

Tutorial No. 7 - Statistical Technique

Calculate the coefficient of correlation from the following data.

x	65	66	67	68	69	70	71	
y	67	68	66	69	72	72	69	

x	y	xy	x^2	y^2	
65	67	4355	4225	4489	
66	68	4488	4356	4624	
67	66	4422	4489	4356	
68	69	4692	4624	4761	
69	72	4968	4761	5184	
70	72	5040	4900	5184	
71	69	4899	5041	4761	
$\Sigma x =$	$\Sigma y =$	$\Sigma xy =$	$\Sigma x^2 =$	$\Sigma y^2 =$	
476	483	32864	32396	33359	

Coefficient correlation is given by

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}} = \frac{32864}{\sqrt{476 \times 476 \times 483 \times 483}} = \frac{32864}{229,908}$$

$$r = 0.145$$

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$$\bar{X} = \frac{\sum X_i}{n} = 68 \quad \bar{Y} = \frac{\sum Y_i}{n} = 69$$

X	Y	$x = X_i - \bar{X}$	$y = Y_i - \bar{Y}$	$x \cdot y$	x^2	y^2
65	67	-3	-2	6	9	4
66	68	-2	-1	2	4	1
67	66	-1	-3	3	1	9
68	69	0	0	0	0	0
69	72	1	3	3	1	9
70	72	2	3	6	4	9
71	69	3	0	0	9	0
				$\sum x \cdot y =$	$\sum x^2 =$	$\sum y^2 =$
				20	28	32

Coefficient of correlation

$$r = \frac{\sum x \cdot y}{\sqrt{\sum x^2 \sum y^2}} = \frac{20}{\sqrt{28 \times 32}} = \frac{20}{29.93} = 0.668$$

Q.2. Calculate the coefficient of correlation from the following data

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

$$\bar{X} = \frac{\sum X_i}{n} = \frac{120}{6} = 20 \quad \bar{Y} = \frac{\sum Y_i}{n} = \frac{126}{6} = 21$$

x	y	$x = x_i - \bar{x}$	$y = y_i - \bar{y}$	$x \cdot y$	x^2	y^2
10	18	-10	-3	30	100	9
14	12	-6	-9	54	36	81
18	24	-2	3	-6	4	9
22	6	4	-15	-30	4	225
26	30	8	9	54	36	81
30	36	10	15	150	100	225
				$\Sigma xy =$	$\Sigma x^2 =$	$\Sigma y^2 =$
				252	280	630

Coefficient of correlation is given by

$$r = \frac{\Sigma xy}{\Sigma x \Sigma y} = \frac{252}{\sqrt{280 \times 630}} = \frac{252}{420} = 0.6$$

3. Given, covariance = 12.5, $r = 0.5$, variance of $x = 25$ find σ_y .

We know that,

$$r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$$0.5 = \frac{12.5}{\sqrt{25} \times \sigma_y} \quad \text{where } \text{var}(x) = (\sigma_x)^2$$

$$0.5 = \frac{12.5}{5 \times \sigma_y}$$

$$\sigma_y = \frac{12.5}{5 \times 0.5} = \frac{12.5}{2.5} = 5$$

σ_y = standard deviation of $y = 5$

Q.4 Use the following data to obtain the regression equation of y on x and x on y

x	6	2	10	4	8
y	9	11	5	8	7

x	y	xy	x^2	y^2
6	9	54	36	81
2	11	22	4	121
10	5	50	100	25
4	8	32	16	64
8	7	56	64	49
$\Sigma x =$	$\Sigma y =$	$\Sigma xy =$	$\Sigma x^2 =$	$\Sigma y^2 =$
30	40	214	220	340

$$\bar{y} = 8 \quad \bar{x} = 6$$

Regression equation of y on x

$$b_{yx} = \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{\Sigma x^2 / N - (\Sigma x / N)^2}$$

$$= \frac{42.8 - 6 \times 8}{44 - 36}$$

$$= \frac{-5.2}{8}$$

$$= -0.65$$

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$y - 8 = -0.65 (x - 6)$$

$$y - 8 = -0.65x + 3.9$$

$$y = -0.65x + 11.9$$

Regression equation of x on y

$$x - \bar{x} = b_{xy} (y - \bar{y})$$

$$b_{xy} = \frac{\sum xy / N - (\sum x / N)(\sum y / N)}{\sum x^2 / N - (\sum x / N)^2}$$

$$= \frac{42.8 - 48}{68 - 64}$$

$$= \frac{-5.2}{4}$$

$$= -1.3$$

$$x - 6 = -1.3 (y - 8)$$

$$x - 6 = -1.3y + 10.4$$

$$x = -1.3y + 16.4$$

5. Obtain the two regression equation from the following data.

x	2	4	6	8	10	12
y	4	2	5	10	3	6

x	y	xy	x ²	y ²
2	4	8	4	16
4	2	8	16	4
6	5	30	36	25
8	10	80	64	100
10	3	30	100	9
12	6	72	144	36
$\Sigma x =$	$\Sigma y =$	$\Sigma xy =$	$\Sigma x^2 =$	$\Sigma y^2 =$
42	30	228	364	190

$$\begin{aligned}
 b_{yx} &= \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{(\Sigma x^2 / N) - (\Sigma x / N)^2} \\
 &= \frac{38 - 7 \times 5}{60.66 - 49} \\
 &= \frac{3}{11.66} \\
 &= 0.2572
 \end{aligned}$$

$$\begin{aligned}
 b_{xy} &= \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{(\Sigma y^2 / N) - (\Sigma y / N)^2} \\
 &= \frac{38 - 7 \times 5}{31.66 - 25} \\
 &= \frac{3}{6.66} \\
 &= 0.45
 \end{aligned}$$

$$\bar{x} = \sum x / N = 7 \quad \bar{y} = \sum y / N = 5$$

Regression equation of y on x

$$(y - \bar{y}) = b_{yx} (x - \bar{x})$$

$$y - 5 = 0.257 (x - 7)$$

$$y - 5 = 0.257x - 1.799$$

$$y = 0.257x + 3.201$$

Regression equation of x on y

$$(x - \bar{x}) = b_{xy} (y - \bar{y})$$

$$x - 7 = 0.45 (y - 5)$$

$$x - 7 = 0.45y - 2.25$$

$$x = 0.45y + 4.75$$

6. Obtain the two regression equations from the following data and predict the value of x when $y=40$ and value of y when $x=50$

x	16	22	28	19	21	25	18
y	60	65	63	66	68	61	70

Also find the correlation coefficient

x	y	xy	x^2	y^2
16	60	960	256	3600
22	65	1430	484	4225
28	63	1764	784	3969
19	66	1254	361	4356
21	68	1428	441	4624
25	61	1525	625	3721
18	70	1260	324	4900
$\Sigma x =$	$\Sigma y =$	$\Sigma xy =$	$\Sigma x^2 =$	$\Sigma y^2 =$
149	453	9621	3275	29395

$$\begin{aligned}
 b_{yx} &= \frac{\Sigma xy / N - (\Sigma x / N)(\Sigma y / N)}{\Sigma x^2 / N - (\Sigma x / N)^2} \\
 &= \frac{1374.42 - 21.28 \times 64.71}{467.85 - 452.83} \\
 &= \frac{1374.42 - 1377.02}{15.02} \\
 &= \frac{-2.6}{15.02} \\
 &= -0.1731
 \end{aligned}$$

Regression equation of y on x

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$y - 64.71 = -0.1731 (x - 21.28)$$

$$y - 64.71 = -0.1731x + 3.6835$$

$$y = -0.1731x + 68.3935$$

$$\text{when } x = 50$$

$$y = -8.655 + 68.3935$$

$$y = 59.73$$

$$b_{xy} = \frac{\sum xy / N - (\sum x / N)(\sum y / N)}{\sum x^2 / N - (\sum x / N)^2}$$

$$= \frac{1374.42 - 1377.02}{4199.28 - 4187.93}$$

$$= -0.2290$$

Regression equation of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 21.28 = -0.2290(y - 64.71)$$

$$x - 21.28 = -0.2290y + 14.81$$

$$x = -0.2290y + 36.09$$

$$\text{when } y = 40$$

$$x = -9.16 + 36.09$$

$$x = 26.93$$

Q.7. Given, variance of x (σ_x^2) = 9, regression equation are $8x - 10y + 66 = 0$, $40x - 18y + 214 = 0$

Find i) Average value of x and y .

ii) Correlation coefficient between two variables

iii) Standard deviation of y