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Q. Convert to prefix and suffix form.

1) $P \to Q \vee R \vee S$.

→ $(P \underset{3}{\to} ((Q \underset{1}{\vee} R) \underset{2}{\vee} S))$

Prefix : ~~$\to \vee \vee Q R S P$~~ $\quad \vee Q R$ $\boxed{\to P \vee \vee Q R S}$

Suffix : $P S Q R \vee \vee \to$ $\quad P Q R \vee S \vee \to$

2) $\neg\neg P \vee Q \wedge R \vee \neg Q$.

→ $(((\underset{3}{\neg}(\underset{4}{\neg} P)) \underset{1}{\vee} ((Q \underset{3}{\wedge} R)) \underset{1}{\vee} (\underset{2}{\neg} Q)))$

Prefix : ~~$\wedge \neg\neg\neg P Q R$~~ $\vee \vee \neg\neg P \wedge Q R \neg Q$

Suffix : $P \neg\neg Q R \wedge \vee Q \neg \vee$

3) $\neg\neg(P \vee Q) \wedge R \vee \neg Q$.

→ $(((\underset{3}{\neg}(\underset{4}{\neg}(P \underset{5}{\vee} Q))) \underset{2}{\wedge} R) \underset{1}{\vee} (\underset{2}{\neg} Q))$

Prefix : $\vee \wedge \neg\neg \vee P Q R \neg Q$.

Suffix : $P Q \vee \neg\neg R \wedge Q \neg \vee$

Q. Convert to infix:

R to L.

1) $\to \to P Q \to \to Q R \to P R$

→ ~~$(P \to R) \to ($~~

$((P \underset{3}{\to} Q) \underset{2}{\to} ((Q \underset{3}{\to} R) \underset{1}{\to} (P \underset{2}{\to} R)))$

R-L

2) $P \neg P \to P \to P \to$

$(((\neg P \to P) \to P) \to P)$

→

Q. Show the validity using premises.

1) $\neg(P \land \neg Q)$, $\neg Q \lor R$, $\neg R$ ∴ Prove $\neg P$.

→

1)	$\neg R$	P
2)	$\neg Q \lor R$	P
3)	$\neg Q$	T from ① and ②
4)	$\neg(P \land \neg Q)$	P
5)	$\neg P \lor Q$	De morgans law (from ④)
6)	$\neg P$	T from ③ and ⑤.

2) $(P \land Q) \to R$, $\neg R \lor S$, $\neg S$ ∴ Prove $\neg P \lor \neg Q$.

→

1)	$\neg S$	P
2)	$\neg R \lor S$	P
3)	$\neg R$	T from ① and ②
4)	$(P \land Q) \to R$	P
5)	$\neg(P \land Q) \lor R$	T from ④.
6)	$(\neg P \lor \neg Q) \lor R$	T from ⑤
7)	$\neg P \lor \neg Q$.	T from ③ and ⑥.

3) $P, P \to (Q \to (R \land S)) \Rightarrow Q \to S$

→ Consider Q as addition premise.

1)	P	Premise
2)	Q	[CP.]
3)	$P \to (Q \to (R \land S))$	P
4)	$Q \to (R \land S)$	T from ①, ② and ③
5)	$R \land S$	T from ② and ④.

6) S T from ⑤

4) $(P \vee Q) \rightarrow R \Rightarrow (P \wedge Q) \rightarrow R$.

→ consider, $(P \wedge Q)$ as an addition premise.

∴ 1) $(P \vee Q) \rightarrow R$ P.
2) $(P \wedge Q)$ C.P.
3) R T from ① and ②.

Q. Show that following sets of premises are inconsistent.

1) $P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R, P$.

1) P Premise.
2) $P \rightarrow Q$ P
3) Q T from ① and ②
4) $Q \rightarrow \neg R$ P
5) $\neg R$ T from ③ and ④.
6) $P \rightarrow R$ P
~~7)~~ 7) $R \wedge \neg R$ T ⑤ and ⑥.

∴ From ① and ⑤ $R \wedge \neg R$ ~~$P \rightarrow R$~~ is False.
Hence, given sets of premises are inconsistent.

2) $S \rightarrow \neg Q, S \vee R, \neg R, \neg R \leftrightarrow Q \Rightarrow \neg P$

→ consider, $\neg P$ ~~to~~ to be false.
∴ P is true.
Take P as an addition premise.

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1)	P	Premise.
2)	$\neg R$	P.
3)	$S \vee R$	P
4)	S	T from ② and ③.
5)	$S \rightarrow \neg Q$	P
6)	$\neg Q$	T from ④ and ⑤.
7)	$\neg R \leftrightarrow Q$	P
8)	Q	T from ② and ⑦.
9)	$Q \wedge \neg Q$	T from ⑥ and 8.

$\therefore$ $Q \wedge \neg Q$ is a contradiction. Hence, our assumption was wrong $\neg P$ is true.

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Q.1 Write the following formulas in prefix and suffix form. The following precedence is assumed: $\rightleftarrows, \rightarrow, \vee, \wedge, \neg$ ($\neg$ having highest precedence.)

a) $P \rightarrow Q \vee R \vee S$.

$\rightarrow$ $(P \underset{1}{\rightarrow} ((Q \underset{3}{\vee} R) \underset{2}{\vee} S))$

$\therefore$ Prefix $\Rightarrow P \vee \vee Q R S$

Suffix : $P Q R \vee S \vee \rightarrow$

b) $P \wedge \neg R \rightarrow Q \rightleftarrows P \wedge Q$.

$\rightarrow$ $(((P \underset{3}{\wedge} (\underset{4}{\neg} R)) \underset{2}{\rightarrow} Q) \underset{1}{\leftrightarrow} (P \underset{2}{\wedge} Q))$.

Prefix : $\leftrightarrow \rightarrow \wedge P \neg R Q \wedge P Q$

Suffix : $P R \neg \wedge Q \rightarrow P Q \wedge \leftrightarrow$

Q.2 Convert to infix.

a) $\rightarrow \rightarrow P Q \rightarrow \rightarrow Q R \rightarrow P R$.

$\rightarrow$ ~~$(P \rightarrow R) \rightarrow (Q \rightarrow R) \rightarrow (P \rightarrow Q)$~~.

$(P \rightarrow Q) \rightarrow (Q \rightarrow R) \rightarrow (P \rightarrow R)$

b) $P \neg P \rightarrow P \rightarrow P \rightarrow$

$\rightarrow$ $(((\neg P \rightarrow P) \rightarrow P) \rightarrow P)$

Q.3 Show the validity for given premises.

a) $(A \rightarrow B) \wedge (A \rightarrow C)$, $\neg(B \wedge C)$, $D \vee A$. D

$\rightarrow$

1) $(A \rightarrow B) \wedge (A \rightarrow C)$ P

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2) $A \to B$ T from ①
3) $A \to C$ T from ①
4) $\neg(B \wedge C)$ P
5) $\therefore D \vee A$ P
6) D T from ②, ③, ④ and ⑤.

b) $\neg J \to (M \vee N)$, $(H \vee G) \to \neg J$, $H \vee G$. $M \vee N$.

→

1) $(H \vee G) \to \neg J$ P
2) $H \vee G$ P
3) $\neg J$ T from ① and ②
4) $\neg J \to (M \vee N)$ P
5) $M \vee N$ T from ③ and ④.

c) $P \to Q$, $Q \to \neg R$, R, $P \vee (J \wedge S)$ $J \wedge S$.

→

1) R P
2) $Q \to \neg R$ P
3) $P \to Q$ P
4) $P \vee (J \wedge S)$ P
5) $J \wedge S$. T from ①, ②, ③ and ④.

Q.4 Derive using rule CP.

a) $(P \vee Q) \to R \Rightarrow (P \wedge Q) \to R$.

→ Consider, $(P \wedge Q)$ as an additional premise.

$\therefore$ 1) $(P \wedge Q)$ P
2) $(P \vee Q) \to R$ P
3) R T from ① and ②.

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Q.5 Show that following sets of premises are inconsistent.

a) $P \to Q$, $P \to R$, $Q \to \neg R$, P .

1)	P	Premise.
2)	$P \to Q$	P
3)	Q	T from 1 and 2.
4)	$Q \to \neg R$	P
5)	$\neg R$	T from 3 and 4.
6)	$P \to R$	P
7)	R	T from 1 and 6
8)	$R \wedge \neg R$	T from 5 and 7

∴ The given sets of premises are inconsistent.

Q.6 Show the following using indirect method.

a) ~~$P \wedge \neg P \wedge Q \Rightarrow R$~~.

$S \to \neg Q$, $S \vee R$, $\neg R$, $\neg R \Leftrightarrow Q \Rightarrow \neg P$.

→ Consider, $\neg P$ to be false. ∴ P is true.

1)	$\neg R$	P
2)	$S \vee R$	P
3)	S	T from 1 and 2
4)	$\neg R \Leftrightarrow Q$	P
5)	Q	T from 1 and 4
6)	$S \to \neg Q$	P
7)	$\neg Q$	T from 3 and 6.
8)	$Q \wedge \neg Q$	T from 5 and 7.

∴ Hence, our assumption was wrong.

∴ $\neg P$ is true.