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Tutorial. 2

Q.1

a) $\neg(P \rightleftarrows (Q \rightarrow (R \vee P)))$

$\rightarrow$ $\neg(P \rightleftarrows (Q \rightarrow (R \vee P)))$

$\neg(P \rightleftarrows (\neg Q \vee (R \vee P)))$

$\neg(P \leftrightarrow (\neg Q \vee R \vee P))$

$\neg((P \rightarrow (\neg Q \vee R \vee P)) \wedge ((\neg Q \vee R \vee P) \rightarrow P))$

$\neg((\neg P \vee \neg Q \vee R \vee P) \wedge (\neg(\neg Q \vee R \vee P) \vee P))$

$\neg((\neg P \vee \neg Q \vee R \vee P) \wedge (Q \wedge \neg R \wedge \neg P \vee P))$

$\neg(\underline{(\neg P \vee P \vee \neg Q \vee R)} \wedge (Q \wedge \neg R)\cancel{A})$

$\neg(\cancel{\neg Q \vee R \wedge} Q \wedge \neg R)$ $\qquad \neg(Q \wedge \neg R).$

$\neg(\cancel{(\neg Q \wedge Q) \vee (R \wedge \neg R)}) = \cancel{\neg\neg(\neg(\neg Q \wedge Q) \wedge \neg(R \wedge \neg R))}$

$= \cancel{Q \vee \neg Q \wedge \neg R \vee R}$

$= \underline{T}$

Q.2 Show that $\{\wedge, \vee\}$, $\{\vee\}$ and $\{\neg\}$ are not functionally complete.

$\rightarrow$ Consider tautology $(P \vee \neg P)$

The above tautology cannot be expressed using any of the given connectives.

Q.3 Prove that:

$P \vee (Q \bar{\vee} R) \neq (P \vee Q) \bar{\vee} (P \bar{\vee} R)$.

$\rightarrow$ Consider, LHS $= T$.

$\therefore P \vee (Q \bar{\vee} R) = T$

$\therefore$ $\cancel{(F \bar{\vee} F)}$

If $P = T$ then,

LHS $= T$

RHS $= F$ $\therefore$

Hence, $P \vee (Q \bar{\vee} R) \neq (P \vee Q) \bar{\vee} (P \vee R)$

Q. $P \wedge (Q \bar{\vee} R) = (P \wedge Q) \bar{\vee} (P \wedge R)$

→ Consider, $P = T$, $Q = F$, $R = F$

$\therefore T \wedge (F \bar{\vee} F) = (T \wedge F) \bar{\vee} (T \wedge F)$

$T \wedge T = F \bar{\vee} F$

$T = T$.

Hence proved.

Q. Select well formed formulas.

a) $(P \rightarrow (P \vee Q))$

→ It is a well formed formula. The given formula is tautology.

b) $(P \rightarrow (\neg P) \rightarrow \neg P)$

→ It is not a well formed formula. Tautology.

c) $((\neg Q \wedge P) \wedge Q)$

→ Well-formed formula. Contradiction.

Q. Substitution instance. $(P \rightarrow Q)$ for P and $((P \wedge Q) \rightarrow R)$ for Q.

a) $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$

→ $((((P \rightarrow Q) \rightarrow Q) \rightarrow (P \rightarrow Q)) \rightarrow (P \rightarrow Q))$

$((((P \rightarrow Q) \rightarrow ((P \wedge Q) \rightarrow R)) \rightarrow (P \rightarrow Q)) \rightarrow (P \rightarrow Q))$

b) $((P \rightarrow Q) \rightarrow (Q \rightarrow P))$. Q for P and $(P \wedge \neg P)$ for Q.

implies: $\Rightarrow$ $\begin{bmatrix} RHS = F \\ LHS = T \end{bmatrix}$

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$\rightarrow$ $((Q \rightarrow (P \wedge \neg P)) \rightarrow \cancel{(P \Rightarrow} (P \wedge \neg P) \rightarrow Q)))$

$\therefore$ substitution is not equivalent.

Q. Determine substitution instance of the following:

a) $(P \rightarrow (Q \rightarrow P))$
b) $((((P \rightarrow Q) \wedge (R \rightarrow S)) \wedge (P \vee R)) \rightarrow (Q \vee S))$
c) $(Q \rightarrow ((P \rightarrow P) \rightarrow Q))$
d) $(P \rightarrow ((P \rightarrow (Q \rightarrow P)) \rightarrow P))$

Q. Show the following implication.
a) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$.
$\rightarrow$ Let, RHS = F.
$\therefore (P \wedge Q) \Rightarrow T \rightarrow F$
$F \Rightarrow F$.
T $\quad$ $\cancel{T}$

b) $(P \rightarrow (Q \rightarrow R)) \Rightarrow (P \rightarrow Q) \rightarrow (P \rightarrow R)$
$\rightarrow$ consider, RHS = F
$\therefore$ ~~$P \rightarrow (Q \rightarrow R) = T$~~
$\cdot$ ~~$P =$~~ $\therefore P = T, R = F, Q = T$.

LHS $= P \rightarrow (Q \rightarrow R)$
$= T \rightarrow (T \rightarrow F)$
$= T \rightarrow F$
$= F$.
$\therefore$ LHS = RHS.

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Q. Show the following equivalence.

i) $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

$\to$ ~~$P \to (Q \vee R)$~~

~~Let, $P = F$, $Q = F$, $R = F$~~

~~$\therefore P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$~~

~~$F \to (F) \Leftrightarrow T \vee T$~~

~~$T \Leftrightarrow T$~~

Let, $LHS = F$.

$\therefore LHS = F$.

$\therefore P \to (Q \vee R) = F$.

$\therefore T \to (Q \vee R) = F$

$T \to (F \vee F) = F$.

$\therefore P = T$, $Q = F$, $R = F$.

$\therefore RHS = (P \to Q) \vee (P \to R)$.

$= (T \to F) \vee (T \to F)$

$= F \vee F$

$= F$

$\therefore LHS = RHS$.

Let, $LHS = T$.

$\therefore P \to (Q \vee R) = T$

OR

Q. $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

$\neg P \vee (Q \vee R) \Leftrightarrow (\neg P \vee Q) \vee (\neg P \vee R)$

$\neg P \vee Q \vee R \Leftrightarrow \neg P \vee Q \vee R$.