Tutorial No. I

Q. 1. Write down the regular expression for the following languages.

1. $L = \{ab, aab, bab, aabbab, ----\}$ over $\Sigma = \{a, b\}$

→ $RE = (a+b)^* ab$

2. $L = \{11, 110, 1111, 11010, ----\}$ over $\Sigma = \{0, 1\}$

→ $RE = 11(0)^* 11(0)^*$

3. A string with odd number of 1's over $\Sigma = \{0, 1\}$

→ $RE = 0^* 1 0^* (11)^* 0^*$

4. A string containing either ab or bba over $\Sigma = \{a, b\}$

→ $RE = (a+b)^* (ab + bba)(a+b)^*$

Q. 2. If $\Sigma = \{a, b, c\}$ then find the $\Sigma^1, \Sigma^2, \Sigma^3$

→ $\Sigma^1 = \{a, b, c\}$

$\Sigma^2 = \{aa, ab, ac, bb, ba, bc, cc, ca, cb\}$

$\Sigma^3 = \{aaa, aab, aac, abc, acb, abb, acc, bbb, ...\}$

Q. 3. Prove the following using mathematical induction?

$1+2+3+------+n = n(n+1)/2$

→ Step I : Base Step

Let us assum that

$p(n) = 1+2+3+----+n = n(n+1)/2$

For $n=1$ $\quad P(1) = \frac{1(1+1)}{2} = 1 = RHS$, $LHS = 1$

$\therefore LHS = RHS$

For $n=2$ $\quad P(2) = \frac{2(2+1)}{2} = 3 = RHS$

$LHS = 1+2 = 3$ $\quad \therefore LHS = RHS$

It is true for $n=1$ and $n=2$

Step 2: Induction step

For $n=k$

$$1+2+3+----+k = \frac{k(k+1)}{2} \quad \text{———} \quad (1)$$

For $n=k+1$

$$1+2+3+----+k+k+1 = \frac{k+1\,((k+1)+1)}{2}$$

$$LHS = 1+2+3+----+k+k+1$$

$$= \frac{k(k+1)}{2} + (k+1) \quad \text{——— from equat}^n\ 1$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right)$$

$$= (k+1)\left(\frac{(k+1)+1}{2}\right)$$

$$= RHS$$

$\therefore LHS = RHS$

$\therefore$ The given formula is true for every value of n, $(n>0)$.

Q. 4. Find the recursive defination of a. Factorial function b. Fibonacci funct^n

a. Factorial Function

→ Base Step: For $n=0$, $f(0)=1$; $n=1$, $f(1)=1$

Inductive step: $f(n) = n * f(n-1)$

b. Fibonacci Function

→ Base step : For $n = 0, 1$, $f(0) = 0$, $f(1) = 1$

Inductive step : $f(n) = f(n-1) + f(n-2)$ for $n \geq 1$

Q. 5. Find the recursive defination of set $\{1, 3, 6, 10, 15, 21, ----\}$

→ Base step : for $n = 1$ $f(1) = 1$

Recursive step : For $n > 1$, $f(n) = n + f(n-1)$

Q. 6. If $\Sigma = \{0, 1\}$, then find the following languages

a. The language of string of length zero

→ $L = \{\wedge\}$

b. The language of strings of 0's and 1's with equal no. of each.

→ $L = \{01, 0011, 0101, 000111, 010101, ----\}$

c. The language $\{0^n 1^n \mid n \geq 1\}$

→ $L = \{01, 0011, 000111, 00001111, ----\}$

d. The language $\{0^i 1^j \mid 0 \leq i \leq j\}$

→ $L = \{\varepsilon, 0, 00, 000, ----\}$

e. The language of strings with odd no. of 0's & even no. of 1's.

→ $L = \{011, 101, 110, 00011, 01010, 01100, 11000, ----\}$

Q. 7. Define the Kleen Closure, Positive Closure & Power of alphabets

1. Kleen Closure

→ The set of all the strings over an alphabet Σ is called Kleen Closure of Σ and is denoted by Σ^*. Thus, kleen closure is set of all the strings over alphabet Σ with length 0 or more.

$\therefore \Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup ----$

e.g. $A = \{0\}$

$A^* = \{0^n / n = 0, 1, 2, 3, ----\}$

2. Positive Closure

→ The set of all the strings over an alphabet Σ. Except the empty string is called positive closure & is denoted by Σ^+

$\therefore \Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4 \cup ----$

3. Power of alphabet

→ The set of all strings of certain length k from an alphabet is the k^{th} power of that alphabet.

i.e. $\Sigma k = \{W \mid |W| = k\}$

If $\Sigma = \{0, 1\}$ then

$\Sigma^0 = \{\varepsilon\}$

$\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$

Q. 8. Write regular expression for

1. Language containing strings that either starts with or ends with 01.

$(01(1+0)^* + (1+0)^* 01)$

2. Language consisting of strings that have at least two consecutive 0's or 1's

$$(0+1)^*00(0^*)(0+1)^* + (0+1)^*11(1)^*(0+1)^*$$

3. Language consisting of strings which are starting with 1 or ending with 0.

$$1(1+0)^* + (0+1)^*0$$

4. Language consisting of string with even lengths of 0's

→ For $\Sigma = \{0\}$ $L = \{00, 0000, 000000, \ldots\}$

RE = $(00)^*$

5. Language consisting string with length exactly two

→ for $\Sigma = \{0,1\}$ $L = \{00, 01, 10, 11\}$ RE = $(00+01+10+11)$

6. Language consisting string which starts and ends with diff. symbols

→ For $\Sigma = \{a,b\}$ $L = \{ab, aab, aabb, \ldots\}$ RE = $(a(a+b)^*b + b(a+b)^*a)$

7. Strings starts with and ends with same symbol $\Sigma = \{a,b\}$

→ For $\Sigma = \{a,b\}$ $L = \{a, b, aa, bb, aba, bab, \ldots\}$

RE = $(a(a+b)^*a + b(a+b)^*b)$