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Tutorial No. 1:
Advanced Linear Algebra

28/9/22

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Q.1) Apply the Gauss Jordan method

a) $3x+y-z=4$; $-2x+3y-4z=-1$; $x-y+2z=2$

$\longrightarrow$ Given:-

$3x+y-z=4$

$-2x+3y-4z=-1$

$x-y+2z=2$

In matrix form,

$AX=B$

$$\begin{bmatrix} 3 & 1 & -1 \\ -2 & 3 & -4 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$R_2 \longrightarrow R_2+2R_1$, $R_3 \longrightarrow R_3-3R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

$R_1 \to R_1 + R_2$, $R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ -14 \end{bmatrix}$$

$R_3 \to R_3/-7$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

$R_1 \to R_1 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$\therefore$ from $R_1 \to x = 1$
from $R_2 \to y = 3$
from $R_3 \to z = 2$

$\therefore x = 1, y = 3, z = 2.$

b) $3x+2y-2z=4$; $x-2y+3z=6$; $2x+3y+4z=15$

$\to$ Given:-

$3x+2y-2z=4$
$x-2y+3z=6$
$2x+3y+4z=15$

In matrix form:-

$AX = B$

$$\begin{bmatrix} 3 & 2 & -2 \\ 1 & -2 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 15 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -2 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 15 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ $\quad R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 8 & -11 \\ 0 & 7 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -14 \\ 3 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -9 \\ 0 & 7 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -17 \\ 3 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 2R_2$, $R_3 \rightarrow R_3 - 7R_2$

$$\begin{bmatrix} 1 & 0 & -15 \\ 0 & 1 & -9 \\ 0 & 0 & 61 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -28 \\ -17 \\ 122 \end{bmatrix}$$

$R_3 \rightarrow R_3/61$

$$\begin{bmatrix} 1 & 0 & -15 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -28 \\ -17 \\ 2 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 15R_3$, $R_2 \rightarrow R_2 + 9R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ from $R_1 \rightarrow x = 2$

from $R_2 \rightarrow y = 1$

from $R_3 \rightarrow z = 2$

$\therefore x = 2, y = 1, z = 2$

c) $x+y+z = 4$, $4x+3y-z = 12$, $3x+5y+3z = 15$

$\rightarrow$ Given

$x+y+z = 4$

$4x+3y-z = 12$

$3x+5y+3z = 15$

In matrix form,

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 15 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 4R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 3 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & -1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix}$$

$R_2 \rightarrow R_2/2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1.5 \\ -4 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1.5 \\ -2.5 \end{bmatrix}$$

$R_3 \rightarrow R_3/-5$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1.5 \\ 0.5 \end{bmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1.5 \\ 0.5 \end{bmatrix}$$

from $R_1 \to x = 2$

from $R_2 \to y = 1.5$

from $R_3 \to z = 0.5$

$\therefore x = 2,\ y = 1.5,\ z = 0.5$

d) $10x+y+z=12;\ x+10y+z=12;\ x+y+10z=12$

$\to$ Given

$$10x+y+z=12$$
$$x+10y+z=12$$
$$x+y+10z=12$$

In matrix form,

$$AX = B$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 10 & 1 \\ 10 & 1 & 1 \\ 1 & 1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

$R_2 \to R_2 - 10R_1 \quad R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 10 & 1 \\ 0 & -99 & -9 \\ 0 & -9 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -108 \\ 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 10 & 1 \\ 0 & -9 & 9 \\ 0 & -99 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -108 \end{bmatrix}$$

$R_2 \to R_2/-9$

$$\begin{bmatrix} 1 & 10 & 1 \\ 0 & 1 & -1 \\ 0 & -99 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -108 \end{bmatrix}$$

$R_3 \to R_3/-108$

$$\begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3, \quad R_1 \to R_1 - 11R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

from $R_1 \longrightarrow x=1$
from $R_2 \longrightarrow y=1$
from $R_3 \longrightarrow z=1$

$\therefore \; x=1, y=1, z=1$

Q.2) Solve the following equation by factorization method.

a) $3x_1+5x_2+2x_3=8$
$8x_2+2x_3=-7$
$6x_1+2x_2+8x_3=26$

$\longrightarrow$ Given :-
$3x_1+5x_2+2x_3=8$
$8x_2+2x_3=-7$
$6x_1+2x_2+8x_3=26$

In matrix form,
we know that,
$A=LU$

$$\begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 6 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 6 & 2 & 8 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33} \end{bmatrix}$$

$\therefore \; U_{11}=3 \qquad U_{12}=5 \qquad U_{13}=2$
$L_{21}U_{11}=0 \longrightarrow L_{21}=0$
$L_{31}U_{11}=6 \longrightarrow L_{31}=2$
$L_{21}U_{12}+U_{22}=8 \longrightarrow U_{22}=8$
$L_{21}U_{13}+U_{23}=2 \longrightarrow U_{23}=2$
$L_{31}U_{12}+L_{32}U_{22}=2 \rightarrow L_{32}=-1$
$L_{31}U_{13}+L_{32}U_{23}+U_{33}=8 \rightarrow U_{33}=6$

$\therefore$

$AX=B$

$$\begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 6 & 2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 26 \end{bmatrix}$$

Now $AX=B$
$LUX=B$

let $UX==Y$
$\therefore \; LY=B$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\2 & -1 & 1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}8\\-7\\26\end{bmatrix}$$

from $R_1 \to y_1 = 8$

from $R_2 \to y_2 = -7$

from $R_3 \to$

$$2y_1 - y_2 + y_3 = 26$$

$$16 + 7 + y_3 = 26$$

$$y_3 = 3$$

Now $UX = Y$

$$\begin{bmatrix}3 & 5 & 2\\0 & 8 & 2\\0 & 0 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}8\\-7\\3\end{bmatrix}$$

from $R_3 \to 6x_3 = 3$

$$x_3 = 1/2$$

from $R_2 \to$

$$8x_2 + 2x_3 = -7$$

$$8x_2 + 1 = -7$$

$$x_2 = -1$$

from $R_1 \to$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$3x_1 - 5 + 1 = 8$$

$$3x_1 = 12$$

$$x_1 = 4$$

$\therefore$

$\therefore x_1 = 4,\ x_2 = -1,\ x_3 = 1/2$

b) $x + 2y + 3z = 14$

$2x + 3y + 4z = 20$

$3x + 4y + z = 14$

$\to$ In matrix form,

$$AX = B$$

$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 4 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}14\\20\\14\end{bmatrix}$$

Now $A = LU$

$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 4 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}U_{11} & U_{12} & U_{13}\\L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23}\\L_{31}U_{13} & L_{31}U_{12} & L_{31}U_{13}+L_{32}U_{23}+U_{33}\end{bmatrix}$$

$U_{11} = 1,\ U_{12} = 2,\ U_{13} = 3$

$L_{21}U_{11} = 2,\ L_{21}\cdot 2 + U_{22} = 3,\ L_{21}U_{13} + U_{23} = 4$

$\boxed{L_{21} = 2},\ \boxed{U_{22} = -1},\ \boxed{U_{23} = -2}$

$L_{31}U_{11} = 3$, $L_{31}U_{12} + L_{32}U_{22} = 4$, $L_{31}U_{13} + L_{32}U_{23} + U_{33} = 1$

$\boxed{L_{31} = 3}$, $6 + L_{32}(-1) = 4$, $9 - 4 + U_{33} = 1$

$\boxed{L_{32} = 2}$, $\boxed{U_{33} = -4}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{bmatrix}$$

let, $AX = B$

$LUX = B$

let $UX = Y$

$\therefore$ $LY = B$

consider $LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

$\therefore \boxed{y_1 = 14}$

$2y_1 + y_2 = 20$, $3y_1 + 2y_2 + y_3 = 14$

$\boxed{y_2 = -8}$

$42 + (-16) + y_3 = 14$

$\boxed{y_3 = -12}$

consider, $UX = Y$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -12 \end{bmatrix}$$

$x + 2y + 3z = 14$

$-4z = -12$

$\boxed{z = 3}$

$-y - 2z = -8$

$-y - 6 = -8$

$-y = -2$

$\boxed{y = 2}$

$x + 4 + 9 = 14$

$\boxed{x = 1}$

$\therefore$ The solutions is $x = 1, y = 2, z = 3$

c) $2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

Let, $AX = B$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

let, $A = LU$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & U_{13}L_{21} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

$\therefore \boxed{U_{11} = 2}$ $\quad \boxed{U_{12} = 1}$ $\quad \boxed{U_{13} = 1}$

$L_{21} \cdot 2 = 3$ $\quad 3/2 \cdot 1 + U_{22} = 2$ $\quad 3/2 + U_{23} = 3$

$\boxed{L_{21} = 3/2}$ $\quad \boxed{U_{22} = 1/2}$ $\quad \boxed{U_{23} = 3/2}$

$L_{31}U_{11} = 1$ $\quad 1/2 + L_{32}/2 = 4$ $\quad L_{31}U_{13} + L_{32}U_{23} + U_{33} = 9$

$\boxed{L_{31} = 1/2}$ $\quad \boxed{L_{32} = 7}$ $\quad \boxed{U_{33} = -2}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 7 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix}$$

let $AX = B$

$LUX = B$

let $UX = Y$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

$\boxed{y_1 = 10}$

$3/2\, y_1 + y_2 = 18$

$\boxed{y_2 = 3}$

$1/2\, y_1 + 7y_2 + y_3 = 16$

$5 + 21 + y_3 = 16$

$y_3 = 16 - 26$

$\boxed{y_3 = -10}$

let $UX = Y$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ -10 \end{bmatrix}$$

$-2z = -10$

$\boxed{z = 5}$

$1/2\, y + 3/2\, z = 3$

$\boxed{y = -9}$

$2x + y + z = 10$

$2x - 9 + 5 = 10$

$\boxed{x = 7}$

$\therefore$ Solutions are $x=7, y=-9, z=5$

D) $10x+y+z=12$
$2x+10y+z=13$
$x+y+5z=7$

Let $AX=B$

$$\begin{bmatrix}10 & 1 & 1\\ 2 & 10 & 1\\ 1 & 1 & 5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}12\\ 13\\ 7\end{bmatrix}$$

$A=LU$

$$\begin{bmatrix}10 & 1 & 1\\ 2 & 10 & 1\\ 1 & 1 & 5\end{bmatrix}=\begin{bmatrix}U_{11} & U_{12} & U_{13}\\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23}\\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33}\end{bmatrix}$$

$U_{11}=10$ $\quad$ $U_{12}=1$ $\quad$ $U_{13}=1$

$L_{21}U_{11}=2$ $\quad$ $1/5+U_{22}=10$ $\quad$ $U_{23}+1/5=1$

$\boxed{L_{21}=1/5}$ $\quad$ $\boxed{U_{22}=49/5}$ $\quad$ $\boxed{U_{23}=4/5}$

$L_{31}\cdot 10=1$ $\quad$ $L_{31}\cdot U_{12}+L_{32}U_{22}=1$

$\boxed{L_{31}=1/10}$ $\quad$ $1/10+\frac{49}{5}L_{32}=1$

$\boxed{L_{32}=9/98}$

$L_{31}U_{13}+L_{32}U_{23}+U_{33}=5$

$\boxed{U_{33}=473/98}$

$AX=B$
$LUX=B$
put $UX=Y$
$\therefore LY=B$

$$\begin{bmatrix}1 & 0 & 0\\ 1/5 & 1 & 0\\ 1/10 & 9/98 & 1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}12\\ 13\\ 7\end{bmatrix}$$

$y_1=12$

$1/5\, y_1+y_2=13$

$\boxed{y_2=53/5}$

$1/10\, y_1+9/98\, y_2+y_3=7$

$\boxed{y_3=473/98}$

$\therefore UX=Y$

$$\therefore \begin{bmatrix}10 & 1 & 1\\ 0 & 49/5 & 4/5\\ 0 & 0 & 473/98\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}12\\ 53/5\\ 473/98\end{bmatrix}$$

$10x+y+z=12$
$49/5\,y+4/5\,z=53/5$
$473/98\,z=473/98$

$\therefore \; z = 1$

$x = 1$

$y = 1$

$\therefore$ The solution is $x = 1, y = 1, z = 1$.