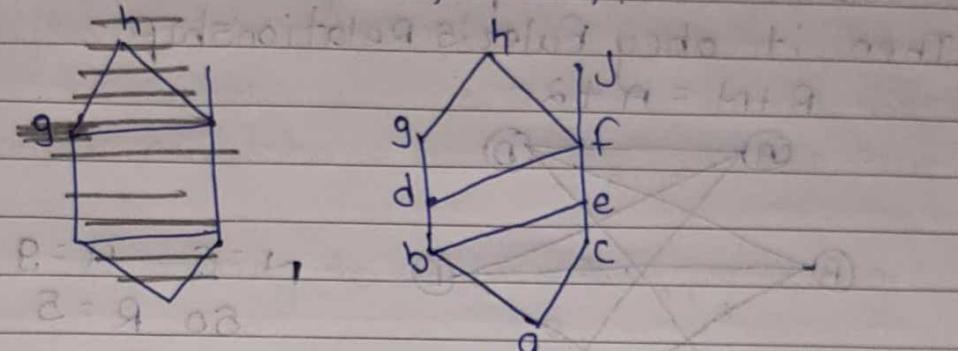


DMS

- Q. Explain lattice as posets. Find the lower upper bonds of the subsets $\{a, b, c\}$, $\{j, h\}$ and $\{a, c, d, f\}$ in the poset with the Hasse diagram shown in fig.

- 1. A lattice is a poset (L, \leq) in which every subset $\{a, b\}$ consisting of two elements has a least lower bound and a greatest lower bound.
- 2. An element c in A is called an upper bound of a and b if $a \leq c$ and $b \leq c$ for all a, b in A .
- 3. An element d in A is called a greatest lower bound of a and b if $d \leq a$ and $d \leq b$ for all a, b in A .
- 4.



The upper bounds of $\{a, b, c\}$ are e, f, j and h and its lower bound is a .

There is no upper bounds of $\{j, h\}$ and its lower bounds are a, b, c, d, e and f .

The upper bounds of $\{a, c, d, f\}$ are f, h and j and its lower bound is a .

- Q. Explain planar graphs with examples? Show whether K_3 , K_5 planar? Show whether K_6 is planar?

1. A graph is a collection of vertices connected to each other through set of edges.

2. The study of graphs is known as graph theory.

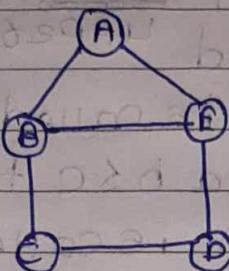


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3. In graph theory, planar graph is a graph that can be drawn in a plane such that none of its edges cross each other.

4. Planar graph means if we draw graph in the plane without edge crossing it is called embedding the graph in the plane.

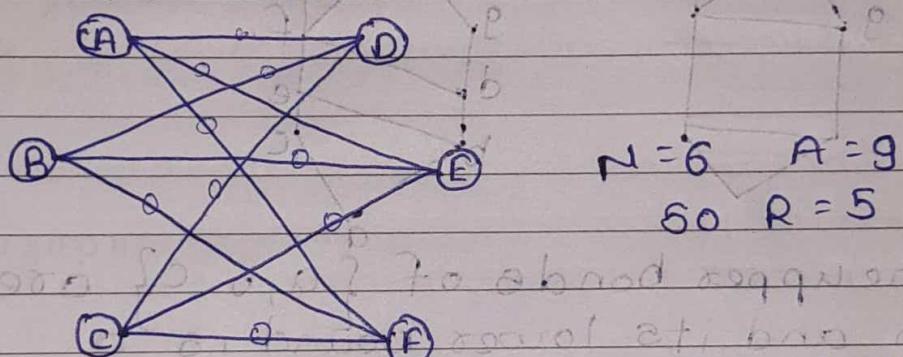
5. ~~Ex. no bounded regions formed and also no 3~~



6. Assume $K_3, 3$ is planar.

Then it obey Euler's Relationship

$$R + N = A + 2$$



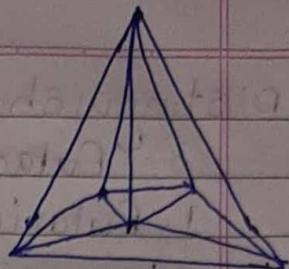
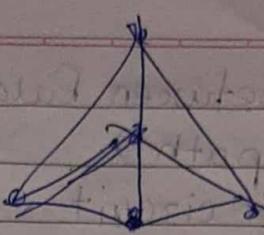
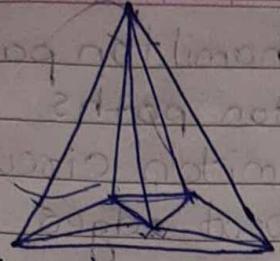
Each region is bounded by at least four arcs. So there are at least $4R$ region boundaries. So those are at least $4R \div 2 = 2R$ arcs. Since each arc is a region boundary.

$$2 \times 5 = 10, \text{ so at least } 10 \text{ arcs.}$$

But there are 9 arcs.

There is a contradiction.

$K_3, 3$ does not obey Euler's Relationship
 $\therefore K_3, 3$ is not planar.



K₅ has 5 vertices each with degree 4, so by the handshaking theorem it has $5 \times 4 / 2 = 10$ edges. By Theorem 1 of Euler's formula every connected planar graph must satisfy $E \leq 3V - 6$, gives $10 \leq 3 \times 5 - 6 = 9$ which is false. So K₅ is not a planar graph.

9. Illustrate pigeon hole principle with example.

There are 280 people in the party. Without knowing anybody's birthday, what is the largest value of n for which we can prove that at least n people must have been born in the same month.

According to Pigeon hole principle,

ratio for at least $\frac{n}{k}$
n people have been born in
same month

given

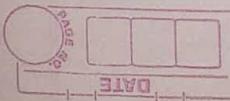
$$n = 280 \text{ and } k = 12$$

$$= \frac{n}{k}$$

$$= \frac{280}{12}$$

$$= 23.3 \\ \approx 24$$

There are at least 24 people those have been born in the same month.



- Q. Distinguish between Euler and hamilton paths.
- | Euler path | Hamilton paths. |
|--|--|
| 1. Eulerian circuit | 1. Hamilton circuit may traverse every edge |
| | repeat edges. |
| exactly once | |
| 2. Eulerian circuit may repeat vertices. | 2. Hamilton circuit visits each vertex exactly once. |
| 3. Path in euler circuit is called Euler path. | 3. Path in hamilton circuit is called hamilton path. |
| 4. Euler circuit always follows Euler's formula $V-E+R=2$ | 4. Hamilton circuit also follows Euler's formula. |
| which graph shown in fig below have an Euler path / circuit and hamilton path / circuit. | |

→ The graph G_1 has an Euler circuit. But as can easily be verified by inspection neither G_2 nor G_3 has an Euler circuit.

G_3 has an Euler path but G_2 has no Euler path.

a. compare permutation and combination solve
i. In how many different ways can the letters of the word CORPORATION be arranged so that the vowels always come together.

ii From a group of 7 men and 6 women Five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?

1. multiple permutations can be derived from a single combination
2. From a single permutation, only a single combination can be derived.
3. They can simply be defined as ordered elements (permutation)
4. They can simply be defined as unordered sets (combination)

5. Write both formulas.
In word CORPORATION, we treat the vowels OOAIO as one letter AEIOU

Thus we have CRPRTN (OOAIO)

This has 7 letters of which R occurs 2 times and rest are different.

Number of ways arranging these letters =

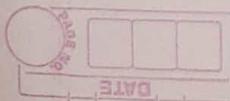
$$\frac{7!}{2!} = 2520$$

$$3!$$

now 5 vowels in which O occurs 3 times and rest are different can be arranged

$$\frac{5!}{3!} = 20 \text{ ways}$$

$$\therefore \text{Required number of ways} = 2520 \times 20 \\ = 50400$$



case 1 3 men 2 women

$$7C_3 \times 6C_2 = 85 \times 12 = 526$$

case 2 4 men 1 women

$$7C_4 \times 6C_1 = 35 \times 6 = 210$$

case 3 5 men 0 women

$$7C_5 = 21$$

$$\text{Total} = 526 + 210 + 21 \\ = 756$$

So it can be done by 756 ways.

Q. What is conditional probability? Explain Bayes theorem and its application with example.

1. The conditional probability of the event is the probability that the event will occur, provided the information that an event A has already occurred.
2. This probability can be written as $P(B|A)$, notation signifies the probability of B given A.
3. Conditional probability is the probability that an event has occurred, taking into account some additional information about outcomes of an experiment.

1. Bayes theorem describes the probability of occurrence of an event related to any condition. It is also considered for the case of conditional probability.

2. Bayes theorem states that the conditional probability of an event A, given the occurrence of another event B is equal to the product of the likelihood of B, given A and the probability of A

3. Bayes theorem also known as the bayes rule or bayes law.

4. Formula for Bayes Theorem

$$P(A|B) = \frac{P(B \cap A)}{P(B)}$$

$$= \frac{\prod_{i=1}^n P(A_i) P(B|A_i)}{\sum_{i=1}^n P(A_i) P(B|A_i)}$$

Application

1. To describe the relationship between the data and the model, the bayes theorem provides a probabilistic model.

2. Simplification of bayes theorem is referred to as the Naive Bayes. It is widely used for classification and predicting models.

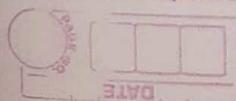
3. The most common application of the bayes theorem in machine learning is the development of classification problems.

4. Spam filtering is another application of bayes theorem.

Q. What is lattice as a poset? Explain the properties of Lattice.

1. A Lattice as a poset (L, \leq) in which every subset $\{a, b\}$ consisting of two elements has a least ~~lower~~ bound and a greatest lower bound.

2. We denote LUB by $a \oplus b$ and GLB by the



$a \vee b$

3 LUB means He join or sum and GLB
means He meet or product.
Properties of lattice

- Q. Define traversal of trees. Find the Inorder, preorder and postorder traversal of following tree.
1. A tree traversal algorithm is a method for systematically visiting every vertex of an ordered rooted tree.
2. Procedure for systematically visiting every vertex of an ordered rooted tree are called as a traversal algorithm.
3. we will describe several important algorithms for visiting all the vertices of an ordered tree.
4. we will describe most commonly used algorithms are

a. Preorder

b. Postorder

c. Inorder



Q. Explain with respect to graph.

i. Planar graph.

1. Planar graph is a graph that can be embedded in the plane i.e. it can be drawn on the plane in such a way that its edges intersect only at their end points.

2. Planar graph is a graph that can be embedded in the plane i.e. it can be drawn on the plane in such a way that its edges

2. A graph or multigraph which can be drawn in the plane so that its edges do not cross is said to be planar.

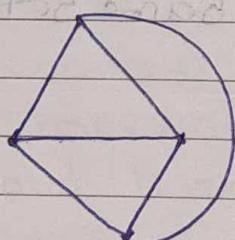
3. In planar graph the arcs have intersect only at nodes.

4. Planar graph can be drawn in the certain way.

5. Planar graph already drawn in the plane without edge intersections is called a plane graph or planar embedding of the graph.

6. Planar graph can be drawn in the plane such that no two edges intersect except at the vertices.

7. Pictorial representation of planar graph G as a plane graph is called a planar representation of G.

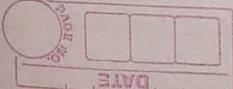


$$\text{no of vertices} - \text{no of edges} + \text{no of faces}$$

$$\text{no of regions} = e - v + 2$$

$$= e - v + (k + 1)$$

↓
components

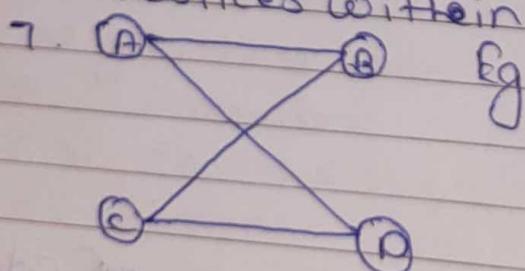


2. Bipartite graph
- A graph G is said to be bipartite if its vertices V can be positioned into two subsets M and N such that each edge of G connects a vertex of M to a vertex of N .
 - By complete bipartite graph, we mean that each vertex of M is connected to each vertex of N .
 - This graph is denoted by the $K_{m,n}$ where m is no of vertices in M and n is no of vertices in N .
 - Any two nodes chosen from the same set are not adjacent but any two nodes chosen one from each set are adjacent. Such graph is called as bipartite graph.

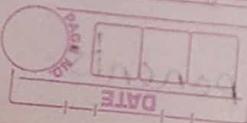
5. A graph is said to complete bipartite graph if it is both bipartite and complete.

6. Properties

Consist two sets of vertices X and Y . The vertices of set X join only with vertices of set Y . If graphs are 2 colorable, vertices within the same set do not join.



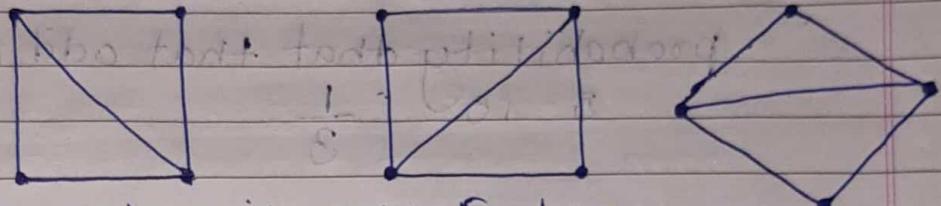
3. Isomorphic graphs.



The isomorphism graph can be described as a fogograph in which a single graph can have more than one form.

2. That means two different graphs can have the same number of edges, vertices and same edges connectivity. These types of graphs are known as isomorphism graphs.
3. Two graphs are isomorphic if they have same number of vertices and same number of edges.

4.



5. Properties it satisfied are,

- a. Number of vertices in both graphs are same
- b. Number of edges are same
- c. Degree of sequence are same!

Two graphs are isomorphic if and only if their complement graphs are isomorphic.

Two graphs are isomorphic if their adjacency matrices are same.

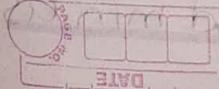
Q: A box contains 4 red, 3 white and 2 blue balls. Three balls are drawn at random. Find out number of ways of selecting 3 balls of different colors?

There are 4 red balls, 3 white balls and 2 blue balls.

3 balls are drawn at random.

case 1

1 red, 1 white and 1 blue



$$4C_1 \times 3C_1 \times 2C_1 = 24$$

24 Different no of ways.

iii. When a certain dice is rolled the numbers from 1 to 6 will appear on the top of the dice, what is the probability that the odd number is on the top and that the number is 5.

→ For dice roll sample space is

$$\{1, 2, 3, 4, 5, 6\}$$

now probability that odd number on top is $\frac{3}{6} = \frac{1}{2}$

probability that that odd no is 5 is $\frac{1}{6}$

Q. Explain distributive lattice with its properties and examples.

1. A Lattice L is called distributive lattice if for any element a, b and c of L it satisfies following distributive properties

2. Properties are
- $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
 - $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

3. If the Lattice is not satisfies the above two properties then it is called as non-distributive lattice.

4. A distributive lattice is a lattice in which operations of join and meet distribute over each other.

5. Every boolean algebra is a distributive lattice

6. Every totally ordered set is a distributive lattice with max as join and min as meet.

7 The lattice ordered vector space is a distributive lattice.

8 Every Heyting algebra is distributive lattice

⑧ Define Boolean function. Solve to obtain SOP form of:

$$1. (x+y) * (x * y)$$

$$2. (x * y) + (x * y') + (y * z)$$

1. A special mathematical function with n degrees and where $x = \{0, 1\}$ is the boolean domain with a being a non-negative integer.

2. The boolean function helps in describing the way in which the boolean output is derived from the boolean inputs.

$$3. (x+y) * (x+y) = x + (y * x)$$

3. Let $x(x_1, x_2, \dots, x_n)$ be a boolean expression. A function of the form $f(x_1, x_2, \dots, x_n) = x(x_1, x_2, \dots, x_n)$ is called boolean function.

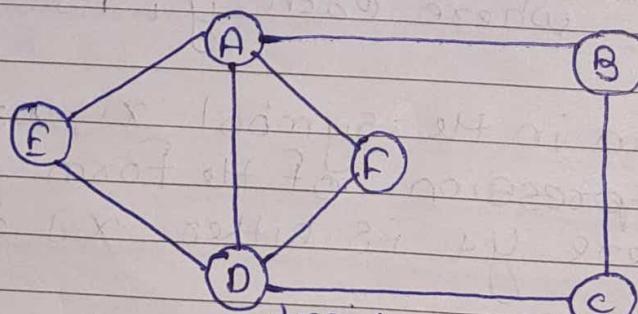
4. A min term in the symbol x_1, x_2, \dots, x_n is a boolean expression of the form $y_1 \cdot y_2 \cdot \dots \cdot y_n$ where each y_i is either x_i or \bar{x}_i .

5. Max term in the symbol x_1, x_2, \dots, x_n is boolean expression of the form $y_1 \cdot y_2 \cdot \dots \cdot y_n$ where y_i is either x_i or \bar{x}_i .



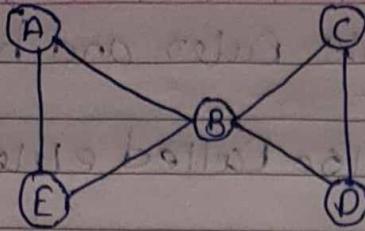
Euler graph

1. A graph is a collection of vertices connected to each other through a set of edges.
2. Any connected graph is called as an Euler graph if and only if all its vertices are of even degree.
3. Euler graph means a connected graph that contain an Euler circuit.
4. Euler path is also known as Euler trail or Euler walk.
5. If there exist a trail in the connected graph that contains all the edges of the graph ; then that trail is called as an Euler trail.
6. Euler circuit is also known as Euler cycle or Euler tour.
7. A closed Euler trail is called as an Euler circuit.
8. A graph will contain an Euler circuit if and only if all its vertices are of even degree.
9. If there exist a circuit in the connected graph that contains all the edges of the graphs , then that circuit is called as an Euler circuit.
- 10.

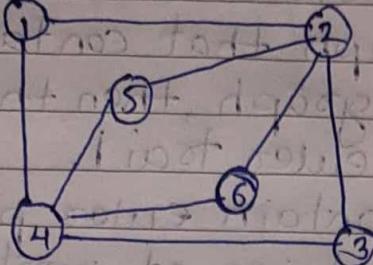


Euler graph.

11. Euler ^{trail} circuit contains the same vertex at the start and end of the trail then that type of trail known as Euler circuit.



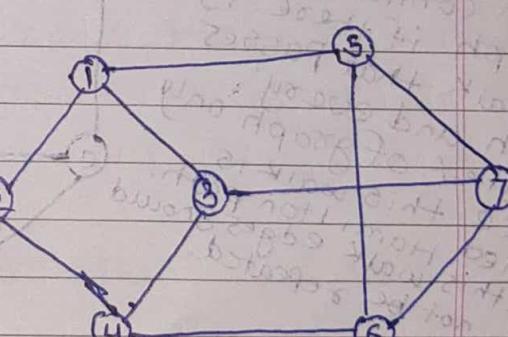
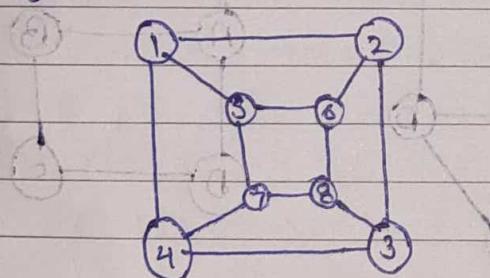
euler graph



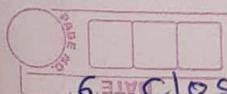
• Hamilton graphs

1. If there exist a closed walk in the connected graph that visits every vertex of the graph exactly once.
2. If there exist a path in the connected graph that contains all the vertices of the graph, then such a path is called as hamiltonian path.
3. A hamiltonian path starts and ends at the same vertex is called as a hamiltonian circuit.
4. Any connected graph that contains a hamiltonian circuit is called as hamiltonian graph.

Ex.



This graph contains both hamiltonian path and circuit.



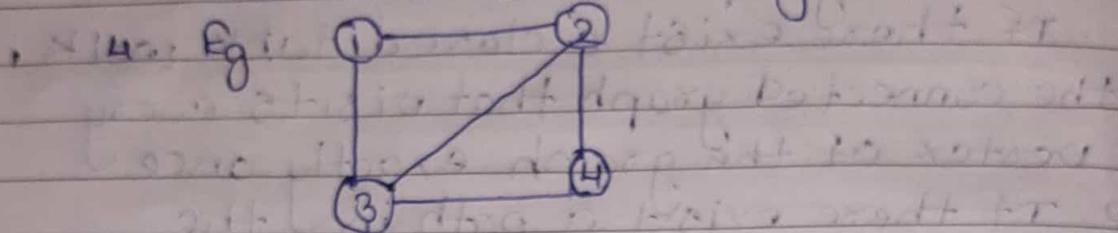
Closed Hamilton path is hamilton circuit.

i) Explain with example Euler's and hamilton paths?

1. Euler's path is also called Euler's trail or Euler's walk

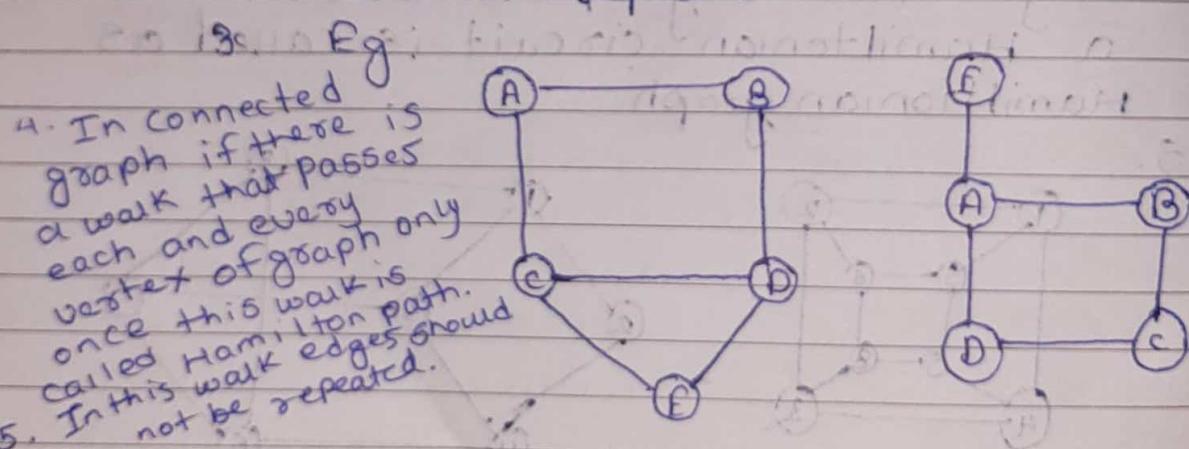
2. If there exists a trail in the connected graph that contains all the edges of the graph, then that trail is called as an Euler's trail.

3. A graph will contain Euler's path if and only if it contains at most two vertices of odd degree.

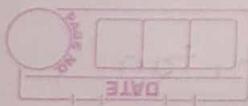


If there exist a path in the connected graph that contains all the vertices of the graph, then such a path is called as hamiltonian path.

2. In Hamiltonian path, all the edges may or may not be covered but edges must not repeat.

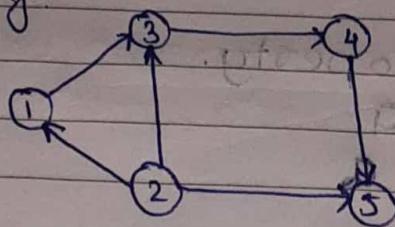


i). Define with example i) indegree and outdegree of nodes ii) connectedness of graphs.



1. The out degree of a node is the number of edges going outside from that node or in other words

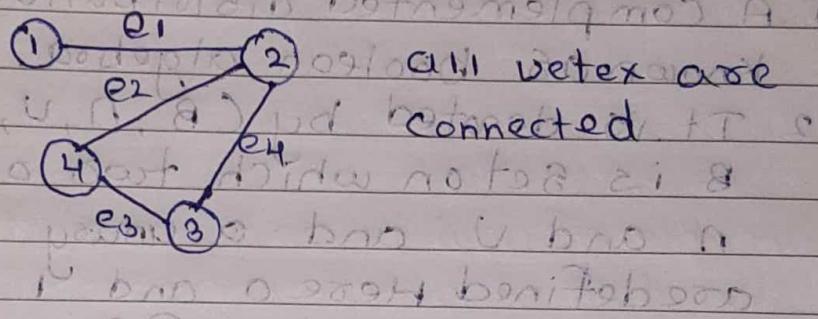
2. Eg.



outdegree of 1 is '3'

1. A graph G is said to be connected if there is at least one path between every pair of vertices in G. otherwise, G is disconnected

Eg.



Q. Define the lattice as algebraic systems. List the properties of lattice.

1. A lattice is an algebraic system (L, \vee, \wedge) with two binary operations \vee and \wedge defined on L, if and only if both operations satisfies the following three properties for any three elements of L ie a, b, c

E.L.

2. Properties are $(a \vee b) \vee c = a \vee (b \vee c)$

a. commutative property

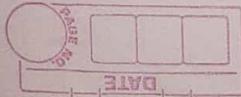
$$a \vee b = b \vee a$$

$$a \wedge b = b \wedge a$$

$$0 = 1 + 0$$

$$1 = 1 \cdot 0 + 0$$

$$0 = 1 \cdot 0 \cdot 0$$



b. associative property

$$a \vee (b \vee c) = (a \vee b) \vee c$$

$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

c. Absorption property.

$$a \vee (b \wedge c) = a$$

$$a \wedge (b \vee c) = a$$

Ques 2: Define Boolean algebra. Explain the properties of Boolean algebra.

Ans: Define Boolean algebra. Explain the properties of Boolean algebra.

1. A complemented distributive lattice is known as Boolean algebra.

2. It is denoted by $(B, \wedge, \vee, 0, 1)$ where B is set on which two operations \wedge and \vee and a unary operation are defined. Here 0 and 1 are two distinct elements of B .

Properties of Boolean algebra:

Properties that should be satisfied by it:

a. Commutative properties

$$a + b = b + a$$

$$a * b = b * a$$

b. Distributive properties:

$$a + (b * c) = (a + b) * (a + c)$$

$$a * (b + c) = (a * b) + (a * c)$$

c. Identity properties

$$a + 0 = a$$

$$a * 1 = a$$

d. Complemented law

$$a + a' = 1$$

$$a * a' = 0$$