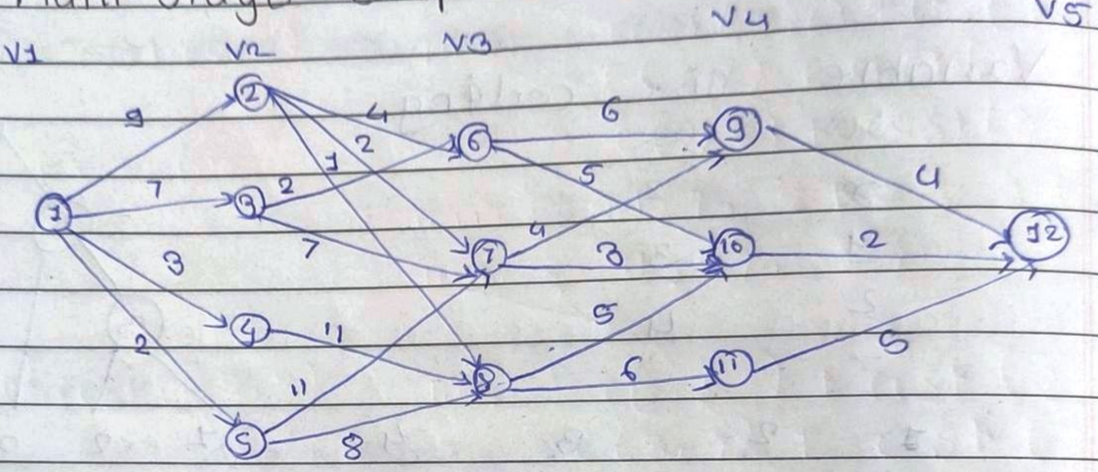
total cost size msg = size msg + sit
= 19 + 41
= 60

Unit 3 :

1) Multi·Stage Graph :-



V	1	2	3	4	5	6	7	8	9	10	11	12
cost	~~cost~~$_k$	7	9	18	15	7	5	7	4	2	5	0
d	~~d~~$_{2,3}$	7	6	8	8	10	10	10	12	12	12	12

Multiple vertices of Graph spred over the stages.

G = (V, E)

V = vertices of given graph spred over multiple stages

E = Edges

Edges of the graph they are represented weighted directed edges.

vertex is represented using 2 parameter (stage, Name of vertex)

Used principle of optimization every stage take sequence of decision

Traverse source to destination.

There is no self loop to vertex 12 : 0

1] cost (4, 9) = c (9, 12) + cost (5, 12)
= 4 + 0
= 4

2] cost (4, 10) = c (10, 12) + cost (5, 12)
= 2 + 0 = 2

3] cost (4, 11) = c (11, 12) + cost (5, 12)
= 5 + 0 = 5

4] cost (3, 6) = min { c (6, 9) + cost (4, 9),
c (6, 10) + cost (4, 10) }

= min { 6 + 4, 5 + 2 }
= min { 10, 7 }
= 7

5] cost (3, 7) = min { c (7, 9) + cost (4, 9),
c (7, 10) + cost (4, 10) }

= min { 4 + 4, 3 + 2 }
= min { 8, 5 }
= 5

6) cost (3,8) = min $\left\{ \begin{array}{l} c(8,10) + cost(4,10), \\ c(8,11) + cost(4,11) \end{array} \right\}$

= min { 5 + 2 , 6 + 5 }

= min { 7, 11 }

= 7

7) cost (2,2) = min $\left\{ \begin{array}{l} cost(2,6) + cost(3,6), \\ c(2,7) + cost(3,7), \\ c(2,8) + cost(3,8) \end{array} \right\}$

= min { 4 + 7, 2 + 5 , 1 + 7 }

= min { 11, 7, 8 }

8) cost (2,3) = min $\left\{ \begin{array}{l} c(3,6) + cost(3,6), \\ c(3,7) + cost(3,7) \end{array} \right\}$

= min { 2 + 7, 7 + 5 }

= min { 9, 12 }

9) cost ($\cancel{9}$, 4) = min $\left\{ c(4,8) + cost(3,8) \right\}$

= min { 11 + 7 }

= min { 18 }

10) cost(2,5) = min $\left\{ \begin{array}{l} c(5,7) + cost(3,7) \\ c(5,8) + cost(3,8) \end{array} \right\}$

= min { 11 + 5, 8 + 7 }

= min { 16, 15 }

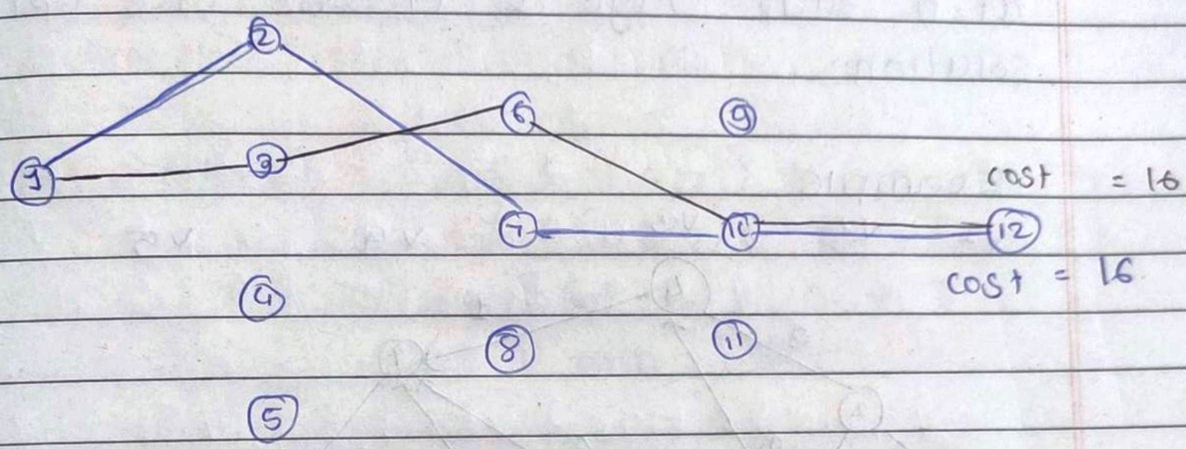
= 15

11) cost (1,1) = min $\left\{ \begin{array}{l} c(1,2) + cost(2,2) \\ c(1,3) + cost(2,3) \\ \cancel{c(1,4) + cost(2,4)} \\ c(1,5) + cost(2,5) \end{array} \right.$

$= \min(9+7,\ 7+9,\ 3+18,\ 2+15)$

$= \min\{16,\ 16,\ 21,\ 17\}$

$= 16$



Minimum Path from source (vertex 1) to destination (vertex 12).

$1 \xrightarrow{9} 2 \xrightarrow{2} 7 \xrightarrow{3} 10 \xrightarrow{2} 12$

cost = 16 $\quad 9+2+3+2 = 16$

$1 \xrightarrow{7} 3 \xrightarrow{2} 6 \xrightarrow{5} 10 \xrightarrow{2} 12$

cost = 16 $\quad 7+2+5+2 = 16$

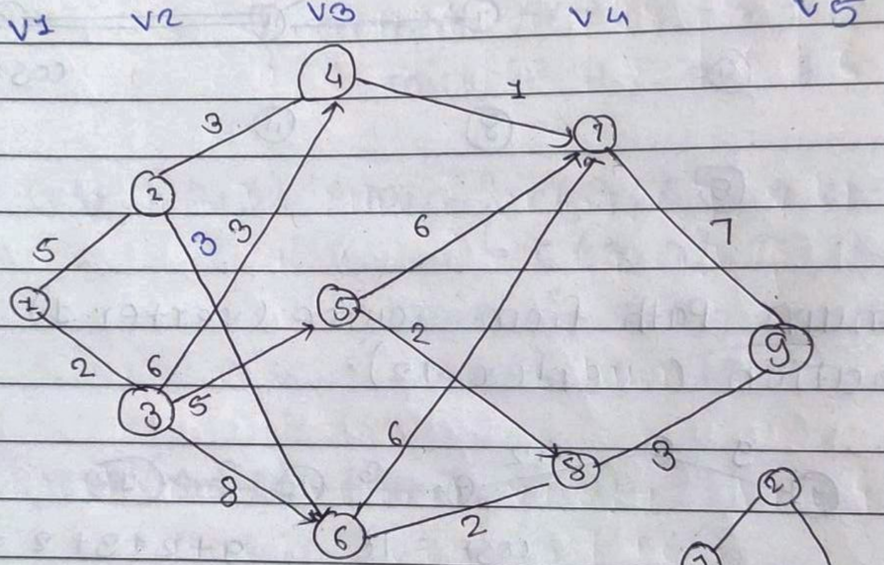
Formula -

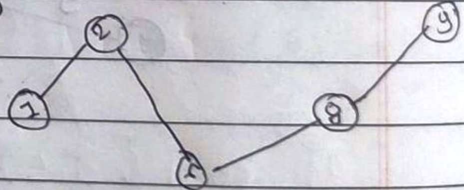
$$\underset{\text{stage}}{\text{cost}(i},\ \underset{\text{vertex}}{j)} = \min_{l,j \in E} \left\{ c(j,l) + \text{cost}(i+1, l) \right\}$$

Dynamic Programming

It is based on principle of optimization in dynamic programming principle of optimization it say's that take a sequence of decision at a each stage & choose one optimal solution.

Example





V	1	2	3	4	5	6	7	8	9
V	12	8	10	8	5	5	7	3	0
cost	3	6	5	7	8	8	9	9	9

There is no self loop to vertex 9 = 0

cost (~~4~~ 7) = c(7, 9) + cost(5, 9)

= 7 + 0

= 7

$\text{cost}(\cancel{3}4, 8) = c(8, 9) + \text{cost}(5, 9)$
$= 3 + 0$
$= 3$

$\text{cost}(3, 4) = c(4, 7) + \cancel{t4}\ \text{cost}(4, 7)$
$= 1 + 7$
$= 8$

$\text{cost}(3, 5) = \min\begin{cases} c(5, 7) + \text{cost}(4, 7), \\ c(5, 8) + \text{cost}(4, 8) \end{cases}$
$= \min\{6 + 7, 2 + 3\}$
$= \min\{5\}$

$\text{cost}(3, 6) = \min\begin{cases} c(6, 7) + \text{cost}(4, 7), \\ c(6, 8) + \text{cost}(4, 8) \end{cases}$
$= \min\{6 + 7, 2 + 3\}$
$= \min\{5\}$

$\text{cos}(2, 2) = \min\begin{cases} c(2, 4) + \text{cost}(3, 4), \\ c(2, 6) + \text{cost}(3, 6) \end{cases}$
$= \min\{3 + 8, 3 + 5\}$
$= 8$

$\text{cost}(2, 3) = \min\begin{cases} c(3, 4) + \text{cost}(3, 4), \\ c(3, 5) + \text{cost}(3, 5), \\ c(3, 6) + \text{cost}(3, 6) \end{cases}$
$= \min\{3 + 8, 5 + 5, 8 + 5\}$
$= \min\{10\}$

$\text{cost}(1, 1) = \min\begin{cases} c(1, 2) + \text{cost}(2, 2), \\ c(1, 3) + \text{cost}(2, 3) \end{cases}$
$= \min\{5 + 8, 2 + 10\} = 12$

* 0/1 knapsack problem

1 $m = 8$ $p = \{1, 2, 5, 6\}$

0 $n = 4$ $w = \{2, 3, 4, 5\}$

p	wi		0	1	2	3	4	5	6	7	8
1	2	0	0	0	0	0	0	0	0	0	0
2	3	1	0	0	1	1	1	1	1	1	1
5	4	2	0	0	1	2	2	3	3	3	3
6	5	3	0	0	1	2	35	5	6	7	7
		4	0	0	1	2	5	6	6	7	8

$v[i, w] = \text{Max}\{v[i-1, w], v[i-1, w - w[i]]\}$

$v[3, 6]$: max $\{v[3-1, 6], v[2, 6-2]+ p[i]\}$

1, 6 2-

1, [2, 5] + 3 1, 2 + 3

1, 5

i = row w = column

$v[4, 1] = \text{Max}\{v[3, 1], v[3, 1-5] + 6\}$

$= \text{Max}\{0, v[3, -5] + 6\}$

$v[4, 5] = \text{Max}\{v[3, 5], v[3, 5-5] + 6\}$

$= \text{Max}\{5, 6\}$

$v[4, 6] = \text{Max}\{v[3, 6], v[3, 6-5] + 6\}$

$= \text{Max}\{6, 0+6\}$

$= 6$

$v[4, 7] = \text{Max}\{v[3, 7], v[3, 7-5] + 6\}$

$= \{7, 1+6\}$

$= 7$

$i = 2 \quad j = 6 \qquad i = 1$

$\omega = 5$

2/2 $T(2,6) = \{T(2-1), 6) + 1 + T(2-1, 6-5)$

$(1,6) + 1 + (1, 1)$

$1 + 1 + 0$

$x_1 \quad x_2 \quad x_3 \quad x_4$

$(0 \quad 1 \quad 0 \quad 1)$ $1,1$ 8 is determined by x_4 object.

$x_4 : 8$

Now subtract profit of 4th object from 8

$= 8 - 6 \qquad (2,1) = \{T(2-1), 1) + 2 + T(2-1) \cancel{1} - 5)$

$= 2 \qquad = (1,1) + 2 + (1, 2)$

$x_2 = 2 \qquad P(x_2 = 2) \quad 1\,2 + 0$

$= \cancel{2/2 = 1} \qquad 1, 2.$

$= 2 - 2$

$\Rightarrow 0$

$\therefore m_1 = 0 \qquad x_1 \quad x_2 \quad x_3 \quad x_4$

$\therefore m_0 = 1 \qquad 0 \quad 1 \quad 0 \quad 1$

$m = 8 \qquad P = \{2, 3, 1, 4\}$

$n = 4 \qquad \omega = \{3, 4, 6, 5\}$

P	wi		0	1	2	3	4	5	6	7	8
		0	0	0	0	0	0	0	0	0	0
2	3	1	0	0	0	2	2	2	2	2	2
3	4	2	0	0	0	2	3	3	3	5	5
1	6	3	0	0	0	2	3	3	3	5	5
4	5	4	0	0	0	0	0	4	4	5	5

$V[3,6] = \{V[2,6], V[2, 6-3] + 1\}$

$= \{3, 2+1\}$

$= 3$

$V[3,7] = \max\{V[2,7], V\{2, 7-3] + 1\}$

$= \max\{5 + 3 + 1\}$

$= \max\{5\}$

$v[4,1] = max\{v[3,1], v[3, 1-5] + 4\}$
$= max\{0, v[3,-4] + 4\}$

$v[4,5] = max\{v[3,5], v[3, 5-5] + 4\}$
$= max\{3, 0+4\}$
$= max [4]$

$[4,6] = max\{v[3,6], v[3, 6-5] + 4\}$
$= max\{3, 0+4\}$
$= max\{4\}$

Using Set method

$m = 8$ $\quad$ $P = \{1 \; 2 \; 5 \; 6\}$

$n = 4$ $\quad$ $W = \{2 \; 3 \; 4 \; 5\}$

set :- (P, W) $\quad$ $S^0 = \{(0,0)\}$

$S^0_1 = \{(1,2)\}$

add (2,3) in $\rightarrow S^1 = \{(0,0), (1,2)\}$

~~$S^1_1 = \{(0,0), (1,2), (2,3)\}$~~

$S^1_1 = \{(2,3), (3,5)\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S^2_1 = \{(5,4), (6,6), (7,7), (8,9)\}$

$S^3 = \{(0,0), (1,2), (2,3), (3/5), (5,4), (6,6), (7,7), (8/9)\}$

Here (3,5) is killed because when profit is increased then weight is also increased but here weight is decreased in next set.

(8,9) is killed bcz size over of sack.

$S^3_1 = \{(6,5), (7,7), (8,8), (11,9), (12,11), (12,12)\}$

$S^4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), (6,6), (7,7), (8,8)\}$

(6,6) is killed because the weight of this sack is greater than previous means (6,5)

Here our aim to reduce the weight of set.

x_1	x_2	x_3	x_4
0	1	0	1

$(x,8) \in S^4$

$x^4 = 1$

minus weight of x_4 object weight from x_4 weight same in case of profit.

$8-6 \qquad 8-5$

$(2,3) \in x_2$

$(2,3) \notin x_3$

$2-2 \qquad 3-3$

$0 \qquad 0$

$(0,0)$

x_1	x_2	x_3	x_4
0	1	0	1

Example:-

② $m = 8 \qquad p = \{2, 3, 1, 4\}$

$n = 4 \qquad w = \{3, 4, 6, 5\}$

$p = (p, w)$

$S_0^0 = \{(0,0)\}$

$S_1^0 = \{(2,3)\}$

$S^1 = \{(0,0), (2,3)\}$

$S_1^1 = \{(3,4), (5,7)\}$

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$S^2 = \{(0,0), (2,3), (3,4), (5,7)\}$
$S_1^2 = \{(1,6), (3,9), (4,10), (6,13)\}$

$S^3 = \{(0,0), (2,3), (3,4), \cancel{(1,6)}, \cancel{(3,9)}, \cancel{(4,10)}, (5,7), \cancel{(6,13)}\}$

$S^4 = \{(0,0), (2,3), (3,4), (5,7)\}$

x_1	x_2	x_3	x_4
1	0	0	1

$(5,7) \in 2$

$S^5 = \{(4,5), (6,8), (7,9), (9,12)\}$
$S^4 = \{(0,0), (2,3), (3,4), (4,5), (5,7), (6,8)\}, \cancel{(7,9)}, \cancel{(9,12)}$

maximum $= (6,8)$

$(6,8) \in S4 \quad (6,8) \notin x^3$

$x^4 = 1$

$6-4 \quad 8-5$
$(2,3)$
$(2,3) \in x^1 \quad (2,3) \notin x^2 \quad (2,3) \notin x^3$
$2-2 \quad 3-3$
$0 \quad 0 \quad (0,0) \notin x^1$
$(0,0) \notin x^2$

x_1	x_2	x_3	x_4
1	0	0	1

⑨ $P = \{1, 2, 3\}$ $m = 9$

$w = \{4, 5, 6\}$

$p_i(P, w)$

$S_0 = \{(0, 0)\}$

$S^0_1 = \{(0 1/4)\}$ $\{(1, 4)\}$

$S^1 = \{(0,0), (1/4)\}$ $\{(0,0)\ (1,4)\}$

$S^1_1 = \{(2/5), (3, 9)\}$ $\{(2,5), (9,9)\}$

$S_1 = \{(0,0), (1,4), (2,5), (3,9)\}$

$S_1 = \{(0,0), (1,4), (2,5), (3/9)\}$ 2

x_1	x_2	x_3	x_4
1	1	0	0

Reliability Design.

To increase reliability of system by taking multiple copy of each device.
Reliability is nothing but probability of failure free time of device. Each device has its own cost and reliability.

	D1	D2	D3	D4
r	0.9	0.9	0.9	0.9

$\Pi r_i = (0.9)^4 = 0.65$

failure time $= 1 - 0.65 = 0.35$

probability lies between 0 to 1

failure time $= \boxed{1 - r}$

Because of any reason, your system may get failure by 0.35 %

D1	D2	D3	D4
D1	D2	D3	D4
D1			D4

0.9

Reliability of device $D_1 = 0.9$

Hence failure time of D_1 (single copy) $= 1 - 0.9$
$= 0.1$

So failure time of 3 copies of $D_1 = (1-0.9)^3$
$= 0.001$

Reliability of device D_1 (3 copies) $= 0.999$

hence $0.9 < 0.999$

hence $0.9 < 0.999$

So we have proved that if we take multiple copies of devices, reliability will be improved

Eg.

D_i	c_i	r_i	u_i
D1	30	0.9	2
D2	15	0.8	3
D3	20	0.5	3

$C = 105 \qquad \Sigma c_i = c_1 + c_2 + c_3 = 30 + 15 + 20 = 65$

$C - \Sigma c_i = 105 - 65 = 40$

$D_1 \Rightarrow u_1 = \left[\dfrac{C - \Sigma c_i}{c_1}\right] + 1 = \left[\dfrac{40}{30}\right] + 1 = 1 + 1 = 2$

~~D2~~

$D_2 \Rightarrow u_2 = \left[\dfrac{C - \Sigma c_i}{c_2}\right] + 1 = \left[\dfrac{40}{15}\right] + 1 = 2 + 1 = 3$

$D_3 \Rightarrow u_3 = \left[\dfrac{C - \Sigma c_i}{c_3}\right] + 1 = \left[\dfrac{40}{20}\right] + 1 = 2 + 1 = 3$

Set Method

(R, C)

$S^0 = \{(1, 0)\}$

Consider Device D1

$S_1^1 = \{(0.9, 30)\}$

$S_2^1 = (1 - 0.9)^2$

$= 1 - (1 - 0.9)^2 = 1 - 0.01 = 0.99$

$S_2^1 = \{0.99, 60)\}$

$S^1 = \{(0.9, 30), (0.99, 60)\}$

Consider Device D2

$S_1^2 = \{(0.9 * 0.8, 30 + 15), (0.99 * 0.8, 60 + 15)\}$

$= \{(0.72, 45), (0.792, 75)\}$

$S_2^2 = 1 - (1 - 0.8)^2 = 1 - (0.2)^2 = 0.96 \quad (0.96, 30)$

$S_2^2 = \{(0.96 * 0.9, 60), (0.96 * \cancel{0.99}, 90)\}$

$= \{(0.864, 60)\}$

$S_3^2 = 1 - (1 - 0.8)^3 = 1 - (0.2)^3 = 1 - 0.008 = 0.992, 45$

$S_3^2 = \{(0.992 * 0.9, 60) \times 15, (0.992 * \cancel{0}.99, 105)\}$

$= \{(0.8928, 75)\}$

$S^2 = \{(0.72, 45), (0.7\cancel{9}2, 75), (0.864, 60), (0.8928, 75)\}$

Remove the pair with maximum cost

Consider Device D3

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.448\cancel{8}, 95)\}$

$S_2^3 = \{(0.54, 85), (0.648, 100)\}$

$S_3^3 = \{(0.63, 105)\}$

$S^3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.6\cancel{3}, 105), (0.648, 100)\}$

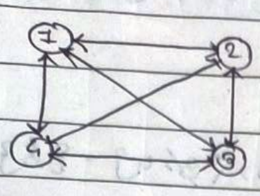
D3 = 2

D2 = 2

D1 = 1

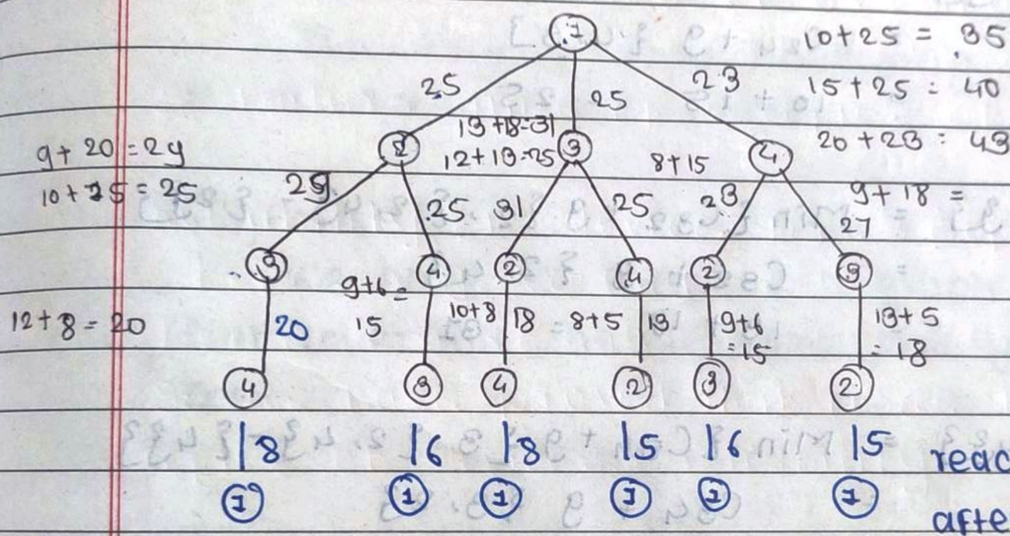
12

Travelling Sales person problem :-



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 9 & 10 \\ 3 & 6 & 13 & 0 & 12 \\ 4 & 8 & 8 & 9 & 0 \end{array}$$

Traverse from any node and cover all nodes & reach back to the starting node. we will get minimum cost.



reach starting node after traversing all nodes

$G - g(1, \{2, 3, 4\}) = \min_{k \in \{2,3,4\}} \{ C_{1k} + g\{k, \{2,3,4\} - \{k\}\} \}$

Formula :

$g(i, \{s\}) = \min_{k \in \{s\}} \{ C_{ik} + g\{k, \{s\} - \{k\}\} \}$ remaining

s - remaining node set

$g(2, \emptyset) = 5$

$g(3, \emptyset) = 6$

$g(4, \emptyset) = 8$

$g\{2\{3\}\} = \min \{ C_{23} + g\{3; \{3\} - \{3\}\} \}$

$= C_{23} + g\{3, 0\} = 9 + 6 = 15$

$g\{2\{4\}\} = \min \{ C_{24} + g\{4, \{4\} - \{4\}\} \}$

$= C_{24} + g\{4, 0\} = 10 + 8 = 18$

$g(3,\{2\}) = 18$ $g(3\{4\}) = 20$ $g(4,\{2\}) = 15$

$g(4,\{3\}) = 18$

$g\{2,\{3,4\}\} = \text{Min}\{C_{23} + g\{3,\{\cancel{3},4\} - \{3\}\}$

$= C_{23} + g\{3,4\}\}$

$= 9 + 20 = 29$

$g\{2,\{3,4\}\} = \text{Min}\{C_{24} + g\{4\{3,4\} - \{4\}\}$

$= C_{24} + g\{4,3]$

$= 10 + 15 = 25$

$g\{3\{2,4\}\} = \text{Min}\{C_{32} + g\{2,\{2,4\} - \{2\}\}$

$= C_{32} + g\{2,4\}$

$= 13 + 18 = 31$

$g\{3\{2,4\}\} = \text{Min}\{C_{34} + g\{3\{2,4\} - \{4\}\}$

$= C_{34} + g\{3,2\}$

$= 12 + 18 = 25$

$g\{4,\{2,3\}\} = \text{Min}\{C_{42} + g\{4\{2,3\} - \{2\}\}$

$= C_{42} + g\{4,3\}$

$= 8 + 15 = 23$

$g\{4,\{2,3\}\} = \text{Min}\{C_{43} + g\{4\{2,3\} - \{3\}\}$

$= 9 + \{4,2\}$

$= 9 + 18 = 27$

$g\{1,\{2,3,4\}\} = \text{Min}\{C_{12} + g\{1\{2,3,4\} - \{2\}$

$C_{12} + g\{2,\{3,4\}\}$

$10 + 25 = 35$

$g\{1,\{2,3,4\}\} = \text{Min}\{C_{13} + g\{3,\{2,3,4\} - \{3\}\}$

$= 15 + g\{3,\{2,4\}\}$

$= 15 + 25 = 40$

$g\{1,\{2,3,4\}\} = \text{Min}\{C_{14} + g\{4,\{2,3,4\} - \{4\}\}$

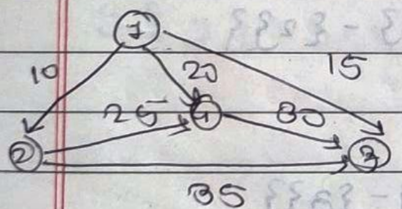
$= C_{14} + g\{4,\{2,3\}\}$

$= 20 + 23 = 43$

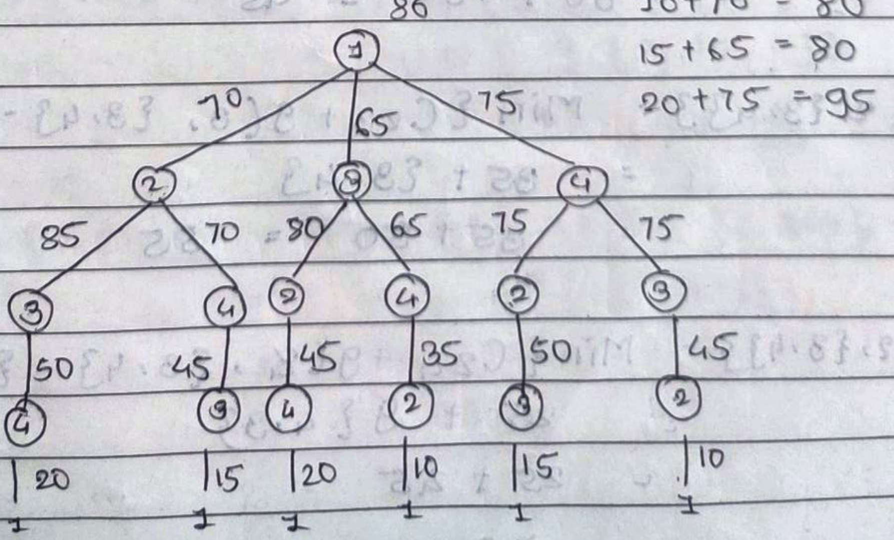
You may start a traversing from any vertese of a node a pro graph.

Objective:-

We have to traverse a graph in a such way that cover all the vertices and you in minimum cost.



$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 10 & 0 & 35 & 25 \\ 3 & 15 & 35 & 0 & 30 \\ 4 & 20 & 25 & 30 & 0 \end{array}$$



$10 + 70 = 80$

$15 + 65 = 80$

$20 + 75 = 95$

15
65
80

35 | 35 | 30
15 | 45 | 45
50 | 80 | 75

50
35

25
45

$g(2, \emptyset) = 10$
$g(3, \emptyset) = \cancel{20}15$
$g(4, \emptyset) = 20$

$g(2\{3\}) = Min\{C_{23} + g\{3, \{3\} - \{3\}\}$
$= 35 + \{3, 0\} = 35 + 15 = 50$

$g(2\{4\}) : Min\{C_{24} + g(4, \{4\} - \{4\}\}$
$= 25 + g\{4, 0\} = 25 + 20 = 45$

$g(3\{2\}) = Min\{C_{32} + g(2, \{2\} - \{2\}\}$
$= 35 + \{2, 0\} = 35 + 10 = 45$

$g(3\{4\}) = Min\{C_{34} + g(4, \{4\} - \{4\}\}$
$= 30 + \{4, 0\} = 30 + 20 = 50$

$g(4\{2\}) = Min\{C_{42} + g(2, \{2\} - \{2\}\}$
$= 25 + 10 = 35$

$g(4\{3\}) = Min\{C_{43} + g(3, \{3\} - \{3\}\}$
$= 30 + 15 = 45$

$g\{2, \{3, 4\}\} = Min\{C_{23} + g(3, \{3, 4\} - \{3\}\}$
$= 35 + \{3, 4\}$
$= 35 + 50 = 85$

$g\{2, \{3, 4\}\} = Min\{C_{24} + g(\cancel{2}, \{3, 4\} - \{4\}\}$
$= 25 + g\{4, 3\}$
$= 25 + 45$
$= 70$

$g\{3\{2,4\}\} = \text{Min}\{C_{32} + g\{2,\{2,4\}-\{2\}\}$
$= 35 + g\{2,4\}$
$= 35 + 45 \quad = 80$

$g\{3\{2,4\}\} = \text{Min}\{C_{34} + g\{4,\{2,4\}-\{4\}\}$
$= 30 + g\{4,2\}$
$= 30 + 35 \quad = 65$

$g\{4\{2,3\}\} = \text{Min}\{C_{42} + g\{2,\{2,3\}-\{2\}\}$
$= 25 + g\{2,3\}$
$= 25 + 50 \quad = 75$

$g\{4\{2,3\}\} = \text{Min}\{C_{43} + g\{3,\{2,3\}-\{3\}\}$
$= \cancel{25}\ 30 + g\{3,2\}$
$= 30 + 45 \quad = 75$

$g\{1\{2,3,4\}\} = \text{Min}\{C_{12} + g\{2\{2,3,4\}-\{2\}\}$
$= 10 + g\{2\{3,4\}\}$
$= 10 + 70 \quad = \underline{80}$

$g\{1\{2,3,4\}\} = \text{Min}\{C_{13} + g\{3\{2,3,4\}-\{3\}\}$
$= 15 + g\{3,\{2,4\}\}$
$= 15 + 65 \quad = \underline{80}$

$g\{1\{2,3,4\}\} = \text{Min}\{C_{14} + g\{4\{2,3,4\}-\{4\}\}$
$= 20 + g\{4,\{2,3\}\}$
$= 20 + 75$
$= \underline{\underline{95}}$

$n = 4 \quad m = 8$

W ~~P~~ -	2	3	1	4
P ~~W~~ -	3	4	6	5

$S_0^0 = \{(0,0)\}$

$S_1^0 : \{(3,2)\}$

$S1 = \{(0,0)\ (3,2)\}$

$S_1^1 = \{(3,2)\ (6,4)\}$ $\quad S_1^1 = \{(34,3)\ (7,5)\}$

$S^2 = \{(0,0)\ (3,2)\ (4,3)\ (7,5)\}$

$S_1^2 = \{(6,1)\ (9,3)\ \underline{(10,4)}\ (13,6)\}$

$S3 = \{(0,0)\ (3,2)\ (4,3)\ (6,1)\ (7,5)\ (9,3)\ (10,4)\ (13,6)\}$

$S3 = \{(0,0)\ (6,1)\ (9,3)\ (10,4)\ (13,6\}$

~~xo~~

$S^4 = \{(5,4)\ (11,5)\ (14,7)\ (15,8)\ (18,10)\}$

$S^4 : \{(0,0)\ (5,4)\ (6,1)\ (9,3)\ (10,4)\ (11,5)\ (13,6)\ (14,7)\ \underline{(15,8)}$
$(18,10)\}$

$= \{(0,0)\ (6,1)\ (9,3)\ (10,4)\ (11,5)\ (13,6)\ (14,7)\ (15,8)\}$

max : $(15,8)$

$(15,8) \in \boxed{S4}$

x_1	x_2	x_3	x_4
0	1	~~0~~1	1

$15 - 5 \quad 8 - 4$

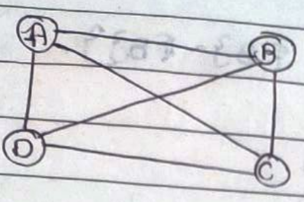
$(10,4) \in \boxed{S3}$ $\quad (10,4) \notin S2$ $\quad$ ~~$(10,4) \notin S1$~~

$10 - 6 \quad 4 - 1$

$(4,3) \in S2$

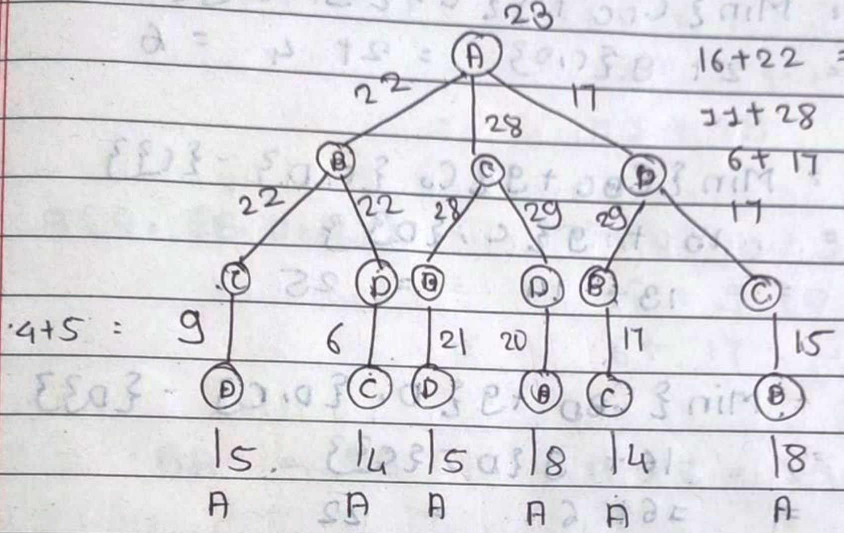
$4 - 4 \quad 3 - 3$

$(0,0) \notin S1$



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 16 & 11 & 6 \\ B & 8 & 0 & 13 & 16 \\ C & 4 & 7 & 0 & 9 \\ D & 5 & 12 & 2 & 0 \end{array}$$

$22 + 16 = 38$ $\quad$ $11 + 28 = 39$ $\quad$ $17 + 16 = 29$



$16 + 22 = 38$

$11 + 28 = 39$

$6 + 17 = 23$

$g(B, \emptyset) = 8$

$g(C, \emptyset) = 4$

$g(D, \emptyset) = 5$

$g(B\{C\}) = \text{Min}\{C_{BC} + g\{C\{C\} - \{C\}\}$

$= 13 + g\{C, \emptyset\}$

$= 13 + 4 = 17$

$g(B\{D\}) = \text{Min}\{C_{BD} + g\{D\{D\} - \{D\}\}\}$

$= 16 + g\{D, \emptyset\} = 16 + 5 = 21$

$g(C\{B\}) = \text{Min}\{C_{CB} + g\{B, \{B\} - \{B\}\}\}$

$= 7 + g\{B, \emptyset\} = 7 + 8 = 15$

$g(C\{D\}) = \text{Min}\{C_{CD} + g\{D, \{D\} - \{D\}\}\}$

$= 9 + g\{D, \emptyset\}$

$= 9 + 5 = 14$

$g\{D\{B\}) = \text{Min}\{C_{DB} + g\{B, \{B\} - \{B\}\}$
$= 12 + g\{B, 0\}$
$= 12 + 8 = 20$

$g\{D\{C\}) = \text{Min}\{C_{DC} + g\{C, \{C\} - \{C\}\}$
$= 2 + g\{C, 0\} = 2 + 4 = 6$

$g(B\{C, D\}) = \text{Min}\{C_{BC} + g\{C, \{C, D\} - \{C\}\}$
$= 13 + g\{C, \{D\}\}$
$= 13 + 14 = 25$

$g(B\{C, D\}) = \text{Min}\{C_{BD} + g\{D, \{D, C\} - \{D\}\}$
$= 16 + g\{D, \{C\}\}$
$= 16 + 6 = 22$

$g(C\{D, B\}) = \text{Min}\{C_{CB} + g\{B\{B, D\} - \{B\}\}$
$= 7 + g\{B, D\}$
$= 7 + 25 = 32$

$g(C\{B, D\}\} = \text{Min}\{C_{CD} + g\{D, \{B, D\} - \{D\}\}$
$= 9 + g\{D\{B\}\}$
$= 9 + 20 = 29$

$g(D\{B, C\}) = \text{Min}\{C_{DB} + g\{B, \{B, C\} - \{B\}\}$
$= 12 + g\{B, \{C\}\}$
$\begin{array}{r} 17 \\ \underline{12} \\ 9 \end{array}$ $= 12 + 17 = 29$

$g(D\{B, C\}) = \text{Min}\{C_{DC} + g\{C, \{B, C\} - \{C\}\}$
$= 2 + g\{C\{B\}$
$= 2 + 15 = 17$

$g(A, \{B, C, D\}) = \text{Min}\{C_{AB} + g\{B, \{B, C, D\} - \{B\}\}$
$= 16 + g\{B \{C, D\}\}$
$= 16 + 22 = 38$

$g(A, \{B, C, D\}) = \text{Min}\{C_{AC} + g\{\cancel{B}\{B, C, D\} - \{C\}\}$
$= 11 + g\{C, \{B, D\}\}$
$= 11 + 29 = 39$

$g(A, \{B, D, D\}) = \text{Min}\{C_{AD} + g\{D\{B, C, D\} - \{D\}\}$
$= 6 + g\{D \{B, C\}\}$
$= 6 + 17 = 23.$

$AB = 38 \quad A \to C = 39 \quad A \to D = 23$

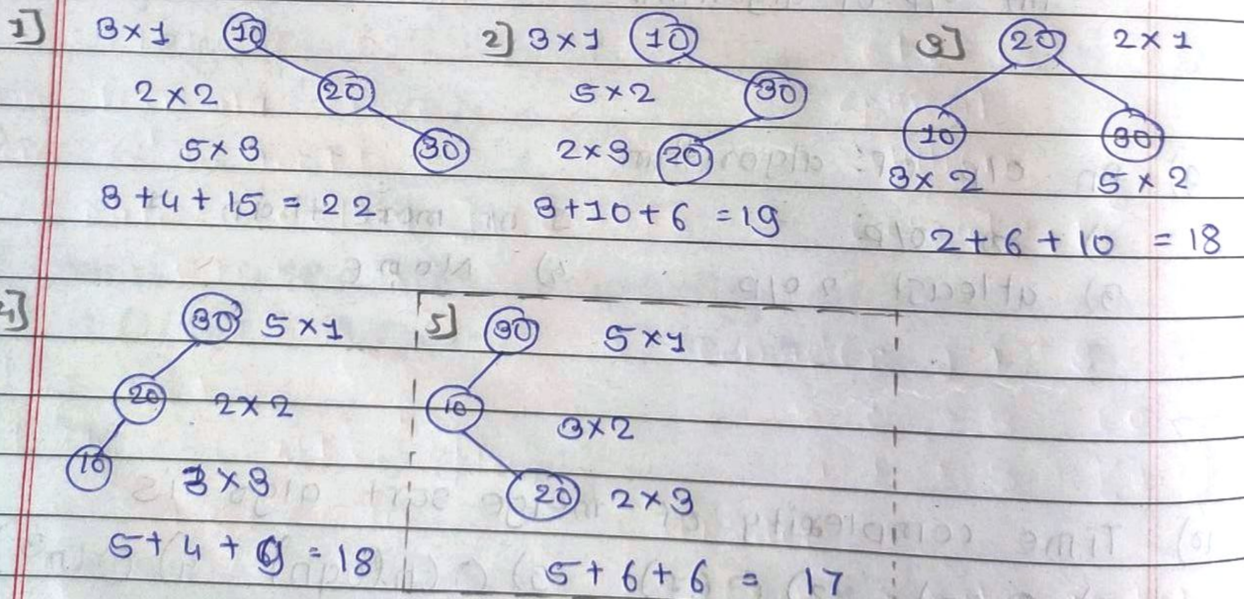
1) n step of algorithm should be clear
2) Step of algo anambigous
3) algorithm must terminate after a finite
4 no. step
all of the above

- Optimal Binary Search tree

key	10	20	30
frequency	3	2	5

- We are creating Binary tree to search particular element.

Step 1 : Create all possible binary tree.



Optimal Solution

Feasible Solution :- tree 1, 2, 3, 4
Optimal solution :- tree 5

For tree 3 we got 5 Binary tree, considering binary tree & determining cost of each Binary tree. So the reduce the time we will apply dynamic programming

Example - 2

	1	2	3	4
key ->	10	20	30	40
frequency ->	4	2	6	3

Here total Four 4 numbers are given.

Formula :-

$$c[i,j] = \min_{i<k<j} \{ c[i,k-1] + c[k,j] + w[i,j] \}$$

	0	1	2	3	4
0	0	4	8^1	20^3	26^3
1		0	2	10^3	16^3
2			0	6	12^3
3				0	3
4					0

Consider $j-1=0$

0 - 0 = 0 [0, 0]
1 - 1 = 0 [1, 1]
2 - 2 = 0 [2, 2]
3 - 3 = 0 [3, 3]
4 - 4 = 0 [4, 4]

Considering $j-i=1$

1 - 0 = 1 [0, 1]
2 - 1 = 1 [1, 2]
3 - 2 = 1 [2, 3]
4 - 3 = 1 [3, 4]

c[0, 1] = 4 c[1, 2] = 2

c[2, 3] = 6 c[3, 4] = 3

K = Root Node.

Considering $j - i = 2$ *

$2 - 0 = 2 \quad [0,2]$

$3 - 1 = 2 \quad [1,3]$

$4 - 2 = 2 \quad [2,4]$

$C[0,2] = 0 + 0 \; 2 =$

i j $\quad \boxed{0, 1, 2}$

Here we assume $j - i = 2$

So only consider $\underline{1, 2}$

$1 : 10 \quad 2 : 20$

10 1×4 → 20 2×2 ; 20 2×1 → 10 2×4

$4 + 4 = 8 \qquad = 2 + 8 = 10$

optimal sol^n

$C[0,2] = 8^1$ from 1^{st} node.

Using Formula Prove :-

from 1^{st} as root :-

$[0,2] = \text{Min} \{ C[0,0] + [1,2] + 6 \}$

$0 + 2 + 6 = 8$

2^{nd} as root :-

$[0,2] = \text{Min} \{ C[0,1] + C[\overset{k}{2},2] + 6 \}$

$4 + 0 + 6 = 10$.

Consider

$C[1,3]$ = value in bet^w

$1, 2, 3$ consider $\boxed{2,3}$

20 - 2 $\qquad$ 30 - 3

1×2 20 → 30 2×6 ; 30 6×1 → 20 2×2

$2 + 12 = 14 \qquad 6 + 4 = 10$

optimal sol^n.

$C[1,3] = 10^3$

$C[2,4]$: value in bet^w

$2, 3, 4$ consider $\boxed{3,4}$

30 - 3 $\qquad$ 40 - 4

30 1×6 → 40 2×3 ; 40 1×3 → 30 2×6

$6 + 6 = 12 \qquad 3 + 12 = 15$

optimal solution

$C[2,4] = 12^3$

Consider $j - i = 3$

$3 - 0 = 3 \quad [0,3]$

$4 - 1 = 3 \quad [1,4]$

Consider

$C[0,3]$ = value in bet^w

$\boxed{0, 1, 2, 3}$ consider $\boxed{1,2,3}$

10 - 1 $\qquad$ 20 - 2 $\qquad$ 30 - 3

10 → 30 → 20 ; 10 → 20 → 30

Consider

c [0, 3] = value in betw 0, 1, 2, 3 consider $\boxed{1,2,3}$

c [0, 3] = Min { c[0, 0], c [1, 3] + w [12] }
i j

0 + 10 + 12 =

= Min { c[0, 0] + c [1, 3] + 12, c [0, 1] + c [2, 3] + 12, c [0, 2] + c [3, 3] + 12 }

= 0 + 10 + 12 , 4 + 6 + 12 , 8 + 0 + 12

= 22 , 22 , 20

1 2 3

optimal solution : 3rd node.

C [0, 3] = 20^3

c[1, 4] : 1, 2, 3, 4 consider 2, 3, 4

c [1, 4] = Min { c [1, 1] + c [2, 4] + 11, c [1, 2] + c [3, 4] + 11 , c [1, 3] + c [4, 4] + 11 }
i j

= 0 + 12 + 11 , 2 + 3 + 11, 10 + 0 + 11

= 23 , 16 , 21

3

optimal solution = 3rd node

c [1, 4] = 16^3

Consider j - i = 4

4 - 0 = 4 [0, 4]

c[0, 4] = value in betw 0, 1, 2, 3, 4

consider [1, 2, 3, 4]

$c[0,4] = \min\{c[0,0] + c[1,4] + 15,\ c[0,1] + c[2,4] + 15,\ c[0,2] + c[3,4] + 15,\ c[0,3] + c[4,4] + 15\}$

i j

$= \{0 + 16 + 15,\ 4 + 12 + 15,\ 8 + 3 + 15,\ 20 + 0 + 15\}$

$= 31,\ 31,\ 26,\ 35$

1 2 3 4

Optimal solution = 3rd node

$c[0,4] = 26$

$r[0,4] = 3$

put K = 3 in formula.

$c[0,4] = \min\{c[0,2] + c[3,4] + 15\}$

left right

3

c[0,2] c[3,4]

1 4

c[0,0] c[1,2] c[3,3] c[4,4]

2