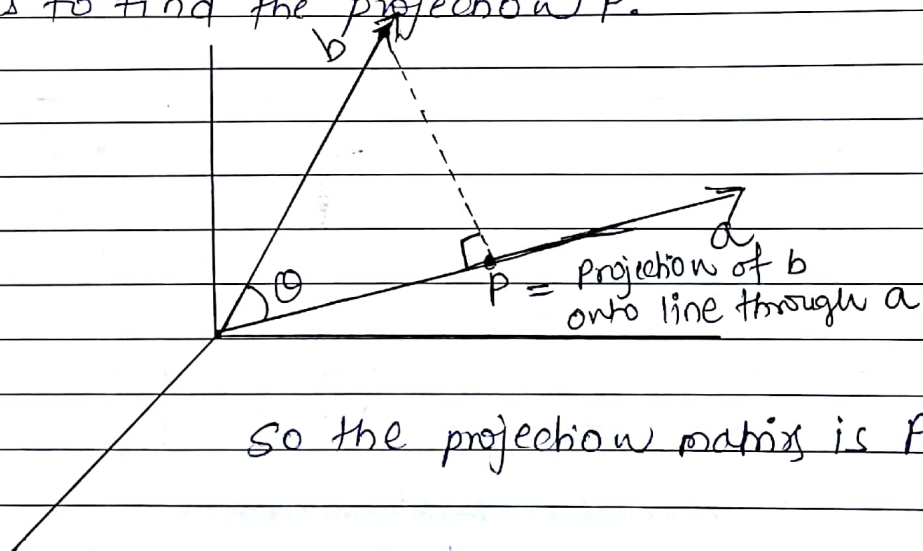


Cosines & projections onto lines

Suppose we want to find the distance from a point b to the line in the direction of the vector a . We are looking along that line for the point p closest to b . The line connecting b to p is perpendicular to a . This fact will allow us to find the projection p .



So the projection matrix is $P = \frac{a^T a}{\|a\|^2}$

To project b onto a at point p ,

$$p^T = p b^T$$

Example 1 :- Find the projection matrix that projects onto the line through $a = (1, 1, 1)$. Hence find the projection point p that projects $b = (2, 3, 4)$ onto a .

→ Given $a = (1, 1, 1)$ $b = (2, 3, 4)$
projection matrix $p = \frac{a^T a}{\|a\|^2}$

$$= \frac{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}}{1+1+1}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

∴ projection point p that projects b onto a $= p \cdot b^T$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \downarrow$$

$$= \frac{1}{3} \begin{bmatrix} 2+3+4 \\ 2+3+4 \\ 2+3+4 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 9 \end{bmatrix} \frac{1}{3}$$

$$= \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Thus projection point $p = (3, 3, 3)$

Ex (2) Find the projection matrix that projects onto the line through $a = (0, 1, 4)$. Hence find projection point p that projects $b = (-1, 3, -2)$ onto a .

⇒ Given $a = (0, 1, 4)$ $b = (-1, 3, -2)$

$$\text{projection matrix } p = \frac{a^T a}{||a||^2}$$

$$= \frac{\begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 \end{bmatrix}}{0 + 1 + 16}$$

$$= \frac{1}{17} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & 16 \end{bmatrix}$$

∴ projection point p that projects b onto a is $p = p \cdot b^T$

$$= \frac{1}{17} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & 16 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

$$= \frac{1}{17} \begin{bmatrix} 0 \\ -5 \\ -20 \end{bmatrix}$$

Thus projection point $p = (0, -5/17, -20/17)$