x_1 \oplus x_2')$

→ Product of sum →

$x_1 * (x_1 \oplus x_2')$

$= (x_1 \oplus 0) * (x_1 \oplus x_2')$

$= (x_1 \oplus (x_2 * x_2')) * (x_1 \oplus x_2')$

$= (x_1 \oplus x_2) * (x_1 \oplus x_2') * (x_1 \oplus x_2')$

$= (x_1 \oplus x_2) * (x_1 \oplus x_2')$

$= * \max 2, 3$

2) Find sum of Product canonical form $(x_1 \oplus x_2)' * x_3$

→ $(x_1 \oplus x_2)' * x_3$

$= (x_1' * x_2') * x_3$

$= \min 1$

3) $x_1 \oplus x_2$ → sum of product

→ $x_1 \oplus x_2$

$= (x_1 * 1) \oplus (x_2 * 1)$

$= (x_1 * (x_2 \oplus x_2')) \oplus (x_2 * (x_1 \oplus x_1'))$

$= (x_1 * x_2) \oplus (x_1 * x_2') \oplus (x_2 * x_1) \oplus (x_2 * x_1')$

$= (x_1 * x_2) \oplus (x_1 * x_2') * (x_2 * x_1')$

$= \oplus \min 1, 2, 3$

$x_1 \oplus x_2$

→ $x_1 \oplus x_2$

$= (x_1 * 1) \oplus (x_2 * 1)$

$= (x_1 * (x_2 \oplus x_2')) \oplus (x_2 * (x_1 \oplus x_1'))$

$= (x_1 * x_2) \oplus (x_1 * x_2') \oplus (x_2 * x_1) \oplus (x_2 * x_1')$

$= (x_1 * x_2) \oplus (x_1 * x_2') \oplus (x_2 * x_1')$

$= [(x_1 * x_2) * 1] \oplus [(x_1 * x_2') * 1] \oplus [(x_2 * x_1') * 1]$

$= [(x_1 * x_2) * (x_3 \oplus x_3')] \oplus [(x_1 * x_2') * (x_3 \oplus x_3')]$

$\oplus [(x_2 * x_1') * (x_3 \oplus x_3')]$

min7

$= [(x_1 * x_2 * x_3) \oplus (x_1 * x_2 * x_3') \oplus (x_1 * x_2' * x_3) \oplus$

$(x_1 * x_2' * x_3') \oplus (x_2 * x_1' * x_3) * (x_2 * x_1' * x_3')$

$= \oplus \min 2, 3, 4, 5, 6, 7$

* Representation of Boolean Algebra / Function

1) Cube Method
2) K-map method
3) Circuit Method.

$* \to$ meet $\Rightarrow \cdot$

$\oplus \to$ join $\Rightarrow +$

$' \to$ complement $\Rightarrow -$

e.g. $(x_1 * x_2') \oplus x_3'$

$x_1 \cdot \overline{x_2} + \overline{x_3}$

1) Cube Method -

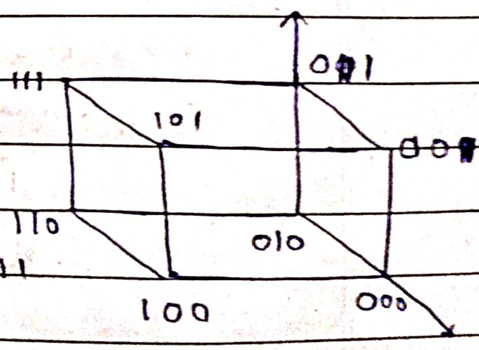
- It is used for only 3 variables

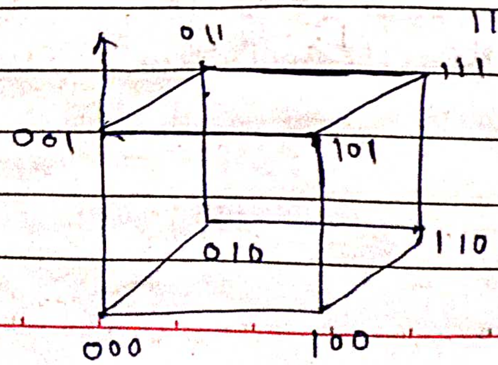
$x_1 \cdot \overline{x_2} + \overline{x_1}$

$x_1 \cdot \overline{x_2} \cdot x_3 + x_1 \cdot x_3 + x_2 \cdot x_3$

$101 + 101 + 011$

$101 + 011$

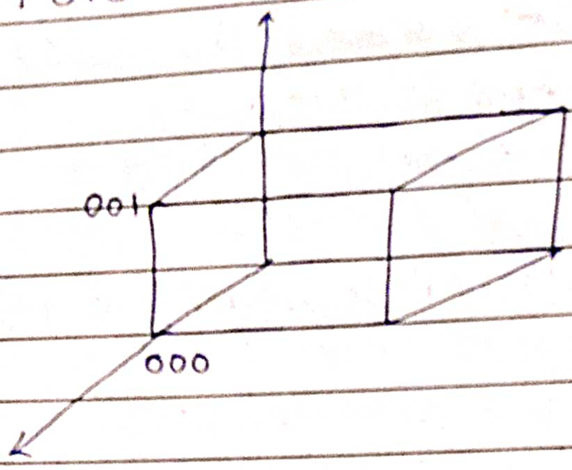




$$x_1\cdot\bar{x}_1 + x_2\cdot\bar{x}_3 + x_1\bar{x}_2\cdot x_3 + \bar{x}_1\cdot\bar{x}_2\cdot\bar{x}_3 + x_1\cdot(x_2+\bar{x}_3)$$

$$= x_1\cdot\bar{x}_3 + x_2\cdot\bar{x}_3 + x_1\bar{x}_2\cdot x_3 + \bar{x}_1\cdot\bar{x}_2\,\bar{x}_3 + x_1\cdot x_2 + \bar{x}_3\cdot x_1$$

$$= 100 + 010 + 101 + 000 + 110 + 011$$



2) K-map

$x_1 \begin{cases} 0 & \bar{x}_1 \\ 1 & x_1 \end{cases}$ $\qquad$ $x_2 \begin{cases} 0 & \bar{x}_2 \\ 1 & x_2 \end{cases}$

x_1	$\bar{x}_1$
1	0

$x_1 \backslash x_2$	0	1
0	00 $\bar{x}_1\bar{x}_2$	01 $\bar{x}_1 x_2$
1	10 $x_1\bar{x}_2$	11 $x_1 x_2$

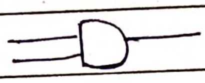
$x_1 \backslash x_2 x_3$	00 $\bar{x}_2\bar{x}_3$	01 $\bar{x}_2 x_3$	10 $x_2\bar{x}_3$	11 $x_2 x_3$
0	000	001	010	011
1	100	101	110	111

100 + 010 + 101 + 000 + 110 +

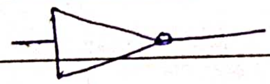
$x_1 \backslash x_2 \cdot x_3$	$\bar{x}_2\bar{x}_3$	$\bar{x}_2 x_3$	$x_2\bar{x}_3$	$x_2 x_3$
	1 000	001	1 010	011
	1 100	1 101	1 110	111

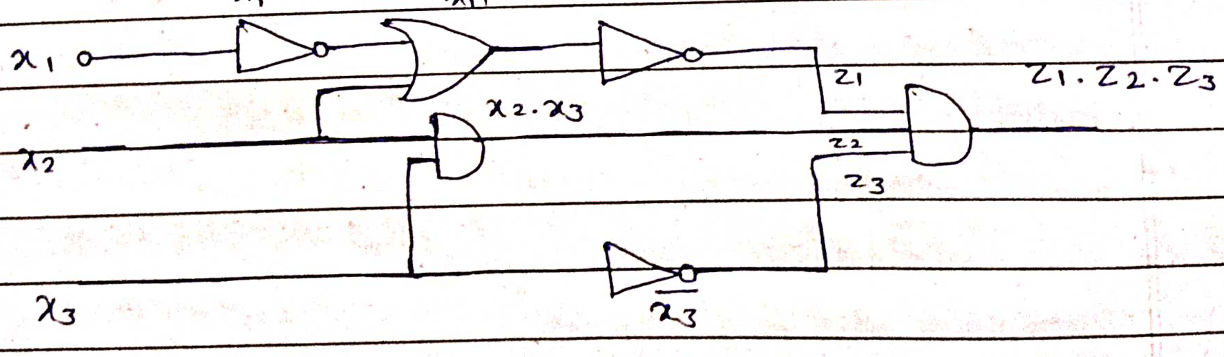
x_1x_2 \ x_3x_4	$\bar{x}_3\bar{x}_4$	$\bar{x}_3x_4$	$x_3\bar{x}_4$	x_3x_4
$\bar{x}_1\bar{x}_2$	0000	0101	0110	0011
$\bar{x}_1x_2$	0100	0101	0110	0111
$x_1.\bar{x}_2$	1000	1001	1010	1011
$x_1.x_2$	1100	1101	1110	1111

3) Circuit Method -

$*$ → meet → $\cdot$ → AND → 

$\oplus$ → join → + → OR → 

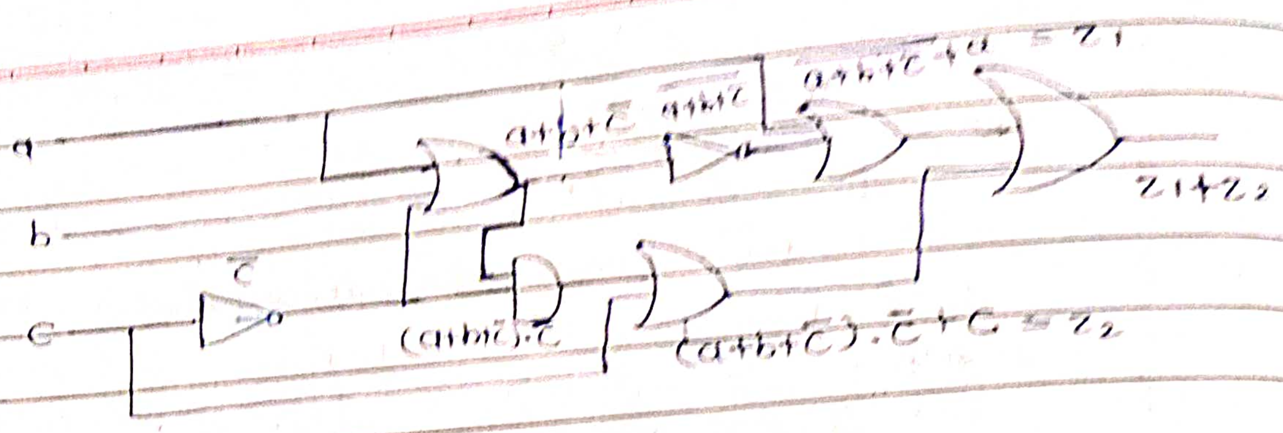
$'$ → complement → $-$ → NOT → 



output: $y = \overline{\bar{x}_1+x_2} \cdot x_2 \cdot x_3 \cdot \bar{x}_3$

$= \overline{\bar{x}_1+x_2} \cdot x_2 \cdot 0$

$= \overline{\bar{x}_1+x_2}$



$$y = z_1 + z_2$$
$$= [\overline{(a+b+\bar{c})} + a] + [(a+b+\bar{c})\cdot\bar{c} + c]$$
$$= \bar{a}\bar{b}c + a + a + b + \bar{c}$$
$$= \bar{a}\bar{b}\,c + a + b + \bar{c}$$

$\lceil a \rceil \Rightarrow$ ceil
$\lfloor a \rfloor \Rightarrow$ floor

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5. Permutations & Combinations

- Basic of counting

1] sum rule -
If a task can be done either in one of 'n' ways or in one of 'm' ways, where none of the set of 'n' ways is the same as any of the set of 'm' ways, then there are n+m ways to do the task

2] Product rule-
suppose that a procedure can be broken down into a sequence of tasks (two). If there are 'm' ways to do the first task & 'n' ways to do the second task. then there are n×m ways to do the task.

- floor (smallest)

It gives the nearest integer of the specified value.

e.g. 22.4 = 22
-22.4 = -23
22.6 = 22

- ceil

Round a number upto the next highest integer. while floor function rounds a number down to the next lowest integer.

e.g. 22.4 = 23 -22.4 = -22
22.6 = 23

- set - All the elements are unique in nature and follow some order.

e.g. $A = \{1, 2, 3, 4\}$ → set

$D = \{1, 2, 3, 1\}$ → not a set, it is a list.

- List -
  It is group of elements in which the elements may repeat & they may not follow any order.

- Pegionhole Principle / Box principle
  If a flock of 20 pegions flys into 19 pegionholes, then atleast one of these 19 pegionholes must have atleast 2 pegions in it.
  In short if there are more pegions & then pegionholes then there must be atleast 1 pegionhole with 2 pegions in it.

Let 'n' be number of holes & 'K' be the number of pegions.

E.g. If there are 3 bikes & 7 peoples to ride on then what is the maximum no. of people that bike can take.

→ It is calculated with the formula

$\lceil K/n \rceil$

Here, $n = 3$, $K = 7$

$\therefore \lceil 7/3 \rceil = \lceil$ [illegible] $\rceil = 3$

2) consider, there are 10 peoples & 3 Aut rickshaws apply pegionhole Principle & find out the max. no people that auto rickshaw can contain.

→ Here, $n = 3$, $k = 10$

$\therefore \lceil k / n \rceil = \lceil 10/3 \rceil = \lceil 3.3 \rceil = 4$

⊙ permutations

Permutation relates to the act of arranging all the members of a set into some sequence.

E.g. The permutations made with letters a, b, c taking all at a time ~~are~~ abc, acb, bac, bca, cab, cba

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

E.g. 3 awards to be assigned for 7 students.

→ Total no. of permutations $= \frac{7!}{(7-3)!}$

$= \frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}}$

$= 210$

1) Find the total no. of permutations in the folln scenario.

i) Ther are 10 students 1 has to be as president & second VP & 3rd whatever.

→ Here, $n = 10$, $r = 3$

$\therefore {}^{10}P_{3} = \frac{10!}{7!} = 10 \times 9 \times 8 = 720$

ii) A hotel menu contains 10 items Find the permtution for 3 of your favorite desert

→ $^{10}P_3 = \frac{10!}{7!} = 10 \times 8 \times 7 = 720$

Combination

A combination is a way of selecting items from a collection such that the oreder of selection does not matter.

E.g. Various groups of 2 out 4 persons A, B, C, D are
AB, AC, AD, BC, BD, CD

Formula : $\boxed{^nC_r = \frac{n!}{r!(n-r)!}}$

E.g. Create a team of 3 People out of total 7 People.

→ Here, $n = 7, \quad r = 3$

$\therefore \; ^nC_r = \; ^7C_3$

$= \frac{7!}{3!(7-3)!}$

$= \frac{7!}{3! \times 4!} = \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{6} \times \cancel{4!}} = 7 \times 5 = 35$

$= 35$

- Find the total no. of combinations to choose 3 deserts from the menu. Total no. of items in the menu is 10

→ Here, $n = 10, \quad r = 3$

$^{10}C_3 = \frac{10!}{3!\,7!} = \frac{10 \times 9 \times 8 \times \cancel{7!}}{6 \times \cancel{7!}} = \frac{90 \times 4}{3} = 30 \times 4 = 120$

1] out of 5 consonants & 3 vowels how many words of 3 consonants & 2 vowels can be formed.

→ No. of ways of choosing 3 constonants from 5 is $^5C_3 = \frac{5!}{3!\,2!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}\,2!} = 5 \times 2 = 10$

No. of ways of choosing 2 vowels from 3 is

$^3C_2 = \frac{3!}{2!\,1!} = 3$

$\therefore$ Total words $= {^5C_3} \times {^3C_2}$

$= 10 \times 3$

$= 30$

It means that we can have 30 groups where each group contains the total of 5 letters.

No. of ways of arranging 5 letters among themselves is $5! = 120$

$\therefore$ Required no. of ways $= 30 \times 120 = 3600$

2] out of 6 consonants & 3 vowels how many expressions of 2 consonants & 1 vowel can be paired.

→ Here, No. of consonants = 6

No. of vowels = 3

No. of ways of choosing 2 consonants & 1 vowels from 6 consonants & 3 vowels is.

$^6C_2 = \frac{6!}{2!\,4!} = \frac{6 \times 5 \times \cancel{4!}}{2 \times \cancel{4!}} = 15$

$^3C_1 = \frac{3!}{1!\,2!} = 3$

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Total words = $^6C_3 \times {}^3C_1$

= 15 × 3

= 45

This means that we can have 45 groups where each groups contains the total no. of 3 letters.

∴ No. of ways of arranging 3 letters among themselves is 3! = 6

∴ Required no. of ways = 45 × 6

= 270

3) In how many distinct forms can the letters of the term 'PHONE' be organized so that the vowels consistently come together.

→ Tho word 'PHONE' has 5 letters in which 'O' & 'E' are vowels.

The requirement is that the vowels should appear together. so they can consider 'OE' as a single letter.

∴ we can take total 4 letters which are distinct.

∴ Total no. of methods to organise this letters is 4! = 24

No. of ways to arrange the 2 vowels (OE) among themselves is 2! = 2

∴ Required no. of ways are - 24 × 2

- 48

- For consonants -

No. of ways to arrange 3 consonants (PHN) among themselves is 3! = 6

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∴ Required no. of ways are - 6×6
= 36

4) The word 'optical' needs to be rearranged so that the vowels always come together.

→ The word 'OPTICAL' has 7 letters in which 3 vowels & 4 consonants.
The requirement is that vowels should appear togeter. so they can consider 'OIA' as a single letter.

∴ No. of methods to organize this letter is
5! = 120

∴ No. of ways to arrange 3 vowels (OIA) among themselves is 3! = 6
∴ Required no. of ways are = 120×6
= 720

5) Find no. of ways in which the word 'DETAIL' can be arranged, so that vowels only occupy odd position.

→ The word 'DETAIL' has 6 letters in which there are 3 odd positions & 3 even positions.

The requirement is that vowels only occupy odd position = 3! = 6
Arrangement of consonants = 3! = 6

∴ Required no. of ways are = 6×6
= 36

6) How many 6 digit codes could be generated from the digits 0 to 9.

a) if Repetition is allowed.

b) if Repetition is not allowed.

→ There are 10 digits (0 to 9)

a> Repetition is allowed -

9	10	10	10	10	10

∴ To form 6digit code total ways are in which Retition is allowed $= 9 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 9,00,000$

b) Repetition is not allowed.

4	6	7	8	9	10

∴ To form 6 digit code total ways are in which Repetition is not allowed $= 4 \times 6 \times 7 \times 8 \times 9 \times 10$

$= 120960$

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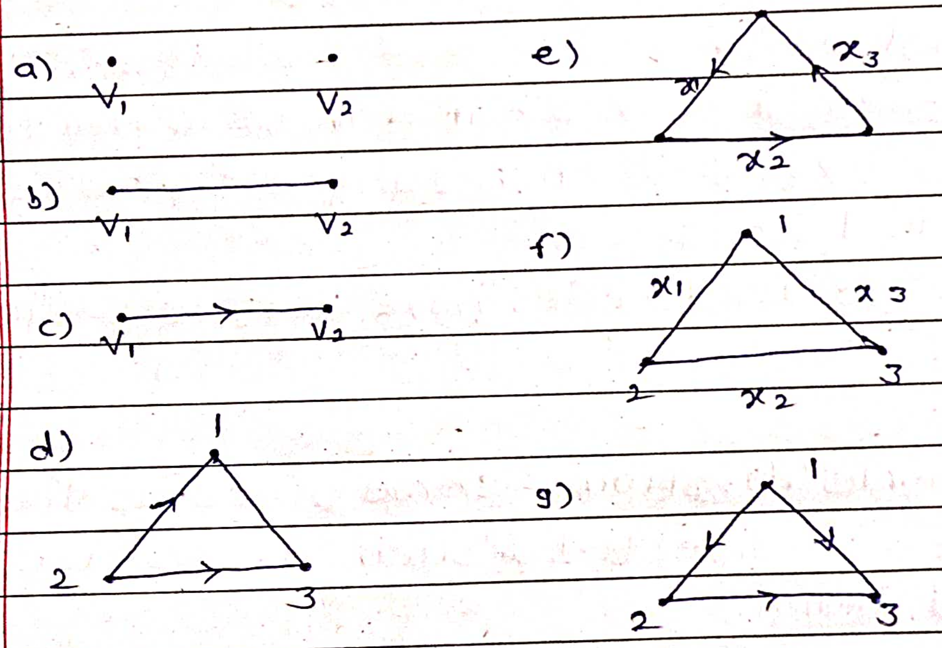
5. Graph Theory

⊙ Basic concepts

A Graph $G = \langle V, E, \Phi \rangle$ consist of a nonempty set V called as the set of nodes (pts. or vertices) of the graph.
'E' is said to be set of edges & 'Φ' is a mapping from the set of edges E to set of pairs of elements of V.



- Any pair of nodes which are connected by an edge are called <u>adjacent nodes</u>.

- An edge can be associated with an ordered or unordered pair of vertices

⊙ Directed graph

In a graph $G = \langle V, E, \Phi \rangle$ & edge which is associated with an ordered pair of $V \times V$ is called a directed edge of G.

Whereas an edge which is associated with an unordered pair of nodes is called undirected edge.

A Graph in which every edge is directed is called directed graph (digraph).

- A Graph in which no edge is directed is called as undirected graph.

Isolated Node

In a graph $G = \langle V, E \rangle$ if a node is not connected to any other node then such nodes are called as isolated node.

Usually, isolated nodes have very less significance in graph theory.

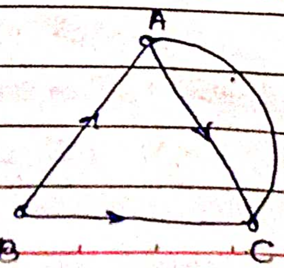
Note- The example 'd)' shows some edges as directed & some are undirected & such graphs are called as mixed graph.

Loop

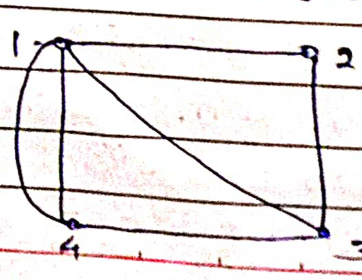
An edge of a graph which joins a node to itself is called a loop (slinge).

- Parallel Edges -

A pair of nodes joined by more than one edge such edges are called parallel edges.



$\langle A, C \rangle$ $\langle C, A \rangle$

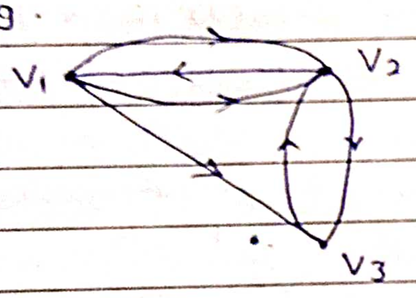


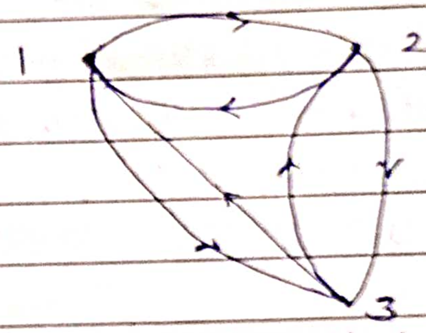
$\langle 1, 4 \rangle$

- **Multi Graph -**
  Any graph which contains some $\|^{el}$ edges is called a multigraph.

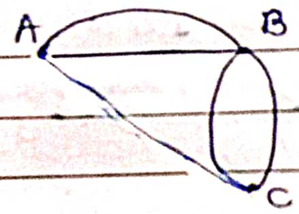
- **Simple Graph -**
  If there is no more than one edge bet^n a pair of nodes (no more than one directed edge in case of diagram) then such a graph is called simple graph.

E.g.

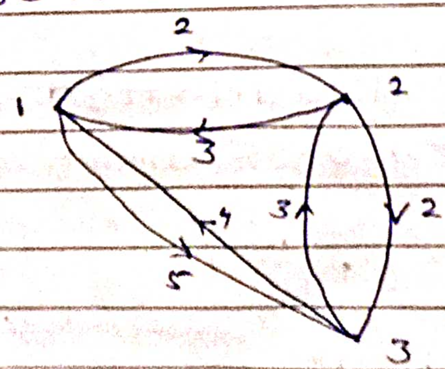




No parallel edges.



- **Weighted graph -**
  A graph in which weights are assigned on every edge is called weighted graph.



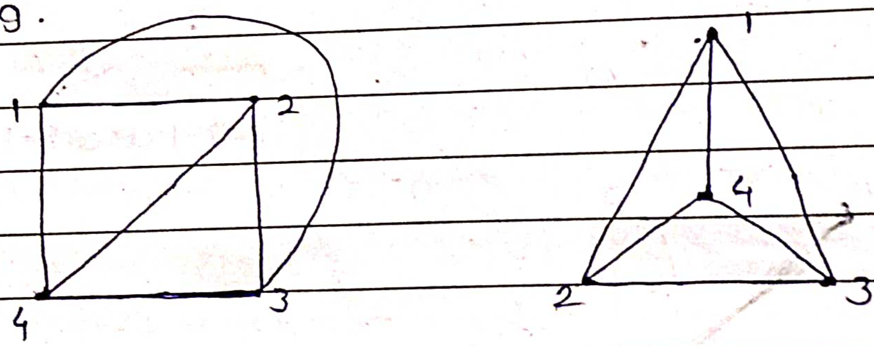
• Null Graph -

A Graph which contains only isolated nodes is called a null graph.

In other words a set of edges in a null graph is zero/empty.

* The Definition of a graph contains no reference to the length or a shape & positioning of the edge joining any pair of nodes, nor does it describing any ordering of positions of the nodes. This may indicate that, 2 diagrams which look entirely different from one another may represent the same graph.

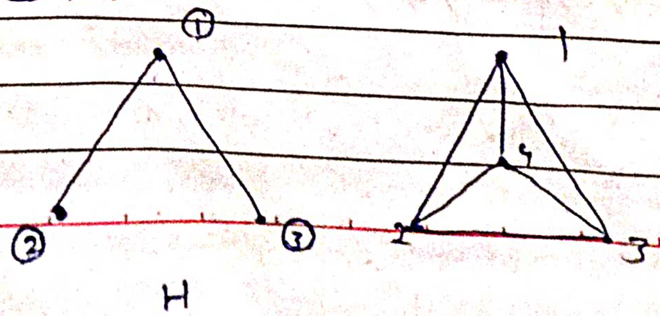
E.g.



subgraph -

Let, V(H) be the set of nodes of graph H & V(G) be the set of nodes of graph G such that $V(H) \subseteq V(G)$.

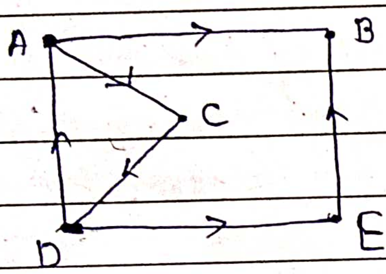
If every edge of H is also an edge G then graph edge is called subgraph of G, which is denoted as $H \subseteq G$.



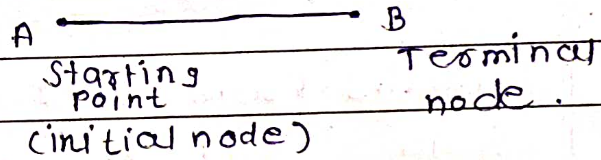
Path -

Any sequence of edges of a diagram such that the terminal node of any edge in the sequence is the initial node of edge, if any appearing in the sequence defining path of graph.

The no. of edges appearing in the sequence of the path is called length of path.



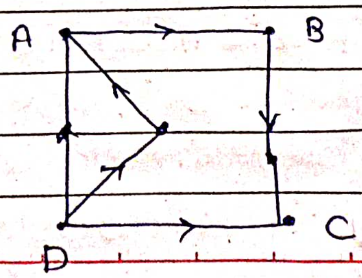
length of Edge - 3
i.e. < A, B >, < A, D >
< A ; C >.



* Indegree & outdegree

- In a directed graph for any node V the no. of edges which have V as their initial node is called the outdegree of node V.
The no. of edges which have V as their terminal node is called indegree of node V.

- The sum of outdegree & indegree is called total degree of node V.



For Node A -:
indegree - 2
outdegree - 1
Total degree - 3

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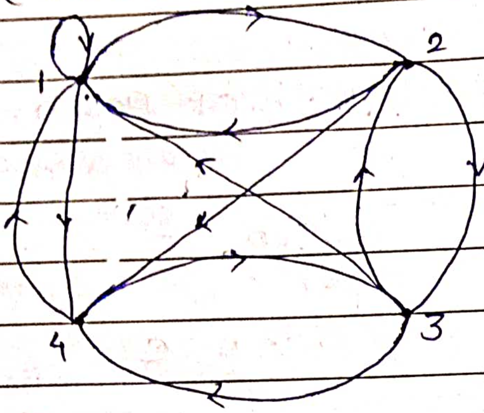
- Simple path (Edge simple) -
A path in a diagram in which the edges are all distinct is called a simple path.

A path in which all the nodes through which it traverlses are distinct is called an elementry path. (node simple).



Paths from 1 to 3 →

P_1 - <1, 2>, <2, 3> → S.G.

P_2 - <1, 4>, <4, 3> → S.G

P_3 - <1, 2>, <2, 4>, <4, 3> → S.G.

P_4 - <1, 2>, <2, 4>, <4, 1>, <1, 4>, <4, 3> → Not S.G

Any node should not traverse more than once.

Elementary path — P_1, P_2, P_3 are elementary paths.

- All the simple paths are elementary paths.

* cycle
- A path which orginates & ends in the same node is called cycle.

- A cycle is called simple if it's path is simple. (distinct edges.)

- A cycle is called elementary if it does not traverse through any node more than once.

E.g. simple cycle → $\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,4 \rangle \langle 4,1 \rangle$

Elementary cycle → $\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,4 \rangle, \langle 4,1 \rangle$

Reachability -

A node v of a simple diagram is said to be reachable (accessable) from the node u of the same diagram if their exist a path from u to v.

The concept of reachability is independent of the no. of alternate path from u to v & also their lengths.

If a node v is reachable from the node u then a path of minimum length from u to v is called geodesic.

The length of geodesic from the node u to node v is called distance & is denoted as $d(u,v)$

It is assumed that $d(u,u) = 0$.

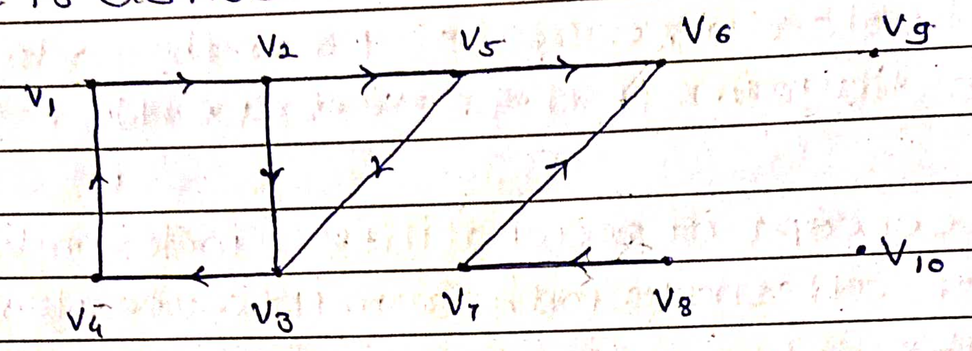
* connectedness
* Reachable set of nodes
* connectedness with respect to
    - Undirected graph.
    - Directed graph (weakly connected)
    - unilaterally connected.
    - strongly connected
    - components.

Reachable set-

For a directed graph $G = \langle V, E \rangle$ the set of nodes which are reachable from a given node v is said to be reachable set of v.

It is denoted as $R(v)$.

E.g.



Reachable set- $R(V_1) = \langle V_1, V_2, V_3, V_4, V_5, V_6 \rangle$

$R(V_2) = \langle V_2, V_1, V_3, V_4, V_5, V_6 \rangle$

$R(V_3) = \langle V_3, V_1, V_2, V_4, V_5, V_6 \rangle$

$R(V_4) = \langle V_4, V_1, V_2, V_3, V_5, V_6 \rangle$

$R(V_5) = \langle V_5, V_3, V_2, V_1, V_4, V_6 \rangle$

$R(V_6) = \langle V_6 \rangle$

$R(V_7) = \langle V_7, V_6 \rangle$

$R(V_8) = \langle V_7, V_6, V_8 \rangle$

Node Base-

In a diagram $G = \langle V, E \rangle$ as subset $X \subseteq V$ is called a node base if its ~~it is~~ reachable set is V & no proper subset of X has this property.

In the above diagram $\{V_1, V_8, V_9, V_{10}\}$ is a node base & so is the set $\{V_5, V_8, V_9, V_{10}\}$.

Every isolated node of a diagram must be present in node base. Any node whose indegree zero must be present in node base.

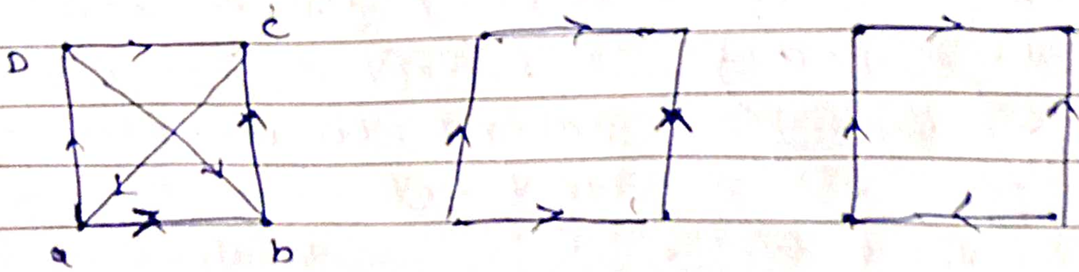
No node in the node base is reachable from another node in the node base.

- connectivity w.r.t undirected graph.
  An undirected graph is said to connected if for any pair of nodes for the graph the 2 nodes are reachable from one another.

- A diagraph is called connected (weekly connected) if it is connected as an undirected graph in which the direction of the edges is neglected.

- A simple graph is called as unilaterally connected if for any pair of nodes of the graph atleast one of nodes of the pair is reachable from the other node.

Strongly connected-
If for any pair of nodes of the graphs both the nodes of the pair are reachable from one another. Then graph is called strongly connected.



(a) strongly connected.

weekly / not unilaterally connected

unilaterally connected not strongly connected

- component -

For a simple diagraph a maximal a strongly connected subgraph is called strong component.

similarly the maxium unilaterally conected or maximal weakly connected subgraph is called unilateral / weak component resp.

- Matrix representation of graphs -

Let $G = \langle V, E \rangle$ be a simple digraph in which $V = \{V_1, V_2, ----V_n\}$ & the nodes are assumed to be ordered from V_1 to V_n.

A nxn matrix A whose elements a_{ij} are given by

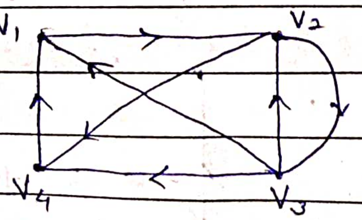
$$a_{ij} = \begin{cases} 1 & \text{if } \langle V_i, V_j \rangle \in E \\ 0 & \text{otherwise} \end{cases}$$

Adjecency matrix is same as relation matrix or the incidence matrix.

Any element of adjecency matrix is either 0 or 1.

such matrices are also called as bit matrix or boolean matrix.

Q.1] Represent folln graph as a matrix.



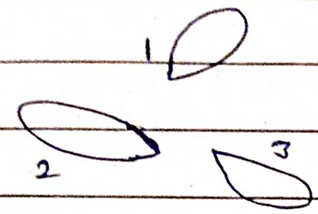
Matrix A : consider order of nodes V_1, V_2, V_3, V_4

Matrix B : consider order of nodes V_2, V_3, V_1, V_4

$$A = \begin{array}{c|cccc|} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 0 & 1 & 0 & 0 \\ V_2 & 0 & 0 & 1 & 1 \\ V_3 & 1 & 1 & 0 & 1 \\ V_4 & 1 & 0 & 0 & 0 \\ \hline \end{array}$$

$$B = \begin{array}{c|cccc|} & V_2 & V_3 & V_1 & V_4 \\ \hline V_2 & 0 & 1 & 0 & 1 \\ V_3 & 1 & 0 & 1 & 1 \\ V_1 & 1 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 1 & 0 \\ \hline \end{array}$$

Identity matrix



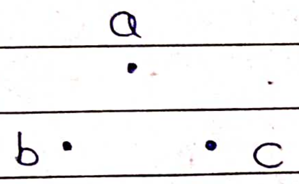
$$\begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ \hline \end{array}$$

If their are loops at each node but no other edges in the graph then adjecency matrix is called as identity matrix / unit matrix.

NULL Matrix

If a graph consist of n no. of nodes but no edges then adjecency matrix will have all its elements as zero.

E.g.



$$\begin{array}{c|ccc|} & a & b & c \\ \hline a & 0 & 0 & 0 \\ b & 0 & 0 & 0 \\ c & 0 & 0 & 0 \\ \hline \end{array}$$

Multigraph / weighted graph

In the case of multigraph / weighted graph we write $a_{ij} = w_{ij}$. where w_{ij} denotes either the multiplicity of the edge $\langle V_i, V_j \rangle$ if $\langle V_i, V_j \rangle \notin E$ or the weight of the edge

then $w_{ij} = 0$

Transpose of matrix

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix Multiplication -

$$\underset{A}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}} \times \underset{A^T}{\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- **Indegree & outdegree**

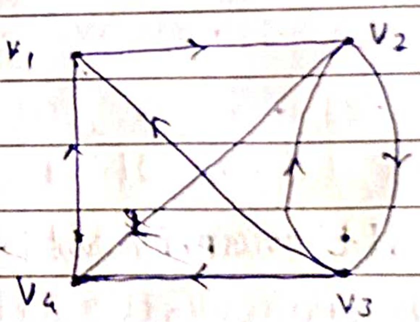
Given a graph $G = \langle V, E \rangle$, using the matrix representation of the G, The indegree & outdegree of all the nodes of G can be computed.

Indegree

For a graph A find the transpose of A (A^T). Then compute the product $A^T A$ in which the diagonal elements show the indegree of all the nodes

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad A^T = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$



outdegree

To find out the outdegree for a graph A, compute transpose of A (A^T). The outdegree of nodes is shown as the diagonal elements of the product matrix AA^T.

$$AA^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

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Paths of length (n)

G

Let A be an adjacency matrix of a diagraph the element in the i^{th} row of j^{th} column of A^n, Cn is a +ve integer = no. of paths of length n = from i^{th} node to the

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad A^3 = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 2 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Find the total no. of paths betn any 2 pair of nodes where maximum length (Path, $1 \le 3$)

$$B_3 = A^1 + A^2 + A^3 = \begin{bmatrix} 2 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Path matrix (Reachability Matrix.

Let $G = \langle V, E \rangle$ be a simple diagraph in which $|V| = n$ & the nodes of G are assumed to be ordered. An nxn matrix P whose elements are given by

$$P_{ij} = \begin{cases} 1 & \text{if exist path from } V_i \text{ to } V_j \\ 0 & \text{if there exist path from } V_i \text{ to } V_j \end{cases}$$

is called path marix / reachability matrix.

e.g. $B_3 = A_1 + A^2 + A^3$

$$\begin{bmatrix} 3 & 2 & 1 & 2 \\ 3 & 3 & 2 & 3 \\ 4 & 4 & 2 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

- Trees

1) Directed tree
2) Leaf node, Branch node.
3) Level of node.
4) Degree of node.
5) Forest, descendants and sons
6) M-ary tree, full M-ary trees.
7) Prefix code
8) Representing trees.

1) Directed Tree

indegree / outdegree

A directed tree is an acyclic diagraph which has one node called root with indegree zero while other nodes have indegree 1.

2) Leaf Node

In a directed tree any node which has indegree or outdegree zero is called leaf node / terminal node. All other nodes are called branch node.

3) Level of node

Level of node is length of its path from root node.

Note. 1) The level of root of directed tree is always zero whereas the level of any node is equal to distance from the root node.

2) All the paths in directed tree are elementary

4) Degree of Node -

Degree of node in a tree is the total no. of subtrees that can be found after delting their root node.

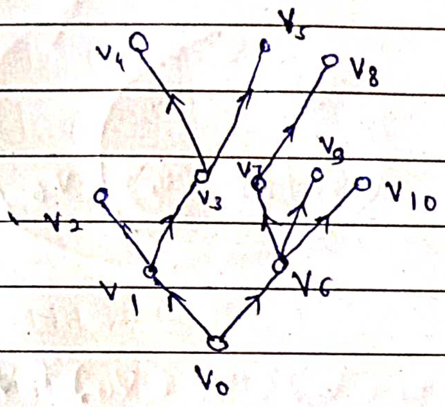


fig (A)

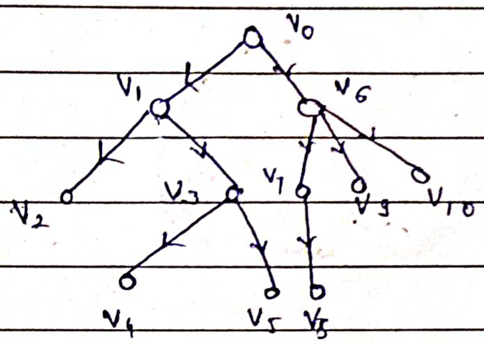


fig (B)

Forest.

~~The group of all~~ If we delete a node & the edges connecting

A set of disjoint trees is calle d forst

descendants -

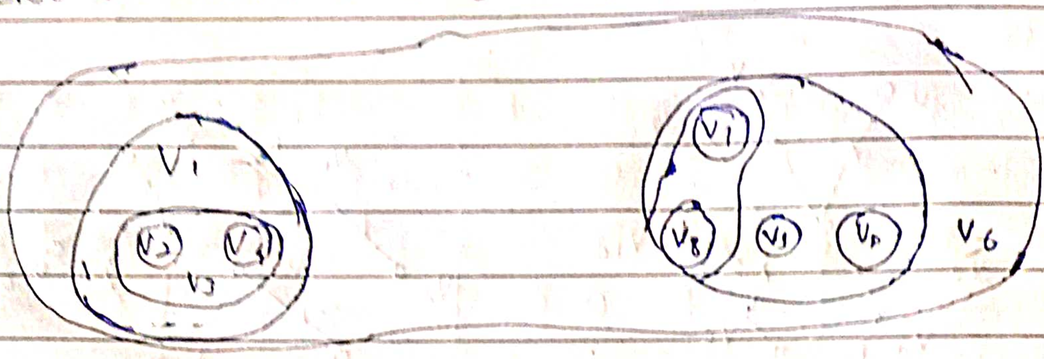
Every node that is reachable from a given node u is called a desended of u.

Also the nodes which are reachable from u given node u thr a single edge are called as sons of. u.

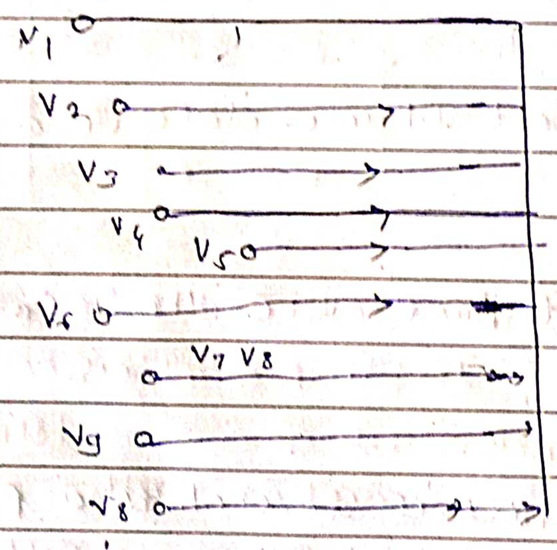
M-ary tree -

Representation of tree.

1) venn diagram
2) Nesting paranthesis
3) List of contacts or look.



Nesting paranthesis - (V0 (V1 (V2) (V3 (V4) (V5)) V6 (V7 (V8)) (V9, V10

3) 

1) consider the following expression draw a tree for the same.

1) $V_2 V_1 + \left(V_4 + \frac{V_5}{V_8} \right) * V_3$

→