Tutorial No. 2

Advanced Linear Algebra

Q.1) Determine the largest given values and the corresponding eigen vector of the matrix using power method.

a) $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ Taking initial eigen vector $X = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

→ Given:-

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

1st iteration:-

$$AX_0 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} 0.33 \\ 1 \end{bmatrix}$$

For 1st iteration $[\lambda = 3]$ and eigen vector

$$X_1 = \begin{bmatrix} 0.33 \\ 1 \end{bmatrix}$$

No. :
Date :

2nd iteration :-

$$AX_1 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0.33 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.32 \\ 3.33 \end{bmatrix}$$

$$= 3.33 \begin{bmatrix} 0.70 \\ 1 \end{bmatrix}$$

For 2nd iteration eigen value $\boxed{\lambda = 3.33}$ and eigen vector $X_2 = \begin{bmatrix} 0.70 \\ 1 \end{bmatrix}$

3rd iteration :-

$$AX_2 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0.70 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.8 \\ 3.7 \end{bmatrix}$$

$$= 3.8 \begin{bmatrix} 0.97 \\ 1 \end{bmatrix}$$

For 3rd iteration eigen value, $\boxed{\lambda = 3.8}$ and eigen vector $X_3 = \begin{bmatrix} 1 \\ 0.97 \end{bmatrix}$

No. :
Date :

4th iteration :-

$$AX_3 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.97 \end{bmatrix}$$

$$= \begin{bmatrix} 4.97 \\ 3.91 \end{bmatrix}$$

$$= 4.97 \begin{bmatrix} 1 \\ 0.79 \end{bmatrix}$$

For 4th iteration $\boxed{\lambda = 4.97}$ and eigen vector $X_4 = \begin{bmatrix} 1 \\ 0.79 \end{bmatrix}$

For 5th iteration :-

$$AX_4 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.79 \end{bmatrix}$$

$$= \begin{bmatrix} 4.79 \\ 3.37 \end{bmatrix}$$

$$= 4.79 \begin{bmatrix} 1 \\ 0.70 \end{bmatrix}$$

∴ For 5th iteration eigen value is 4.79 and eigen vector is $\begin{bmatrix} 1 \\ 0.70 \end{bmatrix}$

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No. :
Date :

b) $A=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}$ Taking initial eigen vector $X=\begin{bmatrix}0\\0\\1\end{bmatrix}$

→ Given :-

$$A=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}, \quad X_0=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

1st iteration,

$$AX_0=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$=\begin{bmatrix}2\\-1\\5\end{bmatrix}$$

$$=5\begin{bmatrix}0.4\\-0.2\\1\end{bmatrix}$$

For 1st iteration :- $\underline{\lambda=5}$ and eigen vector $X_1=\begin{bmatrix}0.4\\-0.2\\1\end{bmatrix}$

No. :
Date :

2nd iteration :-

$$AX_1=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.4\\-0.2\\1\end{bmatrix}$$

$$=\begin{bmatrix}3\\-0.2\\6.8\end{bmatrix}$$

$$=6.8\begin{bmatrix}0.44\\-0.02\\1\end{bmatrix}$$

For 2nd iteration :- $\underline{\lambda=6.8}$ & eigen vector is $X_2=\begin{bmatrix}0.44\\-0.02\\1\end{bmatrix}$

3rd iteration :-

$$AX_2=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}\begin{bmatrix}0.44\\-0.02\\1\end{bmatrix}$$

$$=\begin{bmatrix}2.5\\0.68\\7.58\end{bmatrix}$$

$$=7.58\begin{bmatrix}0.33\\0.09\\1\end{bmatrix}$$

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(An Autonomous Institute)

No. :
Date :

for 3rd iteration $\lambda = 7.58$ & eigen vector is $X_3 = \begin{bmatrix} 0.33 \\ 0.09 \\ -1 \end{bmatrix}$

for 4th iteration :-

$$AX_3 = \begin{bmatrix} 01 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.33 \\ 0.09 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.06 \\ 0.68 \\ 7.25 \end{bmatrix}$$

$$= 7.25 \begin{bmatrix} 0.28 \\ 0.09 \\ 1 \end{bmatrix}$$

for 4th iteration $\lambda = 7.25$ & eigen vector is ~~7.25~~ $X_4 = \begin{bmatrix} 0.28 \\ 0.09 \\ 1 \end{bmatrix}$

No. :
Date :

5th iteration :-

$$AX_4 = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0.28 \\ 0.09 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.01 \\ 0.48 \\ 6.95 \end{bmatrix}$$

$$= 6.95 \begin{bmatrix} 0.29 \\ 0.06 \\ 1 \end{bmatrix}$$

∴ for 5th iteration $\lambda = 6.95$ & eigen vector is $\begin{bmatrix} 0.29 \\ 0.06 \\ 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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No. :
Date :

c) $A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}$ Taking initial eigen vector $X=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

→ Given:-

$$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=X_0$$

1st iteration,

$$AX_0=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 6 \\ 14 \end{bmatrix}$$

$$=14\begin{bmatrix} 0.07 \\ 0.42 \\ 1 \end{bmatrix}$$

No. :
Date :

For 1st iteration $\underline{\lambda=14}$ & $X_1=\begin{bmatrix} 0.07 \\ 0.42 \\ 1 \end{bmatrix}$

2nd iteration,

$$AX_1=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}\begin{bmatrix} 0.07 \\ 0.42 \\ 1 \end{bmatrix}$$

$$=\begin{bmatrix} -0.28 \\ 5.77 \\ 10.05 \end{bmatrix}$$

$$=10.05\begin{bmatrix} -0.02 \\ 0.57 \\ 1 \end{bmatrix}$$

For 2nd iteration $\underline{\lambda=10.05}$ & $X_2=\begin{bmatrix} -0.02 \\ 0.57 \\ 1 \end{bmatrix}$

3rd iteration:-

$$AX_2=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix}\begin{bmatrix} -0.02 \\ 0.57 \\ 1 \end{bmatrix}$$

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$$= \begin{bmatrix} -0.61 \\ 6.16 \\ 10.08 \end{bmatrix}$$

$$= 10.08 \begin{bmatrix} -0.06 \\ 0.61 \\ 1 \end{bmatrix}$$

For IIIrd iteration $\lambda = 10.08$ & eigen vector $X_3 = \begin{bmatrix} -0.06 \\ 0.61 \\ 1 \end{bmatrix}$.

4th iteration :-

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{bmatrix} -0.06 \\ 0.61 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.73 \\ 6.28 \\ 10.04 \end{bmatrix}$$

$$= 10.04 \begin{bmatrix} -0.073 \\ 0.62 \\ 1 \end{bmatrix}$$

For 4th iteration, $\lambda = 10.04$ & $X_4 = \begin{bmatrix} -0.07 \\ 0.62 \\ 1 \end{bmatrix}$

5th iteration :-

$$AX_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 5 \\ 3 & 2 & 9 \end{bmatrix} \begin{bmatrix} -0.07 \\ 0.62 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.76 \\ 6.31 \\ 10.03 \end{bmatrix}$$

$$= \begin{bmatrix} 0.075 \\ 0.62 \\ 1 \end{bmatrix} 10.03$$

For 5th iteration, eigen value $\lambda = 10.03$ & eigen vector is $X_5 = \begin{bmatrix} 0.075 \\ 0.62 \\ 1 \end{bmatrix}$

No. :
Date :

d) $A=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}$ Taking initial eigen vector $X=\begin{bmatrix}1\\0\\0\end{bmatrix}$

→ Given:-

$$A=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}, \quad X_0=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

1st iteration

$$AX_0=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$=1\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$=\lambda X_1$$

for 1st iteration $\lambda=1$ & $X_1=\begin{bmatrix}1\\1\\0\end{bmatrix}$

2nd iteration :-

$$AX_1=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$=\begin{bmatrix}7\\3\\0\end{bmatrix}$$

$$=7\begin{bmatrix}1\\0.42\\0\end{bmatrix}=\lambda X_2$$

For 2nd iteration, $\lambda=7$ & $X_2=\begin{bmatrix}1\\0.42\\0\end{bmatrix}$

3rd iteration :-

$$AX_2=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1\\0.42\\0\end{bmatrix}$$

$$=\begin{bmatrix}3.52\\1.84\\0\end{bmatrix}=3.52\begin{bmatrix}1\\0.52\\0\end{bmatrix}=\lambda X_3$$

For 3rd iteration $\lambda=3.52$ & $X_3=\begin{bmatrix}1\\0.52\\0\end{bmatrix}$

4th iteration :-

$$AX_3 = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.52 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4.12 \\ 2.04 \\ 0 \end{bmatrix}$$

$$= 4.12 \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = \lambda X_4$$

for 4th iteration $\lambda = 4.12$

& $X_4 = \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$

5th iteration :-

$$AX_4 = \begin{bmatrix} 1 & 6 & 1 \\ 1.2 & 0 & \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix} = \lambda X_5$$

for 5th iteration, $\lambda = 4$ & $X_5 = \begin{bmatrix} 1 \\ 0.5 \\ 0 \end{bmatrix}$

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