

5. Backtracking.

PAGE NO.:

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Backtracking Techniques

- Provide a solution to the problems.
- Backtracking solve problem recursively step by step or incrementally.
- objective is determine all possible solution.

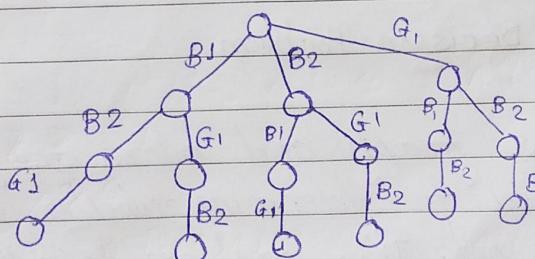
- Backtracking is an algorithmic technique. Use to solve a problem recursively And trying to build solution incrementally, step by step one piece or part a solution at a time and remaining those solution that failed to satisfies the constraints of the problem are removed.

objective

- Is to determine all possible solution.
- In Backtracking problem is solved using state space tree.

P : B₁, B₂, G₁, chairs: □□□

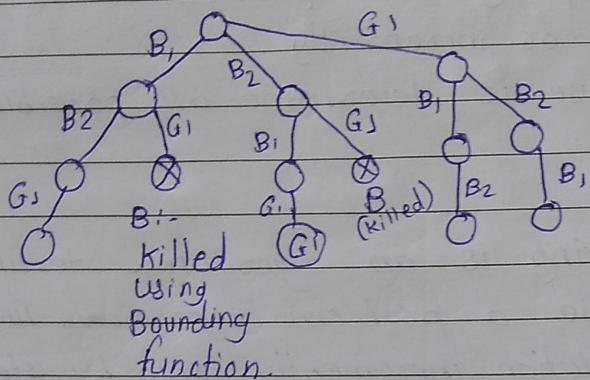
Constraint :-
G₁ &
cond:- should not sit on 2nd chair.



No. of Possible Solution :-

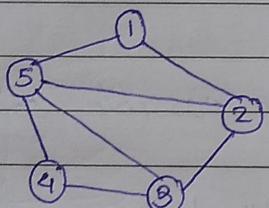
$$3! = 6$$

Now check the condition and remove those solution which not satisfies the condition



- ## i) Graph Colouring Problem.

m = indicating no. of colours.



These adjacent nodes should not give same colours.

- i) MC colouring Decision problem
 - ii) MC colouring Optimization problem

 - i) MC colouring Decision Problem

In this graph and colors are given to you and you need to determine whether that graph is coloured or not. This problem is called as MCelosing decision problem.

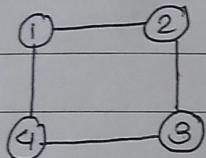
2) Mcoloring optimization Problem

Only graph is given and you need to decide minimum how many colors to required to color the given graph, so this problem is called mColoring optimization problem.

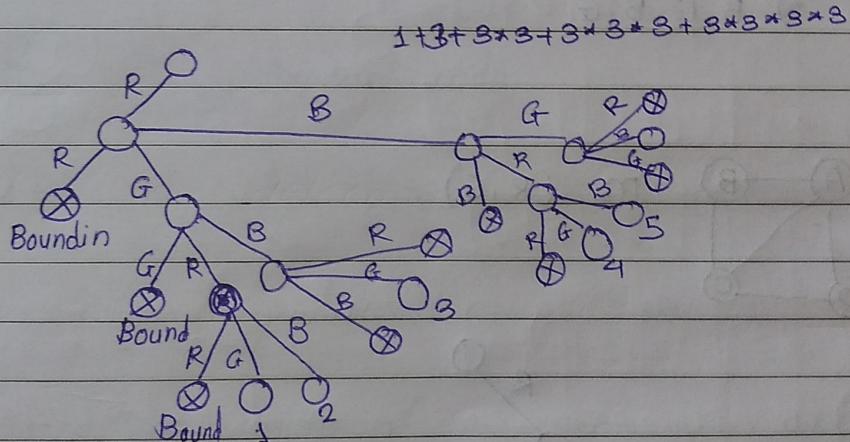
Example:-

i)

$$m = 3 \quad (\text{RGB})$$



start with vertex ①.



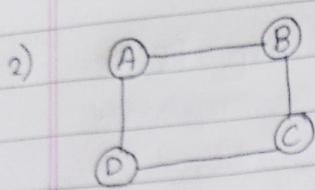
- 1) RGRG
 - 2) RG RB
 - 3) RG BG
 - 4) RBRG
 - 5) RB RB

$$1 + 3 + 3^2 \times 3 + 3^2 \times 3^2 \times 3 + 3^2 \times 3^2 \times 3^2 \times 3$$

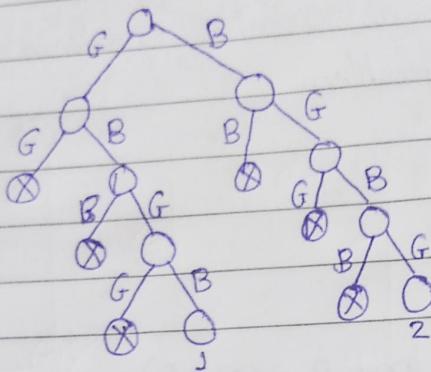
$$1 + 3^2 + 3^2 + 3^3 + 3^4 = \frac{3^{4+1}-1}{3}$$

$$\therefore O(3^{n+1}) = \boxed{C^{n+1}}.$$

when we killed it using Bounding function.
 $\therefore O(g^n)$



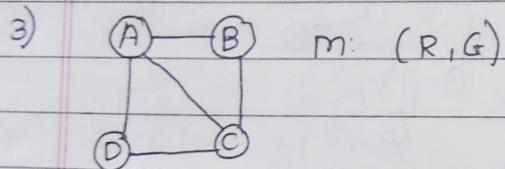
$m: (G, B)$



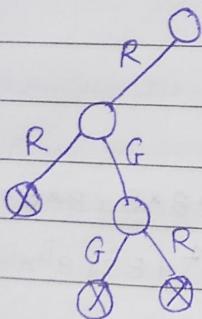
• Feasible Solution:-

1) G, B, G, B

2) B, G, B, G

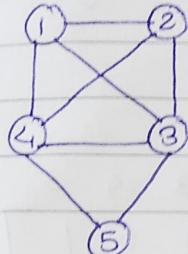


$m: (R, G)$

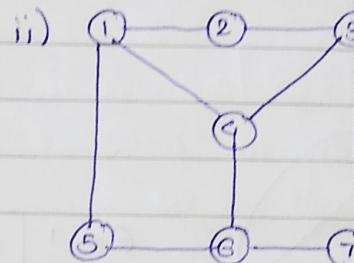


▶ Hamiltonian Cycle.

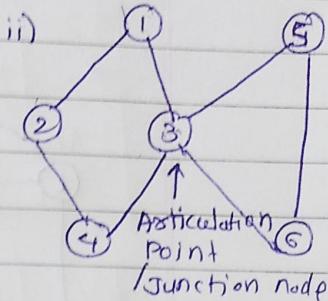
i)



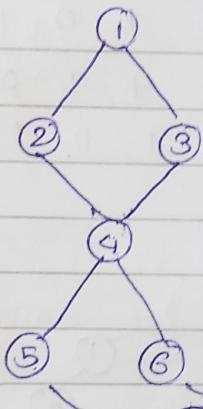
ii)



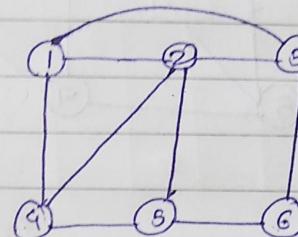
iii)



iv)



v)



Pendant node.

- A graph may be directed or undirected but it must be connected, in disconnected graph there is no possibility of hamiltonian cycle.
- Start from any vertex of a graph and visit all remaining nodes or vertices exactly once and return back to starting vertex.

Possible cycles are present in the v graph.

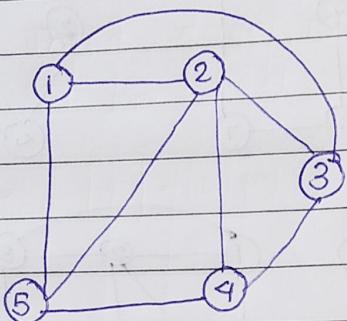
v) i - 1, 2, 3, 7, 5, 6, 1

1, 2, 6, 5, 7, 3, 1

1, 6, 5, 7, 3, 2, 1

2, 3, 1, 5, 6, 1, 1, 2 ← This cycle is same as

If graph is given and there are articulation points or pendant nodes are present then there is not possible of Hamiltonian cycle.



	1	2	3	4	5
1	0	1	1	0	1
2	1	0	1	1	0
3	1	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

take all 0's. $\alpha = [0 \ 0 \ 0 \ 0 \ 0]$

first vertex (1) $\alpha = [1 \ 0 \ 0 \ 0 \ 0]$

add all
vertex

$$\alpha = [1 \ 2 \ 0 \ 0 \ 0]$$

$$\alpha = [1 \ 2 \ 0 \ 0 \ 0]$$

$$\alpha = [1 \ 2 \ 3 \ 4 \ 0]$$

$$\alpha = [1 \ 2 \ 3 \ 4 \ 5]$$

Now check any edge
present in 3 to 2 yes -

$$\alpha = [1 \ 2 \ 4 \ 0 \ 0]$$

$$\alpha = [1 \ 2 \ 4 \ 3 \ 0]$$

$$\alpha = [1 \ 2 \ 4 \ 3 \ 5]$$

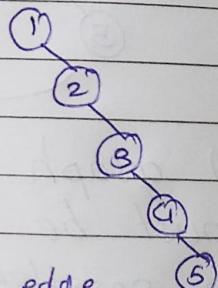
Now, after 5 these
no any vertex
so place 0 of 5.

$$\alpha = [1 \ 2 \ 3 \ 4 \ 0]$$

We got 0 to
go one step
back, and
check these any
edge is present

$$\alpha = [1 \ 2 \ 3 \ 0 \ 0]$$

5 to 3 these no edge so
place 0 to 5



1	2	3	4	3	5
---	---	---	---	---	---

8 to 5 no direct edge so place 0.

x =	1	2	4	3	0	-
-----	---	---	---	---	---	---

x =	1	2	4	5	0
-----	---	---	---	---	---

x =	1	2	4	5	3
-----	---	---	---	---	---

x =	1	2	4	5	0
-----	---	---	---	---	---

→ place 0 to 5 bcz no edges to present over 5.

x =	1	2	4	0	0
-----	---	---	---	---	---

How get 0 so go one step back and change the 4 in 5 bcz we already use 3

x =

x =	1	2	5	0	0
-----	---	---	---	---	---

How check there any edge 5 to 2, if yes go next step.

x =	1	2	5	3	0
-----	---	---	---	---	---

Here no edge betn 5 to 3 so place 0.

x =	1	2	5	0	0
-----	---	---	---	---	---

How go one step back bcz we got 0.

* There is present edge in 5 to 2 so go next step

x =	1	2	5	3	0
-----	---	---	---	---	---

No edge betn 5 to 3

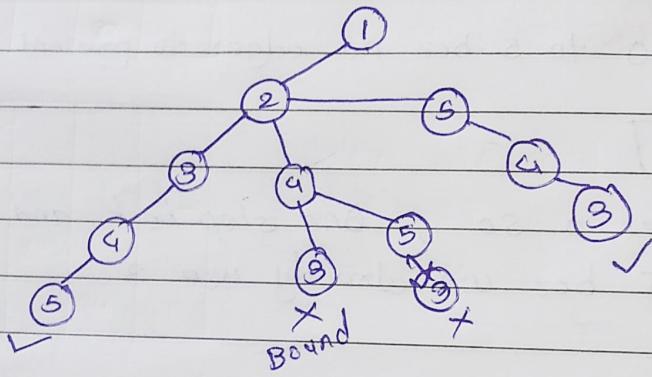
x =	1	2	5	0	0
-----	---	---	---	---	---

* B to 2 these edge is present so replace 2 to 0. bcz we can't replace with 3 bcz putting we can't got cycle.

$$x = [1 | 2 | 5 | 4 | 0]$$

These edge is present between 5 to 4.
so go next step.

$$x = [1 | 2 | 5 | 4 | 3]$$

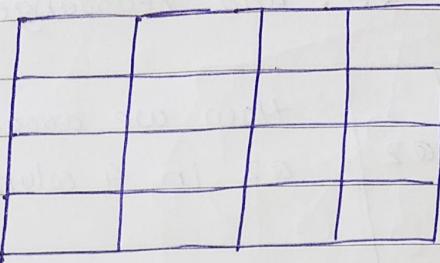


Then Process of Hamilton cycle.

- When these no next vertex present put 0.
- When 0 is come then using backtracking go back one step and change it.
- check if these edge is present then go next step and change it.

N-Queen Problem. 8/4.

chessboard -



4-queen Problem.

4×4

In this we will have 4 queens from Q_1 to Q_4 and we need to place the queens on chessboard such that no two queens are under attack.

When Queens are under attack?

When they are in same row, same column and same diagonal.

No of possible: ${}^{16}C_4$ - It is time consuming so we apply row, column, and diagonal.

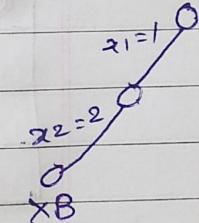
Q_1			
Q_2			

- First placed Q_1 in 1st row, column.

- Now place Q_2

- Here Q_1 and Q_2 are in diagonal.

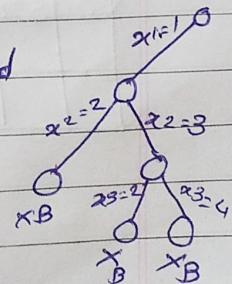
so killed it.



Q_1			
Q_2	Q_3		

- Now we place Q_2 in second row and third column using backtracking.

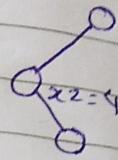
- Place Q_3 .



But here we can't place Q_3 because of ~~Q_2~~ is
So Remove Q_3 , and rearrange Q_2 .

Q_1			
		Q_2	

How we arrange
 Q_2 in 4 column.



Q_1			
		Q_2	
	Q_3		
		Q_4	

Put Q_3 in 2nd block.

Q_1			
		Q_2	
	Q_3		
		Q_4	

Here we put Q_4 in 3rd block
but Q_3 and Q_4 are in under attack.

Q_1			
		Q_2	
	Q_3		
		Q_4	

When Q_1 is in block 1
Then we can't place queens without attack.

Q_1			

So, we put Q_1 in 2nd block.

	Q1		
Q2			

diagonal that comes
in under attack.

	Q3		
	Q2		

That also diagonal
in under attack.

	Q1		
	Q2		

So there that two
queens are not in
under attack.

Now place Q3.

	Q1		
		Q2	
Q3			

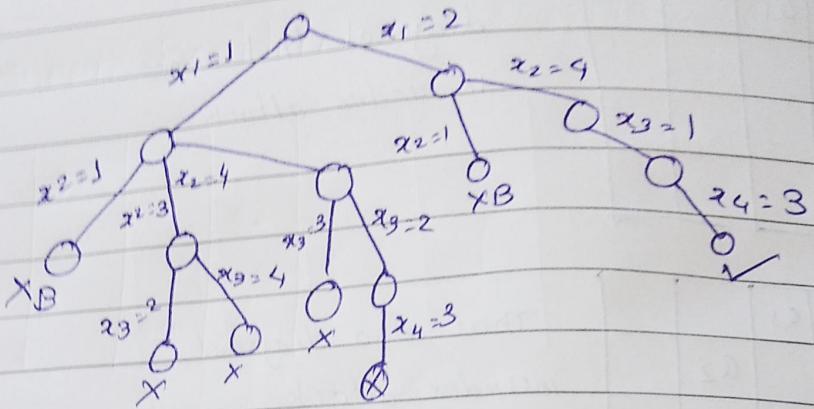
Place Q3 in first
column so therefore
they are not in attack.

Now place Q4

	Q1		
		Q2	
Q3			
	Q4		

This is perfect solution to 4'

Here first check the ^{all} possible solutions and then place
the queens.



Solution

α_1	α_2	α_3	α_4	
2	4	1	3	← column

13-may-2023
Saturday.

► Sum of Subsets :-

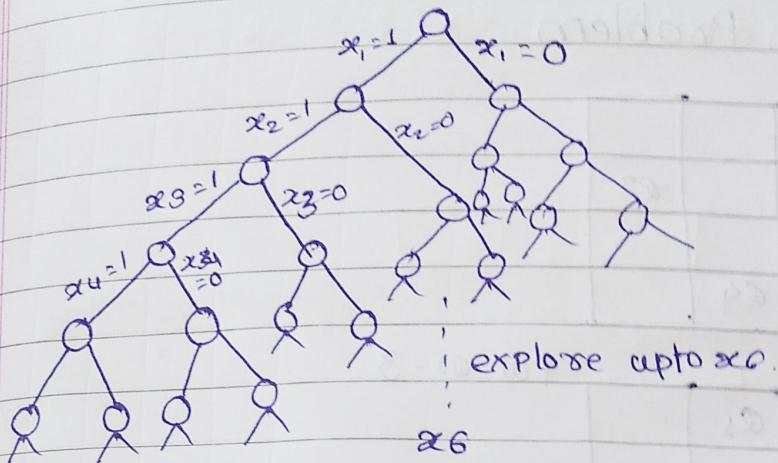
w = weight

w[1:6] = {5, 10, 12, 13, 15, 18}

Total weight is = 73

Constraints : Get only or exactly ~~at~~ 80 weight from set.

$\Delta e = 1/0 \quad \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{matrix}$



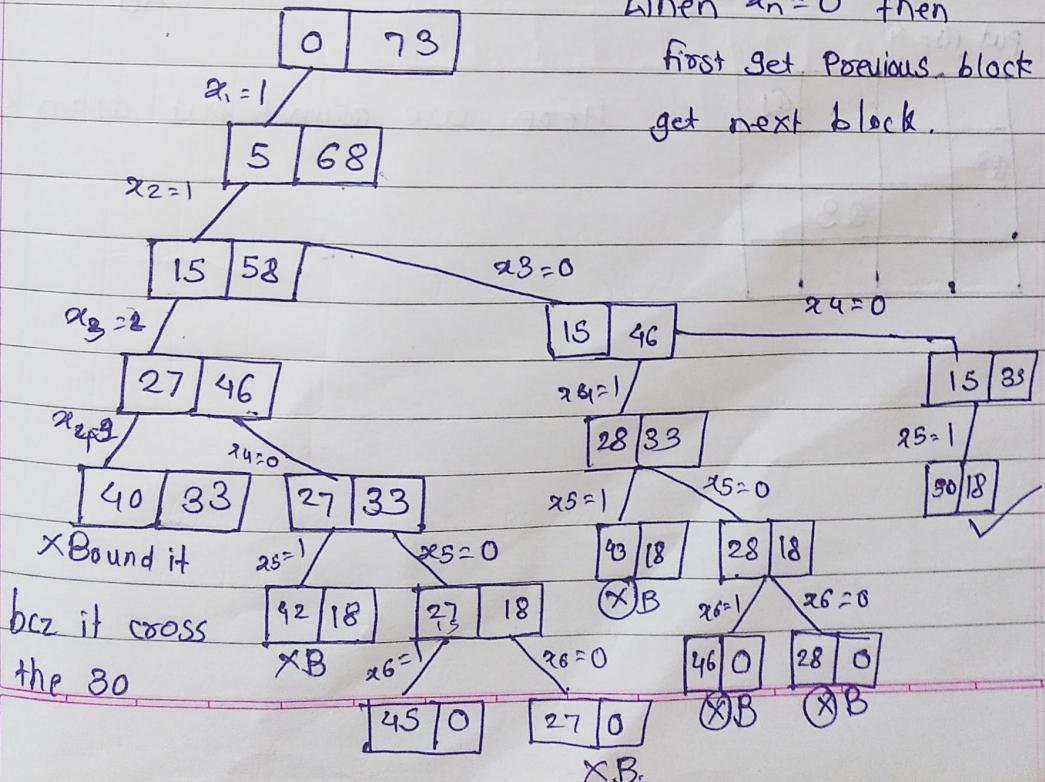
Total no of levels/ height = 7.

Possible solutions = 2^6
 $\therefore 6 = \text{level} - 1$.

If value is more large and therefore it is time consuming so we apply backtracking.

When $x_n = 0$ then

first get previous block then
get next block.



N-Queen Problem

	Q1		
.	.	.	
		Q2	
Q3			

put $Q_1 = 3$

		Q1	
Q2	.	.	.
.	.	.	
Q4			Q3

	Q1		
		Q2	

It is diagonal so in
under attack

$$x_1 = 3$$

$$x_2 = 4$$

X_B

put $Q_1 = 4$

			Q1
Q2			
.			
	Q3		

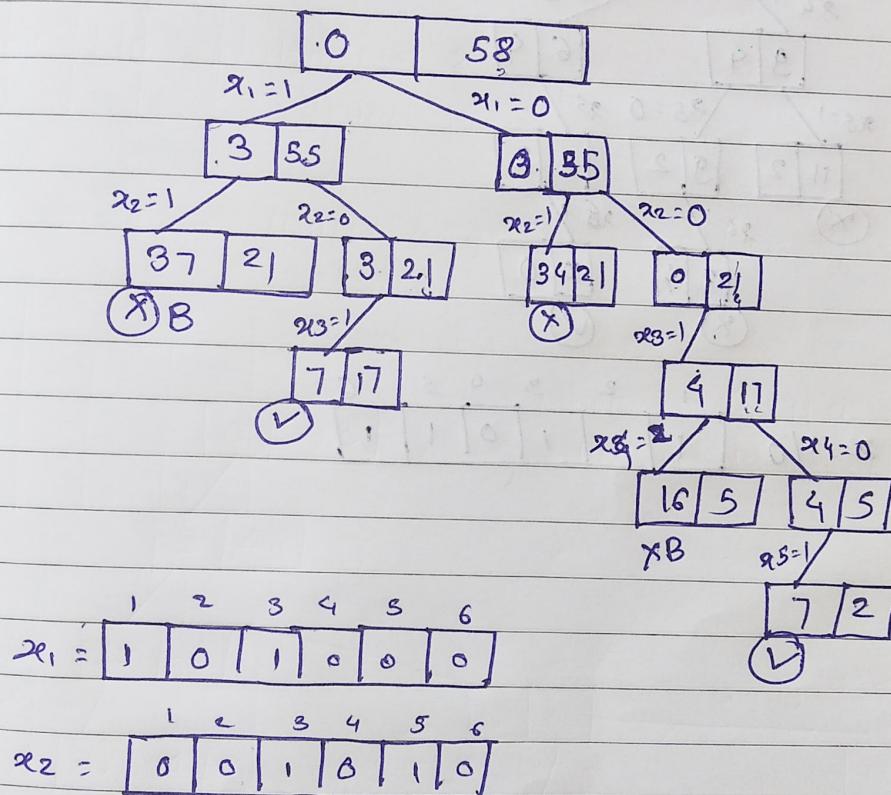
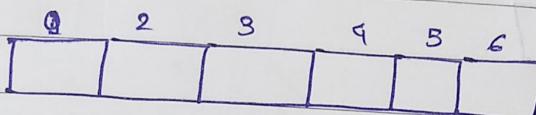
Here we can't put Q_1 in 4 block

Sum of subset - Example.

$$W[3:6] = \{3, 34, 4, 12, 3, 2\}$$

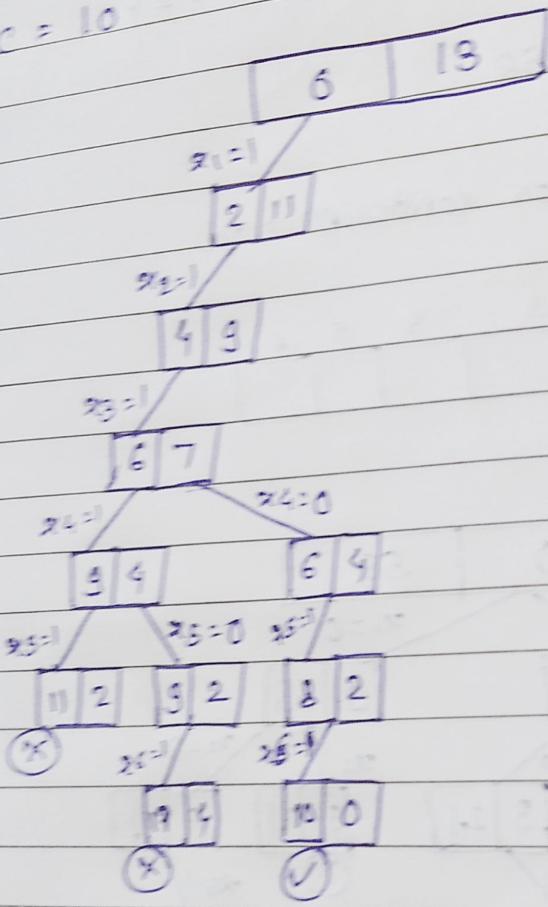
Total weight = 58

Sum or condition or constraint = 7.



$$\text{an } D[6] = \{2, 4, 2, 5, 2, 2\}, = 13$$

$$c = 10$$



	1	2	3	4	5	6
$x = r/0$	1	1	1	0	1	1

$$\text{cof}[1:7] = \{7, 3; 8, 5, 4, 6, 2\} = 35$$

Sum = 18

