Unit 6 : Graph

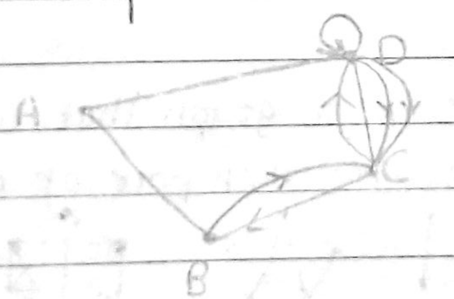
- used to represent some real time scenario.
- loops are used in graphs / computer network to check the activeness of router.

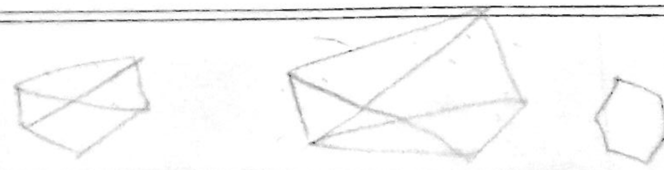
- A simple graph is a graph in which there is no loops.

★ Graph Terminology :-

Type	Edges	Multiple Edges Allowed?	Loops Allowed?
1) Simple graph	Undirected	NO	NO
2) Multigraph	Un	Y	N
3) Pseudograph	Un	Y	Y
4) Simple directed	Directed	N	N
5) Directed Multi-	Directed	Y	Y
6) Mixed graph	Directed & Undirected	Y	Y

→ Mixed Graph :-

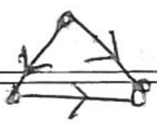


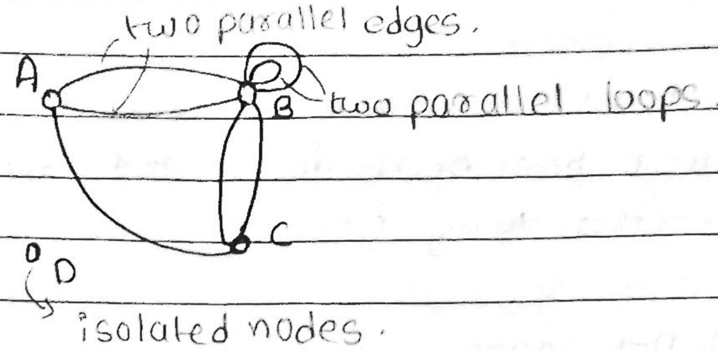


a loop (sling).

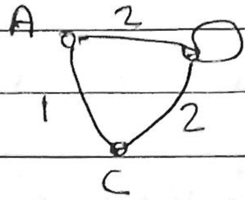
Multigraph :- any graph which contains some para
edges is called a multigraph.

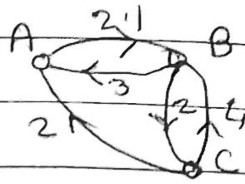
Simple Graph :- no more than one edge b/w a pair o
nodes, then such a graph is called as -





* Weighted graph :- A graph in which weights are assigned to every edge is called a weighted graph.





- a node which is not adjacent to any other node is called an isolated node.
- A graph containing only isolated nodes is called a null graph.

* Isomorphism in Graphs :- two graphs are isomorphic if there exists a one-to-one correspondence between the nodes of the two graphs which preserves adjacency of the nodes as well as the directions of the edges, if any.

* Indegree, Outdegree & total degree :-

In directed graph, for any node v the no. of edges which have v as their initial node is called the out degree & v as their terminal node is called the indegree.
& indegree + outdegree = total degree of that node.

total degree of isolated node is 0.

* Subgraph :- let $V(H)$ be the set of nodes of a graph H & $V(G)$ be the set of nodes of a graphs G such that $V(H) \subseteq V(G)$.

If every edge of H is also an edge of G, then the graph H is called a subgraph of the graph G, which is expressed by writing $H \subseteq G$.

* Path of the graph :- Any sequence of edges of a digraph such that the terminal node of any edge in the sequence is the initial node of the edge, if any, appearing next in the sequence defines a path of the graph.

- Path length → no. of edges appearing in the sequence of path.

distinct edges → simple path
distinct nodes → elementary path.

- A path in a digraph in which the edges are all distinct is called a simple path.
- A path in which all the nodes through which it traverse are distinct is called an elementary path.
  ↳ every elementary path in diagraph is also simple.

* cycle or circuit :- A path which originates & ends in the same node is called a cycle or circuit.

Page No.
Date

Note :- In a cycle the initial node appears at least twice even if it is an elementary cycle.

* Connectivity of Graphs :-

An undirected graph is said to be connected if for any pair of nodes of the graph the two nodes are reachable from one another.

— Unilaterally connected :- for any pair of nodes of the graph at least one of the nodes of the pair is reachable from the other node

— Strongly connected :- If any pair of nodes of the graph both the nodes of the pair are reachable from one another, then the graph is called strongly connected.

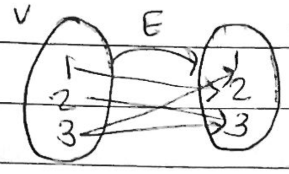
— Unilaterally connected digraph is weakly connected, but a weakly connected digraph is not necessarily unilaterally connected.

* An undirected graph has an even number of vertices of odd degree.
* initial & terminal vertex of loop are the same.
* in-degree of a vertex → $deg^-(v)$ [v as the terminal vertex]
* out-degree of a vertex → $deg^+(v)$ [v —"— initial vertex]

→ $G < V, E >$

Graph is a set of ~~st~~ vertices and edges.

E is the relation in set $V \to V$



* let $G = (V, E)$ be an undirected graph with m edges. Then,

$$2m = \sum_{v \in V} \deg(v)$$

(This applies even if multiple edges & vertices are present).

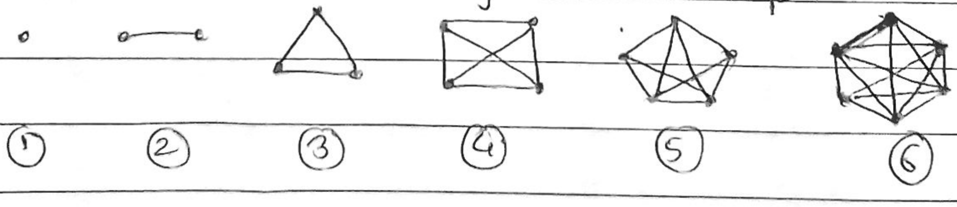
* $+v \to$ out degree $\quad -v \to$ indegree.

* let $G = (V, E)$ be a graph with directed edges. then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

total no. of edges / cardinality of E.

* Complete Graph :- simple graph that contains exactly one edge b/w each pair of distinct vertices.



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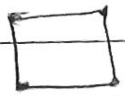
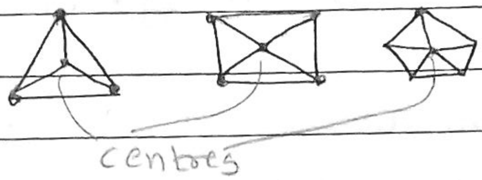
complete Graphs.

* Cycle graph : There is a single ~~graph~~ path in cyclic graph.

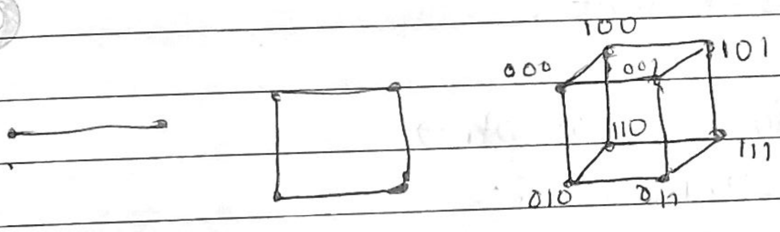


* wheel Graphs :



* connected graphs :-

*



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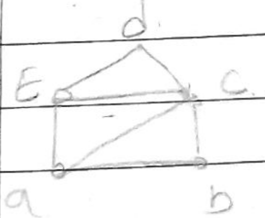


Adjacency list :- a → b, c, E
b → c, ~~b~~ → a
c → E, c → a, c → b, c → d.
d → E, d → c.
e → d, e → c, e → a

matrix :

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right]_{n \times n} \end{array}$$

→ Symmetric matrix

It is called dense when it has large no. / max. no. of 1's.

adjancency list :-

a → | b | c | e |

b → | a | c |

c → | d | b | a | e |

d → | e | c |

e → | a | c | d |

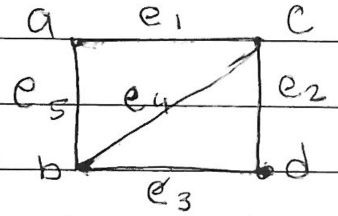
$2 \times E + n = |\text{list}|$

& when graph has less no. of 1's then the graph is called sparse.

→ In case of sparse graph adjacency list method is efficient. & in case of dense graph adj. matrix method is efficient.

★ Representation of graph →

3] Incidence Matrices :-

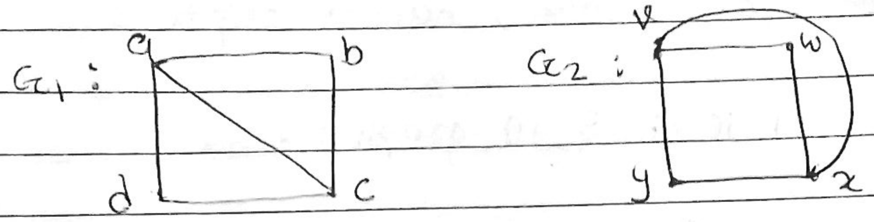


(it is not a weighted graph)

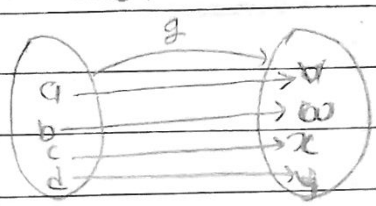
$$\begin{array}{c c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} & \\
\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} a/v \\ b/v \\ c/v \\ d/v \end{array}
\end{array}$$

Page No.
Date

* Isomorphism in Graphs :-



- two graphs are not isomorphic when no. of edges are not same in both the graphs.
- ($g(a \oplus b) = g(a) * g(b)$ --- (for any $a, b \in A$)
 in $\langle A, \oplus \rangle$, $\langle B, * \rangle$)
 one to one-onto.

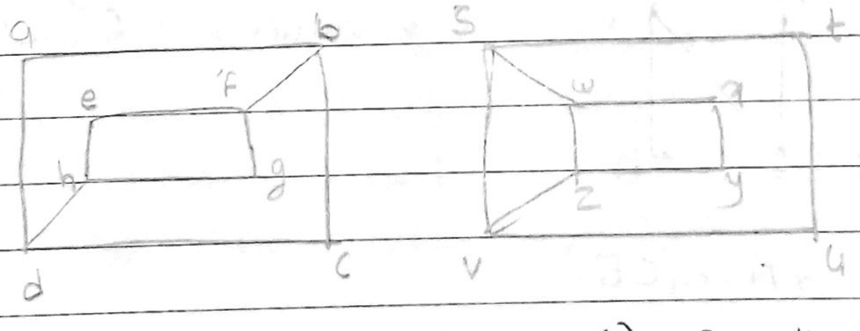


a is map to v (because a has 3 degree & v also has degree = 3)
($\because$ if degree of a $\neq$ degree of v then a not mapping to v.)

If degree of both nodes is not same then it is called as adjacency not maintained.

* To check isomorphism the no. of vertices & edges must be equal.

Q.

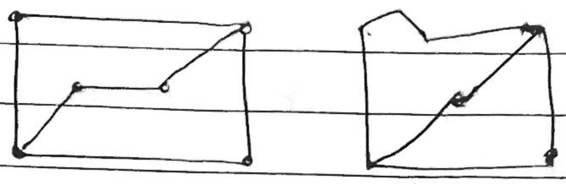


Not isomorphism. 1) s with two adjacent nodes (w & v) having degree 3.

and in 1st graph there is no element which has two ~~nodes~~ adjecent nodes having degree 3.

2) no cycle of length 8 in graph 1.

Q.2]



→ total no. of edges for K nodes is,

$$no.(edges) = \frac{K(K-1)}{2}$$

* Connected components :-

It is a maximal subgraph of graph which is connected.

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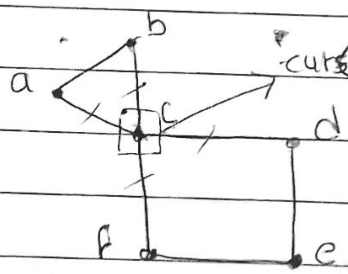
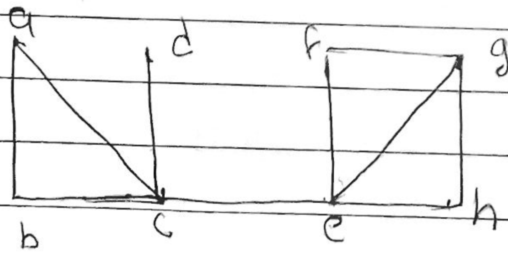


(cut ~~s~~ articulation point / cut vertex)

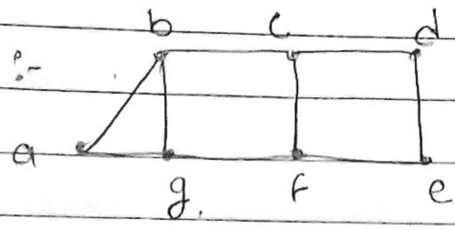
*

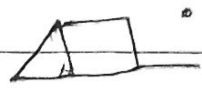


cut vertices → € {c, e}

cut edges → Age(CE)

* Edge Cut :-

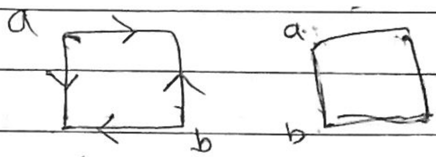


edge cuts ⇒ 1) {(c,d), (d,e)} 

2) {(b,c), (g,f)} 

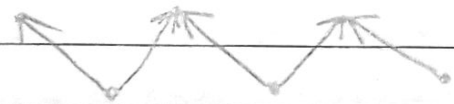
3) {(c,d), (f,e)}

* weakly connected directed graph :-



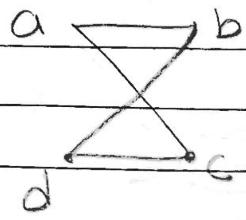
A - here, there is no path b/w a and b.
∴ It is weakly connected graph.

~~If~~ When you treat the graph / directed graph as undirected & ~~then~~ it becomes connected then this graph is called as weakly connected.



weakly connected graph.

* $$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix} = A^2$$



1) a-b, b-a, a-b
2) a-c, c-a, a-b
3) a-b, b-d, d-b
4) a-c, c-d, d-b.
~~5)~~

a to a   a → b → d → c

∴ $R^{+} = R \cup R^2 \cup R^3 \cup R^4 \ldots$
↳ it shows wheather their is path b/w two vertices or not.

- Path matrix is transitive closure.

- $A^3(i,j) = 5$
5 paths between i and j of length 3.

- $A^5 = A^8$
If there is path of length 5 b/w two nodes then their is path of length 8 b/w same nodes & vise versa.

Page No.			
Date			

* Euler & Hamilton Paths :- (related to edges).

A path or circuit is called simple if it does not contain the same edge more than once.

- Euler circuit is circuit which visits simple circuit.
- or which traverses all the edges.
- cycle is special path where starting & ending node is same.

- Euler Path :- It is path which traverses all the edges of graph at only once.

* If degree of each node in given multigraph is even then their is Eular circuit. iff each vertices have even degree iff there are exactly one path ~~which~~ with odd degree then their is Eular path not euler circuit.

→ The connected graph/multigraph has an Euler path but not an Euler circuit iff it has exactly two vertices of odd degree.

* Hamilton Circuit :- (related to vertices)

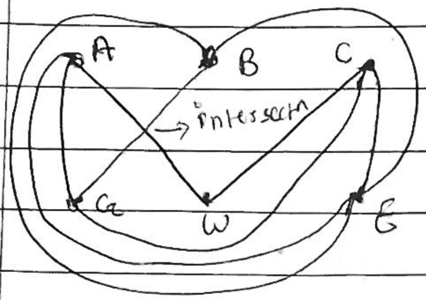
Thm : If there are n ~~devices~~ vertices in given simple graph (undirected) & degree of each vertices is atleast $(n/2)$ then it is called as Hamilton circuit.

degree of each vertex $\geq n/2$

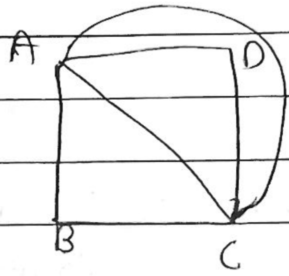
Page No.
Date

* ORE's Theorem :- If G is a simple graph with n vertices with $n \geq 3$ such that $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u & v in G, then G has a Hamilton circuit.

- A graph in which edges are intersecting is called as non-planar graph.



non-planner.



Planner.

* Graph colouring :-

no. of colours required for any graph is 4. (atleast · n no. are (max-no.) ↙ ↳ chromatic no.

- tree is a connected undirected graph with no simple circuit.
(multigraph is not a tree).

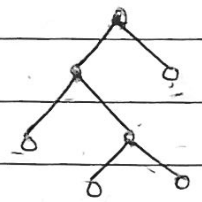
- An undirected graph is a tree if & only if there is a unique simple path between any two of its vertices.

* Trees :- it is a connected undirected graph with no simple circuits.

⇒ An undirected graph is a tree if & only if there is a unique simple path b/w any two of its vertices.

⇒ rooted tree : it is a tree in which one node/vertex has been designated as the root & every edge is directed away from the root.

* Full binary tree → every internal vertex has exactly 2 childrens.



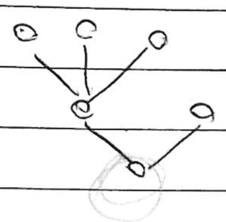
* a rooted tree is called m-ary tree if every internal vertex has no more than m children.

* Ordered rooted tree :- rooted tree where children of each internal vertex are ordered.

- tree with n-vertices has n-1 edges.
- A full m-ary tree with i internal vertices contains $n = mi+1$ vertices.

* A full m-ary tree with
i) n vertices has $i = (n-1)/m$ internal vertices & $l = [(m-1)n+1]/m$ leaves.

* A rooted m-ary tree of height h is balanced if all leaves are at levels h or h-1.

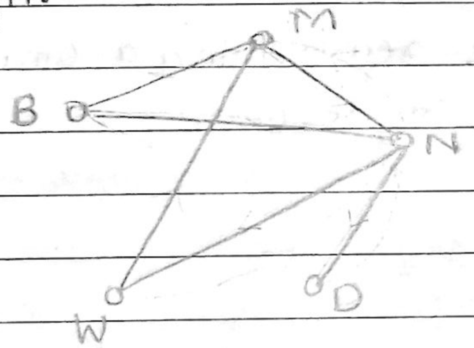


Page No.
Date 23 | 12 | 22

Tutorial No. 11

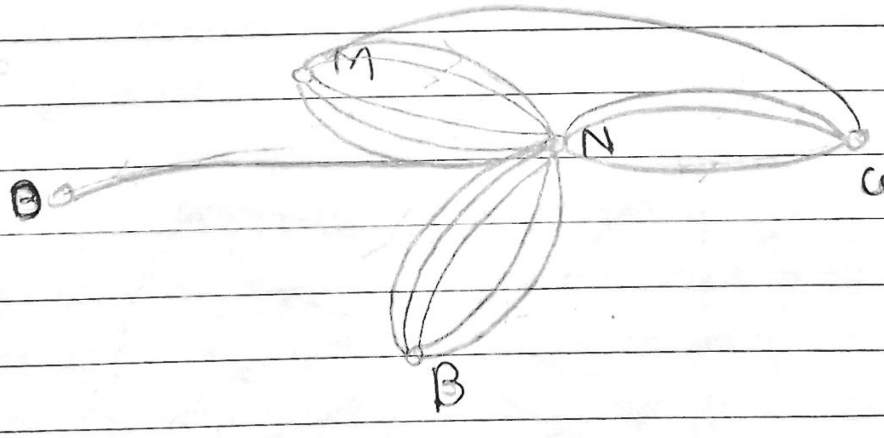
1] Draw Graph models, stating the type of graph (from Table) used; to represent airline routes where every day there are four Flights From Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Network; Three from Newark to washington, two flights from washington to Newark & one flight from washington to miami with -

a) An edge between vertices representing cities that have a flight.

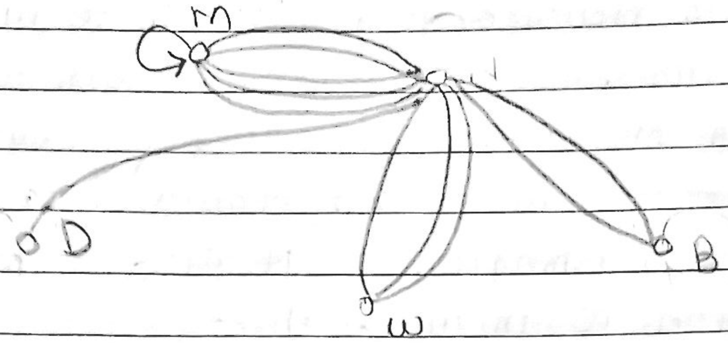


	M	B	N	D	W
M	0		2		
B		0	4		
N	3	2	0	1	3
D			2	0	
W	1		2		0

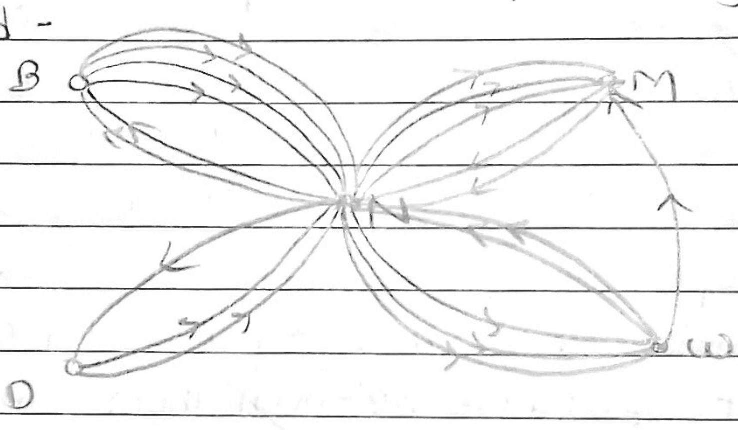
b) an edge between vertices representing cities for each flight that operates between them (in either direction).



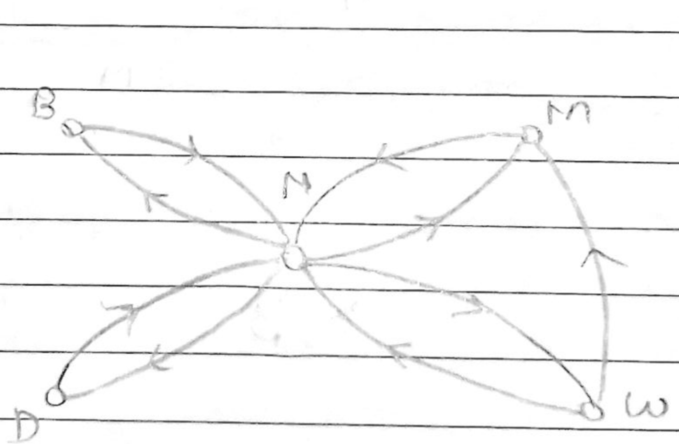
c) an edge bet^n vertices representing cities for each Flight that operates between them (in either $direct^n$) plus a loop for a special sightseeing trip that takes & lands in Miami.



d) An edge from a vertex representing a city where a flight starts to vertex representing the city where it end -



e]

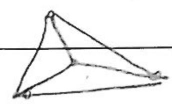


Q.2] write no. of vertices and edges for following cases.

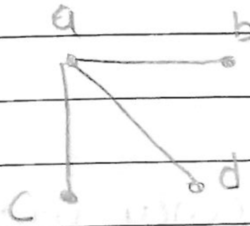
1) K_n :- $K_n \rightarrow \frac{n(n-1)}{2}$ = no. of edges. , vertices = n.

2) C_n :- cycle with n-nodes.
n-edges
n-vertices.

3) W_n :-
n+1 -vertices.
2n edges.



Q.3] How many subgraphs in this graph :-

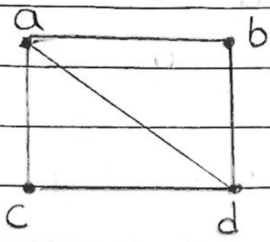


Subsets of V => $2^4 = 16$
Subsets of E => $2^3 = 8$.

total subgraphs = 35

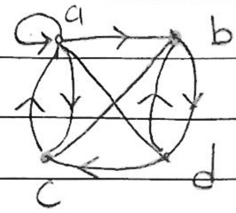
~~G~~	V	E
0	0	0
4	1	0
6	2	0
3	2	1
4	3	0
7	3	1
3	3	2
1	4	0
3	4	1
3	4	2
1	4	3

Q. 4] Draw adjacency matrix & list ~~no.~~ ~~1of~~ to represent given graph.

1] 

adjacency Matrix $\longrightarrow$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

2] 

adjacency list :-

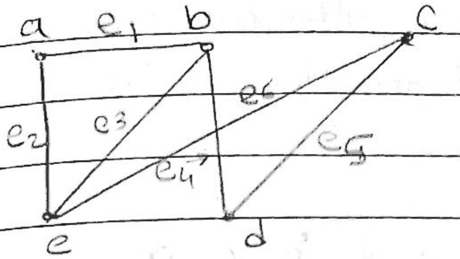
a	$\rightarrow$ a, b, c, d
b	$\rightarrow$ c, d
c	$\rightarrow$ a, b, d
d	$\rightarrow$ a, b, c.

3] Draw the graph of given adjacency matrix.

$\longrightarrow$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

Q.5] Draw incidence matrix.



$$\begin{array}{cc} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \end{array}$$

Q.6] Is every zero-one square matrix that is symmetric & has zeros on the diagonal the adjacency matrix of a simple graph?

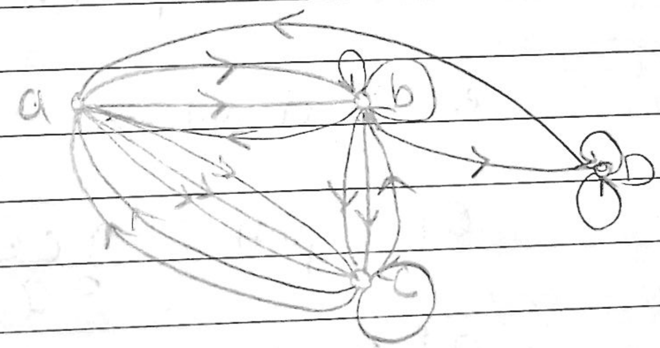
→ correct / Yes.

Q.7]

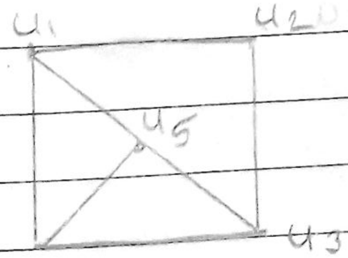
$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \end{array}$$

draw the graph represented by the given adjacency matrix.

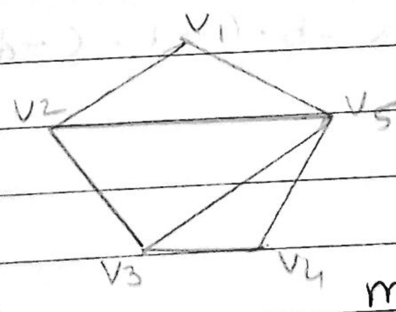
→



Q.8] Isomorphic or not :-



max. degree = 3

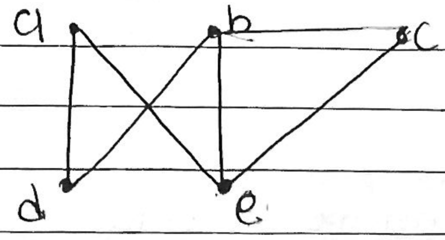


max. degree = 4

∴ not isomorphic.

Page No.	
Date	

Q.9] Does each of these lists of vertices form a path in the following graph? Which paths are simple? which are circuits? what are the lengths of those that are paths?

1) 

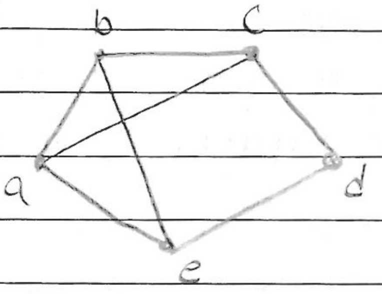
i) a e a d b c a

- No path

ii) e b a d b e

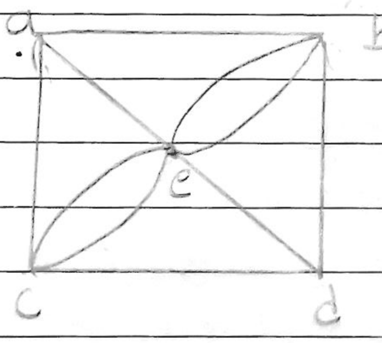
- No path

Q.10] Euler circuit or Path?

1) 

All nodes degree's are not even.

- No euler circuit.
- No euler path.

2) 

Euler circuit X

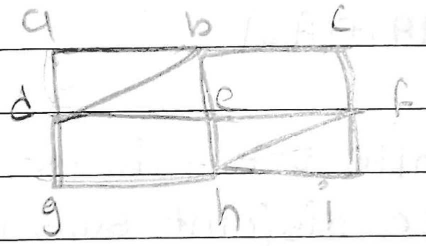
Euler path :- a c e b e c d b a e d

a → b → e → c → e → b → d → c → a → e → d.

d - e - c - e - b - e - a - c - d - b - a.

Conditions for graph isomorphism :-

1) equal no. of edges, vertices.
2) equal amount of degree sequence
3) formatⁿ of cycles



Circuit :-

a b d e f h i f c b e h g d a.