1. A random variable X is a **fair** coin tossing, 1 for head and 0 for tail. What is **P(X=1)** or P(head)?

P(X=1) = P(head) = 1/2

2. Suppose a **biased coin** comes up **heads** with probability **0.3**. What is the probability of having 0, 1, 2, …, 6, 7, 8 heads in **8 tosses**?

P(k) = $\binom{n}{k}p^k\ (1 - p)^{n-k}$

k=heads , n=8 tosses p=0.3

<u>k=0</u>

P(0) = $\binom{8}{0}0.3^0\ (1 - 0.3)^{8-0}$

$\binom{8}{0}$= $\frac{8!}{0!\,8!}$ = 1

1·1·$(0.7)^8$ ≈ 0.0576

<u>k=1</u>

P(1) = $\binom{8}{1}0.3^1\ (1 - 0.3)^{8-1}$

$\binom{8}{1}$= $\frac{8!}{1!\,7!}$ = 8

8·0.3·$(0.7)^7$ ≈ 0.1977

<u>k=2</u>

P(2) = $\binom{8}{2}0.3^2\ (1 - 0.3)^{8-2}$

$\binom{8}{2}$= $\frac{8!}{2!\,6!}$ = 28

28·0.3^2 ·$(0.7)^6$ ≈ 0.2965

k=3

P(3) = $\binom{8}{3}0.3^3\ (1 - 0.3)^{8-3}$

$\binom{8}{3}$= $\frac{8!}{3!\,5!}$ = 56

56·0.3^3 ·$(0.7)^5$ ≈ 0.2541

k=4

P(4) = $\binom{8}{4}0.3^4\ (1 - 0.3)^{8-4}$

$\binom{8}{4}$= $\frac{8!}{4!\,4!}$ = 70

70·0.3^4 ·$(0.7)^4$ ≈ 0.1361

k=5

P(5) = $\binom{8}{5}0.3^5\ (1 - 0.3)^{8-5}$

$\binom{8}{5}$= $\frac{8!}{5!\,3!}$ = 56

56·0.3^5 ·$(0.7)^3$ ≈ 0.0467

k=6

P(6) = $\binom{8}{6}0.3^6\ (1 - 0.3)^{8-6}$

$\binom{8}{6}$= $\frac{8!}{6!\,2!}$ = 28

28·0.3^6 ·$(0.7)^2$ ≈ 0.0100

k=7

P(7) = $\binom{8}{7}0.3^7\ (1-0.3)^{8-7}$

$\binom{8}{7}$= $\frac{8!}{7!\,1!}$ = 8

$8 \cdot 0.3^7 \cdot (0.7)^6 \approx$ 0.0012

k=8

P(8) = $\binom{8}{8}0.3^8\ (1-0.3)^{8-8}$

$\binom{8}{8}$= $\frac{8!}{8!\,0!}$ = 1

$1 \cdot 0.3^8 \cdot (0.7)^0 \approx$ 0.00007

3. Suppose the proportion X of surface area in a randomly selected quadrant that is covered by a certain plant has a beta distribution with α=5 and β=2.

Calculate P(0.2≤X≤0.4)

f(x) = $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$

$B(\alpha,\ \beta)$ = $\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ where $\alpha = 5$ and $\beta = 2$

$B(5,\ 2)$ = $\frac{\Gamma(5)\Gamma(2)}{\Gamma(7)}$ = $\frac{4!\,1!}{6!} = \frac{24\cdot 1}{720} = \frac{1}{30}$

f(x) = $\frac{x^4(1-x)^1}{\frac{1}{30}} = 30x^4(1-x)$

P(0.2≤X≤0.4) = $\int_{0.2}^{0.4} 30x^4(1-x)dx$

= $\int_{0.2}^{0.4} 30x^4 - 30x^5dx$

= $[\ 6x^5 - 5x^6\ ]_{0.2}^{0.4}$

$= [6(0.4)^5 - 5(0.4)^6] - [6(0.2)^5 - 5(0.2)^6]$

$= (0.06144 - 0.02048) - (0.00192 - 0.00032)$

$= 0.04096 - 0.0016$

≈ 0.0394