Unit

## - Graph -

- A Graph G = (V, E) consists of a set of vertices (also called nodes) and a set E of edges.

- If an edge connects to a vertex we say the edge is incident to the vertex and say the vertex is an endpoint of the edge.

- If the edge has only one endpoint then it is called a loop edge.

- If two or more edges have the same endpoints then they are called multiple or parallel edges.

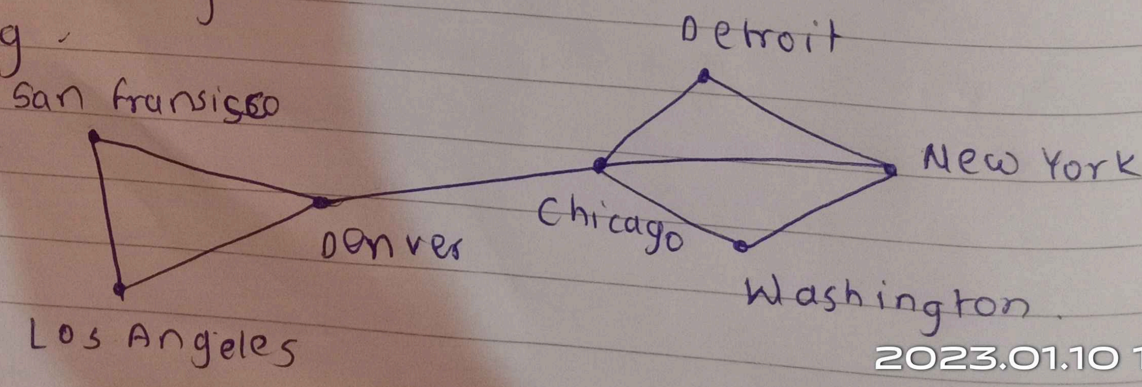
- Two vertices that are joined by an edge are called adjacent vertices.

- A pendant vertex is a vertex that is connected to exactly one other vertex by a single edge.

- **Simple Graph -**

A Graph. G = (V, E) consist of V, a non-empty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

eg.

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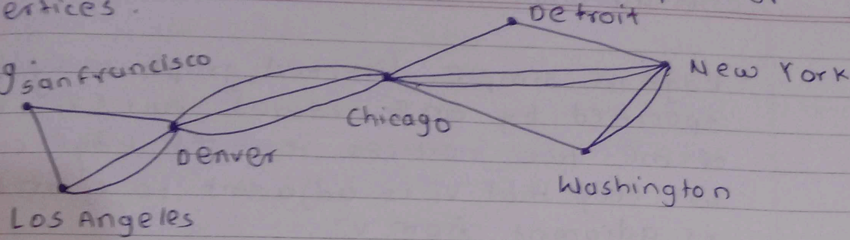
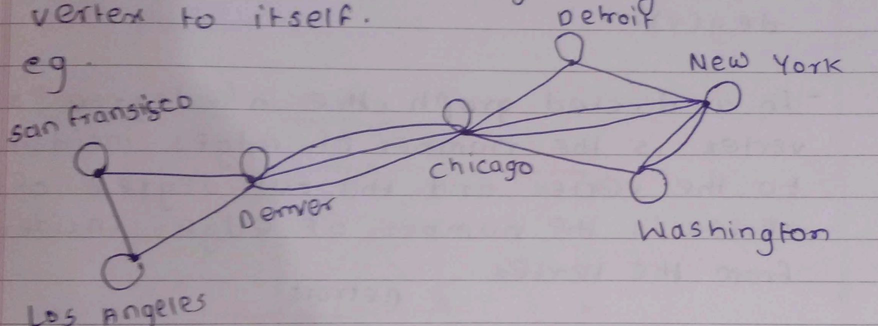
- <u>A Multigraph</u> –
  In a multigraph $G = (V, E)$ two or more edges may connect the same pair of vertices.

eg:

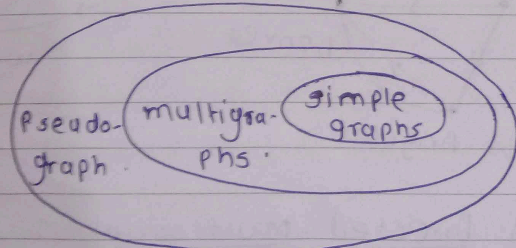



- <u>pseudograph</u> :
  In a pseudograph $G = (V, E)$ two or more edges may connect the same pair of vertices, and in addition, an edge may connect a vertex to itself.

eg.




- <u>Undirected</u> <u>graph</u> :



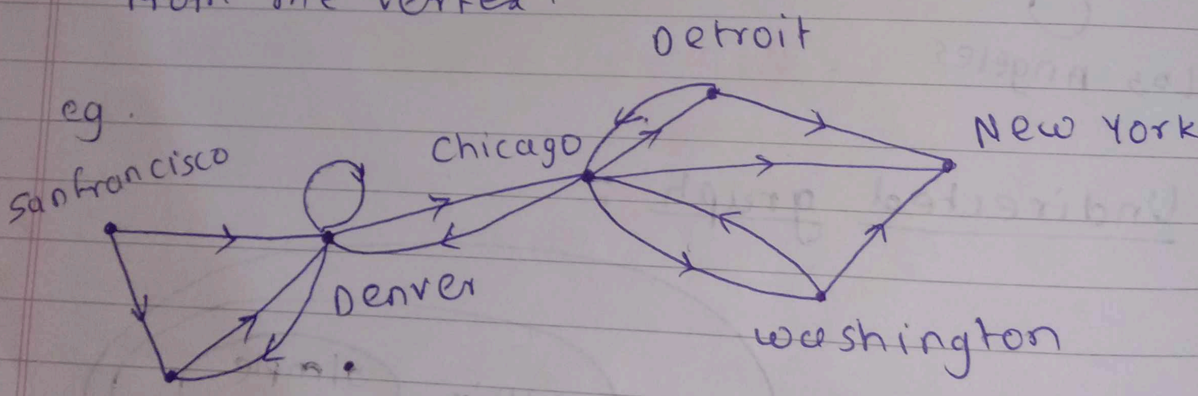
- **Directed Graph :**
  - A directed graph is a graph in which the edges may only be traversed in one direction.

  - Edge in simple directed graph may be specified by an ordered pair $(v_i, v_j)$ of the two vertices that the edge connects. We say that $v_i$ is adjacent to $v_j$ and $v_j$ is adjacent from $v_i$.

- **Degree of vertex :**
  - The degree of a vertex is the number of edges incident to the vertex and is denoted deg(v).

  - In a directed graph, the in-degree of a vertex is the number of edges incident to the vertex and the out-degree of vertex is the number of edges incident from the vertex.

eg.

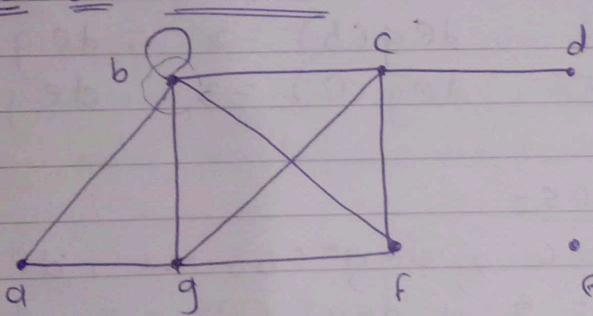



- **A Directed Multigraph :**
  - In a directed multigraph $G = (V, E)$ the edges are ordered pairs of vertices, and in addition there may be multiple edges.

# Types of Graphs

| Type | Edges | Multiple Edges Allowed. | loops Allowed. |
|---|---|---|---|
| simple graph | undirected | NO | NO |
| Multigraph | undirected | yes | NO |
| pseudograph | undirected | Yes | yes. |
| Directed graph | directed | No | ~~Yes~~ NO. |
| directed multigraph | directed | yes | Yes. |

- **Degree of vertex.**




find the degrees of all the vertices :

deg(a) = 2, deg(b) = 6, deg(c) = 4
deg(d) = 1, deg(e) = 0, deg(f) = 3
deg(g) = 4.

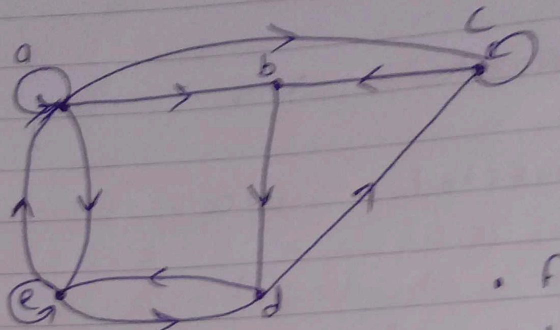
- **In-degree of a vertex v -:**

- The number of vertices adjacent to v (the number of edges with v as their terminal vertex.
- Denoted by deg(v):

- Out-degree of a vertex v -
  - The number of vertices adjacent from v (the number of edges with v as their initial vertex).

- A loop at a vertex contributes 1 to both the indegree and out degree.

eg.




- Find the in-degrees and out-degrees of this digraphs.

In-degrees -

$deg(a) = 2$, $deg(b) = 2$, $deg(c) = 3$,
$deg(d) = 2$, $deg(e) = 3$, $deg(f) = 0$

out-degrees -

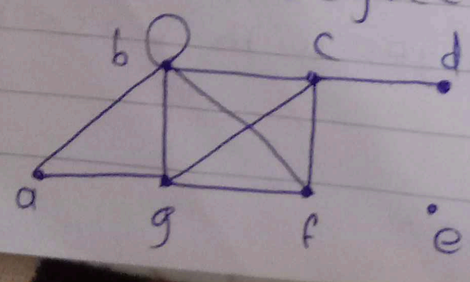
$deg^+(a) = 4$, $deg^+(b) = 1$, $deg^+(c) = 2$,
$deg^+(d) = 2$, $deg^+(e) = 3$, $deg^+(f) = 0$

- Degree of undirected graph:

The degree of a vertex is an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.

eg.

- Find the degree of all the other vertices



$deg(a) = 2$ $deg(c) = 4$ $deg(f) = 3$

$deg(g) = 4$ $deg(b) = 6$ $deg(d) = 1$

$deg(e) = 0$



Total of degrees $= 2+4+3+4+6+1+0 = 20$

- **Handshaking Theorem :**

- In an undirected graph,

$$|E| = \frac{1}{2} \sum_{e \in E} deg(e) \quad = \quad 2E = \sum_{v \in V} deg(v)$$

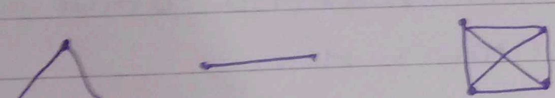
- In directed graph -

$$|E| = \sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}()$$

- **Complete graph -**

- The complete graph on n vertices ($K_n$) is the simple graph that contains exactly one edge betn each pair of distinct vertices.

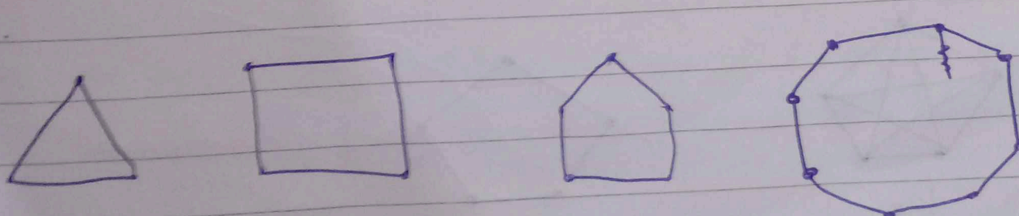
eg



- **cycle -**

- The cycle $C_n$ ($n \geq 3$), consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$.

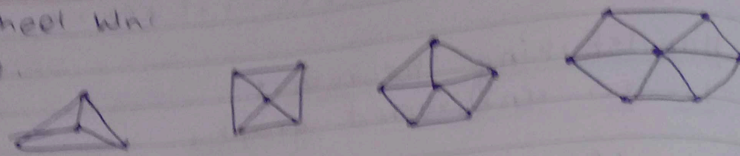
eg.



- **Wheel** -

When a new vertex is added to a cycle $C_n$ and this new vertex is connected to each of the $n$ vertices in $C_n$, we obtain a wheel $W_n$.

eg.



- **Connected graph** -

A graph is connected if there is a walk between every pair of distinct vertices in the graph. / if there is a path betn every pair of distinct vertices of the graph.

- **Subgraph** -

A graph H is a subgraph of a graph G if all vertices and edges in H are also in G.

- **Connected component** :

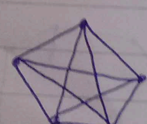
A connected component of G is is a connected subgraph H of G such that no other connected subgraph of G contains H.

- **Subgraph** -

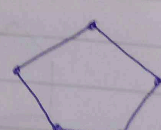
A subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$.

eg.

① $C_5$ is a subgraph of $K_5$



$K_5$



$C_5$

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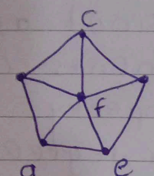
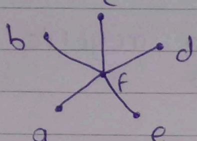
- **Union :** The union of 2 simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. The union is denoted by $G_1 \cup G_2$

eg.

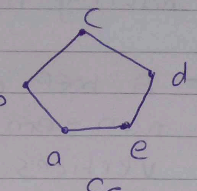
$W_5$ is the union of $S_5$ and $C_5$




$W_5$



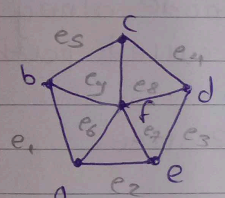

$S_5$




$C_5$

$S_5 \cup C_5 = W_5$

- **Adjacency Matrix :**




$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

- **Euler's Theorem :**

Let G be an connected, undirected graph. G has an Eulerian circuit if and only if every vertex in G has even degree.

G has an Eulerian trail if and only if G has exactly two vertices with odd degree.

- **Hamiltonian Circuits and Trails:**

  Given a connected graph G, does G have a circuit in which every vertex is visited exactly once, is called Hamiltonian Circuit.

- **Tree**

  A tree is an acyclic graph or graph having no cycle.

- A tree or general trees is defined as a non-empty finite set of elements called vertices or nodes having the property that each node can have minimum degree 1 and maximum degree n.

**Tree Terminology**

① **Path**: Traversal from node to node along the edges results in a sequence called path.

② **Root**:- Node at the top of the tree.

③ **Parent** - Any node, except root has exactly one edge running upward to another node. The node above it is called parent.

④ **child** - Any node may have one or more lines running downwards to other nodes. Nodes below are children.

⑤ **Leaf**: A node that has no children.

⑥ **Subtree**: Any node can be considered to be the root of a subtree, which consists of its children and its children's children and so on.

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(7) Traversing - To traverse a tree means to visit all the nodes in some specified order.

(8) Levels - The level of a particular node refers to how many generations the node in is from the root.

- Root is assumed to be level ( ).

(9) Keys - key value is used to search for the item or perform other operations on it.

- Finite number of elements called nodes & finite set of directed lines called branches, that connect the nodes.

- Branch directed towards a node is an In-degree Branch.

- Branch directed away from the node is out-degree Branch.

- Sum of the number of In-degree and out-degree node branches = degree of node.

- Nodes that are neither root nor a leaf are known as Internal nodes.

- Children of the same Parent are siblings.

- Path is a sequence of nodes in which each node is adjacent to the next one.

- level of a node is its distance from the root.

- Height (depth) of the Tree is the level of the ~~level of the~~ leaf in the longest path from the root.

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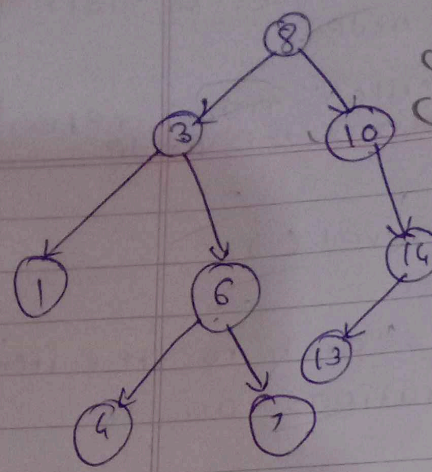


- No. of nodes = 9
- (node) Height = 4
- (edge) Heighest level = 3
- Root Node = 8
- Leaves = 1, 4, 7, 13
- Interior nodes = 3, 10, 6, 14
- Ancestors of 6 = 3, 8.
- Descendents of 10 = 14, 13
- sibling of 1 = 6

- Ancestors of a node are all the nodes along the path from the root to the node

- <u>Ordered Trees</u> -
  - If in a tree at each level, an ordering is defined, then such a tree is called an ordered tree.

- <u>Rooted Treees</u> -
  - If a directed tree has exactly one node or vertex called root whose incoming degrees is 0 and all other vertices have incoming degree one, then the tree is called rooted tree.

<u>Note</u>: 1) A tree with no nodes is a rooted tree.
2) A single node with no children is a rooted tree.