

Tutorial - 5

Q.1 Give power sets of the following.

a) $\{a, \{b\}\}$

→ Power sets = $\{a\}, \{\{b\}\}, \{\emptyset\}, \{a, \{b\}\}$

b) $\{1, \emptyset\}$

→ Power sets = $\{1\}, \{\emptyset\}, \{1, \emptyset\}, \emptyset$

Cardinality (No of elements)

$$\emptyset = 0$$

$$\{\emptyset\} = 1$$

Q.2 Show that $(R \subseteq S) \wedge (S \subseteq Q) \Rightarrow R \subseteq Q$.

→ $(R \subseteq S) \wedge (S \subseteq Q) \Rightarrow R \subseteq Q$

$$\therefore R \subseteq S = (x)(x \in R \rightarrow x \in S)$$

$$S \subseteq Q = (x)(x \in S \rightarrow x \in Q \wedge S \neq Q)$$

$$\therefore (x)(x \in R \rightarrow x \in S) \wedge (x)(x \in S \rightarrow x \in Q \wedge S \neq Q)$$

$$\rightarrow (x)(x \in R \rightarrow x \in Q) \wedge R \neq Q$$

OR.

$$\left[\begin{array}{l} \text{If } (A \subseteq B) \wedge (B \subseteq C) \text{ then } (A \subseteq C) \\ \text{If } (A = B) \wedge (B \subseteq C) \text{ then } (A \subseteq C) \end{array} \right]$$

$$\text{If } (A \subseteq B) \wedge (B \subseteq C) \text{ then } (A \subseteq C)$$

$$(A \subseteq B) \wedge (B \subseteq C) \rightarrow A \subseteq C$$

$$\text{OR } (A = B) \wedge (B \subseteq C) \rightarrow A \subseteq C$$

Q.3 Show that $A \cap B \subseteq A$

→ $A \cap B = (x)(x \in A \wedge x \in B) \Rightarrow (x)(x \in A)$

Q. $A = \{2, 3, 4\}$, $B = \{1, 2\}$, $C = \{4, 5, 6\}$

→ $A + B = \{~~2~~, 1, 3, 4\}$

$A + B + C = \{1, 3, 5, 6\}$

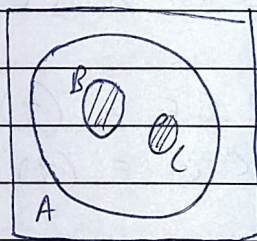
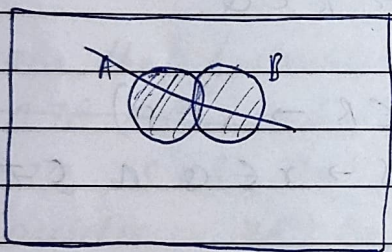
$(A+B) + (B+C) = \{1, 3, 4\} + \{1, 2, 4, 5, 6\}$
 $= \{2, 3, 5, 6\}$

OR,

$(A+B) + (B+C) = A + \underline{B+B} + C$
 $= A + \phi + C$
 $= A + C$
 $= \{2, 3, 5, 6\}$

Q. Give examples of sets A, B, C such that,
 $A \cup B = A \cup C$ but, $B \neq C$.

→ $A \cup B = A \cup C$



$\{\phi, \{\phi\}\} - \{\phi\} = \{\{\phi\}\}$

Q. $A = \{2\}$, $B = \{a, b\}$, $C = \{2, 3\}$. Find $A \times B$,
 A^2 , B^2 , $B^2 \times C$

→ $A \times B = \{\langle 2, a \rangle, \langle 2, b \rangle\}$

1 Pair. $A^2 = \{\langle 2, 2 \rangle\}$

4 Pairs. $B^2 = \{\langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle, \langle b, b \rangle\}$

8 Pairs. $B^2 \times C = \{\langle \langle a, a \rangle, 2 \rangle, \langle \langle a, a \rangle, 3 \rangle, \langle \langle a, b \rangle, 2 \rangle, \langle \langle a, b \rangle, 3 \rangle, \langle \langle b, a \rangle, 2 \rangle, \langle \langle b, a \rangle, 3 \rangle, \langle \langle b, b \rangle, 2 \rangle, \langle \langle b, b \rangle, 3 \rangle\}$
OR $\{\langle a, a, 2 \rangle, \dots\}$

Q. Show that for any two sets A and B,
 $P(A) \cup P(B) \subseteq P(A \cup B)$

$$\begin{aligned} \rightarrow P(A) \cup P(B) &\Leftrightarrow (x)(x \in P(A) \vee x \in P(B)) \\ &\Leftrightarrow (x)(x \subseteq A \vee x \subseteq B) \\ &\Leftrightarrow (x)(x \subseteq A \cup B) \\ \therefore P(A) \cup P(B) &\subseteq P(A \cup B) \end{aligned}$$

Q. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

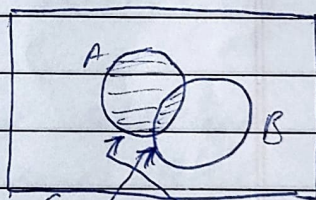
→

Q. Prove that $(A \cap B) \cup (A \cap \sim B) = A$.

$$\begin{aligned} \rightarrow (A \cap B) \cup (A \cap \sim B) \\ \Rightarrow (x)((x \in A \cap x \in B) \vee (x)(x \in A \cap x \in \sim B)) \\ \Rightarrow (x)(x \in A) \\ \Rightarrow A. \end{aligned}$$

OR

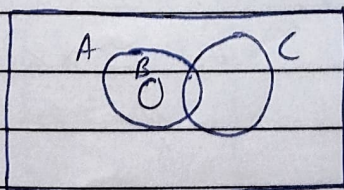
$$\begin{aligned} &= A \cap (B \cup \sim B) \\ &= A \cap E \\ &= A \end{aligned}$$



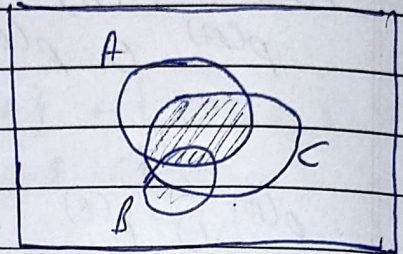
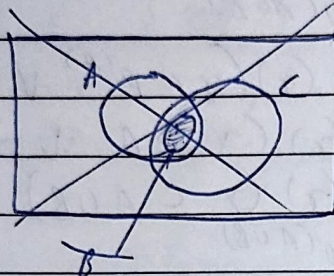
$$\therefore (A \cap B) \cup (A \cap \sim B) = A$$

Q. Draw venn diagram.

1) $A \cup B \subseteq A \cup C$, $B \not\subseteq C$



2) $A \cap B \subset A \cap C$, $B \not\subset C$



Q. Prove that $(A \cap B) \times (C \times D) = (A \times C) \cap (B \times D)$

	3	4	5	6	7	8	9	10
-3	00	01	02					
-2								
-1								27
0								
1								
2								
3								
4								
5								
6								

$$\begin{aligned}
 A[2, 7] &= b_0 + (-1 + 4 \cdot 1) \cdot 8 + (10 - 3) \\
 &= b_0 + (2 \times 8) + 7 \\
 &\text{Step 2 done}
 \end{aligned}$$