10/11/22

Assignment No: 3

Q1) Define the following with examples

1) Power set

Ans: The power set of a set S is the set of all subsets of S, including the empty set and S itself and it is denoted by P(S).

Eg: $P(S) = \{\phi, \{0\}, \{1\}, \{0,1\}\}$ $S = \{0,1\}$

$|S|$ $|P(S)| = 4$

Let $T = \{0, 1, 2\}$

$P(T) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}$.

$P(\phi) = \{\phi\}$ 2^n is formula for P(S).

2) Emptyset / Null set

Ans: A Empty set is a set which has no member is also called Null set.

$\phi = \{\} \longrightarrow \{\phi, 1, 2, 3, x\}$ it is valid set

$\phi \neq \{\phi\}$

$H = \{$ Number of dinosaurs on earth $\}$

$H = \{\}$

3) Singleton set

Ans: A singleton is a set that contains exactly one element. also known as unit set.

It can be written as $\{a\}$.

Note that $\{\phi\}$ is a singleton set.

eg: $S = \{x \mid x \in N, 7 < x < 9\}$

$\Rightarrow S = \{8\}$

4) Equal and equivalent sets

Ans: Two sets are equal if they have the same elements.

→ $\{1,2,3,4,5\} = \{5,4,3,2,1\}$

Equivalent set :- Two sets are equivalent if they hav same number of elements

Eg: $F = \{2,4,6,8,10\}$ and $G = \{10,20,30,40,50\}$

$n(F) = n(G)$

$\therefore$ F and G are equivalent.

Q2) Given $A = \{2,3,4\}$ $B = \{1,2\}$, $C = \{4,5,6\}$
find $A+B$, $B+C$, $A+B+C$, $(A+B)+(B+C)$

Ans: $A+B = (A \cup B) - (A \cap B)$

$= \{1,2,3,4\} + \{2\}$

$= \{1,3,4\}$

$B+C = (B \cup C) - (B \cap C)$

$= \{1,2,4,5,6\} - \{0\}$

$= \{1,2,4,5,6\}$

$A+B+C = (A+B)+C$

$= \{1,3,4\} + \{4,5,6\}$

$= \{1\ 3,5,6\}$

$(A+B)+(B+C) = \{1,3,4\} + \{1,2,4,5,6\}$

$= \{2,3,5,6\}$

Q3) What are ordered pairs give examples

Ans: ① An ordered pair <a, b> is a pair of objects.
② The order in which the objects appear in the pair is significant.
③ The ordered pair <a, b> is different from the ordered pair <b, a> unless $a = b$
④ ordered pair consist of 2 objects in given pair note that order set is not set of consisting two elements

$A \times B = \{\langle x, y \rangle \mid x \in A \wedge y \in B\}$

$\langle 1, 3 \rangle \quad \langle 2, 1 \rangle \quad \langle 1, 2 \rangle \quad \langle 2, 1 \rangle$

⑤ ordered triple is ordered pair who's first member is itself considered to be order pair.

$\langle (x, y), z) \rangle \mid \{\langle x, y \rangle \in A, z \in B\}$

Q4) What are cartesian product of A and B. Give Notation for cartesian product.

Ans: A cartesian product is a set of all ordered 2-tuple where each "part" is from is given set.
It is Denoted by $A \times B$

eg: 2-D Cartesian coordinates are set of all ordered pairs $Z \times Z$

- Recall Z is set of all integers

Eg: Given $A = \{a, b\}$ and $B = \{0, 1\}$

$\therefore C = A \times B$

$= \{(a, 0), (a, 1), (b, 0), (b, 1)\}$

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Q5) Define Binary relation, explain properties of binary relation with example & with notation.

Ans: A binary relation between two sets A and B is a rule R which decides, for any elements, whether a is in relation R to b. If so, we write a R b. If a is not in relation R to b, then we shall write a/Rb.

Any set of ordered pairs defines a binary relation.

eg: $S = \{(1,2), (3,a), (b,a), (b, Joe)\}$

* Properties of Binary Relations

1) Reflexive

A binary relation R in a set X is reflexive if, for every $x \in X$, xRx, That is $(x,x) \in R$ or $x\ o(x)(x \subset X ® xRx)$. $\langle 1,1 \rangle$ or xRx

eg: ① The relation equality of set is also reflexive

② The relation is parallel in set lines in a plane

2) Symmetric: A binary relation R in set is symmetric if for every $(x,y) \in X$, $\langle x,y \rangle \in R$

R is symmetric in X.

$(x)(y) | x \in X \wedge y \in X \wedge xRy - yRx)$

eg: xRy then yRx or $\langle 1,3 \rangle$ or $\langle 3,1 \rangle$

3) Transitive: A binary relation R in a set X is transitive if for every x & y in X, whenever $xRy \wedge yRz$ then xRz

R is transitive in X.

$(x)(y)(z) \mid (x \in X \wedge y \in X \wedge xRy \rightarrow xRz \wedge yRz)$

eg:- $\langle 1,2 \rangle \wedge \langle 4,3 \rangle \Rightarrow \langle 1,3 \rangle$

4) Anti-symmetric - A relation R in a set X is a antisymmetric if for any x and y in X, whenever xRy & yRy then $(x)(y) \mid (x \in X \wedge y \in X \wedge xRy \wedge y R x$

eg:- $\langle 1,3 \rangle \wedge \langle 3,1 \rangle \Rightarrow 1 = 3$

Q6) Given $S = \{1,2,3,4\}$ & relation R on set defined by $R = \{\langle 1,2 \rangle, \langle 4,3 \rangle, \langle 2,2 \rangle \langle 2,1 \rangle \langle 3,4 \rangle\}$ show that R is transitive.

Ans:- According Transitive property the condition is $xRy \wedge yRx \Rightarrow xRz$.

$\therefore x = 1 \quad y = 2 \quad z = 2$

$\langle 1,2 \rangle \quad \langle 2,2 \rangle \Rightarrow \langle 1,2 \rangle$

which is $\in R$

$\therefore$ R is Transitive.

Q7) Given $S = \{1,2, \ldots\ldots, 10\}$ & relation R on S where $R = \langle x,y \rangle$ such that $x+y=10$

Ans:- ordered pairs :-

$\{\langle 1,9 \rangle, \langle 2,8 \rangle, \langle 3,7 \rangle, \langle 4,6 \rangle, \langle 5,5 \rangle$
$\langle 6,4 \rangle, \langle 7,3 \rangle, \langle 8,2 \rangle, \langle 9,1 \rangle\}$

Here $\langle 5,5 \rangle$ - Reflexive

$\langle 1,9 \rangle \langle 9,1 \rangle$ - Symmetric

Non transitive $\rightarrow x = 1, y = 9, z = 1$

$x+y=10$

$y+z=10$

$x+z \neq 10$

It is non transitive.

∴ satisfies three properties.

Q8) What is equivalence Relation.

Ans: A relation R in a set A is called an equivalence relation if

- aRa for every ie R is reflexive
- aRb ⇒ bRa for every a, b ⊂ A ie R is symmetric
- aRb and PbRC ⇒ aRc for every a, b, c ∈ A ie R is transitive.

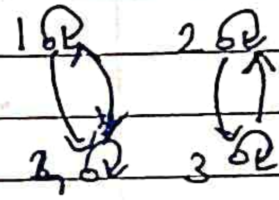
Eg: The relation equality of numbers on set of real numbers.

- It satisfies three properties: Reflexive, Transitive and symmetric.

X = {1, 2, 3, 4}

R = {<1,1> <1,4> <4,1> <4,4> <2,2> <2,3> <3,2> <3,3>}

	1	2	3	4
1	1	0	0	1
2	0	1	1	0
3	0	1	1	0
4	1	0	0	1



∴ It is an equivalence Relation.

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