Homework 1

EX 2.3. Y is a random variable that can take any positive integer value. The likelihood of these outcomes is given by the Poisson pdf

$$p(y) = \frac{\lambda^y}{y!}\exp\{-\lambda\}.$$

By using the fact that for a discrete random variable the pdf gives the probabilities of the individual events occurring and that probabilities are additive, (a) compute the probability that $Y \leq 4$ for $\lambda = 5$, i.e. $P(Y \leq 4)$. (b) Using the result of (a) and the fact that one outcome has to happen, compute the probability that $Y > 4$. (Hint, one of the two events, $Y \leq 4$ and $Y > 4$, *has* to happen.)

a)

$P(Y\leq4) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4)$

$\lambda=5$, $P(Y=y) = \frac{5^y}{y!}e^{-5}$

$$P(Y=0) = \frac{5^0}{0!}e^{-5} = e^{-5}$$

$$P(Y=1) = \frac{5^1}{1!}e^{-5} = 5e^{-5}$$

$$P(Y=2) = \frac{5^2}{2!}e^{-5} = \frac{25}{2}e^{-5}$$

$$P(Y=3) = \frac{5^3}{3!}e^{-5} = \frac{125}{6}e^{-5}$$

$$P(Y=4) = \frac{5^4}{4!}e^{-5} = \frac{625}{24}e^{-5}$$

$P(Y\leq4) = (1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24})e^{-5}$

$P(Y\leq4) \approx 0.4405$

b)

$P(Y>4) = 1 - P(Y\leq4)$

$P(Y>4) = 1 - 0.4405 = 0.5595$

EX 2.9. Assume that a dataset of N binary values, $x_1, \ldots, x_N$, was sampled from a Bernoulli distribution. Compute the maximum likelihood estimate for the Bernoulli parameter.

$$P(x_i | p) = p^{x_i}(1-p)^{1-x_i}$$

L(p)= $\prod_{i=1}^{N} p^{x_i}(1-p)^{1-x_i}$

log L(p) = $\sum_{i=1}^{N} x_i log(p) + \sum_{i=1}^{N}(1-x_i)log(1-p)$

$$\frac{d}{dp}\left[\sum_{i=1}^{N} x_i log(p) + \sum_{i=1}^{N}(1-x_i)log(1-p)\right]$$

$$\sum_{i=1}^{N}\frac{x_i}{p} - \sum_{i=1}^{N}\frac{1-x_i}{1-p} = 0$$

$$p = \frac{\sum_{i=1}^{N} x_i}{N}$$