$[A]^T = A$

$\sim A = A$

converse = Transpose.

Tutorial - 6

Q.1 Consider a 2-D array A whose subscript limits are $-3 \leq i \leq 6$, $3 \leq j \leq 10$. Give the addressing function for element A[i, j] where the storage representation is row wise order. (Assume size of element = 1)

→ $-3 \leq i \leq 6 \quad = \quad 10$

$3 \leq j \leq 10 \quad = \quad 8.$

1) Row major:

$A[i,j] = b_0 + (i+4-1) \times 8 \times 1 + (j-2-1)(1).$

2) Column Major:

$A[i,j] = b_0 + (j-2-1) \times 10 \times 1 + (i+4-1) \times 8 \times (1)$

Q.2 Show that if a relation R is reflexive the R converse is also reflexive. Also, similar remark holds if R is transitive, irreflexive, symmetric and anti-symmetric.

→ R is reflexive.

$\therefore (x)(xRx \rightarrow x\tilde{R}x)$

Consider,

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \text{Converse } (\tilde{R}) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$\therefore R = \tilde{R}$

Consider,

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\therefore \tilde{R} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Converse of irreflexive is irreflexive.

i.e. $(x)\,(x \not R x \rightarrow x \not\tilde{R} x)$

3) Symmetric :

Consider,

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore R^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ ...... (converse of R)}$$

$\therefore \tilde{R}$ is equal to symmetric R.

i.e. $(y)(x)\,(x\tilde{R}y \Rightarrow y\tilde{R}x)$

$\because$ From symmetricity,

$(y)(x)\,(yRx \rightarrow xRy)$

4) Antisymmetric,

$(x)(y)\,(x \in X \wedge y \in X \wedge xRy \wedge yRx \Rightarrow x = y)$

$\therefore (x)(y)\,(x \in X \wedge y \in X \wedge y\tilde{R}x \wedge x\tilde{R}y \Rightarrow y = x)$

Date ________
Page ________

Q.3 $M_R = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 1 & 1 & 0 \\ c & 1 & 1 & 1 \end{array}$. Find $\tilde{R}$, R^2, R^3, $R \cdot \tilde{R}$

→

1) $\tilde{R} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

2) $R^2 = R \cdot R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$M_{R^2} = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

3) $R^3 = R^2 \cdot R = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore M_{R^3} = \begin{bmatrix} 5 & 3 & 4 \\ 4 & 2 & 3 \\ 7 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

4) $R \cdot \tilde{R} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$M_{R \cdot \tilde{R}} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore R^+ = R \cup R^2 \cup R^3 \ldots$

$R^+ = 9.$

\# Covering = $\{\overline{1, 2, 3}, \overline{4, 5, 6}\}$ — (Not disjoint Partitions).

\# Partition = $\{\overline{1, 2, 3}, \overline{4, 5, 6}\}$ — (disjoint)

Date

Page

Q. Given $S = \{1, 2, 3, 4, 5\}$. Find equivalence relation which generate a partition $\{\overline{1, 2}, \overline{3}, \overline{4, 5}\}$. Draw the graph of relation.

→

$\{3\}$

$\{1, 2\}$

$\{4, 5\}$