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Assignment No: 2

Q1) What are Normal Forms?

Ans:- The problem of determining in a finite number of steps, whether a given statement formula is a tautology or a contradiction or at least satifiable is known as a decision problem. obviously, the construction of truth tables involves a finite number of steps, and as such, a decision problem in the statement calculus always as a solution. Similarly, decision problems can be posed for other logical systems, particularly, for the predicate calculus. However, in the latter case, the solution of the decision problem may not be simple.

a) Principle of disjunctive normal form (PDNF)

Ans: ① A statement which consists of a sum of products canonical form.

② For a given formula, an equivalent formula consisting of disjunctions of minterms only. This form is not unique for given statement.

Minterms

① Let P and Q be two propositional variables.

② All possible formulas which consist of product of P or its negation and Product of Q or its negation but should not contain both the variable and its negation in any one of formula are called minterms of P and Q eg: For two variables P and Q, there are $2^2 = 4$ minterms

$P \wedge Q$ TT

$P \wedge \neg Q$ TF

$\neg P \wedge Q$ FT

$\neg P \wedge \neg Q$ FF

procedure: 1: For every truth table T in the truth table of the given statement choose the minterm which also has the value T for the same combination of truth values of p and q. The sum of these minterms will then be equivalent to the given statement

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

→ All Three true values considered as PDNF.

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$ - PDNF from truth

obtain the PDNF of $\neg P \vee Q$

$\neg P \vee Q$ — Boolean sum

Sum of product

$\neg P \to Q$, $Q \to P$

$[\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$

$(\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$

PDNF $= \Sigma$ min1, min0, min3

$= \Sigma\, 0, 1, 3$

eg: $(P \to Q) \land (\neg P \land Q)$

$(\neg P \lor Q) \land (\neg P \land Q)$
$= (\neg P \land Q) \land (\neg P \lor Q)$
$= (\neg P \land Q \land \neg P) \lor (\neg P \land Q \land Q)$
$= (\neg P \land \neg P \land Q) \lor (\neg P \land Q \land Q)$
$= (\neg P \land Q) \lor (\neg P \land Q)$

b) Principal Conjunctive normal form (PCNF)

Ans: ① product of sums canonical form.
② For a given formula, an equivalent formula consisting of conjunctions of maxterms only. form is not unique for the given Stateme

Maxterms
① Let P and q be two Propositional variables
② All possible formulas which consist of sum of or its negation and sum of Q or its negat but should not contain both the variabl and its negation in any one of the formula called Maxterms of P and Q.

eg. For two variables P and Q there are $2^2 = 4$
① $P \lor Q$
② $P \lor \neg Q$
③ $\neg P \lor Q$
④ $\neg P \lor \neg Q$

Procedure 1: For every truth table F in truth table of given statement, choose the maxterm which also has the value F for the same combination of the truth values of P and q. The product of these maxterms will then be equivalent to the given statement.

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

All three false values will considered as PCNF

$(\neg P \vee Q) \wedge (P \vee \neg Q) \wedge (P \vee Q)$ — PCNF

Q2) write equivalent forms for following formula and obtain PCNF.

1) $\neg(P \rightarrow Q)$

ANS $\neg(\neg P \vee Q)$

$P \wedge \neg Q$

$[P \vee (Q \wedge \neg Q)] \wedge [\neg Q \vee (P \wedge \neg P)]$

$(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg Q \vee P) \wedge (\neg Q \vee \neg P)$

2) $\neg(P \vee Q)$

ANS $\neg P \wedge \neg Q$

$\neg P \vee (Q \wedge \neg Q) \wedge \neg Q \vee (P \wedge \neg P)$

$(\neg P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (\neg Q \vee P) \wedge (\neg Q \vee \neg P)$

3) $\neg(P \Leftrightarrow Q)$

ANS: $\neg((P \rightarrow Q) \wedge (Q \rightarrow P))$

$\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$

$P \wedge \neg Q \vee Q \wedge \neg P$

$P \vee (Q \wedge \neg Q) \wedge (\neg Q \vee (P \wedge \neg P) \wedge (Q \vee (P \wedge \neg P)$

$\wedge (\neg P \vee (Q \wedge \neg Q))$

$= P \vee Q \wedge P \vee \neg Q \wedge (\neg Q \vee P) \wedge (\neg Q \wedge \neg P) \wedge$

$(Q \vee P) \wedge (Q \vee \neg P \wedge (\neg P \vee Q) \wedge (\neg P \vee \neg Q)$

Q3) Obtain product of Cannonical Normal Forms for the following

1) $(P \wedge Q \wedge R) \vee (\neg P \wedge R \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

ANS:	P	Q	R	¬P	¬Q	¬R	P∧(Q∧R)	¬P∧R∧R	¬P∧¬Q∧¬R	A
M0	T	T	T	F	F	F	T	F	F	T
M1	T	T	F	F	F	T	F	F	F	F
M2	T	F	T	F	T	F	F	F	F	F
M3	T	F	F	F	T	T	F	F	F	F
M4	F	T	T	T	F	F	F	T	F	T
M5	F	T	F	T	F	T	F	F	F	F
M6	F	F	T	T	T	F	F	T	F	T
M7	F	F	F	T	T	T	F	F	T	T

$\prod (M_1, M_2, M_3, M_5)$ .. Product of sum

$M_1 \rightarrow (\sim P \vee \sim q) \vee r) \wedge$

$M_2 \rightarrow (\sim P \vee q \sim r) \wedge$

$M_3 \rightarrow (\sim P \vee q \vee r) \wedge$

$M_4 \rightarrow (P \vee \neg q \vee r)$

2) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$

	P	Q	¬P	¬Q	P∧Q	¬P∧Q	P∧¬Q	A
M_0	T	T	F	F	T	F	F	T
M_1	T	F	F	T	F	F	T	T
M_2	F	T	T	F	F	T	F	T
M_3	F	F	T	T	F	F	F	F

$\prod_0^7 (M_3)$ — Product of sums

$M_3 = (P \vee Q)$

3) $(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$

	P	Q	R	¬P	P∧Q	¬P∧Q∧R	A
M_0	T	T	T	F	T	F	T
M_1	T	T	F	F	T	F	T
M_2	T	F	T	F	F	F	F
M_3	T	F	F	F	F	F	F
M_4	F	T	T	T	F	T	T
M_5	F	T	F	T	F	F	F
M_6	F	F	T	T	F	F	F
M_7	F	F	F	T	F	F	F

$\prod_0^7 (M_2, M_3, M_5, M_6, M_7)$ product of sum

$M_2 = (\neg P \vee Q \vee \neg R) \wedge$

$M_3 = (\neg P \vee Q \vee R) \wedge$

$M_5 = (P \vee \neg Q \vee R) \wedge$

$M_6 = (P \vee Q \vee \neg R) \wedge$

$M_7 = (P \vee Q \vee R)$

abc

Q4) Obtain the principal disjunctive and conjunctive Normal form formulas

a) $(\neg P \vee \neg Q) \rightarrow (P \rightleftarrows \neg Q)$

Ans:- $\neg(\neg P \vee \neg Q) \vee ((P \rightarrow \neg Q) \wedge (\neg Q \rightarrow P))$

$(P \wedge Q) \vee ((\neg P \vee \neg Q)) \wedge (Q \vee P))$

$(P \wedge Q) \vee ((\neg P \wedge (Q \vee P)) \vee (\neg Q \wedge (Q \vee P)$

$(P \wedge Q) \vee ((\neg P \wedge Q) \vee (\neg P \wedge P)) \vee ((\neg Q \wedge Q) \vee (\neg Q \wedge P)$

$(P \wedge Q) \vee ((\neg P \wedge Q) \vee (\neg Q \wedge P))$ – Sum of Product

Disjunctive Normal form.

$(\neg P \vee \neg Q) \rightarrow (P \rightleftarrows \neg Q)$

Ans:- $\neg(\neg P \vee \neg Q) \vee ((P \rightarrow \neg Q) \wedge (\neg Q \rightarrow P))$

$(P \wedge Q) \vee ((\neg P \vee \neg Q) \wedge (Q \vee P))$

$(P \vee ((\neg P \vee \neg Q) \wedge (Q \vee P)) \wedge (Q \vee ((\neg P \vee \neg Q) \wedge (Q \vee P)$

$(P \vee (\neg P \vee \neg Q)) \wedge (P \vee (Q \vee P)) \wedge ((Q \vee (\neg P \vee \neg$

$\wedge (Q \vee Q \vee P)$

$(P \vee (\neg P \vee \neg Q)) \wedge (Q \vee P) \wedge (Q \vee (\neg P \vee \neg Q)$ – Po

Conjunctive Normal form.

b) $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$

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Ans:- $(\neg P \vee (Q \wedge R)) \wedge (P \vee (\neg Q \wedge \neg R)$

$(\neg P \vee Q) \wedge (\neg P \vee R) \wedge (P \vee \neg Q) \wedge (P \vee \neg R)$

– product of sum.

conjunctive Normal form

$(\neg P \vee Q \wedge R) \wedge (P \vee (\neg Q \wedge \neg R)$

$(\neg P \wedge R) \vee (\neg P \wedge R) \vee (\neg P \wedge \neg R) \vee (Q \wedge \neg Q)$

$(\neg P \wedge R) \vee (\neg P \wedge \neg R)$

Disjunctive Normal form – Sum of Product

Q5) obtain PDNF/PCNF of using Truth Table

1) $\neg(P \wedge Q)$

Ans:- A)

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$
m_0	T	T	T	F
m_1	T	F	F	T
m_2	F	T	F	T
m_3	F	F	F	T

All True values considered as PDNF

$(P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

PDNF $= \sum m_1, m_2, m_3$

∴ It is the Principle disjunctive Normal Form.

B)

	P	Q	$(P \wedge Q)$	$\neg(P \wedge Q)$
M_0	T	T	T	F
M_1	T	F	F	T
M_2	F	T	F	T
M_3	F	F	F	T

All False values considerd as PCNF

$(\neg P \vee \neg Q)$

PCNF $= \prod (M_0)$

It is the principle conjunctive Normal form.

2) $\neg P \wedge Q$

Ans:

	P	Q	$\neg P$	$\neg P \wedge Q$
M_0	T	T	F	F
M_1	T	F	F	F
M_2	F	T	T	F
M_3	F	F	T	F

All False values Consider as PCNF

$(\neg P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (P \vee \neg Q) \wedge (P \vee Q)$

PCNF $= \prod (M_0, M_1, M_2, M_3)$

It's principle conjunctive Normal form.

For PDNF - No. of True values is 0.

$\therefore$ PDNF $= 0$

Q6) obtain PDNF of

1) $\neg P \vee Q$

Ans: $\neg P \wedge (Q \vee \neg Q) \vee Q \wedge (P \wedge \neg P)$

$(\neg P \wedge Q) \vee (\neg P \vee \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$

2) $P \vee (\neg P \wedge Q)$

Ans: $(P \vee \neg P) \wedge (P \vee Q)$

$P \wedge (\neg P \vee P) \vee P \wedge (Q \vee \neg Q)$

$(P \wedge \neg P) \vee (P \wedge P) \vee (P \wedge Q) \vee (P \wedge \neg Q)$

$(P \wedge \neg P) \vee T \vee (P \wedge Q) \vee (P \wedge \neg Q)$

Ans: $(P \wedge (Q \vee \neg Q)) \vee (\neg P \vee Q)$

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$

3) $P \rightarrow (P \wedge (Q \rightarrow P))$

Ans: $P \rightarrow P \wedge (\neg Q \vee P)$

$P \rightarrow (P \wedge \neg Q) \vee P \wedge P)$

$P \rightarrow (P \wedge \neg Q)$

$(\neg P \vee (P \wedge \neg Q))$

$((\neg P \wedge (Q \vee \neg Q) \vee (P \wedge \neg Q))))$

$(\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge \neg Q)$

Q3) obtain PCNF of

1) $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

Ans: $(P \vee R) \wedge (Q \rightarrow P) \wedge (P \rightarrow Q)$

$(P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$(P \vee R) \vee (Q \wedge \neg Q) \wedge (\neg Q \vee P) \vee (R \wedge \neg R) \wedge (\neg P \vee Q) \vee (R \wedge \neg R)$

$\Rightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R) \wedge$
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$\Rightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee \neg P) \wedge$
$(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

2) $(Q \rightarrow P) \wedge (\neg P \wedge Q)$

$\Rightarrow (\neg Q \vee P) \wedge (\neg P \wedge Q)$

$(\neg Q \vee P) \wedge \neg P \wedge (Q \vee \neg Q)$

$\cancel{\neg Q \vee P \wedge \neg P \wedge Q\ \neg P \wedge \neg Q)}$

$(\neg Q \vee P) \wedge (\neg P \vee \neg Q) \wedge (\neg P \vee Q)$

3) $\sim (P \leftrightarrow Q)$

Ans: $\neg((P \rightarrow Q) \wedge (Q \rightarrow P))$

$\neg(\neg P \vee Q) \vee (\neg Q \vee P)$

$(P \wedge \neg Q) \vee (Q \wedge \neg P)$

$P \vee (Q \wedge \neg Q) \wedge (\neg Q \vee (P \wedge \neg P) \wedge (Q \vee (P \wedge \neg P) \wedge$
$(\neg P \vee (Q \wedge \neg Q)$

$\Rightarrow (P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg Q \vee P) \wedge (\neg Q \wedge \neg P) \wedge$
$(Q \vee P) \wedge (Q \vee \neg P \wedge (\neg P \vee Q) \wedge (\neg P \vee \neg Q)$

Q4) Show the following

1) $\neg(P \uparrow Q) \longleftrightarrow \neg P \downarrow \neg Q$

$\neg(P \uparrow Q) \Longleftrightarrow \neg P \downarrow \neg Q$

$\Rightarrow \neg(P \uparrow Q) \Longleftrightarrow \neg P \downarrow \neg Q$

2) $\neg(P \downarrow Q) \longleftrightarrow \neg P \uparrow \neg Q$

$\Rightarrow \neg(P \downarrow Q) \Rightarrow \neg P \uparrow \neg Q$

$\neg P \uparrow \neg Q \Rightarrow \neg(P \downarrow Q)$

from ① and ②

$\neg(P \downarrow Q) \longleftrightarrow \neg P \uparrow \neg Q$

Q5) write in prefix & suffix form

~~1) $P \wedge \neg R \rightarrow Q \rightleftarrows P \wedge Q$~~

~~Ans: Su~~

2) $\neg\neg P \vee Q \wedge R \vee \neg Q$

$= \sim\sim P \vee Q \wedge R \vee \neg Q$

$= \neg(\neg P \vee Q) \wedge (R \vee \sim Q)$

$= \neg(\neg P \vee Q) \wedge R \vee \neg Q)$

Q5) write in prefix and suffix form

1) $P \land \neg R \rightarrow Q \leftrightarrow P \land Q$

Ans: $= (P \land \neg R) \rightarrow (Q \rightarrow (P \land Q)) \land ((P \land Q) \rightarrow Q)$

$= (P \land \neg R) \rightarrow ((\neg Q \lor (P \land Q)) \land (\neg (P \land Q)$

$= \neg \land P R \land ((\lor \neg \land P Q Q) (\lor \neg \land P Q Q))$ - prefix

$= (P R \land \neg) \; (P Q Q \lor \neg \land) \; (P Q Q \lor \neg \land) \land$ - postfix

2) $\neg \neg P \lor Q \land R \lor \neg Q$

Ans: $= \neg \neg P \lor Q \land R \lor \neg Q$

$= \neg (\neg P \lor Q) \land (R \lor \neg Q)$

$= \neg (\neg P \lor Q) \land R \lor \neg Q)$

$= \neg ((\neg P \lor Q) \land (R \lor \neg Q))$

$= \neg \land \; (\neg \lor P Q) (\neg \lor R Q)$ - prefix

$= (P Q \neg \lor) (R Q \neg \lor) \land \neg$ - Suffix

Q6) Convert following prefix and suffix into complete parathesize.

1) $\rightarrow \rightarrow P Q \rightarrow \rightarrow Q \leftrightarrow P \land Q$

Ans: $= (P \rightarrow Q) \rightarrow (Q \rightarrow R) \rightarrow (P \rightarrow R)$

2) $\neg P \lor Q \leftrightarrow P \neg S$

Ans: $= (\neg P \lor Q) \rightarrow (R \leftrightarrow \neg S)$

3) $P \neg P \rightarrow P \rightarrow P \rightarrow$

Ans. $= (P \rightarrow \neg P) \rightarrow (P \rightarrow P)$

5) $PQ \rightarrow RQ \rightarrow \wedge PR \vee \wedge Q \rightarrow$

Ans: $= (P \rightarrow Q) \rightarrow ((R \rightarrow Q) \wedge (Q \wedge (P \vee R))$

$= (P \rightarrow Q) \rightarrow [(R \rightarrow Q) \wedge (Q \wedge (P \vee R))]$

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