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Tutorial 8

Q1) If fuzzy set $A(x) = \frac{x}{x+3}$, $x = [0, 1, 2, \ldots 10]$
then find,

i) α-cut of A for $\alpha = 0.6$

we find α-cut of fuzzy set A as,

$$\alpha_A = \{x \in X \mid A(x) \geq \alpha\}$$

$$A(x) = \frac{x}{x+3} \qquad x = [0, 1, 2, \ldots 10]$$

$$A = \left\{ \frac{\frac{0}{3}}{0}, \frac{\frac{1}{4}}{1}, \frac{\frac{2}{5}}{2}, \frac{\frac{3}{6}}{3}, \frac{\frac{4}{7}}{4}, \frac{\frac{5}{8}}{5}, \frac{\frac{6}{9}}{6}, \frac{\frac{7}{10}}{7}, \frac{\frac{8}{11}}{8}, \frac{\frac{9}{12}}{9}, \frac{\frac{10}{13}}{10} \right\}$$

$$\alpha_A = \{x \in X \mid A(x) \geq \alpha\}$$

$$A = \left\{ \frac{0}{0}, \frac{0.25}{1}, \frac{0.40}{2}, \frac{0.50}{3}, \frac{0.57}{4}, \frac{0.625}{5}, \frac{0.66}{6}, \frac{0.7}{7}, \frac{0.72}{8}, \frac{0.75}{9}, \frac{0.76}{10} \right\}$$

$$0.6_A = \{x \in x \mid A(x) \geq 0.6\}$$

$$0.6A = \{5, 6, 7, 8, 9, 10\}$$

ii) Strong α-cut of A for $\alpha = 0.7$

$$\alpha_A^+ = \{x \in X \mid A(x) > \alpha\}$$

$$0.7_A^+ = \{x \in X \mid A(x) > 0.7\}$$

$$0.7_A^+ = \{8, 9, 10\}$$

3) level set of fuzzy set A:

$$\Lambda A = \{\alpha \mid A(x) = \alpha \text{ for some } x \in X\}$$

$$\Lambda A = \{0, 0.25, 0.40, 0.50, 0.57, 0.625, 0.66, 0.7, 0.72, 0.75, 0.76\}$$

4) Support of fuzzy set A:

we defined support of fuzzy set A as,

$$0^+_A = \{x \in X \mid A(x) > 0\}$$

$$0^+_A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

5) Core of fuzzy set A.

we defined core of fuzzy set as,

$$^1A = \{x \in X \mid A(x) = 1\}$$

$$^1A = \{\phi\} = \phi$$

6) height of fuzzy set A:

we defined height of fuzzy set A as:

$$h(A) = \sup_{x \in X} A(x)$$

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$h(A) = 0.76$

hence, fuzzy set is subnormal.

7) Cross over point of fuzzy set A.

Cross over point of fuzzy set A is a point $x \in X$ at which $A(x) = 0.5$.

here, $x = 3$ is crossover point of fuzzy set A (equilibrium point)

8) cardinality of fuzzy set A.

The no. of elements in a set

$|X| = 11.$

9) Scalar Cardinality of fuzzy set A.

we defined scalar cardinality of fuzzy set A.

$$|A| = \sum_{x \in X} A(x)$$

$$= 0 + 0.25 + 0.40 + 0.50 + 0.57 + 0.625 + 0.66 + 0.7 + 0.72 + 0.75 + 0.76\}$$

$$|A| = 5.92$$

10) Relative cardinality of fuzzy set A.

$$\|A\| = \frac{|A|}{|X|} = \frac{5.92}{11} = 0.538$$

Q.2) Determine α-cut and strong α-cut of fuzzy set A for $\alpha = 0.2, 0.5, 0.8$

where,

$$A(x) = \begin{cases} \frac{x+3}{3} & \text{if } -3 < x \leq 0 \\ \frac{3-x}{3} & \text{if } 0 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

⟹ Given fuzzy set,

$$A(x) = \begin{cases} \frac{x+3}{3} & \text{if } -3 < x \leq 0 \\ \frac{3-x}{3} & \text{if } 0 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

we know that α-cut of fuzzy set A,

$${}^{\alpha}A = \{x \in X \mid A(x) \geq \alpha\}$$

Consider,	consider,
$A(x) \geq \alpha$	$A(x) \geq \alpha$
$\frac{x+3}{3} \geq \alpha$	$\frac{3-x}{3} \geq \alpha$
$x+3 \geq 3\alpha$	$3-x \geq 3\alpha$
$x \geq 3\alpha - 3$ — ①	$3-3\alpha \geq x$ — ②

from equation ① & ②

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given, $\alpha_A = [3\alpha-3,\ 3-3\alpha]$
$\alpha = 0.2, 0.5, 0.8$
putting values of α,

$0.2_A = [-2.4, 2.4]$

$0.5_A = [-1.5, 1.5]$

$0.8_A = [-0.6, 0.6]$

for strong α-cut,

$$\alpha_A^+ = \{x \in X \mid A(x) > \alpha\}$$

consider,

$A(x) > \alpha$

$\frac{x+3}{3} > \alpha$

$x+3 > 3\alpha$

$x > 3\alpha - 3$ — ③

consider,

$A(x) > \alpha$

$\frac{3-x}{3} > \alpha$

$3-x > 3\alpha$

$3-3\alpha > x$ — ④

from eqn ③ & ④,

$\cancel{0.2}\ \alpha_A^+ = [3\alpha-3,\ 3-3\alpha]$

given, $\alpha = 0.2, 0.5, 0.8$

$\therefore 0.2_A^+ = [-2.4, 2.4)$

$\therefore 0.5_A^+ = (-1.5, 1.5)$

$0.8_A^+ = (-0.6, 0.6)$

$(D_1 \cap D_2)(x) = \min \{D_1(x), D_2(x)\}$

$\overline{A(x)} = 1 - A(x)$

Q.4) Verify Commutative law for the fuzzy sets.

$$D_1 = \left\{ \frac{1}{1} + \frac{0.75}{1.5} + \frac{0.3}{2}, \frac{0.15}{2.5} + \frac{0}{3} \right\}$$

$$D_2 = \left\{ \frac{1}{1}, \frac{0.6}{1.5} + \frac{0.2}{2} + \frac{0.1}{2.5} + \frac{0}{3} \right\}$$

$$(D_1 \cup D_2)(x) = \max \{D_1(x), D_2(x)\}$$

$$= \left\{ \frac{1}{1} + \frac{0.75}{1.5} + \frac{0.3}{2} + \frac{0.15}{2.5} + \frac{0}{3} \right\}$$

$$(D_2 \cup D_1)(x) = \max \{D_2(x), D_1(x)\}$$

$$= \left\{ \frac{1}{1} + \frac{0.75}{1.5} + \frac{0.3}{2} + \frac{0.15}{2.5} + \frac{0}{3} \right\}$$

$$\therefore (D_1 \cup D_2)(x) = (D_2 \cup D_1)(x)$$

Q.5) Consider fuzzy sets,

$A(x) = \frac{2x}{2x+5}$, $B(x) = \frac{x}{x+1}$ for $x \in \{6, 7, \dots 10\}$

find i) $A \cup (\overline{A \cap B})$

ii) α-cut of $A \cup (\overline{A \cap B})$ for $\alpha = 0.5, 0.7, 0.9$

iii) Scalar cardinality of $A \cup (\overline{A \cap B})$.

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i) $A \cup (\overline{A \cap B})$

$A \cap B(x) = \min \{A(x), B(x)\}$

$A = \left\{ \frac{12/17}{6}, \frac{14/19}{7}, \frac{16/21}{8}, \frac{18/23}{9}, \frac{20/25}{10} \right\}$

$= \left\{ \frac{0.705}{6} + \frac{0.73}{7} + \frac{0.761}{8}, + \frac{0.782}{9} + \frac{0.8}{10} \right\}$

$B = \left\{ \frac{6/7}{6}, \frac{7/8}{7}, \frac{8/9}{8}, \frac{9/10}{9}, \frac{10/11}{10} \right\}$

$= \left\{ \frac{0.857}{6} + \frac{0.875}{7} + \frac{0.888}{8} + \frac{0.9}{9} + \frac{0.909}{10} \right\}$

$(A \cap B)(x) = \min \{A(x), B(x)\}$

$= \left\{ \frac{0.705}{6} + \frac{0.73}{7} + \frac{0.761}{8} + \frac{0.782}{9} + \frac{0.8}{10} \right\}$

$(\overline{A \cap B})(x) = \left\{ \frac{0.295}{6} + \frac{0.27}{7} + \frac{0.24}{8} + \frac{0.22}{9} + \frac{0.2}{10} \right\}$

$\cancel{A \cup B \cup}$ $(A \cup (\overline{A \cap B}))(x) = \max \{A(x), (\overline{A \cap B})(x)\}$

$(A \cup (\overline{A \cap B}))(x) = \left\{ \frac{0.705}{6} + \frac{0.73}{7} + \frac{0.761}{8} + \frac{0.782}{9} + \frac{0.8}{10} \right\}$

ii) α cut of $A \cup (\overline{A \cap B})$ for $\alpha = 0.5, 0.7, 0.9$

we defined α-cut as,

$$^{\alpha}A = \{x \in X \mid A(x) \geq \alpha\}$$

i) $\alpha = 0.5$

$$^{0.5}A = \{x \in X \mid A(x) \geq 0.5\}$$

$$^{0.5}A = \{6, 7, 8, 9, 10\}$$

ii) $\alpha = 0.6$

$$^{0.6}A = \{x \in X \mid A(x) \geq 0.6\}$$

$$^{0.6}A = \{6, 7, 8, 9, 10\}$$

iii) $\alpha = 0.7$

$$^{0.7}A = \{x \in X \mid A(x) \geq 0.7\}$$

$$^{0.7}A = \{6, 7, 8, 9, 10\}$$

iv) $\alpha = 0.9$

$$^{0.9}A = \{x \in X \mid A(x) \geq 0.9\}$$

$$^{0.9}A = \phi$$

iii) scalar cardinality of $A \cup (\overline{A \cap B})$

special fuzzy set $= \alpha_A{}^A$

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we defined scalar cardinality of fuzzy set A,

$$|A| = \sum_{x \in X} A(x)$$

$$= 0.705 + 0.73 + 0.761 + 0.782 + 0.8$$

$$|A| = 3.778$$

Q.6) find α-cuts for distinct values of α of the fuzzy set A and hence find special fuzzy set where $A(x) = \left\{ \frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.4}{-1} + \frac{0.6}{0} + \frac{0.4}{1} \right\}$ that the standard union of these special fuzzy set is exactly the original fuzzy set A.

Speial fuzzy set $= \alpha . \alpha_A$

for $\alpha = 0.1$

special fuzzy set:

$$= 0.1 \cdot 0.1_A$$

$$= 0.1 \cdot \{x \in X \mid A(x) \geq 0.1\}$$

$$= 0.1 \left\{ \frac{1}{-3} + \frac{1}{-2} + \frac{1}{-1} + \frac{1}{0} + \frac{1}{1} \right\}$$

$$= \left\{ \frac{0.1}{-3} + \frac{0.1}{-2} + \frac{0.1}{-1} + \frac{0.1}{0} + \frac{0.1}{1} \right\}$$

for $\alpha = 0.3$

$$= 0.3 \cdot 0.3_A$$

subset hood degree $S(A,B) = \frac{|A \cap B|}{|B|}$

$= 0.3 \left\{ \frac{0}{-3} + \frac{1}{-2} + \frac{1}{-1} + \frac{1}{0} + \frac{1}{1} \right\}$

$= \left\{ \frac{0}{-3} + \frac{0.3}{-2} + \frac{0.3}{-1} + \frac{0.3}{0} + \frac{0.3}{1} \right\}$

for $\alpha = 0.4$

$= 0.4 \cdot {}_{0.4}A$

$= 0.4 \left\{ \frac{0}{-3} + \frac{0}{-2} + \frac{1}{-1} + \frac{1}{0} + \frac{1}{1} \right\}$

special fuzzy set $= \left\{ \frac{0}{-3} + \frac{0}{-2} + \frac{0.4}{-1} + \frac{0.4}{0} + \frac{0.4}{1} \right\}$

for $\alpha = 0.6$

$= 0.6 \cdot {}_{0.6}A$

$= 0.6 \left\{ \frac{0}{-3} + \frac{0}{-2} + \frac{0}{-1} + \frac{1}{0} + \frac{0}{1} \right\}$

$= \left\{ \frac{0}{-3} + \frac{0}{-2} + \frac{0}{-1} + \frac{0.6}{0} + \frac{0}{1} \right\}$

standard union of special fuzzy set,

${}_{0.1}A \cup {}_{0.3}A \cup {}_{0.4}A \cup {}_{0.6}A = \left\{ \frac{0.1}{-3} + \frac{0.3}{-2} + \frac{0.4}{-1} + \frac{0.6}{0} + \frac{0.4}{1} \right\}$

Q.3) find the degree of subset hood $S(\bar{A}, \bar{B})$ for fuzzy sets,

$A(x) = \frac{x}{x+3}$, $B(x) = 5^{-x}$, $x \in \{0, 1, 2, \ldots 5\}$

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degree of subsethood $S(\bar{A}, \bar{B}) = \frac{|\bar{A} \cap \bar{B}|}{|\bar{B}|}$

$A = \left\{ \frac{0}{0} + \frac{0.25}{1} + \frac{0.4}{2} + \frac{0.5}{3} + \frac{0.571}{4} + \frac{0.625}{5} \right\}$

$\bar{A} = \left\{ \frac{0}{0} + \frac{0.75}{1} + \frac{0.6}{2} + \frac{0.5}{3} + \frac{0.429}{4} + \frac{0.375}{5} \right\}$

$B = \left\{ \frac{1}{0} + \frac{0.2}{1} + \frac{0.04}{2} + \frac{0.008}{3} + \frac{0.0016}{4} + \frac{0.00032}{5} \right\}$

$\bar{B} = \left\{ \frac{0}{0} + \frac{0.8}{1} + \frac{0.96}{2} + \frac{0.992}{3} + \frac{0.9984}{4} + \frac{0.99968}{5} \right\}$

$\bar{A} \cap \bar{B} = \min(\bar{A}(x), \bar{B}(x)\}$

$\bar{A} \cap \bar{B} = \left\{ \frac{0}{0} + \frac{0.75}{1} + \frac{0.6}{2} + \frac{0.5}{3} + \frac{0.429}{4} + \frac{0.375}{5} \right\}$

scalar cardinality of $\bar{A} \cap \bar{B}$,

$|\bar{A} \cap \bar{B}| = \{0 + 0.75 + 0.6 + 0.5 + 0.429 + 0.375\}$

$= 2.654$

Scalar cardinality of $\bar{B}$,

$|\bar{B}| = 0 + 0.8 + 0.96 + 0.992 + 0.9984 + 0.99968$

$= 4.75008$

degree of subsethood $= 2.654 / 4.75008 = 0.5587$

Q.7) Let the membership grade function at fuzzy set A define on $X = [0, 1, 2 \cdots 10]$ be given by $A(x) = 2^{-x}$ $f : X \to N$ such that $y = f(x) = x^2$ $\forall x \in X$ use the extension principle and find $f(A)$.

$\Longrightarrow$ Given, $A(x) = 2^{-x}$ on $X = [0, 1, 2 \cdots 10]$

Let $y = f(x) = x^2 = [0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]$

we know that,

$$f(A) f(x) = \sup_{x / y = f(x)} [A(x)]$$

$$f(A)(0) = \sup_{x=0 / y=0} 2^{-x} = 1$$

$$f(A)(1) = \sup_{x=1 / y=1} 2^{-x} = 2^{-1} = 0.5$$

$$f(A)(2) = \sup_{x=2 / y=4} 2^{-x} = 2^{-2} = 0.25$$

$$f(A)(3) = \sup_{x=3 / y=9} 2^{-x} = 2^{-3} = 0.125$$

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$f(A)(4) = \sup_{x=4/Y=16} 2^{-x} = 2^{-4} = 0.0625$

$f(A)(5) = \sup_{x=5/Y=25} 2^{-x} = 2^{-5} = 0.3125$

$f(A)(6) = \sup_{x=6/Y=36} 2^{-x} = 2^{-6} = 0.015625$

$f(A)(7) = \sup_{x=7/Y=49} 2^{-x} = 2^{-7} = 0.0078125$

$f(A)(8) = \sup_{x=8/Y=64} 2^{-x} = 2^{-8} = 0.00390625$

$f(A) = (9) = \sup_{x=9/Y=81} 2^{-x} = 2^{-9} = 0.001953125$

$f(A) = (10) = \sup_{x=10/Y=100} 2^{-x} = 2^{-10} = 0.0009765625$

Hence,

$$f(A) = \left\{ \frac{1}{0} + \frac{0.5}{1} + \frac{0.25}{4} + \frac{0.125}{9} + \frac{0.0625}{16} + \frac{0.3125}{25} + \frac{0.015625}{36} + \frac{0.0078}{49} + \frac{0.0039}{64} + \frac{0.001953}{81} + \frac{0.000976}{100} \right\}$$