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Tutorial - 8.

Q.1 Determine which properties are satisfied by following algebric systems.
a) All odd integers $\langle O, +, \times \rangle$.
b) $\langle N, +, \times \rangle$
c) $\langle Z_6, +_6, \times_6 \rangle$.
→

a) For any $a, b \in O$, (+)
$a+b \neq b+a$. ∵ $a+b \notin O$.
∴ i) Commutative.
ii) For any $a, b, c \in O$,
$(a+b)+c \;\cancel{\neq}= a+(b+c)$
∴ Associative.
iii) ~~Ide~~

(×)

i) Commutative.
ii) Associative.
iii) Identity element. = 1.
iv) Cancellation.

b) $\langle N, +, \times \rangle$.
1) (+)
i) Commutative.
ii) Associative.
iii) Identity element = 0.
2) (×)
i) Commutative.
ii) Associative.
iii) Cancellation.
iv) Identity element
v) Zero element.

c) $\langle Z_6, +_6, \times_6 \rangle$

$Z_6 = \{[0], [1], [2], \ldots [5]\}$.

$+_6$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[6]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

$\therefore$ 1) $(+_6)$:

i) Commutative.

ii) Associative.

iii) Cancellation.

2) $(\times)$:

$\times_6$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]
[2]	[0]	[2]	[4]	[0]	[2]	[4]
[3]	[0]	[3]	[0]	[3]	[0]	[3]
[4]	[0]	[4]	[2]	[0]	[4]	[2]
[5]	[0]	[5]	[4]	[3]	[2]	[1]

$\boxed{[R]_6} = f$

2) i) Commutative.

ii) Associative.

iii) Zero element.

iv) Identity element.

v) Cancellation.

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Q.2 Show that if $g : A \to B$,
$\langle A, * \rangle$ onto $\langle B \to \Delta \rangle$.
$\langle A_1, * \rangle$ is subalgebra of $\langle A, * \rangle$ then,
$\langle B_1, \Delta \rangle$ is subalgebra of $\langle B, \Delta \rangle$

→ $g : A \to B$.
∴ For any $a, b \in A$,
$g(a * b) = g(a) \Delta g(b)$.
∴
$\langle A_1, * \rangle$ is subalgebra of $\langle A, * \rangle$.
∴ A_1 is closed under $*$.

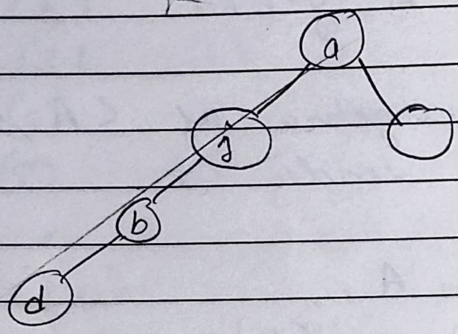
∴ For any $a, b \in A_1$,
$g(a * b) = g(a) \Delta g(b)$.
$g(a * b) \in B_1$

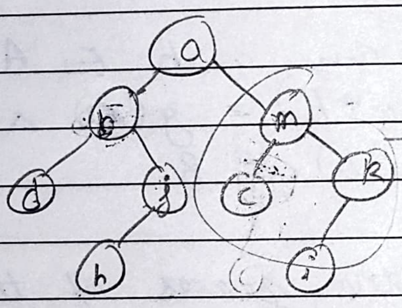
Q.3 Find the zeros of the semigroups. $\langle P(x), \cap \rangle$
$\langle P(x), \cup \rangle$ where, x is any given set
and $P(x)$ is its powerset. Are these monoids? If so, what are the identities?
→ Let, $x = \{a, b, c\}$.
$\langle P(X), \cap \rangle = \phi$ (zero element).
$= x$ (identity element).
$\langle P(x), \cup \rangle = X$ (zero element).
$= \phi$ (identity element).

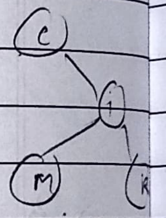
Q.4 Show that the set N of natural numbers is a semigroups under the operation
$x * y = \max\{x, y\}$. Is it a monoid?
→ $N = \{0, 1, 2, 3, \ldots\}$.
∴ $\langle N, * \rangle$ is a monoid.
∴ $\langle N, *, 0 \rangle$

$$x*y*z = (x*y)*z = x*(y*z)$$
$$= ((10*5)*12 = 10*(5*12)$$
$$= 10*12 = 10*12$$
$$= 12 = 12$$

* $\underbrace{dbhj}_{l}$ a $\underbrace{cmik}_{R}$. Inorder l v r.







2) dhj b m k i c a: l r v.