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203-002

§1.9: The matrix of a linear transformation

Def: The columns of the $n \times n$ identity matrix I_n are denoted $\vec{e_1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \vec{e_2} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \vec{e_n} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$.
They are called the standard basis vectors for $\mathbb{R}^n$.

Key Fact: Any vector in $\mathbb{R}^n$ is a linear combo. of $\vec{e_1}, \ldots, \vec{e_n}$.

Ex1. Write $\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ as a linear combo. of $\vec{e_1}, \vec{e_2}, \vec{e_3}$

Sol:

$$\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + (-3)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= 2\vec{e_1} + 7\vec{e_2} - 3\vec{e_3}.$$

Fact: In general

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = a_1\vec{e_1} + a_2\vec{e_2} + \cdots + a_n\vec{e_n}.$$